Excitation of the Mg and Mg⁺ resonance lines by electron impact on Mg atoms*

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We have measured the optical excitation functions and polarizations of the Mg resonance line (2852 Å) and of the Mg⁺ resonance lines (2796 Å, 2803 Å, unresolved) arising from ionizing excitation of magnesium atoms, for electron-impact energies from the excitation thresholds to 1400 eV. In our crossed-beam apparatus, the electron-beam energy resolution was ≈ 0.25 eV for energies below 10 eV, and the atom beam was optically thin. The excitation function of the ionic lines was normalized to that of the atomic line by relative intensity measurements. The 2852-Å excitation function, when normalized to Born theory in the high-energy limit, has a maximum cross section of $17.3 \pi a_0^2 \pm 3\%$ at 18.5 eV. This excitation function is graphically compared with the resonance-line excitation functions of Na, Ca, and Li previously measured in this laboratory. The 2852-Å polarization function is consistent with the theoretical threshold limit of +100%, and has a sharp feature near 5 eV which cannot be due to cascading.

I. INTRODUCTION

Electron-impact excitation functions of the sodium, calcium, and lithium resonance lines have been reported in previous papers from this laboratory,¹⁻³ and we are studying other excitation functions of metal atoms by the same crossedbeam apparatus. Here we report normalized optical excitation functions and polarizations for the Mg $3 {}^{1}P$ - $3 {}^{1}S$ resonance line (2852 Å) and for the unresolved Mg⁺ $3 {}^{2}P$ - $3 {}^{2}S$ resonance doublet (2796, 2803 Å) excited by electron impact on neutral magnesium atoms.

Previous measurements of these two excitation functions have been reported,^{4,5} one measurement without normalization,⁵ and the other an absolute measurement with reported $\pm 35\%$ uncertainty in the two absolute cross-section scales.⁴ These papers disagree about the shapes and relative magnitudes of the two excitation functions and give no information about the polarizations. Several theoretical calculations of the Mg 3 ¹P excitation cross section have been performed,⁶⁻⁹ using either the Born approximation^{7,8} or close coupling with a $\{3 {}^{1}S, 3 {}^{3}P, 3 {}^{1}P\}$ basis set.^{6,9} Two of these calculations yielded the polarization as well.^{8,9}

Our measurements covered the energy range from the 3 ${}^{1}P$ threshold to 1400 eV. We used crossed low-density beams of electrons and magnesium atoms, so that uncertainties arising from spacecharge depression of the electron-beam energy and entrapment of the Mg resonance radiation were less than 1%. The 2852-Å excitation function nearly converged to the Born energy dependence within the experimental energy range (up to 320 times the threshold energy); we have therefore normalized the 2852-Å excitation function to Born theory in the high-energy limit, using the calculations of Ref. 8 supported by the accurately known optical oscillator strength for the resonance transition.⁴⁰⁻¹² We normalized the excitation function of the ionic lines to that of the atomic line by comparing the radiation intensities at fixed impact energies near the maxima of the excitation functions. We estimate the uncertainty of the normalized cross-section scales to be about $\pm 3\%$ for the atomic line and $\pm 7\%$ for the ionic lines, the former uncertainty being dominated by uncertainty of the theoretical cross section.

Because the apparatus used for the present experiment has been described in detail elsewhere (its basic design and testing in Ref. 1, modifications in Ref. 2), it is described only briefly in this paper. In Sec. II we define the terms used to express our results, and summarize the observations and assumptions that yielded the results presented in Sec. III. Then we discuss the Born normalization procedure, which yields an absolute cross section for emission of the 2852-A line, the cascade contributions at high impact energies being estimated from theoretical and experimental data in the literature.^{4,8} In Sec. V we compare our normalized results with previous measurements, and with theoretical calculations of Mg $3^{1}P$ excitation. The latter comparisons are incomplete; because cross sections for the dominant cascade contributions to the 2852-Å line are currently available only from Born calculations, we do not attempt to extract a 3 ¹P-level excitation cross section from the present experimental data.

