

Suggested method for coherent x-ray production by combined x-ray and low-energy photon pumping

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Under certain stated conditions, the electron population can be inverted in a variety of targets by a combination of x-ray (e.g., originating in a storage ring) and low-energy photon (e.g., from a laser) pumping. Coherent (mainly soft) x rays could then be produced by stimulated emission.

Recently, it was suggested that coherent x-ray production could be achieved by pumping a suitable target with x-ray photons emerging from a storage ring, thereby preparing an inverted electron population.¹ The advantages of pumping with x rays have been discussed earlier.²⁻⁴ The reason for suggesting x-ray photons emitted in the form of synchrotron radiation is that such photons are well collimated and come in short bursts; this results in locally high instantaneous pumping power.

The purpose of the present paper is to point out that one can increase the coherence length, decrease the required pumping x-ray intensity, and widen the choice of usable target materials by incorporating two new ideas:

(1) Instead of inverting the electron population in a single step by pumping with x rays only, achieve the final inversion in successive steps by pumping with a combination of x-ray and low-frequency photons.

(2) Eliminate Auger processes by stripping the atom of additional electrons by further photoionization.

To illustrate (1), consider a Li gas target. Suppose one tries to induce a *K*-shell vacancy in some of the Li atoms in a single step by pumping with x-ray photons of energy just above the *K* edge in Li, and coming from a storage ring as at the Stanford University Synchrotron-Radiation Project (SSRP). Most of the resulting Li ions will have an electron configuration $1s2s$. Then Auger processes are impossible. The radiative decay $1s2s \rightarrow 1s^2$ is first forbidden, the radiative lifetime is $\tau_R \approx 5.10^{-4}$ sec. On the one hand, this relatively long τ_R is desirable, because only that fraction of the x-ray photons is useful for pumping which is delivered within a time interval τ_K , where τ_K is the total lifetime of the *K* vacancy and is $< \tau_R$. If the synchrotron photons come in bursts lasting τ_X long, then all of them can be used for pumping, provided that $\tau_X \leq \tau_K$. On the other hand, if τ_K is too long, then the cross section σ_s for stimulated emission (neglecting Doppler effects) decreases with increasing τ_K , because

$$\sigma_s = (\lambda^2/2\pi)\omega_K, \quad (1)$$

where the fluorescence yield is $\omega_K = \tau_K/\tau_R$. If τ_R is long, then τ_K will be determined by the collisional deexcitation time in the Li gas. It turns out⁵ that $\omega_K < 1$, because $\tau_K < \tau_R$, unless the Li ion density is $\leq 0.5 \times 10^{11}$ cm⁻³. However, a laser filled with such a dilute gas would have to be at least several hundred meters long before it could produce significant amplification. In order to reduce the length of the laser, one needs to reduce τ_R . To utilize synchrotron photons efficiently for pumping, one needs $\tau_R \approx \tau_X$. Thus one aims for $\tau_R \approx \tau_X$. At the SSRP, $\tau_X \approx 10^{-10}$ sec.

To achieve $\tau_R \approx 10^{-10}$ sec, the inverted electron production should be produced not in one step as described above, but in two successive steps.

First, before creating any *K*-shell vacancies, one should irradiate the Li gas with optical radiation with 3.80 eV energy. This radiation will induce the transition $1s^22s \rightarrow 1s^23p$ and, if it is intense enough to saturate the Li gas, then, as a result, half of the Li atoms will be in the $1s^23p$ state. The rest of the Li atoms will be in the $1s^22s$ state.

