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Magnetohydrodynamics of axisymmetric reactors from the drift-kinetic equation

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The magnetohydrodynamic drift-kinetic equation has been derived by means of the small-gyroradius expansion. The small-gyroradius expansion derived here employs the magnetohydrodynamic ordering. Using this small-gyroradius expansion, all the relevant transport coefficients are systematically evaluated. These results include both diagonal and cross coefficients for the particle fluxes, heat flux, and electric current. By combining the transport coefficients with appropriate moments of the drift equation, a closed set of equations which accurately summarizes the significance of these equations, in particular with regard to recent axisymmetric reactor experiments, provides an extension of the Chew-Goldberger-Low equations, now valid when the thermal velocity is comparable to the electric drift. The generalized equations may be used to investigate the macroscopic stability theory of doublet plasmas.

I. INTRODUCTION

Drift-kinetic equations have formed the basis for numereous recent studies of equilibrium, transport, and instabilities in fusion plasmas.[1-8] However, previous treatments of the problem have considered only the limiting cases where the electrostatic phenomena play an important role. There is considerable theoretical interest in the magnetohydrodynamic description of the plasma confined in axisymmetric systems, e.g., tokamaks, doublets, and divertors, where the Chew-Goldberger-Low fluid-type equation may not be appropriate to apply. This is especially true because the drift mechanism becomes dominant in such systems, and thus the primary objective of the present analysis is to obtain a more general expression in complete agreement with the most exact magnetohydrodynamic equations, including magnetic viscosity.

In a previous paper[2] the drift-kinetic equation was derived by the small-gyroradius expansion theory from the Vlasov equation, with the severe requirement that the drift velocity $V=cE/B$ be much smaller than the thermal velocity of the species in an axisymmetric plasma. The present paper is a continuation of this work, in which we now assume $E_\perp \sim (v_T/c)B$. As in the previous paper, we assume $E_\parallel \ll (v_T/c)B$, since otherwise the acceleration of electrons along the magnetic field line dominates the behavior of the plasma, and its cohesive properties are lost; we neglect collisions throughout the present work. We also assume that the motion is nonrelativistic, i.e., $v_T/c \ll 1$, which would obviously hold when $E_\perp \ll B$. However, if $E_\perp \gtrsim B$, the analysis given here even for drift velocity $V \gtrsim c$ holds if the initial particle velocity is nonrelativistic and the time intervals are small. The approximation technique which we use to obtain the magnetohydrodynamic drift-kinetic equation is more analogous to the Chapman-Enskog method, which has been developed in its complete form by Cheung and Horton,[2] Hastie *et al.*,[9] and several other authors,[10-14] than to the Bogoliubov method,[15] employing the recursion on the Vlasov equation. Like the former authors, we consider electromagnetic fields in the plasma that vary on the large space-time scale (L, T), henceforth referred to as the macroscopic scale. The macroscopic scales by assumption contain many gyroradii $(\rho_T \ll L)$ and gyroperiods $(\Omega^{-1} \ll T)$ of the average particle. In particular, our work has proceeded to higher orders of the small expansion parameter $\epsilon = \rho_T/L \sim 1/\Omega T$.

Frieman, Davidson, and Langdon (FDL)[16] have given a similar analysis and have obtained a first-order drift equation. However, from a dimensional analysis it can be seen that the dimensions of their equation are inconsistent from term to term. Without getting over this obstacle FDL have concluded that the kinetic theory is not in agreement with particle orbiting theory. A study of the one-dimensional flow in a straight magnetic field is also exhibited in this paper. The differences between the present article and FDL's work are given throughout the paper. An additional discussion on the different points of these two papers is presented at the end of the article.

II. DERIVATION OF THE MAGNETOHYDRODYNAMIC DRIFT-KINETIC EQUATION

Here we exhibit in detail the derivation of the magnetohydrodynamic drift-kinetic equation through use of the Vlasov equation with a consistent order in small parameter ϵ equivalent to mc/e. In the usual magnetohydrodynamic ordering, one assumes both terms $\vec{E}_\perp \cdot \nabla_{\vec{v}} f$ and $(1/c)\vec{v}\times\vec{B}\cdot\nabla_{\vec{v}} f$ of the Vlasov equation to be of the same order. Hence we must perform a transformation of coordinates to a frame of reference moving with the $\vec{E}\times\vec{B}$ drift velocity before expanding in powers of the small gyroradius in order to be able to treat the electric field term in the order ϵ with the additional assumption that $E_{\parallel}$ is sufficiently small. Thus a new velocity of the averaged particle is defined as $\vec{c}=\vec{v}-\vec{V}$, in which $\vec{V}=\vec{E}\times\hat{n}/cB$. Furthermore, we transform the Vlasov equation to the appropriate variables, i.e., from $(\vec{x},\vec{c},t)$ to $(\vec{x},c_\perp,c_\parallel,\zeta,t)$, by introducing the locally defined parallel and perpendicular random velocities $(c_\parallel,c_\perp)$ and the gyrophase ζ as $c_\parallel=\hat{n}\cdot\vec{c}$, $\vec{c}_\perp=\vec{c}-\hat{n}(\hat{n}\cdot\vec{c})$ and $\vec{c}_\perp=c_\perp(\hat{e}_1\cos\zeta+\hat{e}_2\sin\zeta)$, where $\hat{n}(x)=\vec{B}/B$ is the unit tangent vector field, and $\hat{e}_1$ and $\hat{e}_2$ are two orthogonal vectors in the plane perpendicular to the magnetic field.

There are now two variables which play the role of cyclic variables, the gyrophase ζ and the parallel coordinate s. The distribution function must, of course, be single valued in each of these coordinates. Thus in each order the procedure will involve deriving consistency conditions analogous to the Chapman-Enskog derivation of hydrodynamics from the requirements that these variables be single valued. It is also worth noting that this requirement for s is somewhat different for trapped and circulating particles. The circulating particles go all the way around; thus the interpretation is straightforward. On the other hand, for trapped particles, the condition arises because the number of left-bounded particles passing through an arbitrary point of the orbit must be equal to the number of right-bounded particles.

Following the above mathematical prescription, the Vlasov equation is reduced to a set of equations for which, to the lowest order,

$$-\Omega\frac{\partial f^{(0)}}{\partial\zeta}=0,$$

which implies that $f^{(0)}$ is independent of the gyrophase variable, and for the general nth-order equation