II. MEASUREMENTS AND CORRECTIONS

The electric-dipole radiation emitted by an atom after excitation by an electron beam may be regarded as due to three incoherent dipoles, one dipole parallel to the electron-beam direction and two equal dipoles perpendicular to this direction and

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to each other.^{13, 14} We define Q_{\parallel} and Q_{\perp} to be the cross sections for emission (in all directions) of radiation from the parallel dipole and from one of the perpendicular dipoles, respectively. The polarization components I_{\parallel} and I_{\perp} (in the notation of Refs. 13 and 14) of the radiation intensity propagating at right angles to the electron beam are, of course, proportional to Q_{\parallel} and Q_{\perp} . Then the total cross section Q_T for emission of a given line or multiplet (i.e., the optical excitation function) is

$$Q_{T} = Q_{\parallel} + 2Q_{\perp} \propto I_{\parallel} + 2I_{\perp},$$

and the polarization P is

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$$P = (Q_{\parallel} - Q_{\perp}) / (Q_{\parallel} + Q_{\perp}) = (I_{\parallel} - I_{\perp}) / (I_{\parallel} + I_{\perp}).$$

Having observed I_{\parallel} and I_{\perp} for the given Mg lines, we obtained P and Q_T , not just the apparent excitation function observed at right angles to the electron beam as in Refs. 4 and 5. The quantities Q_{\parallel} and Q_{\perp} , which are convenient for the discussion of polarization in Sec. V, can be obtained by using $Q_{\parallel} = Q_T (1+P)/(3-P)$ and $Q_{\perp} = Q_T (1-P)/(3-P)$.

In the present experiment, a steady beam of magnesium atoms from an oven was crossed at right angles by a focused electron beam, and the impact radiation was collected in a cone along the direction orthogonal to both beam axes. This light was collimated and passed at normal incidence through a dichroic-film polarization analyzer and an interference filter for either the atomic or the ionic resonance lines. (These two filters had half-bandwidths of 23 Å; minor consequences of incomplete spectral isolation are discussed later.) Light of the selected wavelength was then focused onto one end of a light pipe and thereby conducted to a cooled photomultiplier tube, the light being spread across the entire photocathode. The detected photoelectrons were counted into one of two channels according to the orientation of the polarizer axis, which was positioned alternately parallel and perpendicular to the electron-beam axis. Thus, we measured I_{\parallel} and I_{\perp} , and the electron-beam current, as a function of energy. The atom-beam density was not measured but was estimated to be typically $\sim 3 \times 10^9$ cm⁻³. We evaluated the uncertainty of the crossed beams' overlap and possible spatial variations of the photodetector efficiency by the methods of Ref. 1, and found these uncertainties to be $\pm 1\%$ or less.

From these measurements, we determined the polarizations and excitation functions given in Table I and Figs. 1-5. We took into account the finite electron-beam and photodetector solid angles and the imperfect polarizer. The latter correction was about 25% of P, but introduced negligible uncertainty because we measured the polarizer's principal transmittances for the wave-

lengths of the experiment. The instrumental polarization was (0.0 ± 0.1) %. Periodically we measured the electron-beam energy distribution using a retarding-potential analyzer. For energies below 32 eV, we corrected the energy scale by reference to the 3 ¹P excitation threshold; the correction to the energy value ascertained by retardation analysis was usually less than 0.2 eV. Further details of the data-handling procedure can be found in Ref. 2; the apparatus description given there also applies, except that we installed f/2.6 quartz optics and a different polarizer for the present experiment.

The polarization of the Mg resonance line was observed to decrease by about 1% of P when the magnesium-beam density was doubled from its typical operating value. We attributed this depolarization to entrapment of the radiation, and corrected for it by extrapolating P linearly to zero atom-beam density. (Similar effects are discussed in more detail in Refs. 1 and 3.)

