PHYSICAL REVIEW A VOLUME 14, NUMBER 1 JULY 1976

Differential cross section for positronium formation in positron–atomic-hydrogen collisions

Richard J. Drachman and Kazem Omidvar
Theoretical Studies Group, Goddard Space Flight Center, Greenbelt, Maryland 20771

J. H. McGuire
Department of Physics, Kansas State University, Manhattan, Kansas 66505
(Received 18 September 1975; revised manuscript received 8 March 1976)

We have combined the $L = 0$ and 1 partial-wave amplitudes obtained by a two-state coupled static approximation with correlation with the $L \geq 2$ Born amplitudes to obtain the differential cross section for positronium formation in positron–atomic-hydrogen collisions. For positron energies of 0.64 and 0.75 Ry, minima at the scattering angles of 57° and 51° are found. Total cross sections for positronium formation for low and intermediate impact energies are given. Measurement of the differential cross section for the process $e^+ + \mathrm{He} \rightarrow \mathrm{Ps} + \mathrm{He}^+$ for the detection of possible minima is suggested.

I. INTRODUCTION

We consider the formation of a positronium atom in its ground state due to collision of a positron and a hydrogen atom. Aside from purely theoretical interest, the formation cross section is of interest in a number of astrophysical problems. A case to be mentioned is the formation and annihilation of positronium in the sun following an energetic solar flare. The emitted γ rays are expected to provide sensitive probes of conditions in the annihilation region.[1]

The first calculation of the positronium formation cross section was done by Massey and Mohr,[2] using the Born approximation. This calculation was repeated later, and errors in the numerical values of the cross section were corrected.[3] Chen and Kramer[4] have calculated the differential cross section for positronium formation in both the Born and the Faddeev-Watson multiple-scattering approximations. Similarly, Sural and Barman[5] have performed an approximate form of the second Born approximation calculation.

For low impact energies a number of elaborate calculations have been performed by several authors.[6–8] These include both the hydrogen and positronium ground states, as in the coupled-static approximation, while representing polarization and distortion either by the addition of correlation terms or by the inclusion of effective potentials.

To obtain a reliable low-energy differential cross section, partial-wave amplitudes for all values of the angular momentum quantum number L are needed. In the absence of any better calculation for the differential cross section we have combined the seemingly accurate $L=0$ and 1 partial-wave amplitudes of Refs. 6 and 7, which are based on the two-state coupled static approximation with correlation, with the $L \geqslant 2$ Born amplitudes to obtain the differential cross section. This cross section for a number of low impact energies will be presented. In addition, the total cross section for low and intermediate impact energies will be given.

The first Born approximation meets with some difficulties for the similar process of proton–atomic-hydrogen electron transfer. It has been shown that in the range of relatively high impact energies of a few MeV, the second Born terms are comparable in magnitude to the first Born terms.[9] As the energy increases the contribution to the cross section from the second-order terms dominates the cross section.

The (p, H) process has two characteristics which are absent in the (e^+, H) process: (i) The (p, H) process is a resonance charge exchange collision, with zero energy transfer. As a result of this effect the collision amplitude has a different analytic form compared to the nonresonance charge exchange amplitude. (ii) In the (p, H) process, because of their heavy masses, the motion of the projectile and the nucleus can be treated classically. It can then be shown that in the limit of high impact energies the role of the projectile-nucleus interaction is to introduce only a phase in the total wave function which depends on the relative velocity of the two particles. This leads to the fact that this interaction has a negligible effect on the exact transition probability, provided the initial and final wave functions are made orthogonal to each other.[10]

Because of these differences it is difficult to draw any conclusion for the (e^+, H) process. The second Born calculation of Ref. 5 does not show the domination of the second Born terms, although the calculation is approximate, and it is not extended to high-enough incident energies.

In this paper we have concerned ourselves with the low-energy positronium formation cross section. We have made the assumption that as L increases the partial-wave first Born amplitudes approach the true values. A test for the validity of this assumption is that as the more accurate partial-wave amplitudes become available in the future, they should converge to the first Born amplitudes.

II. METHOD OF CALCULATION AND RESULTS

Using the R matrix of Chan and Fraser[6] for $L=0$ and the R matrix of Chan and McEachran[7] for $L=1$, the corresponding T matrix can be calculated. Similarly, a partial-wave expansion is made of the total T matrix according to the Born approximation given analytically by Omidvar and Puget.[11] The T matrices for different L are then combined to obtain the differential cross section.

The justification for using the Born approximation for $L \geq 2$ is that in going from $L=0$ to $L=1$ the discrepancy between the Born approximation and the more accurate approximation of Refs. 6 and 7 is substantially reduced. In going from $L=1$ to $L=2$ it is hoped that a similar reduction will take place.