Second, one should irradiate the Li gas with synchrotron x-ray photons. Let the pumping radiation have an energy spectrum which is constant from $E = 66$ to 70 eV and is zero elsewhere. Appropriate mirrors and windows may be used to achieve such a step-function-like energy spectrum. The pumping radiation induces $1s^23p \rightarrow 1s3p$ transitions by further photoionization of the Li ions with an average (over the 66–70 eV energy range) photoionization cross section of about $\bar{\sigma}_{Pa} = 6.10^{-18}$ cm². It also induces $1s^23p \rightarrow 1s^2$ transitions but with an average (over the same energy range) photoionization cross section⁶ of only $\bar{\sigma}_{PK} = 3.10^{-19}$ cm². Thus, about 95% of all Li II ions will have a *K*-shell vacancy. The remaining 5%, which do not have such a vacancy, will be in the $1s^2$ level. The $1s^22s \rightarrow 1s^23p$ energy in Li is around 60 eV so that the pumping radiation cannot induce $1s3p \rightarrow 1s^2$ transitions.

The lifetime of the $1s3p^1P_{J=1}$ inverted state is $\tau_R \approx 10^{-10}$ sec, as desired, while the $1s3p^3P_{J=0,1,2}$ state is metastable. To have $\omega_K \approx 1$, the ion density must be $\leq 10^{18}$ cm $^{-3}$, about seven decades higher than in the earlier-discussed case when optical photon pumping was not used. The length of a laser can be reduced by the same factor. This shows one of the advantages of pumping with a combination of x rays and low-energy photons.

The other advantage is that the angular momentum of the inverted state is correlated with the angular momentum of the low-energy pumping photons. In the example under discussion, the angular momentum of the $^1P_{J=1}$ inverted state is aligned parallel to the angular momentum of the pumping low-energy photon. If the latter are polarized, and provided that coherent deexcitation occurs before collisions will disorient the angular momenta, then all $^1P_{J=1}$ states can participate in the laser action.

To illustrate point (2), consider Be atoms. First irradiate them with low-energy photons to raise one of the $2s$ electrons into an appropriate state, such as $5p$. Second, select two energy bands of pumping photons, one just above the K edge to remove a $1s$ electron, the other to remove a $2s$ electron. The resulting ions with an electron configuration $1s5p$ have $\tau_R \approx 3.10^{-10}$ sec, and can be treated as Li was treated above.

Next we state conditions to be satisfied by a general x-ray laser.

Choose a rectangular target whose sides are parallel to the coordinate axes, and are d_x, d_y, d_z long, respectively (see Fig. 1). We label so that $d_y \leq d_x \leq d_z$. It is convenient to select a long target: $d_y, d_x \ll d_z$.

To be specific, let us consider the case when the induced coherent radiation of interest is emitted in a radiative transition which fills a K -shell vacancy. We use the following notation: The pumping x-ray pulse which creates the K -shell vacancy is a step function in time; at the target it starts at $t=0$ and ends at $t=\tau_X$. The photon which triggers the coherent stimulated emission has energy E_2 , momentum \bar{k}_2 parallel to z , wavelength in vacuum λ_2 , and arrives at the target at $t=\Delta t$. We assume that before $t=0$, all electrons except one are removed from all shells higher than the K shell.⁷ This one remaining electron (the "outer electron") remains in a level with energy E_1 or E_2 above the K level. We refer to these levels as 1 and 2, respectively. Then the trigger photon with energy E_2 can induce an electron transition from level 2 to the K shell.

When a K vacancy exists, then the radiative lifetime of the two levels is τ_{K1R} and τ_{K2R} , respectively, and their total lifetimes are τ_{K1} and τ_{K2} .

We shall assume $\tau_{K1R} \gg \tau_{K2R}$. In the example discussed earlier, levels 1 and 2 are, respectively, the $2s$ and $3p$ levels in Li, $\tau_{K1R} \approx 5 \times 10^{-4}$ sec, and $\tau_{K2R} \approx 10^{-10}$ sec. We will refer to those atoms in which the stimulated transition of interest can take place as "active atoms." In the example discussed earlier, the active atoms are Li atoms. Those active atoms which have a K vacancy and have the outer electron in level 1 (level 2), we will call "K1 atoms" ("K2 atoms"). Active atoms with no K vacancy and which did not lose their outer electron are referred to as "a atoms" (ignoring thereby the difference among such atoms owing to the fact that the outer electron may be in level 1 or 2). Active atoms which have no K vacancy but have lost their outer electron we call "a0 atoms." Let us denote by ρ_{K1} , ρ_{K2} , ρ_a , and ρ_{a0} the density of K1, K2, a, and a0 atoms, respectively.