$$\frac{\partial f^{(n)}}{\partial\zeta}=Df^{(n-1)}, \tag{1}$$

where the operator D is defined by

$$D=\frac{1}{\Omega}\frac{\partial}{\partial t}\bigg|_{\vec{x},c_\perp,c_\parallel,\zeta}+(\vec{V}+\vec{c})\cdot\frac{1}{\Omega}\nabla\bigg|_{c_\perp,c_\parallel,\zeta,t}+\left(\frac{e}{m}\right)E_\parallel\frac{1}{\Omega}\frac{\partial}{\partial c_\parallel}\bigg|_{\vec{x},c_\perp,\zeta,t}$$
$$+\vec{c}\circ\left(\frac{\partial\hat{n}}{\partial t}+(\vec{V}+\vec{c})\circ\nabla\hat{n}\right)\frac{1}{\Omega}\left(\frac{\partial}{\partial c_\parallel}-\frac{c_\parallel}{c_\perp}\frac{\partial}{\partial c_\perp}\right)\bigg|_{\vec{x},\zeta,t}-\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+\vec{c})\cdot\nabla\vec{V}\right)\cdot\frac{1}{\Omega}\left(\hat{n}\frac{\partial}{\partial c_\parallel}+\vec{c}_\perp\frac{\partial}{\partial c_\perp^2/2}\right)\bigg|_{\vec{x},\zeta,t}$$
$$+\left[\frac{c_\parallel}{c_\perp^2}\left(\frac{\partial\hat{n}}{\partial t}+(\vec{V}+\vec{c})\cdot\nabla\hat{n}\right)\cdot\vec{c}\times\hat{n}+\left(\frac{\partial\hat{e}_2}{\partial t}+(\vec{V}+\vec{c})\cdot\nabla\hat{e}_2\right)\cdot\hat{e}_1+\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+\vec{c})\cdot\nabla\vec{V}\right)\frac{\vec{c}\times\hat{n}}{c_\perp^2}\right]\frac{1}{\Omega}\frac{\partial}{\partial\zeta}\bigg|_{\vec{x},c_\perp,c_\parallel,t}. \tag{2}$$

It is observed that the solution of Eq. (1) must be periodic in ζ, and thus additional constraints on the solutions are obtained by integrating equations with respect to ζ over the period of 2π.

From the $n=1$ term of the sequence of equations given by Eq. (1) we have the constraint

$$\frac{\partial f^{(0)}}{\partial t}(\vec{x},c_\perp,c_\parallel,t)+(\vec{V}+c_\parallel\hat{n})\cdot\nabla f^{(0)}+\frac{e}{m}E_\parallel\frac{\partial f^{(0)}}{\partial c_\parallel}+\tfrac{1}{2}c_\perp^2\nabla\cdot\hat{n}\left(\frac{\partial f^{(0)}}{\partial c_\parallel}-\frac{c_\parallel}{c_\perp}\frac{\partial f^{(0)}}{\partial c_\perp}\right)$$
$$-\tfrac{1}{2}c_\perp^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})\frac{\partial f^{(0)}}{\partial c_\perp^2/2}-\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_\parallel\hat{n})\cdot\nabla\vec{V}\right)\cdot\hat{n}\frac{\partial f^{(0)}}{\partial c_\parallel}=0 \tag{3}$$

that $f^{(0)}$ must satisfy. Equation (3) is substantially simplified by transforming the coordinates $(c_\perp,c_\parallel)$ into (μ,ϵ), where $\mu=c_\perp^2/2B$ and

$$\epsilon=\tfrac{1}{2}c_\parallel^2+\tfrac{1}{2}c_\perp^2+\tfrac{1}{2}V^2+(e/m)\phi(\vec{x},t).$$

The quantities μ and ϵ are the magnetic moment per unit mass and the total energy per unit mass, respectively. With these definitions we perform the change of variables in Eq. (3) and obtain

$$\frac{\partial f^{(0)}}{\partial t}(\vec{x},\ \mu,\ \epsilon,\ t)+(\vec{V}+c_\parallel\hat{n})\cdot\nabla f^{(0)}+\left[\frac{\partial(e/m)\phi}{\partial t}+c_\parallel\hat{n}\cdot\frac{e}{m}\vec{E}^{(A)}-c_\parallel\hat{n}\cdot\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_\parallel\hat{n})\cdot\nabla\vec{V}\right)+\vec{V}\cdot\nabla\left(\frac{e}{m}\right)\phi\right.$$
$$\left.-\tfrac{1}{2}c_\perp^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})+\vec{V}\cdot\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_\parallel\hat{n})\cdot\nabla\vec{V}\right)\right]\frac{\partial f^{(0)}}{\partial\epsilon}$$
$$-\left(\mu\frac{\partial B}{\partial t}+\vec{V}\cdot\mu\nabla B+\tfrac{1}{2}c_\perp^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})\right)\frac{1}{B}\frac{\partial f^{(0)}}{\partial\mu}=0, \tag{4}$$

where

$$c_{\parallel}=\pm\{2[\epsilon-\mu B-\tfrac{1}{2}V^2-(e/m)\phi]\}^{1/2}$$

and $\vec{E}=-\nabla\phi(\vec{x},t)+\vec{E}^{(A)}$. At static equilibrium, as far as the spatial dependence is concerned, the dependence of $f^{(0)}$ on parallel spatial variable s is included in $c_{\parallel}$ or ϵ, and $f^{(0)}=f^{(0)}(\vec{x}_{\perp},c_{\perp},c_{\parallel})$ or $f^{(0)}=f^{(0)}(\vec{x}_{\perp},\mu,\epsilon)$, where the $\vec{x}_{\perp}$ must be defined as the perpendicular spatial variables.

Having the distribution function $f^{(0)}$ satisfy this constraint permits one to evaluate the first-order distribution function $f^{(1)}$ given by

$$\begin{aligned}f^{(1)}=&h_1(\vec{x},c_{\perp},c_{\parallel},t)+\frac{\vec{c}\times\hat{n}}{\Omega}\cdot\nabla f^{(0)}+\frac{\hat{n}}{\Omega}\times\left(\frac{\partial\hat{n}}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\hat{n}\right)\cdot\vec{c}\left(\frac{\partial f^{(0)}}{\partial c_{\parallel}}-\frac{c_{\parallel}}{c_{\perp}}\frac{\partial f^{(0)}}{\partial c_{\perp}}\right)-\left(\frac{\vec{c}\times\hat{n}}{\Omega}\cdot\nabla\vec{V}\right)\cdot\hat{n}\frac{\partial f^{(0)}}{\partial c_{\parallel}}\\&-\frac{\hat{n}}{\Omega}\times\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\vec{V}\right)\cdot\vec{c}\frac{\partial f^{(0)}}{\frac{1}{2}\partial c_{\perp}^2}+\frac{1}{\Omega}\int_0^{\zeta}(\vec{c}_{\perp}\vec{c}_{\perp}:\ \nabla\hat{n}-\tfrac{1}{2}c_{\perp}^2\nabla\cdot\hat{n})\,d\zeta\left(\frac{\partial f^{(0)}}{\partial c_{\parallel}}-\frac{c_{\parallel}}{c_{\perp}}\frac{\partial f^{(0)}}{\partial c_{\perp}}\right)\\&-\frac{1}{\Omega}\int_0^{\zeta}[\vec{c}_{\perp}\vec{c}_{\perp}:\ \nabla\vec{V}-\tfrac{1}{2}c_{\perp}^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})]\,d\zeta\,\frac{\partial f^{(0)}}{\frac{1}{2}\partial c_{\perp}^2},\end{aligned}\tag{5}$$

where h_1 is an arbitrary gyrophase-independent function. Transforming to the coordinates (μ,ϵ) and defining the drift velocity V_D, Eq. (5) can be written in the form