The atomic Mg spectrum¹⁵ contains a number of lines too close to the Mg⁺ resonance lines to be completely rejected by our filter for the ionic lines. Figure 4 shows how we corrected the ionizing excitation function for leakage of these unwanted lines. Most of the leakage was found to be due to the 2852-Å line; at 12 eV, this line's contribution to the apparent cross section was 3% of the maximum value of the ionizing excitation function. Using our measured results for the 2852-A line, we removed its contribution to the apparent excitation and polarization functions of the ionic lines for all impact energies. The rest of the leakage, observed in the energy range 7.4-12 eV, contributed an apparent cross section about 1% as large as the maximum of the ionizing excitation function, and was decreasing with energy above 9 eV. The only Mg lines that can account for this leakage arise from the triplet terms marked in Fig. 4, whose spin-forbidden excitation functions are expected to decrease faster than the ionizing excitation function at high energy.^{4,16} We removed the effects of this unknown leakage from the data in the Mg⁺ $3^{2}P$ threshold region, by extrapolating from the observations below 12 eV as shown in Fig. 4; we regarded this leakage as inconsequential at higher energies. The uncertainties due to all the leakages were taken into account, but were smaller than the other uncertainties except for the polarization data below 14 eV. Below 12.5 eV the polarization of the ionic lines could not be ascertained because of uncertainty of the leakages and background counts.

Finally, we measured the filter transmissions of the 2852-, 2796-, and 2803-Å lines individually, using auxiliary apparatus. The filter calibration

TABLE I. Experimental results for electron-impact excitation of the Mg 2852-Å resonance line, including cascade contributions. Q_T is the normalized optical excitation function.^a

Energy ^b (eV)	Polarization (%)	$Q_T (\pi a_0^2)$
4 60(0) ^c	86.5(12) ^c	2 10(6) ^c
4.75(1)	80.5(10)	2.85(4)
4.90(1)	77.8(9)	3.86(4)
5.08(1)	80.0(8)	5.03(5)
5.40(1)	72.6(7)	6.09(6)
5.75(1)	66.0(7)	7.67(8)
6.10(2)	64.9(6)	9.13(10)
6.60(2)	62.1(6)	10.69(12)
7.50(2)	57.9(6)	14.20(15)
8.90(2)	52.4(5)	16.55(18)
10.00(3)	47.4(5)	16.02(17)
12.00(3)	40.1(4)	16.41(17)
15.00(4)	32.1(3)	16.98(17)
18.50(4)	25.2(3)	17.32(17)
24.00(4)	18.3(2)	17.13(17)
30.00(5)	13.7(2)	16.63(16)
37.9(2)	8.5(2)	15.87(16)
62.7(2)	-1.3(2)	13.25(10)
98.1(2)	-9.3(2)	10.62(6)
148.4(2)	-16.2(2)	8.264 (55)
248.8(2)	-23.8(2)	5.799(40)
399.2(3)	-29.5(3)	4.065(27)
599.7(3)	-34.2(3)	2.957 (20)
800.2(4)	-36.6(3)	2.348(16)
1100.6(6)	-39.4(3)	1.808(10)
1401.7(8)	-41.4(3)	1.480(0)

 ${}^{a}Q_{T}$ is defined in Sec. II, and is normalized according to Sec. IV, but only the experimental uncertainties of Q_{T} (relative to the 1400-eV value) are given here. b The mean energy of the incident electrons, corrected

⁶ The mean energy of the incident electrons, corrected by reference to the 3 ¹P excitation threshold (4.345 eV). The electron energy resolution was ≈ 0.25 eV FWHM for energies below 10 eV. Besides the uncertainty given for each point (relative to the 4.6-eV point), there is an additive uncertainty of ± 0.05 eV in the energy scale, discussed in Sec. V.

^cThe number in parentheses gives the uncertainty in the last places of the previous number. In columns 2 and 3, estimated systematic uncertainties have been combined with roughly 2σ statistical uncertainties.

enabled us to normalize the excitation function of the ionic lines to that of the atomic line; for this measurement we assumed the S-13 photomultiplier's quantum efficiency to be constant over the 2% wavelength change involved. Also, a correction to the observed polarization of the Mg⁺ doublet was necessary because the filter's transmission for the 2803-Å line was 32% smaller than for the 2796-Å line. We applied a correction (about a 10% reduction of the apparent polarization) using the *LS*-coupling results that the doublet intensity ratio is 2:1 and that the 2803-Å line is unpolarized.¹³