When the outer electron is in level 2, then the decay width of the K vacancy is⁸

$$\Gamma_{K2} = \frac{\hbar}{\tau_{K2}} = \sum_j \Gamma_{K2j} + \Gamma_{K2A} + \Gamma_{K2R}, \quad (2)$$

where the Γ_{K2j} are widths arising from collisional deexcitation with particles of type j , the Γ_{K2A} and Γ_{K2R} are the widths due to Auger processes and radiative decays respectively. For a collection of atoms, ions, and electrons (except for very low electron densities) the $\sum \Gamma_{K2j}$ is dominated by Γ_{K2e} , the term referring to electrons because they move faster. Let \bar{v}_e be the average electron velocity, $\bar{\sigma}_{K2e}$ the average (over velocities) cross section for collisional deexcitation by electrons, and ρ_e the electron density. To make the ratio $\Gamma_{K2e}/\Gamma_{K2R}$ small, we require

$$\bar{\sigma}_{K2e} \rho_e \bar{v}_e \tau_{K2R} \ll 1, \quad 0 \leq t \leq \tau_X. \quad (3)$$

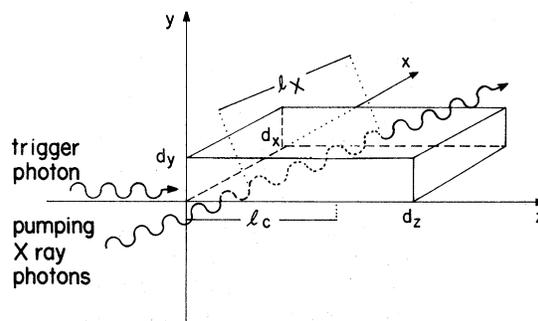


FIG. 1. The dimensions $d_y \leq d_x \leq d_z$ and orientation of the gas target in the x, y, z coordinate frame are shown. The pumping x-ray photons travel a distance l_x in the target. The trigger photon travels along the z axis and, by the time it passes l_c , its photon number reaches the critical value.

Since all electrons except one have been removed from the energy levels above the K shell,⁷ $\Gamma_{K2A} \approx 0$, and if Eq. (3) holds, then $\Gamma_{K2} \approx \Gamma_{K2R}$, so that in this case⁹

$$\omega_{K2} \equiv \Gamma_{K2R}/\Gamma_{K2} \approx 1. \quad (4)$$

The cross section σ_{sK2} for emission of a photon of energy E_2 by stimulated emission from a $K2$ atom is given by Eq. (1) if the atom is at rest. If Eq. (4) holds, then σ_{sK2} approaches its theoretically permitted maximum of $(2\pi)^{-1}\lambda_2^2$. If the atom is moving, then its resonant frequency $\nu_2 = c/\lambda_2$ undergoes a Doppler shift. For a collection of atoms in thermal motion,

$$\sigma_{sK2} = (\lambda_2^2/2\pi)(\ln 2/\pi)^{1/2}(2\tau_{K2R}\Delta_D\nu_2)^{-1}, \quad (5)$$

where $\Delta_D\nu_2$ is the full width at half maximum (FWHM) of the Doppler shift in ν_2 .

A photon with energy E is absorbed by an a atom with a cross section $\sigma_{ta}(E)$, by an $a0$ atom with $\sigma_{ta0}(E)$, and with $\sigma_{tK1}(E)$ by $K1$ atoms, with $\sigma_{tK2}(E)$ by $K2$ atoms; it creates a K vacancy with photoionization cross section $\sigma_{Pa}(E)$ in a atoms. An overbar (as in $\bar{\sigma}_{ta}$) means averaging over the pumping x-ray photon energies.