$$\begin{aligned}f^{(1)}=&h_1(\vec{x},\mu,\epsilon,t)+\frac{\vec{c}_{\perp}\times\hat{n}}{\Omega}\cdot\nabla f^{(0)}+\left\{\vec{c}_{\perp}\cdot\frac{\hat{n}}{\Omega}\times\left[\nabla\frac{e}{m}\phi+\nabla\vec{V}\cdot\vec{V}-\nabla\vec{V}\cdot c_{\parallel}\hat{n}-\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\vec{V}\right)\right]\right.\\&\left.-\frac{1}{\Omega}\int_0^{\zeta}[\vec{c}_{\perp}\vec{c}_{\perp}:\ \nabla\vec{V}-\tfrac{1}{2}c_{\perp}^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})]\,d\zeta\right\}\frac{\partial f^{(0)}}{\partial\epsilon}\\&-\left(\vec{c}_{\perp}\cdot\vec{V}_D+\frac{c_{\parallel}}{\Omega}\int_0^{\zeta}(\vec{c}_{\perp}\vec{c}_{\perp}:\ \nabla\hat{n}-\tfrac{1}{2}c_{\perp}^2\nabla\cdot\hat{n})\,d\zeta+\frac{1}{\Omega}\int_0^{\zeta}[\vec{c}_{\perp}\vec{c}_{\perp}:\ \nabla\vec{V}-\tfrac{1}{2}c_{\perp}^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})\,d\zeta\right)\frac{1}{B}\frac{\partial f^{(0)}}{\partial\mu},\end{aligned}\tag{6}$$

where

$$\vec{V}_D=\frac{\hat{n}}{\Omega}\times\left[\mu\nabla B+\left(\frac{\partial}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\right)(\vec{V}+c_{\parallel}\hat{n})\right].$$

Complications in the evaluation of higher-order components of Eq. (1) is due to the nonorthogonality of the local coordinate system in the first-order neighborhood of the origin. This nonorthogonality arises from the torsion or twisting of the magnetic field which is caused by the parallel plasma current. Thus the calculation of the constraint equation for h_n requires the use of the commutator relation given in Eq. (7) below, i.e.,

$$(\hat{e}_1\cdot\nabla)(\hat{e}_2\cdot\nabla)-(\hat{e}_2\cdot\nabla)(\hat{e}_1\cdot\nabla)=[(\hat{e}_1\cdot\nabla)\hat{e}_2]\cdot\nabla-[(\hat{e}_2\cdot\nabla)\hat{e}_1]\cdot\nabla.\tag{7}$$

For the sake of simplifying the procedure, we reduce the operator D and the function $f^{(1)}$ to the forms

$$D=D_0+\cos\zeta\,D_c+\sin\zeta\,D_s+\cos2\zeta\,D_{2c}+\sin2\zeta\,D_{2s}+(A_0+\cos\zeta\,A_c+\sin\zeta\,A_s+\cos2\zeta\,A_{2c}+\sin2\zeta\,A_{2s})\frac{\partial}{\partial\zeta}\bigg|_{\vec{x},c_{\perp},c_{\parallel},t}$$

and

$$f^{(1)}=h_1(\vec{x},c_{\perp},c_{\parallel},t)-\cos\zeta\,D_sf^{(0)}+\sin\zeta\,D_cf^{(0)}-\tfrac{1}{2}\cos2\zeta\,D_{2s}f^{(0)}+\tfrac{1}{2}\sin2\zeta\,D_{2c}f^{(0)}.$$

Here

$$\begin{aligned}D_0=&\frac{1}{\Omega}\left[\frac{\partial}{\partial t}\bigg|_{\vec{x},c_{\perp},c_{\parallel},\zeta}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla+\frac{e}{m}E_{\parallel}\frac{\partial}{\partial c_{\parallel}}+\tfrac{1}{2}c_{\perp}^2\nabla\cdot\hat{n}\left(\frac{\partial}{\partial c_{\parallel}}-\frac{c_{\parallel}}{c_{\perp}}\frac{\partial}{\partial c_{\perp}}\right)\right.\\&\left.-\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\vec{V}\right)\cdot\hat{n}\frac{\partial}{\partial c_{\parallel}}-\tfrac{1}{2}c_{\perp}^2(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})\frac{\partial}{\frac{1}{2}\partial c_{\perp}^2}\right],\end{aligned}$$

$$\begin{aligned}D_{[c,s]}=&\frac{c_{\perp}}{\Omega}\left[(\hat{e}_{[1,2]}\cdot\nabla)-(\hat{e}_{[1,2]}\cdot\nabla)\vec{V}\cdot\hat{n}\frac{\partial}{\partial c_{\parallel}}+\left(\frac{\partial\hat{n}}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\hat{n}\right)\cdot\hat{e}_{[1,2]}\left(\frac{\partial}{\partial c_{\parallel}}-\frac{c_{\parallel}}{c_{\perp}}\frac{\partial}{\partial c_{\perp}}\right)\right.\\&\left.-\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}+c_{\parallel}\hat{n})\cdot\nabla\vec{V}\right)\cdot\hat{e}_{[1,2]}\frac{\partial}{\frac{1}{2}\partial c_{\perp}^2}\right],\end{aligned}$$

$$D_{[2c,2s]}=\frac{c_\perp^2}{2\Omega}\left[\left[\hat{e}_{[1,2]}\cdot(\hat{e}_1\cdot\nabla)\hat{n}\mp\hat{e}_{[2,1]}\cdot(\hat{e}_2\cdot\nabla)\hat{n}\right]\left(\frac{\partial}{\partial c_\parallel}-\frac{c_\parallel}{c_\perp}\frac{\partial}{\partial c_\perp}\right)+\left[\hat{e}_{[2,1]}\cdot(\hat{e}_2\cdot\nabla)\vec{V}\mp\hat{e}_{[1,2]}\cdot(\hat{e}_1\cdot\nabla)\vec{V}\right]\frac{\partial}{\frac{1}{2}\partial c_\perp^2}\right],$$

$$A_0=\frac{1}{\Omega}\left[\hat{e}_1\cdot\left(\frac{\partial\hat{e}_2}{\partial t}+(\vec{V}+c_\parallel\hat{n})\cdot\nabla\hat{e}_2\right)-\tfrac{1}{2}\hat{n}\cdot\nabla\times(c_\parallel\hat{n}+\vec{V})\right],$$

$$A_{[c,s]}=\frac{1}{\Omega}\left[c_\perp(\hat{e}_{[1,2]}\cdot\nabla)\hat{e}_2\cdot\hat{e}_1-\frac{1}{c_\perp}\hat{e}_{[2,1]}\cdot\left(\frac{\partial}{\partial t}+(\vec{V}+c_\parallel\hat{n})\cdot\nabla\right)(c_\parallel\hat{n}+\vec{V})\right],$$

$$A_{[2c,2s]}=-\frac{1}{2\Omega}\left(\hat{e}_{[2,1]}\cdot(\hat{e}_1\cdot\nabla)\pm\hat{e}_{[1,2]}\cdot(\hat{e}_2\cdot\nabla)\right)(c_\parallel\hat{n}+\vec{V}).\tag{8}$$

