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Positron-hydrogen elastic scattering in the eikonal approximation

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(Received 22 September 1975)

The full eikonal approximation has been used to calculate differential and total cross sections for the elastic scattering of 50-, 100-, and 200-eV positrons from atomic hydrogen. The eikonal results are compared with other theoretical calculations and with the corresponding electron scattering cross sections. The electron and positron cross sections are found to differ as expected, in contrast to the Born and Glauber approximations which predict identical cross sections for both electron and positron scattering. Although no experimental results are available for comparison with our calculated cross sections, our results indicate that the full eikonal method may be useful for the calculation of positron (or proton) -atom scattering cross sections.

In a previous paper,[1] the full eikonal approximation was used to study electron-hydrogen elastic scattering. In that paper (Ref. 1) we reported our results for the differential and total cross sections for the elastic scattering of 50-, 100- and 200-eV incident electrons from atomic hydrogen. In this addendum we report our full eikonal results for the elastic scattering of positrons from atomic hydrogen, also for incident energies of 50, 100, and 200 eV.

The full eikonal scattering amplitude for electron-hydrogen scattering is given by[2]

$$F_{fi}(\vec{q}) = \frac{-2m}{4\pi\hbar^2} \int e^{i\vec{q}\cdot\vec{R}}\, V(R,R') \times \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{Z} V(R,R')\,dZ\right] \times u_f^*(\vec{r})\,u_i(\vec{r})\,d\vec{R}\,d\vec{r}\,, \tag{1}$$

where $\vec{R}$ and $\vec{r}$ are the coordinates of the incident and bound electrons, respectively, $\vec{R}' = \vec{R} - \vec{r}$, and $V(R,R') = e^2(1/R' - 1/R)$ is the interaction potential between the target atom and the incident electron. Here $m\vec{v} = \hbar\vec{k}$ is the incident electron's momentum, $\vec{q} = \vec{k} - \vec{k}'$ is the momentum transfer to the target, and u_f and u_i are the final and initial bound states of the target atom. In the above expression, $\vec{k}$ is along the z axis.

For positron scattering, the potential becomes $V(R,R') = e^2(1/R - 1/R')$. Hence Eq. (1) can be used for positron scattering by letting $V(R,R') \to -V(R,R')$. Making this substitution changes (i) the over-all sign of $F_{fi}(\vec{q})$ and (ii) the sign of the eikonal phase factor. The over-all sign change makes no difference since the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k}\,|F_{fi}(\vec{q})|^2\,. \tag{2}$$

However, the change in sign of the eikonal phase factor in Eq. (1) above is important, and in fact is the reason why the positron and electron cross sections should be different for a given incident energy.

Recently, Gau and Macek[3] showed that the six-dimensional eikonal scattering-amplitude integral in Eq. (1) above can be reduced to a two-dimensional integral by using the following representation of the bound-state wave-function product:

$$u_f^*(\vec{r})\,u_i(\vec{r}) = D(\mu,\vec{\gamma})\,C_{fi}\,e^{-\mu r + i\vec{\gamma}\cdot\vec{r}}\,\big|_{\vec{\gamma}=0}\,. \tag{3}$$

Here, C_{fi} is a normalization constant and $D(\mu,\vec{\gamma})$ is the differential operator which generates the required wave functions when operating on the exponential in Eq. (3) above.

Their double-integral expression for the full eikonal amplitude in Eq. (1) is[4]

$$F_{fi}(\vec{q}) = \frac{-2^{4-i\eta}}{a_0}\,\pi\frac{\Gamma(1-i\eta)}{\Gamma(-i\eta)}\,C_{fi}\,D(\mu,\vec{\gamma})\,\mu\left(\frac{d}{d\mu^2}\right)^2\left[\int_0^\infty d\lambda\,\lambda^{-i\eta-1}\int_0^1 d\chi\,\chi^{-1}[\mathfrak{F}(1,0,0,0)-\mathfrak{F}(1,1,0,1)]\right]\Bigg|_{\vec{\gamma}=0}\,, \tag{4a}$$