Let one plane-wave photon with the induced-emission energy E_2 enter the target at $z=0$. As it travels parallel to the z axis, it will be absorbed (because of $\sigma_{ta}, \sigma_{ta0}, \sigma_{tK2}$) and also emitted (because of σ_{sK2}). The number $N(E_2, \bar{k}_2, z)$ of such photons at z will vary exponentially with z . If this number at $z=d$ is >1 , then after one passage through the target the photon will not be damped, but amplified. Assuming that at time τ_X all densities are constants as a function of z , the amplification will be large, provided that

$$N(E_2, \bar{k}_2, z=d) = \exp(D^{-1}d) \gg 1. \quad (6)$$

Here,

$$\begin{aligned} D^{-1} \equiv & g\rho_{K2}(\tau_X)[\sigma_{sK2} - \sigma_{tK2}(E_2)] \\ & - (1-g)\rho_{K2}(\tau_X)\sigma_{tK2}(E_2) - \rho_{K1}(\tau_X)\sigma_{tK1}(E_2) \\ & - \rho_a(\tau_X)\sigma_{ta}(E_2) - \rho_{a0}(\tau_X)\sigma_{ta0}(E_2) \end{aligned} \quad (6a)$$

characterizes the exponent of the amplification. The photons under consideration can induce stimulated emission only in those atoms in which the total angular momentum of all electrons $J=1$, and is aligned with the angular momentum of the inducing photons; we denote by g that fraction of $K2$ atoms in which these conditions are satisfied.

Denote by l_c the "critical length" at which

$$N(E_2, \bar{k}_2, z=l_c) = d_x d_y \sigma_{sK2}^{-1},$$

i.e., by the time the initial photon under discussion reaches $z=l_c$, the photon number is amplified so much that these photons induce the stimulated

emission of a photon of energy E_2 in essentially every $K2$ atom having the appropriate angular-momentum state. Using Eq. (6),

$$l_c = D \ln(d_x d_y \sigma_{sK2}^{-1}). \quad (7)$$

The laser will not efficiently convert energy into coherent photons of energy E_2 unless $d_z > l_c$, or

$$d_z > D \ln(d_x d_y \sigma_{sK2}^{-1}). \quad (8)$$

Let an (almost) plane wave of wavelength λ travel parallel to the z axis, and be focused at $z=0$ on a spot whose diameters along x and y are a_x and a_y . We denote the full angular divergence of the wave in the x, z plane by $\Delta\Theta_x$ and that in the y, z plane by $\Delta\Theta_y$. From the uncertainty relation, $\Delta\Theta_x \gtrsim \lambda/2\pi a_x$, $\Delta\Theta_y \gtrsim \lambda/2\pi a_y$. We shall denote by l_c the "halving distance," at which the area of the wavefront is $2a_x a_y$, so that the energy density in the wavefront is half of what it is at $z=0$. The plane-wave approximation used in deriving Eqs. (6) and (7) is a good one, provided that

$$d_h \ll l_h. \quad (9)$$

The value of l_h is easily found:

$$l_h = \pi a_x^2 [-1 - f^2 + (1 + 6f^2 + f^4)^{1/2}], \quad (9a)$$

where $f \equiv a_x/a_y$.