By applying the same notation, the constraint on the function $f^{(1)}$ for a single-valued function $f^{(2)}$ is

$$\langle Df^{(1)}\rangle_\zeta=D_0h_1+\tfrac{1}{2}(D_sD_c-D_cD_s)f^{(0)}+\tfrac{1}{2}(A_cD_c+A_sD_s)f^{(0)}+\tfrac{1}{4}(D_{2s}D_{2c}-D_{2c}D_{2s})f^{(0)}+\tfrac{1}{2}(A_{2c}D_{2c}+A_{2s}D_{2s})f^{(0)}=0.\tag{9}$$

With the aid of the identity (7) and the additional relations

$$a_2[(\hat{e}_1\cdot\nabla)\vec{V}]\cdot\hat{n}-a_1[(\hat{e}_2\cdot\nabla)\vec{V}]\cdot\hat{n}=[\vec{V}\cdot\nabla)\hat{n}]\cdot\hat{n}\times\vec{a}-(\vec{V}\cdot\vec{a})(\hat{n}\cdot\nabla\times\hat{n})$$

and

$$\frac{c_\perp^2}{2\Omega}\{\hat{e}_2\cdot\nabla[\vec{V}\cdot(\hat{e}_1\cdot\nabla)\hat{n}]-(\hat{e}_1\cdot\nabla)[\vec{V}\cdot(\hat{e}_2\cdot\nabla)\hat{n}]\}+\frac{c_\perp^2}{2\Omega}\{[\hat{e}_1\cdot(\hat{e}_1\cdot\nabla)\hat{e}_2]\vec{V}\cdot(\hat{e}_1\cdot\nabla)\hat{n}-[\hat{e}_2\cdot(\hat{e}_2\cdot\nabla)e_1]\vec{V}\cdot(\hat{e}_2\cdot\nabla)\hat{n}\}$$
$$+\frac{c_\perp^2}{4\Omega}\{[\hat{e}_1\cdot(\hat{e}_1\cdot\nabla)\hat{n}-\hat{e}_2\cdot(\hat{e}_2\cdot\nabla)\hat{n}][\hat{e}_1\cdot(\hat{e}_2\cdot\nabla)\vec{V}+\hat{e}_2\cdot(\hat{e}_1\cdot\nabla)\vec{V}]$$
$$-[\hat{e}_2\cdot(\hat{e}_1\cdot\nabla)\hat{n}+\hat{e}_1\cdot(\hat{e}_2\cdot\nabla)\hat{n}][\hat{e}_2\cdot(\hat{e}_2\cdot\nabla)\vec{V}-\hat{e}_1\cdot(\hat{e}_1\cdot\nabla)\vec{V}]\}$$
$$=\frac{c_\perp^2}{4\Omega}\{[\nabla\cdot\vec{V}+3\vec{V}\cdot(\hat{n}\cdot\nabla)\hat{n}](\hat{n}\cdot\nabla\times\hat{n})-(\hat{n}\cdot\nabla\times\vec{V})(\nabla\cdot\hat{n})\},$$

we obtain the constraint equation for h_1. Combining this result with the kinetic equation (3), one obtains the final constraint for the gyrophase-averaged distribution function $f(\vec{x},c_\perp,c_\parallel,t)$ correct to the first order in ϵ,

$$\frac{\partial f}{\partial t}\bigg|_{\vec{x},c_\perp,c_\parallel}+\left[\vec{V}+\left(c_\parallel+\frac{mc}{e}\mu\hat{n}\cdot\nabla\times\hat{n}\right)\hat{n}\right]\cdot\nabla f+\vec{V}_D\cdot\nabla f\bigg|_{c_\perp,c_\parallel,t}$$
$$+\left[\frac{e}{m}E_\parallel+\frac{c_\perp^2}{2}\nabla\cdot\hat{n}+\vec{V}\cdot\frac{d\hat{n}}{dt}+\frac{mc}{e}\left(\frac{d\hat{n}}{dt}+(\vec{V}\cdot\nabla)\hat{n}\right)\cdot\hat{n}\times\mu(\hat{n}\cdot\nabla)\hat{n}-\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})\vec{V}\cdot(\hat{n}\cdot\nabla)\hat{n}+\left(\frac{d\hat{n}}{dt}+(\vec{V}\cdot\nabla)\hat{n}\right)\cdot\vec{V}_D\right.$$
$$-\frac{1}{\Omega}(\hat{n}\cdot\nabla\times\hat{n})\vec{V}\cdot\left(\mu\nabla B+\frac{d\vec{u}}{dt}\right)-\frac{mc}{e}\mu\hat{n}\cdot\nabla\times\frac{d\hat{n}}{dt}+\frac{1}{2}\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})[\nabla\cdot\vec{V}+3\vec{V}\cdot(\hat{n}\cdot\nabla)\hat{n}]$$
$$\left.-\frac{1}{2}\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\vec{V})\nabla\cdot\hat{n}\right]\frac{\partial f}{\partial c_\parallel}\bigg|_{\vec{x},c_\perp,t}$$
$$-\left[\frac{c_\perp^2}{2}(\nabla\cdot\vec{u}-\hat{n}\hat{n}:\ \nabla\vec{u})+\frac{mc}{e}\mu\left(\frac{d\hat{n}}{dt}+(\vec{V}\cdot\nabla)\hat{n}\right)\cdot\hat{n}\times\frac{\partial}{\partial c_\parallel}\frac{d\vec{u}}{dt}-\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})\vec{V}\cdot\frac{\partial}{\partial c_\parallel}\frac{d\vec{u}}{dt}\right.$$
$$\left.-\frac{mc}{e}\mu B\hat{n}\cdot\nabla\times\frac{1}{B}\frac{d\vec{u}}{dt}+\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})\hat{n}\cdot\frac{d\vec{u}}{dt}\right]\frac{\partial f}{\frac{1}{2}\partial c_\perp^2}\bigg|_{\vec{x},c_\parallel,t}=0,\tag{10}$$

where $\vec{u}=\vec{V}+c_\parallel\hat{n}$ and $d/dt=\partial/\partial t+(\vec{V}+c_\parallel\hat{n})\cdot\nabla$. Its equivalent form expressed for the (μ,ϵ) dependence is simply given by

$$\frac{\partial f}{\partial t}\bigg|_{\vec{x},\mu,\epsilon}+\left[\vec{V}+\left(c_\parallel+\frac{mc}{e}\mu\hat{n}\cdot\nabla\times\hat{n}\right)\hat{n}\right]\cdot\nabla f\bigg|_{\mu,\epsilon}+\vec{V}_D\cdot\nabla f\bigg|_{\mu,\epsilon,t}$$
$$+\left[\frac{\partial(e/m)\phi}{\partial t}+(\vec{V}\cdot\nabla)\frac{e}{m}\phi+c_\parallel\hat{n}\cdot\frac{e}{m}\vec{E}^{(A)}+\vec{V}\cdot\frac{d\vec{u}}{dt}-\frac{c_\perp^2}{2}(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})+\frac{mc}{e}\left(\frac{d\hat{n}}{dt}+(\vec{V}\cdot\nabla)\hat{n}\right)\cdot\mu\left(\frac{d\hat{n}}{dt}+(\hat{n}\cdot\nabla)\vec{V}\right)\times\hat{n}\right.$$
$$+\frac{mc}{e}2\mu(\hat{n}\cdot\nabla\times\hat{n})\vec{V}\cdot\left(\frac{d\hat{n}}{dt}+(\hat{n}\cdot\nabla)\vec{V}\right)+\frac{mc}{e}\mu\hat{n}\cdot\nabla\times\frac{d\vec{V}}{dt}+\frac{\hat{n}}{\Omega}\times\frac{d\vec{u}}{dt}\cdot\mu\nabla B+\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})\hat{n}\cdot\nabla\frac{e}{m}\phi$$