As we have stated in Figs. 4 and 5, the Mg⁺ $3^{2}D \rightarrow 3^{2}P$ multiplet (2791, 2798 Å) was not rejected by our filter for the Mg⁺ resonance doublet. Although the filter's transmission was different for each line, the effective transmission for the $3^{2}D \rightarrow 3^{2}P$ multiplet was about the same as for the resonance doublet. In our experiment the contributions from the unwanted ionic lines could not be identified and subtracted out as was done for the unwanted atomic lines mentioned above. The presence of the $3^{2}D$ $-3^{2}P$ lines is thought to affect our observed ionizing excitation function by 10% or less, as discussed in Sec. V, and was ignored when we made the polarization correction mentioned in the previous paragraph.

III. EXPERIMENTAL RESULTS

Here we present our results for the optical excitation functions Q_T and polarizations P. Although the excitation functions are given in absolute units,



FIG. 1. Method of normalizing the relative cross section for the 2852-Å line. Born calculations by Robb (Ref. 8) have been used for the $3^{1}P$ level and for the major cascade contributors $3^{1}D$, $4^{1}S$, and $4^{1}P$ (see Fig. 2). A small contribution from many higher levels, based on the experimental data of Ref. 4, is also included in the total cascade estimate. The present total cross section Q_T (dots) is normalized to the sum of the Born $3^{1}P$ plus estimated total cascade cross sections. The present cross sections Q_{\parallel} and Q_{\perp} , for the separate polarization components of the 2852-Å line, are shown by curves (interpolated from the original data).

our measurements have determined only the relative excitation functions and the ratio of their magnitudes. Only the experimental uncertainties of Q_T are given here; the Born normalization and its uncertainty are discussed in the next section.

A. The Mg $3^{1}P-3^{1}S$ line

The present results are given in Table I and in Figs. 1-3. All of the instrumental corrections and uncertainties mentioned in Sec. II have been taken into account, although the data contain an unknown but minor contribution from the Mg multiplet at 2850 Å, discussed in Sec. V. The uncertainty of the relative excitation function is mainly due to uncertainty of the crossed beams' overlap. The dominant uncertainty for the larger polarization values is that of the radiation-entrap-



FIG. 2. Present experimental results (heavy solid curve) for the $2852\text{-}\text{\AA}$ line compared with previous measurements and Born theory. Absolute cross sections by Aleksakhin et al. (Ref. 4): dashed curve. Relative cross sections by Karstensen and Köster (Ref. 5), shown normalized to present results at 80 eV: dotted curve. All the experimental data include cascades, as does the Born cross section to which the present data are normalized. Born cross sections for individual cascade contributions are also given as follows (these are the $3^{1}D$, $4^{1}S$, $4^{1}P$ excitation functions of Ref. 8 multiplied by the respective branching ratios 100%, 100%, 8.3%): The $3^{1}D$ and $4^{1}P$ contributions are plotted, the $4^{i}S$ contribution was $0.1\pi a_0^2$ at 10 eV. Born polarizations, calculated by Robb (Ref. 8) for direct $3^{1}P$ excitation only, are shown by plus symbols.

ment correction; the uncertainties of the smaller polarization values arise mainly from the counting statistics and the uncertainty of the instrumental polarization.

The data were obtained at energy intervals of less than 0.1 eV below 10 eV, at energy intervals of



FIG. 3. Detailed low-energy experimental data (dots) for the 2852-Å line, compared with results of threestate close-coupling calculations. Below 4.6 eV the excitation and polarization functions are not given directly by the data because of limited energy resolution; see Sec. V for interpretation of the crosssection data near the $3^{1}P$ threshold. If one assumes the polarization to behave as the indicated line from 100% at threshold to 86.5% at 4.6 eV, convolution with the electron energy distribution (inset) weighted by the excitation function shows the expected result of our experiment to be the dashed line. Excitation thresholds for higher singlet levels are marked by bars; the experimental data include cascades from these levels. Theoretical results of Van Blerkom (+, Ref. 6) and of Fabrikant (×, Ref. 9) are for direct $3^{1}P$ excitation only.