The number of $K2$ atoms which fill their K vacancy by spontaneous decay during a time $\tau_X \ll \tau_{K2R}$ is $\rho_{K2}\tau_X/\tau_{K2R}$. This number is $\ll \rho_{K2}$, if

$$\tau_X/\tau_{K2R} \ll 1. \quad (10)$$

We shall denote by $\rho_{K2,1}$ the density of $K2$ atoms which have orbital angular momentum $J=1$. Assume that the angular momenta of all such atoms are aligned parallel to each other. The x rays emitted by spontaneously decaying atoms travel in random directions. For most of them, their path length in the target will be on the order of d_y . The number of stimulated photon emissions induced by each of these photons will be of order of $1 - \exp(-\rho_{K2,1}\sigma_{sK2}d_y)$. We require that this number be less than 1, or, to first order

$$\rho_{K2,1}(t)\sigma_{sK2}d_y < 1. \quad (11)$$

Spontaneously decaying atoms will emit approximately

$$\omega_{K2}\rho_{K2,1}(t)d_x d_y d_z (d_x d_y / l_c^2 4\pi) \tau_X / \tau_{K2R}$$

x-ray photons sufficiently parallel to the z axis to travel a distance l_c or more in the target. Such photons can trigger premature induced deexcitation of a significant fraction of all excited $K2$ atoms with $J=1$. To find the total number of such deexcitations, we integrate over the time t . When

$\rho_{K2,1}(t)$ is a linear function of t , then l_c is approximately inversely proportional to t , and the value of the integral is easily obtained in this approximation. We require that its value of

$$\frac{\tau_X}{4\tau_{K2R}} \omega_{K2} \rho_{K2,1} \frac{\tau_X d_x^2 d_y^2 d_z}{4\pi l_c^2} \equiv \epsilon_v \ll 1. \quad (12)$$

Next we find the $\rho_a(t)$, $\rho_{K1}(t)$, and $\rho_{K2}(t)$. We use the approximation

$$\bar{\sigma}_{tK1} \approx \bar{\sigma}_{tK2} \equiv \bar{\sigma}_{tK}.$$

Let us denote by l_x the distance which a pumping x-ray photon travels in the target. For mathematical simplicity we require that only a small fraction, ϵ_x , of the total number of pumping x-ray photons be absorbed during one passage through the target, i.e.,

$$l_x \{ \rho_a(t) \bar{\sigma}_{ta} + \rho_{a0}(t) \bar{\sigma}_{ta0} + [\rho_{K1}(t) + \rho_{K2}(t)] \bar{\sigma}_{tK} \} \equiv \epsilon_x \ll 1, \quad 0 \leq t \leq \tau_X. \quad (13)$$

Then we may neglect the spatial variation due to pumping x-ray photons, of ρ_a , ρ_{a0} , and of $\rho_{K1} + \rho_{K2}$ in the target. The sum $\rho_a(t) + \rho_{a0}(t) + \rho_{K1}(t) + \rho_{K2}(t)$ must equal $\rho_a(t=0)$ and thus be independent of time. During one passage through the target, a pumping x-ray photon will augment $\rho_{K1} + \rho_{K2}$ on the average by $l_x \bar{\sigma}_{Pa} \rho_a$, and decrease ρ_a by the same amount. If N_X is the total number of pumping photons delivered to the target in one pulse, then the number of such photons per unit volume per unit time (during τ_X) is $N_X (d_x d_y d_z \tau_X)^{-1}$. Remembering Eqs. (10), (11), and (12), we neglect spontaneous decays and the stimulated decay events induced by them. Then we find

$$\frac{d\rho_a}{dt} = \rho_a \tau_a^{-1}, \quad (14)$$

where

$$\tau_a^{-1} = -N_X (d_x d_y d_z \tau_X)^{-1} l_x \bar{\sigma}_{Pa}. \quad (14a)$$

The solution is

$$\rho_a(t) = \rho_a(t=0) \exp(-t\tau_a^{-1}), \quad (15)$$

independent of ρ_{K2}/ρ_{K1} .