More explicitly, for an incident energy of 0.64 Ry, for which a calculation of the differential cross section has been carried out here, the T_0 value for the matrix element connecting the incident channel to the Ps-formation channel according to the two-channel, 26-correlation-term calculation of Ref. 6 is $0.042-0.017i$, while this value according to the Born approximation is $0.85i$, fifty times larger than the imaginary part of the more accurate calculation. For $L=1$ and the same incident energy the corresponding value according to Ref. 7, which is obtained as in Ref. 6 but with 56 correlation terms, is $-0.00673-0.323i$, while the Born approximation value is $-0.535i$. The

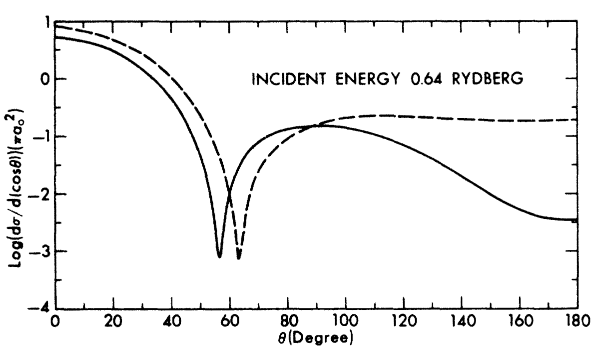

FIG. 1. Differential cross section for positronium formation in positron–hydrogen-atom collisions as a function of the scattering angle θ, and for an incident energy of 0.64 Ry. The dashed line contains the $L=0$ amplitude of Ref. 6 and the Born amplitudes for higher L. The solid line contains the $L=0,1$ amplitudes of Refs. 6 and 7 and the Born amplitudes for higher L.

ratio of the imaginary parts in the two calculations has decreased to 1.7. Furthermore, it should be noted that while the magnitudes of the real and imaginary parts for $L=0$ in the more accurate calculation are comparable, for $L=1$ the real part is two orders of magnitude smaller than the imaginary part, indicating the increased accuracy of the Born approximation.

These considerations suggest that for $L \geq 2$ the Born approximation values which are purely imaginary may not be too far from the true values.

In Fig. 1 we show the differential cross section for an incident energy of 0.64 Ry. The more accurate cross section represented by the solid line shows a deep and narrow minimum at 57°. The value of the cross section drops at this minimum by two orders of magnitudes. The width of the minimum where the cross section has dropped by one order of magnitude is approximately 7°. Angles from zero up to the minimum angle contribute 80% to the total cross section.

In Fig. 2 the similar differential cross section for 0.75 Ry incident energy is shown. The minimum in the more accurate solid line occurs at 51° and is shallower than that in Fig. 1. However, a second broad minimum occurs here at 147°.

As the incident energy increases, the angle at which the minimum occurs becomes smaller, and the percentage of the cross section arising from angles smaller than the minimum angle increases.

In Fig. 3 the differential cross section for an incident energy of 20 Ry is shown. Here we have used the Born approximation to calculate all partial-wave amplitudes. This is justified by noticing that the partial cross sections due to $L=0$ and $L=1$ at this energy in the Born approximation are only 1.48% and 1.47% of the total cross section. The main contribution comes from the higher partial waves, where the Born approximation is ap-

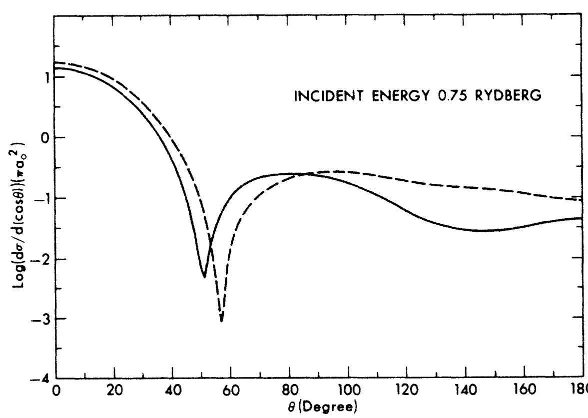

FIG. 2. Same as Fig. 1, but for an incident energy of 0.75 Ry.

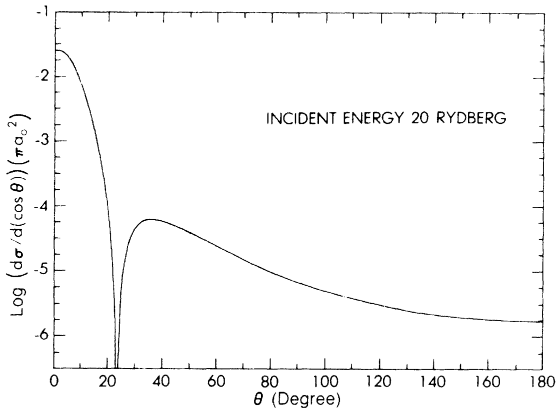

FIG. 3. Same as Fig. 1, but for an incident energy of 20 Ry. Here all amplitudes are given by the Born approximation.