(2-5)% between 10 and 32 eV, and at ten energy values between 37 and 1400 eV. Table I contains all the data above 37 eV, and representative, averaged values below that. The original data below 9 eV can be seen in Fig. 3. Because of the smooth behavior of the high-energy Q_T data shown in Fig. 1, we have represented our data as a continuous curve in subsequent plots.

B. The Mg⁺ $3^{2}P - 3^{2}S$ lines

Our results for the ionic resonance doublet are shown in Figs. 4 and 5. As explained in Sec. II and Fig. 4, the results have been corrected for minor spectral leakage of Mg lines. Contributions from the Mg⁺ 3 ^{2}D -3 ^{2}P lines, which overlap the resonance doublet, could not be removed and are discussed in Sec. V.

The data were obtained at energy intervals of about 1% between 12 and 32 eV, and at ten energy values between 37 and 1400 eV (the latter ten being the same values listed in Table I). To produce our results shown in Fig. 5, we averaged the low-



FIG. 4. Normalized total cross section for ionizing excitation of the Mg⁺ $3^{2}P \rightarrow 3^{2}S$ resonance doublet (2796, 2803 Å) from the ground state of Mg. Spectral leakages from nearby Mg lines contributed to the apparent cross section as indicated: The leakage of the 2852-Å line (short-dashed curve) and that of other Mg lines near 2800 Å (long-dashed curve, extrapolated above 12 eV), was deduced from the raw data (dots) for energies below the $Mg^+ 3^2 P$ threshold (12.1 eV). Subtracting these leakages from the smoothed data yields the corrected cross section (solid curve with dashed extrapolation to threshold). The resonance doublet was unresolved and unseparated from the Mg⁺ $3^{2}D \rightarrow 3^{2}P$ lines (2791, 2798 Å), which therefore are included in the corrected cross sections for energies above the $3^{2}D$ threshold (16.5 eV). Excitation thresholds of Mg levels whose radiation could leak through the interference filter are marked: $3^{1}P$ (a), $(3p^{2})^{3}P^{e}$ and $7^{3}S$ (b), $(3p 3d)^3 D^o$ (c). Thresholds of cascade-producing Mg⁺ levels are marked: 4^2S (d), 3^2D (e), 4^2P (f).

energy polarization data in groups, and plotted a smooth curve through the Q_T data. The uncertainty of the relative excitation function is about $\pm 2\%$ for energies below 32 eV and $\pm 3\%$ above 32 eV, owing to the worsening statistical uncertainties as the cross section diminished rapidly with increasing energy. This uncertainty is refflected by the scatter of the high-energy $Q_{T}E$ data in Fig. 5. The energy scale was corrected by reference to the Mg $3^{1}P$ threshold, not the threshold for the ionic lines. Thus, the agreement seen in Fig. 4 between the apparent threshold and the known excitation energy of the $Mg^+ 3^2P$ term is a check on the consistency of our energy measurements. The uncertainty for energies below 32 eV is ± 0.1 eV.

The measured ratio of the ionizing excitation function at its maximum to the maximum value of the 2852-Å excitation function was 0.071, with



FIG. 5. Normalized total cross section and polarization for the Mg⁺ resonance doublet excited from the Mg ground state. The present cross section (solid curve) is compared with measurements by Aleksakhin et al. (dashed curve, Ref. 4) and by Karstensen and Köster (curve of small dots, Ref. 5). We have adjusted the cross-section scales of Refs. 4 and 5 to agree with our data at 27 eV. All the experimental data include cascades; thresholds for some cascadeproducing terms are indicated. The Mg⁺ $3^2D \rightarrow 3^2P$ lines were not separated from the resonance lines in the present work. The present results have been corrected for spectral leakage of atomic Mg lines as indicated in Fig. 4. The present cross section times energy is plotted against the $Q_T E$ scale on the right; only the measured points are given (large dots). The present polarizations are given as dots; the error bars represent 2σ and are constant where omitted.

 $\pm 6\%$ uncertainty, primarily from uncertainty of the relative filter transmissions and optical detection efficiencies.

