Comments on the paper "Ion-transport theory for a slightly ionized rarefied gas in a strong electric field"

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Criticisms are given of a recent theory by Whealton of ion transport for a slightly ionized rarefied gas in a strong electric field. His prediction of shocklike phenomena in ion swarm experiments is shown to be untenable, being the result of an unphysical assumption.

Whealton[1] has recently published an ion-transport theory which he claims is able to predict the initial evolution of an ion swarm released as a δ function in space, i.e., under conditions in which Fick's law of diffusion is known not to be valid. A principal result of the theory is the prediction of shocklike phenomena in pulsed drift tubes, a feature which has, however, never been experimentally observed. The paper[1] also criticizes recent electron diffusion theories[2,3] treating the deviations from Fick's law, claiming that they are suspect "since their starting points are equations which assume smooth initial data."

The criticism of the electron diffusion theories is unjustified, and is due to the failure to realize that the evolution of an electron swarm takes place on two widely different microscopic time scales, comparable to the times characteristic for momentum and energy transfer, respectively. This is made explicitly clear in the two cited papers,[2,3] and requires no further comment here.

The prediction of shocklike phenomena in pulsed drift tubes is due to an inadmissible truncation of the transport equations, and thus lacks physical significance. The moment equations formed from the Boltzmann equation[4] have been approximated by substituting the steady-state values for the highest moments retained,

$$\frac{\partial}{\partial z}\left(n(z,t)\langle\psi_{lm}^{(r)}(\vec{v};z,t)\rangle\right)\sim\langle\psi_{lm}^{(r)}\rangle_{ss}\frac{\partial n}{\partial z}. \qquad (1)$$

The highest moments are those which weigh the high-energy tail and the details of the velocity distribution most heavily, and thus are the ones which relax slowest towards their steady-state values. The approximation (1) is therefore not justified.

Assuming a constant mean free time, Eq. (1) yields on the "Navier-Stokes level" a transport equation

$$\frac{\partial n}{\partial t}+v_d\frac{\partial n}{\partial z}+\frac{M+m}{m}\tau_M\left(\frac{\partial^2 n}{\partial t^2}-\langle v_z^2\rangle_{ss}\frac{\partial^2 n}{\partial z^2}\right)=0, \qquad (2)$$

v_d being the steady-state drift velocity, m and M the ion and neutral masses, respectively, and τ_M the mean free time for momentum transfer in the c.m. system. This equation is the same as Eq. (25) in Ref. 1, but has been rewritten to make clear the physical meaning of the coefficients involved.

Equation (2) is a hyperbolic equation with characteristics dz/dt given by the steady-state rms speed, $dz/dt=\langle v_z^2\rangle_{ss}^{1/2}$. It is thus unable to describe the propagation of disturbances with velocities larger than the rms steady-state speed, i.e., outside the cone of the characteristics.

The "shock waves" predicted by Whealton[1] represent particles following the characteristics of Eq. (2).

Assume for a moment that the ion swarm had initially been released with the steady-state velocity distribution. The fastest particles would then tend to diffuse most rapidly away from the centroid of the swarm, giving rise to a mean-square velocity $\langle v_z^2\rangle$ varying in space, and being largest furthest away from the centroid. Equation (2) does not, however, allow mean-square velocities larger than the steady-state value. Pictorially, the "shock waves" may therefore in this case be looked upon as due to fast particles which unphysically are hindered in their motion by the characteristics.

Now, let us turn to the initial conditions used by Whealton.[1] These initial conditions were meant to correspond to an ion swarm released with zero initial velocity. However, Eq. (2) was obtained by assuming that the steady-state mean-square velocity could be inserted in the moment equations as an approximation for the actual time- and space-dependent mean-square velocity. Whealton's procedure thus implicitly involves the simultaneous assumption of two conflicting initial conditions, in addition to the unphysical truncation of the transport equations.

We conclude that the results presented in Ref. 1 are basically untenable and not of use for providing an estimate of errors that may arise in the usual analysis of drift data. In particular, the prediction of "shock waves" in ion swarm experiments can be tracked directly back to one unphysical assumption.

[1]J. H. Whealton, Phys. Rev. A 11, 297 (1975).
[2]J. H. Parker and J. J. Lowke, Phys. Rev. 181, 290 (1969).
[3]H. R. Skullerud, Aust. J. Phys. 27, 195 (1974).
[4]J. H. Whealton and E. A. Mason, Ann. Phys. (N.Y.) 84, 8 (1974).