PHYSICAL REVIEW A VOLUME 13, NUMBER 5 MAY 1976

Reply to the Comment by Berge and Skullerud

J. H. Whealton*
Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, Colorado 80302
(Received 14 October 1975)

Berge and Skullerud's criticisms arise from an incomplete reexamination of the lowest nontrivial level of the transport theory under question. A closer examination shows their criticism to be unfounded. However, even if true, the value of their criticism is dubious since the theory would still be at worst asymptotically valid for small density gradients. This would serve to at least delineate the region of validity of the conventional diffusion analysis of drift-tube experiments.

The paper in question[1] is intended to delineate an ion transport theory which does not have the explosive characteristics of the diffusion or Burnett equations[2] at short times. The purpose is to be more diligent in treating the time derivative in Boltzmann's equation in order to solve a set of moment equations simultaneously instead of iteratively.[3] In either method[1,3] the truncation of the moment equations is made by substituting higher-order moments into the steady-state Boltzmann equation. If the truncation were exact, there would be no need for an infinite sequence of approximations as these theories[1,3] require. The scheme compensates for the fallibility of the truncation by coupling the truncated terms to the lower moments (such as the ion density, n) to successively higher powers of $1/N$ (or more precisely, the ratio of the drift distance in one mean free time to a characteristic distance on the scale of the initial density inhomogeneity) in successively higher-order approximations.

Berge and Skullerud[4] point out that the truncation is an approximation and consider the trivial lowest-dispersion relation appearing in Table I of Ref. 1. They erroneously conclude that the method of truncation causes shock waves. This conclusion is immediately dispensed with by noting that the diffusion equation has no shock waves associated with it, and is customarily obtained using the *same* truncation.[3] Shock waves come about by solving the moment equations simultaneously instead of iteratively. Examination of Refs. 1 and 3 shows that, to a given order, less approximation is made with the simultaneous scheme than with the iterative scheme although more labor is involved in the former. (Indeed analytical solutions to the iterative scheme exist at the Burnett level[2] whereas for the simultaneous solution scheme, they exist only at the Navier-Stokes level.[1]) Neglecting the possibility that the poorer approximations give better answers, the shock waves in Ref. 1 are a better approximation than the unconstrained explosive initial behavior of the diffusion equation. This is a moot point, except possibly for very heavy ions in a light gas. For if the convolution of the Green's function of the Navier-Stokes solution [Eqs. (54) and (55) of Ref. 1] over the initial density distribution produces noticeable shock waves, the initial density distribution is too sharp for either the Navier-Stokes or diffusion theory to be accurate.[5]

The moment theory[1] assumes no initial conditions on the velocity distribution as claimed by Berge and Skullerud.[4] As an illustrative example, in the solution of the Navier-Stokes approximation, the second term in the Green's function [Eq. (53) of Ref. 1] was neglected. But that is the only use made of such initial data. Berge and Skullerud's arguments concerning such initial data are thus a non sequitur.

Contrary to Berge and Skullerud's statements,[4] the utility of the low-level analytic solutions[1,2] is that they constrain the region of validity of the diffusion analysis of drift-tube data. For heavy ions in a light gas, Ref. 1 gives the lowest-order corrections; and for light ions in a heavy gas, Ref. 2 gives the lowest-order corrections.

One would guess that the sequence of approximations in either Ref. 1 or 3 is at least asymptotic to small density gradients; and I have conjectured that the sequence of approximations in Ref. 1 even has a finite radius of convergence in the ion density gradients. No remark by Berge and Skullerud sheds light on these questions. One could do better by a more extensive consideration of the dispersion relations or by solving higher-level moment equations. The latter procedure has already been carried out.[5]

My principal objection to either of the truncation schemes in Ref. 1 or 3 is that the property of stationization is lost. This is bad because it seems unnecessary, at least for the kinetic theory of ion cyclotron resonance experiments,[6] and it precludes the method[1,3] from being directly applicable for the analysis of ion-trap experiments.[7] An obvious possibility is to truncate the moment equations by substituting the higher moments into the Boltzmann equation, neglecting space derivatives but keeping time derivatives. This yields transport equations similar to Eqs. (21), (25), (28), and (31) of Ref. 1 but with higher-order time-derivative terms.

---

*Present address: Thermonuclear Div., Oak Ridge National Lab., Oak Ridge, Tenn. 37830.

[1]J. H. Whealton, Phys. Rev. A 11, 297 (1975).
[2]J. H. Whealton, J. Phys. B 7, 1602 (1974).
[3]J. H. Whealton and E. A. Mason, Ann. Phys. (N.Y.) 84, 8 (1974).
[4]T. Berge and H. R. Skullerud, preceding Comment, Phys. Rev. A 13, 1975 (1976).
[5]J. H. Whealton, E. F. Jaeger, and D. S. Burch, Bull. Am. Phys. Soc. 20, 1259 (1975).
[6]L. A. Viehland, E. A. Mason, and J. H. Whealton, J. Chem. Phys. 62, 4715 (1975).
[7]H. G. Dehmelt and F. L. Walls, Phys. Rev. Lett. 21, 127 (1968); F. L. Walls and T. S. Stein, Phys. Rev. Lett. 31, 975 (1973).