$$+\vec{V}_D\cdot\left(\nabla\frac{e}{m}\phi+\nabla\vec{V}\cdot\vec{V}\right)+\left(\frac{d\hat{n}}{dt}+(\vec{V}\cdot\nabla)\hat{n}\right)c_{\parallel}\cdot\vec{V}_D-\frac{c_{\parallel}}{\Omega}(\hat{n}\cdot\nabla\times\hat{n})\vec{V}\cdot\left(\mu\nabla B+\frac{d\vec{u}}{dt}\right)$$
$$+\frac{1}{2}\frac{mc}{e}\mu c_{\parallel}(\hat{n}\cdot\nabla\times\hat{n})[\nabla\cdot\vec{V}+3\vec{V}\cdot(\hat{n}\cdot\nabla)\hat{n}]-\frac{1}{2}\frac{mc}{e}\mu\, c_{\parallel}(\hat{n}\cdot\nabla\times\vec{V})\nabla\cdot\hat{n}\Bigg]\frac{\partial f}{\partial\epsilon}\Bigg|_{\vec{x},\mu,t}$$
$$-\Bigg[\mu\frac{\partial B}{\partial t}+\mu(\vec{V}\cdot\nabla)B+\frac{c_\perp^2}{2}(\nabla\cdot\vec{V}-\hat{n}\hat{n}:\ \nabla\vec{V})+\frac{mc}{e}\left(\frac{d\hat{n}}{dt}+(\vec{V}\cdot\nabla)\hat{n}\right)\cdot\mu\hat{n}\times\frac{\partial}{\partial c_{\parallel}}\frac{d\vec{u}}{dt}$$
$$-\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})\vec{V}\cdot\frac{\partial}{\partial c_{\parallel}}\frac{d\vec{u}}{dt}-\frac{mc}{e}\mu\hat{n}\cdot\nabla\times\frac{d\vec{u}}{dt}+\frac{mc}{e}\mu(\hat{n}\cdot\nabla\times\hat{n})\hat{n}\cdot\left(\mu\nabla B+\frac{d\vec{u}}{dt}\right)\Bigg]\frac{1}{B}\frac{\partial f}{\partial\mu}=0, \quad (11)$$

which, defined as the magnetohydrodynamic drift-kinetic equation, namely *Madeleine* equation I, forms a convenient basis for the study of waves, equilibria, and stability in *general* systems.

In comparison with Eq. (III. 6) given in Ref. 17, obtained via the method of averaging of the equations of motion for a charged particle, the characteristics of the drift-kinetic equation are simply the particle's guiding-center orbits. The function $f(\vec{x}, c_\perp, c_\parallel, t)$ gives the distribution function of the guiding centers and satisfies $df/dt=0$. The flux of the guiding centers is not the same as the particle flux, and we derive the relationship between the two fluxes in Sec. III.

III. PARTICLE FLUX AND OTHER MOMENTS

The solution of the Vlasov equation by the small-gyroradius expansion yields at each order the gyrophase-independent component of the distribution function in terms of the phase-averaged component of the distribution function. Consequently, we can compute the perpendicular moments in terms of moments of $f(\vec{x}, c_\perp, c_\parallel, t)$.

The perpendicular particle flux is defined by

$$N\vec{U}_\perp=\int\vec{v}_\perp f(\vec{x},\vec{v},t)\,d^3v. \quad (12)$$

Using the expression of f and Eq. (5) one obtains

$$N\vec{U}_\perp=N\vec{V}+(\nabla\times\vec{M})_\perp+\int\vec{V}_D f^{(0)}(\vec{x},c_\perp,c_\parallel,t)\,d^3c, \quad (13)$$

where

$$\vec{M}=-(cp_\perp^{(0)}/eB)\hat{n},$$
$$p_\perp^{(0)}=\int\frac{mc_\perp^2}{2}f^{(0)}2\pi\, c_\perp\, dc_\perp\, dc_\parallel,$$
$$p_\parallel^{(0)}=\int m(c_\parallel-C_\parallel^{(0)})^2 f^{(0)}2\pi\, c_\perp\, dc_\perp\, dc_\parallel,$$

and

$$NC_\parallel^{(0)}=\int c_\parallel\, f^{(0)}2\pi\, c_\perp\, dc_\perp\, dc_\parallel.$$

In Eq. (13) we obtain the expected result that the flux of particles and the flux of guiding centers through a surface differ by the boundary effect.

With the space-charge effect taken into account the first-order current given by Eq. (13) is

$$\vec{j}_\perp=\sum_{j=e,i}Ne_j\vec{V}-\left(\nabla\times\frac{cp_\perp^{(0)}}{B}\hat{n}\right)_\perp+\frac{cp_\perp^{(0)}}{B^2}\hat{n}\times\nabla B+\frac{cp_\parallel^{(0)}}{B}\hat{n}\times(\hat{n}\cdot\nabla)\hat{n}+\frac{c}{B}\hat{n}\times\sum_{j=e,i}m_jN\left(C_j^{(0)}\frac{d\hat{n}}{dt}+\frac{d\vec{V}}{dt}\right),$$

which is the current required to maintain force balance across the magnetic field. The force from $\vec{j}_\perp$ is given by

$$\frac{1}{c}\vec{j}\times\vec{B}=-\sum_{j=e,i}Ne_j\,\vec{E}_\perp+(\nabla\cdot\vec{P}^{(0)})_\perp+\sum_{j=e,i}m_jN_j\left(C_{\parallel j}^{(0)}\frac{d\hat{n}}{dt}+\frac{d\vec{V}}{dt}\right)_\perp, \quad (14)$$

where we introduce the tensor $\vec{P}^{(0)}=p_\perp^{(0)}\vec{I}+(p_\parallel^{(0)}-p_\perp^{(0)})\hat{n}\hat{n}$.

From another point of view, Eq. (14) can be transformed to a generalized Ohm's law for the first-order plasma when $\vec{E}_\perp+(1/c)\vec{V}\times\vec{B}=0$ remains valid in the zero-order case; thus

$$\vec{E}_\perp+\frac{1}{c}\vec{U}_\perp\times\vec{B}-\frac{1}{Ne}(\nabla\cdot\vec{P}^{(0)})_\perp-\frac{1}{Ne}\left(C_\parallel^{(0)}\frac{d\hat{n}}{dt}+\frac{d\vec{V}}{dt}\right)_\perp=0,$$

or in an approximate form,

$$\vec{E}_\perp+\frac{1}{c}\vec{U}_\perp\times\vec{B}+\frac{1}{Ne}(\nabla\cdot\vec{P}^{(0)})_\perp-\frac{1}{c}\frac{\vec{j}_\perp\times\vec{B}}{Ne}\approx 0.$$

This implies that the plasma no longer appears Ohmic in this order.