where

$$\mathfrak{F}(m,p,r,s) = \lambda^s(1-\chi)^s\Lambda^{-p}(\Lambda^2+q'^2)^{i\eta-m}(\Lambda-iq'_z)^{-i\eta-r}\,, \tag{4b}$$

$$\Lambda = [\lambda^2(1-\chi)^2+\mu^2\chi+2i\lambda\chi(1-\chi)\gamma_z+\gamma^2\chi(1-\chi)]^{1/2}\,, \tag{4c}$$

and

$$\vec{q}\,' = \vec{q} - i\lambda(1-\chi)\hat{z}+\chi\vec{\gamma}\,. \tag{4d}$$

In the above expression, a_0 is the Bohr radius and $\eta = e^2/\hbar v = 1/k$. When dealing with λ integrals which diverge at $\lambda = 0$, η is given a small imaginary part $i\delta$; one then integrates by parts and sets $\delta = 0$.

For elastic scattering of electrons by hydrogen, we note that

$$u_f^* u_i = \pi^{-1}e^{-2r} \quad (r \text{ in units of } a_0) \tag{5}$$

for $1s\to 1s$ scattering. Thus we have that

$$\mu = 2\,, \tag{6a}$$

$$C_{fi} = \pi^{-1}\,, \tag{6b}$$

$$\vec{\gamma} = 0\,, \tag{6c}$$

and

$$D(\mu,\vec{\gamma}) = 1\,. \tag{6d}$$

The eikonal scattering amplitude for $1s\to 1s$ scattering thus becomes

$$F_{1s\to 1s}(\vec{q}) = \frac{-2^{4-i\eta}}{a_0}\frac{\Gamma(1-i\eta)}{\Gamma(-i\eta)} \times\int_0^\infty d\lambda\,\lambda^{-i\eta-1}\int_0^1 d\chi\,\chi^{-1}\mu\left(\frac{d}{d\mu^2}\right)^2 \times[\mathfrak{F}(1,0,0,0)-\mathfrak{F}(1,1,0,1)]\,, \tag{7a}$$

where

$$\Lambda = [\lambda^2(1-\chi)^2+\mu^2\chi]^{1/2}\,, \tag{7b}$$

$$\vec{q}\,' = \vec{q} - i\lambda(1-\chi)\hat{z}\,, \tag{7c}$$

and μ is set equal to 2 after the derivatives are evaluated. The above expression, Eq. (7a), for electron-hydrogen elastic scattering can be modified for positron scattering by merely letting $\eta\to-\eta$, since the eikonal phase in Eq. (1) can be written as

$$\exp\left[-i\eta\int_{-\infty}^{Z}\left(\frac{1}{R'}-\frac{1}{R}\right)dZ\right]. \tag{8}$$

TABLE I. Positron-hydrogen elastic scattering differential cross section vs positron scattering angle for 50-, 100-, and 200-eV incident positrons.

Positron scattering angle (deg)	$d\sigma/d\Omega$ (a_0^2/sr) 50 eV	100 eV	200 eV
2	10.0	5.14	4.13
3	7.79	3.78	3.31
5	5.24	2.61	2.30
7	3.89	1.96	1.66
10	2.74	1.44	1.10
15	1.67	0.881	0.618
20	1.35	0.575	0.332
30	...	0.276	...
40	0.326	0.150	6.06×10^{-2}
50	...	9.08×10^{-2}	...
60	0.133	5.90×10^{-2}	1.92×10^{-2}
80	8.14×10^{-2}	2.94×10^{-2}	8.29×10^{-3}
100	5.25×10^{-2}	1.71×10^{-2}	3.98×10^{-3}
120	3.74×10^{-2}	1.14×10^{-2}	2.82×10^{-3}