IV. NORMALIZATION AND CASCADES

Our measured 2852-Å excitation function has been normalized to Born theory as shown in Fig. 1. The high-energy form of the Born cross section Q_B for a dipole-allowed excitation is determined mainly by the optical oscillator strength f of the transition^{7,17}:

$Q_B E - A + 1705 (f / \Delta) \log_{10} E$

where Q_B is in units of πa_0^2 , A is constant, E is the impact energy in eV, and Δ is the excitation energy in eV. The Born cross section calculated by Robb⁸ for Mg $3^{1}S-3^{1}P$ excitation has the form $Q_B(3 P)E = -262.0 + 711.8 \log_{10} E$ for E > 100, implying f = 1.814. Because this agrees with the experimental value, taken to be $f = 1.82 \pm 0.05$ from Refs. 10-12, we have used Robb's values for $Q_B(3 P)$ without alteration. Born cross sections for excitation of the $3 {}^{1}D$, $4 {}^{1}S$, $4 {}^{1}P$ levels, also calculated by Robb,⁸ have the behavior $Q_{B}(3 \ ^{1}D)E - 74.5, \ Q_{B}(4 \ ^{1}S)E = 1.7, \ Q_{B}(4 \ ^{1}P)E =$ $8.2 + 59.6 \log_{10} E$ for $E \ge 100$. We have computed branching ratios for these levels to decay via the 3 ^{1}P level, taking transition probabilities from Refs. 10 and 18. These branching ratios and the resulting Born cross sections for cascade contributions to the 2852-A line are given in Fig. 2. The ratios of these cross sections to $Q_{P}(3^{-1}P)$ at 1400 eV are: $(3 \ ^{1}D) \ 3.8\%$, $(4 \ ^{1}S) \ 0.1\%$, $(4 \ ^{1}P) \ 0.8\%$. Cascade contributions from higher levels have been estimated by using the emission cross sections of Ref. 4 for spectral lines terminating on the 3 ¹*P* level, which were measured for $E \leq 30$. If these cross sections, arising mainly from dipole-forbidden excitations, are extrapolated as E^{-1} above 30 eV, they indicate that the cross section Q' for populating the 3 ^{1}P level via cascade transitions from $(n > 4)^{1}S$ and $(n > 3)^{1}D$ levels is $Q'E \approx 8$. This Q' is 0.4% of $Q_B(3 P)$ at 1400 eV, so that great accuracy of the extrapolation is not required for the present purpose. Possible contributions not included above, e.g., from $(n > 4)^{1}P$ excitation followed by decay via 3 D or 4 S, are expected to be less than 1% of the 3 ^{1}P cross section and have been neglected.

The sum \overline{Q} of the Born $3^{1}P$ cross section plus the estimated total cascade cross section has the form $\overline{Q}E = -177 + 716.7 \log_{10}E$ for $E \ge 100$ eV. Our relative 2852-Å excitation function is normalized so that it approaches this form in the highenergy limit. The shape of the measured excitation function appears to have nearly converged to the theoretical energy dependence at 1400 eV (see Fig. 1), so that the uncertainty of the normalization due to the lack of experimental data above 1400 eV appears to be less than 1%. The cascade contributions are either small enough or known well enough that the dominant uncertainty of Q at 1400 eV is that of the Born 3 ^{1}P cross section. Considering the accuracy with which the optical oscillator strength is known, we ascribe an uncertainty of $\pm 3\%$ to our normalized cross-section scale for the 2852-Å line. Our measurement of the excitation function of the Mg⁺ lines relative to that of the Mg resonance line was uncertain to $\pm 6\%$ (Sec. III B); adding the uncertainties in quadrature yields $\pm 7\%$ as the uncertainty of the crosssection scale for the Mg⁺ lines.