The first-order contributions to the components of the pressure tensor and to the components of the heat-flux tensor may be calculated from the inhomogeneous part of $f^{(1)}$ which we have already determined. Hence it gives

$$P^{(1)}_{[xx,yy]}=p^{(1)}_\perp \mp \frac{q_\parallel^{\perp(0)}+p_\perp^{(0)}C_\parallel^{(0)}}{2\Omega}[\hat e_2\cdot(\hat e_1\cdot\nabla)\hat n+\hat e_1\cdot(\hat e_2\cdot\nabla)\hat n]\mp\frac{p_\perp^{(0)}}{2\Omega}\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}+V_x\hat e_2\cdot(\hat e_1\cdot\nabla)\hat e_1+V_y\hat e_1\cdot(\hat e_2\cdot\nabla)\hat e_2\right),$$

$$P^{(1)}_{ss}=p^{(1)}_\parallel$$

$$P^{(1)}_{xy}=P^{(1)}_{yx}=\frac{q_\parallel^{\perp(0)}+p_\perp^{(0)}C_\parallel^{(0)}}{2\Omega}[\hat e_1\cdot(\hat e_1\cdot\nabla)\hat n-\hat e_2\cdot(\hat e_2\cdot\nabla)\hat n]+\frac{p_\perp^{(0)}}{2\Omega}\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}-V_x\hat e_2\cdot(\hat e_2\cdot\nabla)\hat e_1+V_y\hat e_1\cdot(\hat e_1\cdot\nabla)\hat e_2\right),$$

$$P^{(1)}_{[xs,ys]}=P^{(1)}_{[sx,sy]}=\mp\frac{1}{\Omega}\frac{\partial q_\parallel^{\perp(0)}}{\partial[y,x]}\mp\frac{p_\perp^{(0)}}{\Omega}\frac{\partial C_\parallel^{(0)}}{\partial[y,x]}\pm\frac{p_\perp^{(0)}-p_\parallel^{(0)}}{\Omega}\hat e_{[2,1]}\cdot\frac{d\hat n}{dt}\pm\frac{2q_\parallel^{\perp(0)}-q_\parallel^{\parallel(0)}-p_\parallel^{(0)}C_\parallel^{(0)}}{\Omega}\hat e_{[2,1]}\cdot(\hat n\cdot\nabla)\hat n$$
$$\mp\frac{p_\parallel^{(0)}}{\Omega}\hat e_{[2,1]}\cdot(\hat n\cdot\nabla)\vec V\mp\frac{p_\perp^{(0)}}{\Omega}[V_x\hat n\cdot(\hat e_{[2,1]}\cdot\nabla)\hat e_1+V_y\hat n\cdot(\hat e_{[2,1]}\cdot\nabla)\hat e_2]\ , \tag{15}$$

and the tensor components of transverse heat flow *across* the magnetic field line,

$$Q^{(1)}_{[xxx,yyy]}=3Q^{(1)}_{[xyy,xxy]}=\mp\frac{3}{2\Omega}\frac{\partial R_\perp^{(0)}}{\partial[y,x]}\mp\frac{3q_\parallel^{\perp(0)}+3p_\perp^{(0)}C_\parallel^{(0)}}{\Omega}\hat e_{[2,1]}\cdot\frac{d\hat n}{dt}$$
$$\mp\frac{3}{2\Omega}(Z^{(0)}-R_\perp^{(0)}+2q_\parallel^{\perp(0)}C_\parallel^{(0)})\hat e_{[2,1]}\cdot(\hat n\cdot\nabla)\hat n\mp\frac{3p_\perp^{(0)}}{\Omega}\hat e_{[2,1]}\cdot\frac{d\vec V}{dt}$$
$$\mp\frac{3q_\parallel^{\perp(0)}}{\Omega}\hat e_{[2,1]}\cdot(\hat n\cdot\nabla)\vec V,$$

the tensor components of transverse heat flow *along* the magnetic field line,

$$Q^{(1)}_{[xxs,yys]}=q_\parallel^{\perp(1)}\pm\frac{1}{4}\frac{R_\perp^{(0)}-Z^{(0)}-2q_\parallel^{\perp(0)}C_\parallel^{(0)}}{\Omega}[\hat e_2\cdot(\hat e_1\cdot\nabla)\hat n+\hat e_1\cdot(\hat e_2\cdot\nabla)\hat n]$$
$$\mp\frac{1}{2}\frac{q_\parallel^{\perp(0)}}{\Omega}\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}-V_x\hat e_2\cdot(\hat e_1\cdot\nabla)\hat e_1-V_y\hat e_1\cdot(\hat e_2\cdot\nabla)\hat e_2\right),$$

$$Q^{(1)}_{xys}=\frac{1}{4}\frac{R_\perp^{(0)}-Z^{(0)}-2q_\parallel^{\perp(0)}C_\parallel^{(0)}}{\Omega}[\hat e_2\cdot(\hat e_2\cdot\nabla)\hat n-\hat e_1\cdot(\hat e_1\cdot\nabla)\hat n]$$
$$+\frac{1}{2}\frac{q_\parallel^{\perp(0)}}{\Omega}\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}-V_x\hat e_2\cdot(\hat e_2\cdot\nabla)\hat e_1+V_y\hat e_1\cdot(\hat e_1\cdot\nabla)\hat e_2\right),$$

the tensor components of longitudinal heat flow *across* the magnetic field line,

$$Q^{(1)}_{[xss,yss]}=\mp\frac{1}{2\Omega}\frac{\partial Z^{(0)}}{\partial[y,x]}\pm\frac{2q_\parallel^{\perp(0)}-q_\parallel^{\parallel(0)}-p_\parallel^{(0)}C_\parallel^{(0)}}{\Omega}\hat e_{[2,1]}\cdot\frac{d\hat n}{dt}\pm\frac{1}{2}\frac{3Z^{(0)}-R_\parallel^{(0)}+(p_\parallel^{(0)}-2p_\perp^{(0)})C_\parallel^{2(0)}}{\Omega}\hat e_{[2,1]}\cdot(\hat n\cdot\nabla)\hat n$$
$$\mp\frac{2q_\parallel^{\perp(0)}}{\Omega}[V_x\hat n\cdot(\hat e_{[2,1]}\cdot\nabla)\hat e_1+V_y\hat n\cdot(\hat e_{[2,1]}\cdot\nabla)\hat e_2]\mp\frac{p_\parallel^{(0)}}{\Omega}\hat e_{[2,1]}\cdot\frac{d\vec V}{dt}\mp\frac{q_\parallel^{\parallel(0)}}{\Omega}\hat e_{[2,1]}\cdot(\hat n\cdot\nabla)\vec V,$$

and the tensor component of longitudinal heat flow *along* the magnetic field line,

$$Q^{(1)}_{sss}=q_\parallel^{\parallel(1)}=\int m(c_\parallel-C_\parallel^{(0)})^3h_1\,2\pi c_\perp dc_\perp\,dc_\parallel\ , \tag{16}$$

where the subscripts of quantities P_{ij} and Q_{ijk} can be interchanged freely. Here we have defined