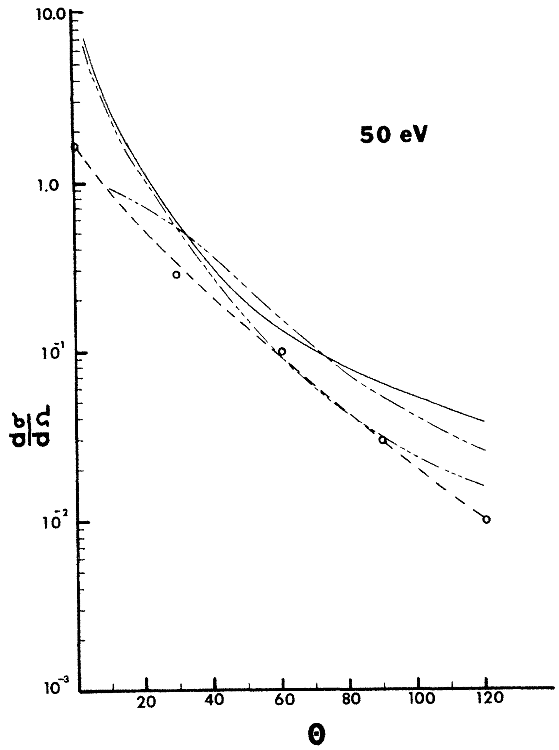

FIG. 1. Differential cross section $d\sigma/d\Omega$ vs scattering angle θ for the elastic scattering of 50-eV positrons from atomic hydrogen. The present full eikonal results (solid line) are compared with the coupled-equation results of Hahn (dashed line) (Ref. 5), the Born results (dash–double-dotted line) (Ref. 6), and the Glauber results (dash–triple-dotted line) (Ref. 7). $d\sigma/d\Omega$ is in units of a_0^2, and θ is in degrees.

We have numerically integrated the eikonal scattering amplitude $F_{1s\to1s}(\vec{q})$ given by Eq. (7a) above (with $\eta\to-\eta$), for incident positron energies of 50, 100, and 200 eV, and scattering angles from 2° to 120°. As in the case of electron-hydrogen scattering, we find that integration of Eq. (7a) by parts is necessary to obtain numerical convergence. Since $k'=k$ for elastic scattering, the differential cross section is given by

$$\frac{d\sigma}{d\Omega}=|F_{1s\to1s}(\vec{q})|^2. \tag{9}$$

In Table I our numerically computed differential cross sections are presented, and in Figs. 1–3 we have plotted $d\sigma/d\Omega$ vs scattering angle θ, along with the coupled-equation results of Hahn,[5] the Born results,[6] the Glauber results,[7] and the eikonal-Born-series results of Byron and Joachain (at 100 eV).[8] Examination of Figs. 1–3 shows that our full eikonal results lie somewhat above all the other theoretical predictions, except for the Born results at intermediate angles. We note here the lack of experimental results with which to compare our calculations.

In Table II, we compare our total cross sections (obtained from the differential cross sections by use of Newton-Cotes open-ended numerical integration[9]) with the corresponding electron scattering cross sections. The above results show that the full eikonal approximation predicts different cross sections for electron and positron scattering, as expected. This is in contrast to the Born and Glauber methods, which predict identical cross sections for both e^--H and e^+-H scattering. Note also that the full eikonal e^--H and e^+-H cross sections tend to each other as the incident