The low-frequency pumping photons of energy $\Delta E = E_2 - E_1$ and wavelength in vacuum λ_Δ , travel parallel to the z axis. They are delivered in a pulse whose time variation is a step function of duration τ_Δ , starting at $t = -(\tau_\Delta - \tau_X)$, and ending at $t = \tau_X$. These photons will interact with the a cross section σ_Δ with atoms which have a K vacancy, raising the outer electron to level 2 if it was in level 1, but lowering it to level 1 if it was in level 2. Let l_Δ be the distance which such a photon travels in the target. For mathematical simplicity, we require that only a small fraction

ϵ_Δ of all such photons be absorbed in the target, i.e.,

$$l_\Delta \sigma_\Delta [\rho_{K2}(t) - \rho_{K1}(t)] \equiv \epsilon_\Delta \ll 1, \quad -\tau_\Delta + \tau_X \leq t \leq \tau_X. \quad (16)$$

We can then neglect the spatial variation of ρ_{K2} and ρ_{K1} arising from photons with energy ΔE . If N_Δ such photons are delivered to the target in one pulse, then their density per volume per unit time (during τ_Δ) is $N_\Delta (d_x d_y d_z \tau_\Delta)^{-1}$. When $\tau_a \gg \tau_\Delta$, then $\rho_{K2}(t)$ satisfies the differential equation

$$\frac{d\rho_{K2}(t)}{dt} = \tau_{\Delta 1}^{-1} \rho_a(t=0) [1 - \exp(-t/\tau_a)] - 2\tau_{\Delta 1}^{-1} \rho_{K2}(t), \quad (17)$$

where

$$\tau_{\Delta 1} \equiv \sigma_\Delta N_\Delta (d_x d_y \tau_\Delta)^{-1}. \quad (17a)$$

$\rho_{K2}(t = -\tau_\Delta + \tau_X) = 0$ by assumption, and $d\rho_{K2}(t)/dt = 0$, if $\rho_{K2}(t) = \rho_{K1}(t)$.

Assume that the pumping x-ray photon energies lie in a sufficiently narrow band above the K edge so that $\bar{\sigma}_{ta} \approx \bar{\sigma}_{Pa}$ and $\bar{\sigma}_{tK} \ll \bar{\sigma}_{ta}$. Note from Eq. (15) that if Eq. (11) holds for $t = \tau_X$, then it holds for $0 \leq t \leq \tau_X$, so choose $t = \tau_X$ in Eq. (11). Similarly, it is sufficient if Eqs. (16) and [assuming ρ_e is highest when $(\rho_{K2} + \rho_{K1})$ is] (3) hold for $t = \tau_X$. Remembering the parameters expected at the SSRP, assume for the pumping x-ray beam $\tau_X = 10^{-10}$ sec.

Consider once more the example of atomic Li gas. The K -shell edge is at $E = 66$ eV. Let the $2s$ level be level 1, and the $2p$ level be level 2, so that $E = 1.8$ eV. The soft x-ray photon emitted during a $(1s2p)^1P \rightarrow 1s^2$ transition has $\tau_R \approx 3 \times 10^{-11}$ sec, energy $E_2 = 60.7$ eV, wavelength $\lambda_2 = 2.04 \times 10^{-6}$ cm, and $\nu_2 = 1.47 \times 10^{16}$ sec $^{-1}$. At 1000° K temperature, $\Delta_D \nu_2 = 2\nu_2 c^{-1}$. $[(2kT \ln 2)/m]^{1/2} = 1.25 \times 10^{11}$ sec $^{-1}$, and $\sigma_{sK2} = 4.2 \times 10^{-14}$ cm 2 . Let the pumping x-ray photon beam have a step-function-like energy distribution that is constant from 66 to 70 eV and zero otherwise. Then $\bar{\sigma}_{tK1} \approx \bar{\sigma}_{tK2} \approx 3 \times 10^{-19}$ cm 2 , and $\bar{\sigma}_{Pa} \approx \bar{\sigma}_{ta} \approx 6 \times 10^{-18}$ cm 2 .¹⁰ Photons with energy E_2 cannot create a K vacancy nor resonantly lift an electron from the K shell to a higher level. So Li atoms with no K vacancy interact with photons of energy E_2 essentially with a cross section $\bar{\sigma}_{tK}$.