$$q_\perp^{\perp(0)}=\int\frac{mc_\perp^3}{2}f^{(0)}2\pi c_\perp dc_\perp dc_\parallel\ ,$$

$$q_\parallel^{\perp(0)}=\int\frac{mc_\perp^2}{2}(c_\parallel-C_\parallel^{(0)})f^{(0)}2\pi c_\perp dc_\perp dc_\parallel\ ;$$

$$R_\perp^{(0)}=\int m(\tfrac{1}{2}c_\perp^2)^2f^{(0)}2\pi c_\perp dc_\perp dc_\parallel\ ,$$

$$R_\parallel^{(0)}=\int m(c_\parallel-C_\parallel^{(0)})^4f^{(0)}2\pi c_\perp dc_\perp dc_\parallel\ ,$$

$$Z^{(0)}=\int mc_\perp^2(c_\parallel-C_\parallel^{(0)})^2f^{(0)}2\pi c_\perp dc_\perp dc_\parallel\ ,$$

and

$$p_\perp^{(1)}=\int\frac{mc_\perp^2}{2}h_1\,2\pi c_\perp dc_\perp dc_\parallel\ ,$$

$$p_\parallel^{(1)}=\int m(c_\parallel-C_\parallel^{(1)})^2h_1\,2\pi c_\perp dc_\perp dc_\parallel\ ,$$

where

$$NC_\parallel^{(1)}=\int c_\parallel h_1\,2\pi c_\perp dc_\perp dc_\parallel\ .$$

Our result corrects the error of the formula calculated some time ago by Thompson[18] and by Roberts and Taylor.[19]

Now it remains to compute the requirement for the maintenance of the parallel force balance. This can be done by forming the moments of the magnetohydrodynamic drift-kinetic equation (10) with m and $mc_{\parallel}$. A straightforward, if somewhat lengthy, calculation, using the expression of first-order pressure $P_{ij}^{(1)}$ reduces the equations to a form analogous to the Chew-Goldberger-Low equations. Thus we have obtained

$$m\frac{\partial N}{\partial t}+\nabla\cdot mN\vec{U}=0\ , \tag{17}$$

$$mN\left(\frac{dC_{\parallel}}{dt}+\hat{n}\cdot\frac{d\vec{V}}{dt}\right)\hat{n}+\frac{1}{\Omega}\frac{d\hat{n}}{dt}\times\left(\nabla p_{\perp}^{(0)}+(p_{\parallel}^{(0)}-p_{\perp}^{(0)})(\hat{n}\cdot\nabla)\hat{n}+mN^{(0)}\frac{d\vec{V}}{dt}\right)+(\nabla\cdot\vec{P}^{(0)})_{\parallel}-Ne\vec{E}_{\parallel}+(\nabla\cdot\vec{P}^{(1)})_{\parallel}=0\ , \tag{18}$$

where the fluid velocity $\vec{U}$ is defined by

$$\vec{U}=C_{\parallel}\hat{n}+\vec{V}+\frac{\hat{n}}{\Omega}\times\left(\frac{1}{mN^{(0)}}\nabla p_{\perp}^{(0)}+\frac{1}{mN^{(0)}}(p_{\parallel}^{(0)}-p_{\perp}^{(0)})(\hat{n}\cdot\nabla)\hat{n}+C_{\parallel}^{(0)}\frac{d\hat{n}}{dt}+\frac{d\vec{V}}{dt}\right)$$

and

$$\frac{d}{dt}=\frac{\partial}{\partial t}+(C_{\parallel}\hat{n}+\vec{V})\cdot\nabla\ ;$$

$$C_{\parallel}=C_{\parallel}^{(0)}+C_{\parallel}^{(1)},\quad N=N^{(0)}+N^{(1)}\ .$$

It is obvious that Eq. (18) is in a higher-order form than Eq. (14). This implies that the aforesaid approach of obtaining the transport equation is incomplete. In fact, a closed set of equations can be obtained by using the moments of the higher-order distribution function $f^{(2)}$; this is presented in Sec. IV. It is also clear that these transport equations provide a generalization of those works done by Stringer[20] and Rosenbluth and co-workers.[21,22] In their works an incomplete analysis was given which led to improper results. For the sake of comparison it is noted that the diamagnetic velocity explicitly shown in their works is implicitly represented here in terms of $C_{\parallel}^{(1)}$.

IV. CONCLUSIONS

In this paper the magnetohydrodynamic drift-kinetic equation has been derived by expansion in powers of the gyroradius and thus produces a magnetohydrodynamic set of equations. It is clear that these transport equations contain less information than the moments of the perturbed distribution function; it has proved more fruitful to develop a theory of equilibrium, and to frame dynamic studies as a perturbation theory about equilibrium. For the analysis of the doublet system we introduce a right-handed set of magnetic coordinates (ψ,χ,θ), where ψ is the poloidal flux function, χ is the orthogonal poloidal coordinate which reduces to magnetic potential at zero β, and θ is the angle about the axis of symmetry. The gradient operator in the (ψ,χ,θ) coordinates is

$$\nabla=\hat{e}_{\psi}RB_{\psi}\frac{\partial}{\partial\psi}+\hat{e}_{\chi}\frac{1}{JB_{\chi}}\frac{\partial}{\partial\chi}+\hat{e}_{\theta}\frac{1}{R}\frac{\partial}{\partial\theta}\ ,$$

where J is determined by the requirement that ψ and χ be orthogonal. Transforming the spatial coordinates in Eq. (11) to (ψ,χ,θ) we obtain a symmetrical equilibrium. However, a treatment of this kind presents serious difficulties and the problem seems to be nonanalytic. In practice, static equilibrium gives significant information without changing the picture of basic investigation of the plasma problem usually of interest. Furthermore, we refer to Refs. 7 and 20, where the authors have stated that with a static radial electric field there could arise a large diffusion in an equilibrium plasma not able to reach the steady state. Henceforth we restrict our study to the case of static equilibrium. Because of the symmetry we obtain

$$F=F^{(0)}(\mu,\epsilon,\psi)+\Delta F^{(1)}(\mu,\epsilon,\psi)-c_{\parallel}\frac{RB_{\theta}}{\Omega}\frac{\partial F^{(0)}}{\partial\psi}-\frac{\mu c_{\parallel}}{\Omega}(n\cdot\nabla\times n)\frac{\partial F^{(0)}}{\partial\mu}\ . \tag{19}$$

The form of Eq.(19), which is our desired result, is not surprising in its complete agreement with the previous one obtained in the study of electrostatic stability theory of tokamaks.[2] That part of the distribution that is constant along the field line, $\bar{F}(\mu,\epsilon,\psi)=F^{(0)}+\Delta F^{(1)}$, can carry an arbitrary divergence-free parallel current of untrapped particles. For the trapped particles the function $\bar{F}$ is independent of the sign of $c_{\parallel}$ and carries no current. The divergent parallel particle and heat flows required to maintain a steady state in an inhomogeneous plasma are given by terms proportional to $c_{\parallel}$ in Eq. (19) with both trapped and untrapped particles contributing.