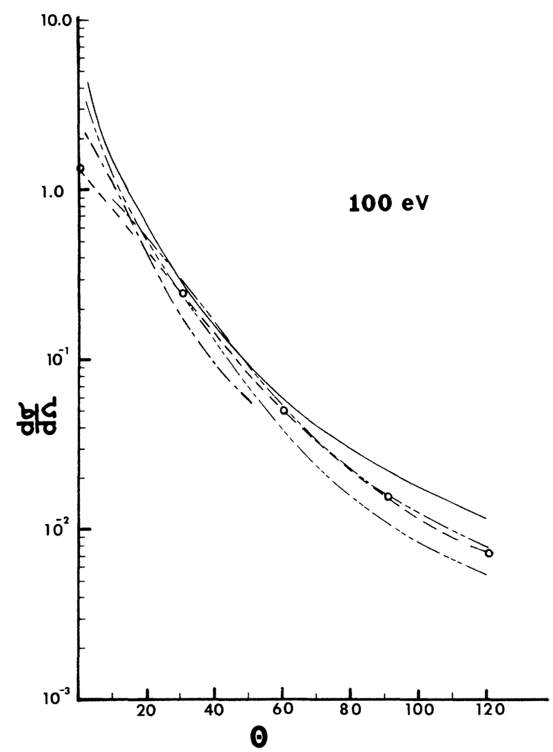

FIG. 2. Differential cross section $d\sigma/d\Omega$ vs scattering angle θ for the elastic scattering of 100-eV positrons from atomic hydrogen. The present full eikonal results (solid line) are compared with the coupled-equation results of Hahn (dashed line) (Ref. 5), the eikonal-Born series results of Byron and Joachain (dash-dotted line) (Ref. 8), the Born results (dash–double-dotted line) (Ref. 6), and the Glauber results (dash–triple-dotted line) (Ref. 7). $d\sigma/d\Omega$ is in units of a_0^2, and θ is in degrees.

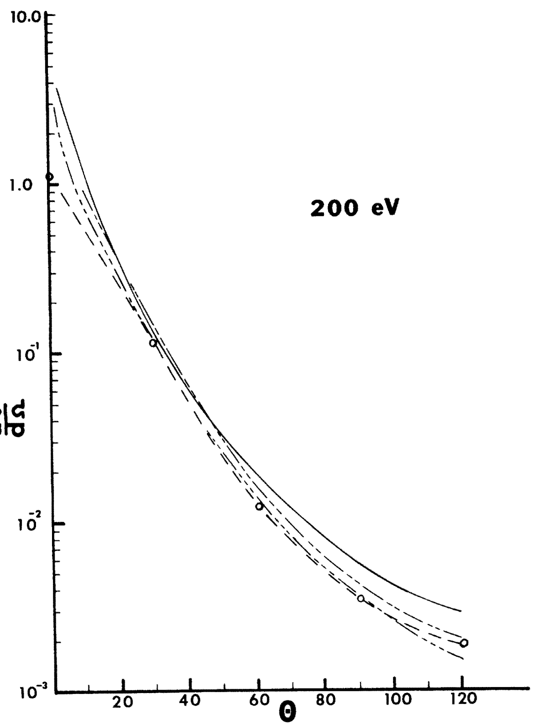

FIG. 3. Differential cross section $d\sigma/d\Omega$ vs scattering angle θ for the elastic scattering of 200-eV positrons from atomic hydrogen. The present full eikonal results (solid line) are compared with the coupled-equation results of Hahn (dashed line) (Ref. 5), the Born results (dash–double dotted line) (Ref. 6), and the Glauber results (dash–triple-dotted line) (Ref. 7). $d\sigma/d\Omega$ is in units of a_0^2, and θ is in degrees.

TABLE II. Total positron-hydrogen and electron-hydrogen elastic scattering cross sections for 50-, 100-, and 200-eV incident positrons and electrons.

Incident energy (eV)	Total scattering cross section (πa_0^2) Positrons	Electrons
50	0.95	0.72
100	0.44	0.39
200	0.24	0.22

energy increases; this result is also expected since the eikonal results should tend to the Born as the incident energy becomes very high. Our full eikonal results for $1s \rightarrow 1s$ scattering are qualitatively similar to those of Byron,[10] who calculated cross sections for $1s \rightarrow 2s$ excitation of hydrogen by electron and positron impact, using the Monte Carlo method to numerically evaluate the full six-dimensional eikonal amplitude.

In conclusion, our full eikonal results for the elastic scattering of positrons from atomic hydrogen show that the full eikonal technique may be useful for the calculation of positron (or proton) -atom scattering cross sections.

We are indebted to the J. Preston Levis Regional Computer Center at the University of Toledo for their assistance.

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