V. DISCUSSION OF RESULTS

A. The Mg $3^{1}P-3^{1}S$ line

Our experimental energy scale has been corrected by using the known excitation energy Δ (=4.345 eV) of the 3 ¹P level. This procedure depends on identifying the position of the threshold with respect to the data shown in Fig. 3(a). The behavior of the excitation and polarization functions for energies E < 4.6 eV was obscured by the energy spread of the electron beam, as shown in Fig. 3(a). We have made various plausible hypotheses about the threshold behavior of the cross section Q_T , convoluted these with the electron energy distribution, attempted to fit the cross-section data, and concluded the following: If the theoretical threshold law¹⁹ $Q_T \propto (E-\Delta)^{1/2}$ holds for 0.155 eV above threshold, then $Q_T = 1.69(8) \pi a_0^2$ at E = 4.5 eV, and the energy scale we have adopted has an uncertainty of ± 0.02 eV. If one assumes only that the excitation function has nonzero slope at threshold and no upward curvature for E < 4.5 eV, then our energy scale is uncertain to ± 0.05 eV, in addition to the uncertainties given for individual energy points in Table I. The form $Q_T = [(E - \Delta)/0.155 \text{ eV}]^{1/2} (1.69 \pi a_0^2)$ for $\Delta \le 4.5$ eV yielded the most satisfactory fit to the data.

In Fig. 2 the present results are compared with earlier measurements by Aleksakhim *et al.*⁴ and by Karstensen and Köster.⁵ Both groups used crossed-beam apparatus and spectrometers; the electron-beam energy resolution was given as 1 eV full width at half-maximum (FWHM) in Ref. 5, somewhat better in Ref. 4. No corrections for anisotropy were applied to the radiation intensities observed at right angles to the electron beam, but the Mg resonance radiation may have been largely depolarized by radiation diffusion since the atombeam intensities were much larger than used for the present work. (Reference 4 states that the ra-

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diation was assumed isotropic; Ref. 5 does not mention the matter.) The excitation function of Ref. 4 has a very different shape, as well as magnitude, from ours; its slower onset in the threshold region cannot be explained merely by the lower energy resolution. The relative excitation function of Ref. 5 agrees with ours below 7 eV, but we did not observe maxima at 8, 12, 16, and 21 eV as listed in Ref. 5 nor the shoulder near 60 eV shown in Ref. 4. The radiation of the Mg 5 ${}^{3}D$ -3 ${}^{3}P$ multiplet (2847, 2848, 2851.7 Å) was unseparated from the resonance line in the present experiment. Karstensen and Köster did not separate the 2851.7-A line from the resonance line, and suggested that the former line might be responsible for the peak they observed at 8 eV.⁵ Aleksakhin *et al.* measured the excitation function of the 2848-Å line and reported its peak value⁴ to be $0.018\pi a_0^2$ at 8.6 eV; according to this, the cross section for the whole multiplet would be insufficient to account for the feature we observed at 9 eV.

Cascading into the 3 ${}^{1}P$ level begins above the 4 ${}^{1}S$ threshold (5.4 eV) and may cause the structure discernible in the present data between 5.4 and 10 eV. According to Born calculations by Robb⁸ (discussed in Sec. IV), the dominant cascade contributors are the 3 ${}^{1}D$ and 4 ${}^{1}P$ levels, whose excitation functions are unmeasured at present. Since considerable departures from the Born cross sections may occur, reliable information about the cascading at low energies is lacking. The comparison in Fig. 3 of the present results for 2852-Å emission with theoretical results for 3 ${}^{1}P$ excitation is therefore incomplete.

The sharp feature of the present data near 5 eV, evident in Fig. 3, cannot be due to cascading but may be related to opening of the 4 ${}^{3}S$ scattering channel at 5.1 eV.

Our polarization data are consistent with theoretical predictions for the threshold and highenergy limits.¹³ In the simple case of $a^{1}S_{0} \rightarrow P_{1}$ excitation followed by ${}^{1}P_{1} \rightarrow {}^{1}S_{0}$ decay with zero nuclear spin, the cross sections Q_{\parallel}, Q_{\perp} (defined in Sec. II) for emission of the separate polarization components of the radiation are the same as the cross sections Q(0), $Q(\pm 1)$ for excitation to the $M_L = 0, M_L = \pm 1$ states of the 1P_1 level. As the impact energy *E* approaches the excitation threshold, $Q(\pm 1)/Q(0) \rightarrow 0$, because angular momentum is conserved, and $P \rightarrow 100\%$. In the Born regime, the dipole-forbidden Q(0) and dipole-allowed $Q(\pm 1)$ behave as $Q(0)E \rightarrow A_0$ and $Q(\pm 1)E \rightarrow A_1 + B_1 \log_{10}E$. Hence in Born theory $Q(\pm 1)/Q(0)$ increases logarithmically without limit as $E \rightarrow \infty$, implying that $P \rightarrow -100\%$ very gradually. These simple predictions are modified slightly for the present case. We make the approximation of ignoring the nuclear spin $(I = \frac{5}{2})$ of ²⁵Mg,