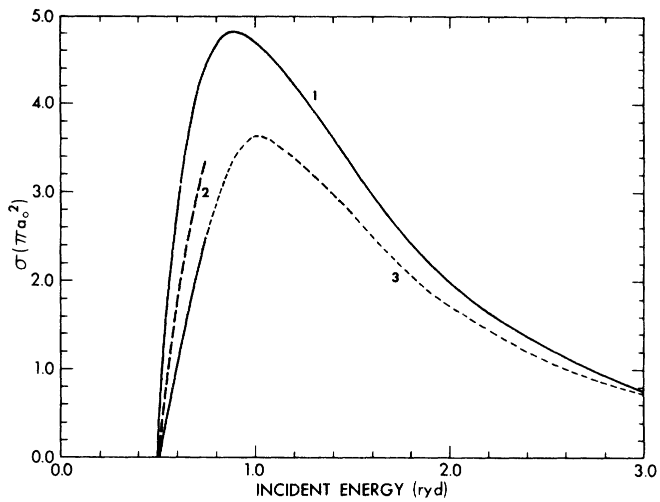

FIG. 4. Total cross section for positronium formation in positron–hydrogen-atom collisions as a function of the incident energy. 1 represents the Born approximation. 2 is obtained by combining the $L=0$ partial-wave cross section of Ref. 6 with higher partial-wave Born cross sections. 3 is obtained by combining the $L=0$ and 1 partial-wave cross sections of Refs. 6 and 7 with the higher partial-wave Born cross sections. The dashed line in 3 is obtained using Eq. (1).

parently reliable. In this case, instead of a minimum in the differential cross section, a zero occurs. This is due to the fact that the Born amplitude is pure imaginary, and the amplitudes due to the attractive and repulsive parts of the potential are equal but opposite in sign. The zero in Fig. 3 occurs at an angle of 23.3°, and 90.9% of the total cross section comes from angles smaller than the minimum angle. The results given in this figure are consistent with the results given in Ref. 4.

In order to see how the cross section peaks in the forward direction for different incident energies, we consider the ratio $4\pi(d\sigma/d\Omega)_{\theta=0}/\sigma$, which for an isotropic scattering is equal to unity. $d\sigma/d\Omega$ and σ are the differential cross section per unit solid angle and the total cross section. For energies of 0.64, 20, and 10^3 Ry this ratio is 16.0, 171, and 197, respectively.

Similar to the calculation of the differential cross section, we have computed the total cross section for a number of impact energies by combining the more accurate $L=0, 1$ partial-wave cross sections of Refs. 6 and 7 with the Born cross section for higher waves. The results are shown in Table I.

TABLE I. Values of the total cross section for positronium formation as a function of the incident energy. σ is obtained by combining the $L=0, 1$ partial-wave cross sections of Refs. 6 and 7 with $L \geq 2$ Born partial cross sections. σ^B is the Born cross section given for comparison. The numbers marked by asterisks are obtained using Eq. (1). The notation $A-B$ stands for $A\times10^{-B}$.

E (Ry)	σ^B (πa_0^2)	σ (πa_0^2)	E (Ry)	σ^B(πa_0^2)	σ (πa_0^2)
0.5041	2.96 − 1	1.63 − 2	3	7.85 − 1	7.09 − 1*
0.5625	1.87	5.45 − 1	4	3.51 − 1	3.24 − 1*
0.64	3.34	1.39	5	1.73 − 1	1.62 − 1*
0.7225	4.28	2.25	6	9.21 − 2	8.70 − 2*
0.75	4.47	2.50	8	3.13 − 2	2.99 − 2*
1	4.74	3.31*	10	1.26 − 2	1.21 − 2*
2	1.97	1.69*	20	5.90 − 4	5.76 − 4*

By studying the cross sections for $L=0$ and 1 given in Refs. 6 and 7, a prescription for approximate computation of the total cross section for energies not given in Refs. 6 and 7 can be found. This study shows that at all impact energies except the lowest one, the $L=0$ cross section is less than 1% of the $L=1$ cross section. With acceptable accuracy we then can neglect the $L=0$ contribution. Furthermore, by taking the ratio of the cross section given in Ref. 7 to the Born cross section for $L=1$, we see that this ratio for impact energies of 0.504, 0.563, 0.64, 0.723, and 0.750 Ry is 0.69, 0.48, 0.38, 0.40, and 0.42, respectively. It may not be a bad approximation if we take this ratio to be 0.4 for higher energies. The following formula for the cross section is then obtained:

$$\sigma \cong \sigma_T^B - \sigma_0^B - 0.6\sigma_1^B \, , \qquad (1)$$

where σ_T^B, σ_0^B, and σ_1^B are the total, $L=0$, and $L=1$ Born cross sections. The total cross sections calculated using (1) are indicated by an asterisk in Table I.