When the pumping x-ray photon ionizes a Li atom, 95% of the time it creates a K -shell vacancy (and thus a K atom) and a slow photoelectron whose average energy is about 2 eV, while about 5% of the time it removes an outer electron, thus creating an $a0$ atom and a fast photoelectron with an average energy of about 64.2 eV. If the low-energy 1.8-eV pumping radiation is intense enough, then $\rho_{K1}(\tau_X) = \rho_{K2}(\tau_X)$, so that one-half of all K atoms

will be $K2$ atoms. One-fourth of these will have their electrons in the $(1s2p)^1P_1$ state. If the low-energy photons are all polarized, then the $(1s2p)^1P_1$ states will all be created with their angular momentum parallel to the photon angular momentum. They will maintain their polarization until depolarized by collisions. Let us assume that the Li gas temperature is 1000°K . We choose the Li gas density to be $\rho_{\text{Li}} = \alpha_{\text{Li}} \times 10^{18} \text{ cm}^{-3}$, where α_{Li} is a parameter to be specified later, and is expected to be of order unity. At time $t = \tau_{K2}$, choose $\rho_{K1} + \rho_{K2} = 10^{-2} \rho_{\text{Li}}$. The number of photoelectrons is approximately equal to the number of K atoms, so that the electron density at τ_{K2} is approximately equal to $\alpha_{\text{Li}} \times 10^{16} \text{ cm}^{-3}$. Assume that $\alpha_{\text{Li}} \leq 5$, and that the cross section for Li-Li collisions, as well as for Li-electron collisions, are both $\leq 5 \times 10^{-16} \text{ cm}^2$. Then the probability that a certain Li atom will collide with any other Li atom during the time interval $\tau_{K2} = 3 \times 10^{-11} \text{ sec}$ is $\leq 1\%$; and the probability that it collides with a 2 eV electron during this time interval is $\leq 7\%$. Under these conditions, the polarization of $(1s2p)^1P_1$ states is maintained to a good approximation throughout τ_{K2} . Ionization of Li atoms cannot occur in collisions with Li atoms at 1000°K , nor with electrons with energy up to about 4 eV, because the ionization potential in the $1s^22p$ state is about 3.6 eV while in the $1s^22s$ state it is 5.4 eV.

So far, we have investigated the effect of collisions between two Li atoms, or between a Li atom and a slow photoelectron. Still to be investigated are the collisions between a Li atom and fast electrons. The fast photoelectrons do have enough energy to ionize a Li atom. However, as noted above, only about 5% of all photoelectrons are fast. Furthermore, these electrons travel about $1.4 \times 10^{-2} \text{ cm}$ during τ_{K2} , which is much greater than the average pathlength of such an electron inside the target. In fact, we will specify the Li gas target dimensions such that a fast electron can spend only about 3×10^{-3} fraction of its time inside the target. Then collisions with fast electrons can be neglected as far as depolarization and ionization are concerned.

Let us choose the target dimensions to be $d_x = 2.5 \times 10^{-3} \text{ cm}$, $d_y = 4.56 \times 10^{-4} \text{ cm}$, and focus the

pumping x-ray beam so that $a_x = d_x$, $a_y = d_y$. When the circulating electron beam has horizontal diameter $3 \times 10^{-2} \text{ cm}$, vertical diameter $< 6 \times 10^{-3} \text{ cm}$ (as is the case in the interaction region in the SPEAR storage ring at Stanford), and the synchrotron radiation is emitted with an angular divergence of about $1.5 \times 10^{-3} \text{ rad}$ (as in SSRP for 66–70-eV photons), then focusing on the desired spot can be achieved with an $f/55$ mirror.¹¹