because this isotope constitutes only 10% of natural magnesium, and because the hyperfine structure of ${}^{25}Mg$ 3 ${}^{1}P_{1}$ is thought to be much smaller than the natural level width. (We concluded the latter from hfs data for the resonance levels of barium¹¹ and calcium²⁰ and a reasonable extrapolation²¹ to the Mg atom.) In this approximation, which was also used in Refs. 8 and 9, the expected threshold polarization is 100%, consistent with the present data as shown in Fig. 3(b). The expected high-energy behavior is $Q_{\parallel}E - A'_0 + B'_0 \log_{10}E$ and $Q_{\perp}E \rightarrow A'_{1} + B'_{1} \log_{10}E$, where the primed constants exceed the foregoing unprimed ones because of cascade contributions. In partucular, B'_{0} is nonzero because of (n > 3) ¹ $P(M_L = \pm 1)$ excitations followed by (multi-step) decay via the $3 P(M_L = 0)$ state. We have calculated the $4^{1}P$ contribution to B'_{0} , using oscillator strengths from Ref. 10 and branching ratios, obtaining $B'_0/B'_1 = 0.5\%$; contributions from (n > 4)¹P levels are expected to be negligible. (Thus the infinite-energy polarization limit of the 2852-Å line in Born theory is about -99%.) The limiting slopes of the $Q_{\parallel}E$ and $Q_{\perp}E$ curves shown in Fig. 1 indicate that $B'_1/B'_0 > 20$. The experimental uncertainties and incomplete convergence to the Born energy dependence below 1400 eV prevent a stronger inference from the data.

In Fig. 6 we have compared the electron-impact excitation functions of the sodium, ¹ calcium, ² lithium, ³ and magnesium resonance lines that have been measured in our laboratory. (The energy resolution in all cases was 0.25-0.35 eV over the range shown.) The dissimilarity of the excitation functions in the threshold region has



FIG. 6. Comparison of emission cross sections for the resonance lines of Na (Ref. 1), Ca (Ref. 2), Li (Ref. 3), and Mg (present work). For each case we have plotted $Q_T \Delta^2/f$, where Q_T is the normalized optical excitation function (emission cross section), Δ the excitation threshold energy, and f the resonance-line optical oscillator strength that was used for the Born normalization of Q_T . Thus, apart from minor differences of the cascades, all the curves merge in the Born regime $(E > 100 \Delta$, not shown).

important consequences for the corresponding excitation rate coefficients if the atoms are excited by a thermal electron distribution having kTless than the threshold energy, as in many stellar atmospheres and plasmas.

B. The Mg⁺ $3^2 P - 3^2 S$ lines

The present excitation function is compared with previous measurements in Fig. 5. In the previous experiments, the doublet lines were resolved and isolated, 4,5 and we have summed the corresponding excitation functions. For the comparison in Fig. 5, we adjusted the cross-section scales of Refs. 4 and 5 to agree with the present data at 27 eV. The ratio of the peak value of the present ionizing excitation function to that of the atomic resonance line is seven times larger than reported in Ref. 4, but four times smaller than in Ref. 5.

Karstensen and Köster also measured the excitation functions of the $Mg^+ 3^2D-3^2P$ lines, which were unseparated from the ionic resonance lines

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in the present work. Their measurements, at four energies, were made relative to the ionic resonance lines; these ratios may be more accurate than their ratios involving the 2852-Å line, because the ionic lines do not suffer radiation entrapment. According to their results, the contribution to the present excitation function due to the unwanted 3 ^{2}D -3 ^{2}P lines is about 10% above 30 eV.

The present ionizing excitation function appears to behave as E^{-1} in the high-energy limit, as shown by the $Q_T E$ plot in Fig. 5. Similar high-energy behavior has been reported for certain ionizing excitation functions of argon.²²

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