The results of Table I up to 3 Ry energy are shown graphically in Fig. 4.

As a test of the consistency of our results, the total cross sections are obtained in two ways, by summing over all partial-wave cross sections, and by integrating the differential cross section with respect to the scattering angles. The discrepancy is about or less than 1%.

III. CONCLUSIONS

With the approximations presented in the text we have found minima in the differential cross sections for 0.64 and 0.75 Ry impact energies. According to the first Born approximation zeros are found in the differential cross section for all impact energies. The approximate form of the second Born approximation of Ref. 5 shows two shallow minima at 12 eV and no minimum at 100 eV impact energies, while the Faddeev-Watson multiple-scattering approximation of Ref. 4 shows no minima in the differential cross section for 200 and 500 eV impact energies.

Zeros or minima were found previously in the differential cross section for the similar (p, H) exchange process in different approximations. The zero appears in the first Born[12] and distorted-wave approximations,[13] while the minimum appears in the second Born approximation.[9] Since the zeros and minima occur at small scattering angles of the order of tenths of a degree, there has been no attempt to substantiate these findings experimentally. For the e^+-H system, however, the minima occur at large scattering angles, and the difficulty of the (p, H) experiment does not arise here, although the present status of low-energy positron-beam technology makes this experiment extremely difficult. Minima similar to those obtained here may also occur in the differential cross section for the process $e^+ + \mathrm{He} \to \mathrm{Ps} + \mathrm{He}^+$, although at smaller angles, because of the increased nuclear charge. A measurement of the differential cross section for this process is more easily realized, and the results will be an aid in discriminating between different theories, although it is difficult to assess the role of correlation of the target electrons.

Note added in proof. Recently C. L. Cocke, J. R. Macdonald, B. Curnutte, S. L. Varghese, and R. Randall [Phys. Rev. Lett. 36, 782 (1976)] have measured the differential cross section for K-electron capture by protons from argon for an incident energy of 6 MeV. No zero or minimum in the differential cross section was seen.

ACKNOWLEDGMENTS

We would like to thank H. M. Eiserike and V. A. Turner for their programming assistance, and Y. F. Chan and R. P. McEachran for providing us with their data prior to publication.

[1] C. J. Crannell, R. Ramaty, and C. Werntz, in Proceedings of the Fourteenth International Cosmic Ray Conference, Munchen, Germany, 1975, Conference Papers (unpublished), p. 1656; C. J. Crannell, G. Joyce, R. Ramaty, and C. Werntz (unpublished).

[2] H. S. W. Massey and C. B. O. Mohr, Proc. Phys. Soc. Lond. A 67, 695 (1954).

[3] I. M. Cheshire, Proc. Phys. Soc. Lond. 83, 227 (1964).

[4] J. C. Y. Chen and P. J. Kramer, Phys. Rev. A 5, 1207 (1972). Also, for a two-state close-coupling approximation, see D. Basu, G. Banerji, and A. S. Ghosh, *ibid.* 13, 1381 (1976).

[5] D. P. Sural and S. Barman, Phys. Rev. 10, 1162 (1974).

[6] Y. F. Chan and P. A. Fraser, J. Phys. B 6, 2504 (1973).

[7] Y. F. Chan and R. P. McEachran, J. Phys. B (to be published).

[8] B. H. Bransden and Z. Jundi, Proc. Phys. Soc. Lond. 92, 880 (1967); M. F. Fels and M. H. Mittleman, Phys. Rev. 182, 77 (1969); Y. Hahn and J. F. Dirks, Phys. Rev. A 3, 1513 (1971); S. E. A. Wakid and R. W. LaBahn, *ibid.* 6, 2039 (1972); J. Stein and R. Sternlicht, *ibid.* 6, 2165 (1972).

[9] P. J. Kramer, Phys. Rev. A 6, 2129 (1972).

[10] D. R. Bates, Proc. R. Soc. Lond. A 247, 294 (1958).

[11] K. Omidvar and J-L. Puget, Astrophys. J. (to be published).

[12] J. D. Jackson and H. Schiff, Phys. Rev. 89, 359 (1953).

[13] R. H. Bassel and E. Gerjuoy, Phys. Rev. 117, 749 (1960). T. Winter (private communication) has verified the existence of zero in the differential cross section found by Bassel and Gerjuoy.