If we choose $\alpha_{\text{Li}} = 5$, then Eq. (4) is satisfied, and we find that $l_c = 1.3 \times 10^{-1} \text{ cm}$. We set $d_z = 0.3 \text{ cm}$, which is consistent with the other target parameters. Equations (6), (8), and (11) are satisfied.¹² The $\epsilon_x \approx 0.6$ and the plasma frequency is $2.2 \times 10^{13} \text{ sec}^{-1}$, so that the low-energy pumping photons are hardly affected by the presence of free electrons; Eqs. (15) and (16) can hold to a good approximation. The number of $K2$ atoms in the target at $t = \tau_x$ will be 1.7×10^{10} , and we need 4.2×10^{10} pumping x-ray photons delivered to the target in one pulse of duration τ_x . About 7×10^9 atoms will radiate coherently. The radiation will be delivered in about 200-W pulses lasting about $3 \times 10^{-11} \text{ sec}$, with an impressive coherence factor of $\Delta\nu/\nu \approx 5 \times 10^5$, suggesting applications in research and industry.

For heavier elements, one may proceed similarly. First, irradiate with photons of energy ΔE and so lift an outer electron to a long-lived state ("level 2"). Next, ionize sufficiently to suppress Auger transitions, and then create a K vacancy into which the coherent decay of level-2 electrons can be induced. Instead of a K vacancy, one may create an L, M , etc. vacancy, leave all lower-lying electrons undisturbed, and proceed as above.

At the SSRP about 3×10^8 photons of energy 66–70 eV are to be delivered per 10^{-10} sec pulse within 2-mrad angle at 100-mA electron current in the ring. This could be increased by using only one electron beam in the ring.¹³ Further increase could be achieved by strong magnetic bending or modulating of the beam¹⁴ or by accepting more photons into the x ray beam. This should be considered in the design of future accelerators, such as PEP.

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⁵Assuming that the gas temperature is 1000 °K, an electron density equal to the Li ion density, and a collisional deexcitation cross section of $0.5 \times 10^{-15} \text{cm}^2$.

⁶The notation alludes to the fact that this cross section is about the same as the total photoionization cross section for Li atoms which have a *K* vacancy, since for such atoms the contribution for the *K* edge is absent.

⁷This may be unnecessary. We only have to remove enough electrons to suppress Auger transitions so that $\Gamma_{K2A} \ll \Gamma_{K2R}$ in the notation of Eq. (2).

⁸We assume that the dimensions of the container are large enough so that deexcitation because of collisions with the walls are negligible.

⁹The Γ_{K1} satisfies an equation similar to Eq. (2), but since $\Gamma_{K1} \ll \Gamma_{K2}$ by assumption, the Γ_{K1j} and Γ_{K1A} may be not be $\ll \Gamma_{K1R}$.

¹⁰W. J. Veigele, E. Briggs, L. Bates, E. M. Henry, and B. Barcewell, Kaman Sciences Corp. Report No. KN-71-431(R) (1971), unpublished. The ratio σ_{Pa}/σ_{tK} is

assumed to be 20 at the *K* edge, which is its measured value for other light elements.

¹¹This spot size is small compared to what is customarily reached with optical wavelengths, because the wavelength and the beam diameters are small compared to the corresponding quantities for optical lasers. For high-power optical lasers the beam divergence is of the order of 10^{-3} rad, but the beam diameter is a few cm. For low-power gas lasers the angular divergence may be as low as 10^{-6} rad, but the diffraction limit is reached around 25 μm spot size.

¹²Equation (12) is not, but will be, if a_x, a_y is left unchanged while d_x and d_y are both reduced by a factor ≥ 6 .

¹³H. Winick, private communication.

¹⁴Paul L. Csonka, Phys. Lett. 24B, 625 (1967); CERN report No. 67-15 (unpublished); University of Oregon preprint N. T. 152/74 (1975); Paul L. Csonka and K. S. Kolbig, Phys. Rev. D 10, 251 (1974).