The study of application to magnetohydrodynamic stability of doublets is presented in a future paper.[23] In consideration of this problem with arbitrary $k_{\perp}\rho_i$ values, a higher-order gyroradius expansion is, indeed, needed, together with a calculation of $f^{(2)}$. With the complicate evaluation as done in Sec. II, we have shown that the higher-order distribution function is given by

$$\begin{aligned} f^{(2)} = {} & h_2(\vec{x}, c_{\perp}, c_{\parallel}, t) - \cos\zeta\,(D_s h_1 + B_0 D_s f^{(0)} + L_s f^{(0)}) + \sin\zeta\,(D_c h_1 + B_0 D_c f^{(0)} + L_c f^{(0)}) \\ & - \tfrac{1}{2}\cos 2\zeta\,(D_{2s} h_1 + A_0 D_{2s} f^{(0)} + L_{2s} f^{(0)}) + \tfrac{1}{2}\sin 2\zeta\,(D_{2c} h_1 + A_0 D_{2c} f^{(0)} + L_{2c} f^{(0)}) \\ & - \tfrac{1}{3}\cos 3\zeta\, L_{3s} f^{(0)} + \tfrac{1}{3}\sin 3\zeta\, L_{3c} f^{(0)} - \tfrac{1}{4}\cos 4\zeta\, L_{4s} f^{(0)} + \tfrac{1}{4}\sin 4\zeta\, L_{4c} f^{(0)} , \end{aligned} \tag{20}$$

where we use the notation defined by

$$B_0 = -\tfrac{1}{2}(\hat{n}/\Omega)\cdot\nabla\times\vec{u} ,$$

$$\begin{aligned} L_{[c,s]} = {} & \pm \frac{1}{\Omega}\left(\frac{1}{B}\frac{dB}{dt} + \tfrac{1}{2}(\nabla\cdot\vec{u} - \hat{n}\hat{n} : \nabla\vec{u})\right) D_{[s,c]} \mp \frac{c_{\perp}}{\Omega^2}\,\hat{e}_{[2,1]}\cdot\frac{d}{dt}\nabla_{\perp} \\ & \pm \frac{c_{\perp}}{\Omega^2}\left[c_{[2,1]}(\hat{n}\cdot\nabla) + \tfrac{1}{2}(c_{\parallel} b_2 + b_4)\hat{e}_{[1,2]}\cdot\nabla \pm \tfrac{1}{2}(c_{\parallel} b_1 + b_3)\hat{e}_{[2,1]}\cdot\nabla\right] \\ & \mp \frac{c_{\perp}}{\Omega^2}\left[\hat{e}_{[2,1]}\cdot\frac{d}{dt}\left(\frac{d\hat{n}}{dt} - \nabla\vec{V}\cdot\hat{n}\right) + c_{[2,1]}\hat{n}\hat{n} : \nabla\vec{V} + \left(\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}\right)\nabla\cdot\hat{n} + a\left[\hat{e}_{[2,1]}\cdot(\hat{n}\cdot\nabla)\hat{n}\right]\right. \\ & \quad - \tfrac{1}{4}c_{\perp}^2 b_{[2,1]}\hat{e}_1\cdot\left[(\hat{e}_2\cdot\nabla)\hat{e}_2 + (\hat{n}\cdot\nabla)\hat{n}\right] \mp \tfrac{1}{4}c_{\perp}^2 b_{[1,2]}\hat{e}_2\cdot\left[(\hat{e}_1\cdot\nabla)\hat{e}_1 + (\hat{n}\cdot\nabla)\hat{n}\right] \pm \tfrac{1}{2}c_{[1,2]}(c_{\parallel} b_2 + b_4) \\ & \quad \left. + \tfrac{1}{2}c_{[2,1]}(c_{\parallel} b_1 + b_3) \pm \frac{c_{\perp}^2 B}{8}\hat{e}_{[1,2]}\cdot\nabla\frac{1}{B}b_2 \mp \frac{c_{\perp}^2 B}{8}\hat{e}_{[2,1]}\cdot\nabla\frac{1}{B}b_1 \mp \tfrac{1}{2}b_2\hat{e}_{[1,2]}\cdot\frac{d\vec{u}}{dt} - \tfrac{1}{2}b_1\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}\right]\frac{\partial}{\partial c_{\parallel}} \\ & \pm \frac{c_{\perp}}{\Omega^2}\left[\hat{e}_{[2,1]}\cdot\frac{d}{dt}\left(\frac{d\vec{u}}{dt}\right) - \frac{c_{\perp}^2}{2}c_{[2,1]}\nabla\cdot\hat{n} + \hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}(\nabla\cdot\vec{u} - \hat{n}\hat{n} : \nabla\vec{u}) + a\left(\frac{\partial}{\partial c_{\parallel}}\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}\right)\right. \\ & \quad - \frac{c_{\perp}^2 B}{8}\hat{e}_{[1,2]}\cdot\nabla\frac{1}{B}(c_{\parallel} b_2 + b_4) \mp \frac{c_{\perp}^2 B}{8}\hat{e}_{[2,1]}\cdot\nabla\frac{1}{B}(c_{\parallel} b_1 + b_3) - \tfrac{1}{4}c_{\perp}^2 b_{[2,1]}(\tfrac{1}{2}c_1 - c_3) \\ & \quad \left. \mp \tfrac{1}{4}c_{\perp}^2 b_{[1,2]}(\tfrac{1}{2}c_2 - c_4) + \tfrac{1}{4}c_{\perp}^2(c_{\parallel} b_2 + b_4)\hat{e}_1\cdot(\hat{e}_{[2,1]}\cdot\nabla)\hat{e}_2 - \tfrac{1}{4}c_{\perp}^2(c_{\parallel} b_1 + b_3)\hat{e}_1\cdot(\hat{e}_{[1,2]}\cdot\nabla)\hat{e}_2\right]\frac{\partial}{\frac{1}{2}\partial c_{\perp}^2} \\ & \mp \frac{c_{\perp}^3}{8\Omega^2} b_2(\hat{e}_{[1,2]}\cdot\nabla)\frac{\partial}{\partial c_{\parallel}} \mp \frac{c_{\perp}}{\Omega^2}\left(a \pm \frac{c_{\perp}^2}{8}b_1\right)(\hat{e}_{[2,1]}\cdot\nabla)\frac{\partial}{\partial c_{\parallel}} \pm \frac{c_{\perp}^3}{8\Omega^2}(c_{\parallel} b_2 + b_4)(\hat{e}_{[1,2]}\cdot\nabla)\frac{\partial}{\frac{1}{2}\partial c_{\perp}^2} \\ & + \frac{c_{\perp}^3}{8\Omega^2}(c_{\parallel} b_1 + b_3)(\hat{e}_{[2,1]}\cdot\nabla)\frac{\partial}{\frac{1}{2}\partial c_{\perp}^2} - \frac{c_{\perp}^3}{8\Omega^2}(b_{[2,1]}c_1 \pm b_{[1,2]}c_2)\frac{\partial^2}{\partial c_{\parallel}\,\partial c_{\parallel}} \\ & + \frac{c_{\perp}^3}{8\Omega^2}\left((c_{\parallel} b_1 + b_3)c_{[2,1]} \pm (c_{\parallel} b_2 + b_4)c_{[1,2]} \pm b_2\hat{e}_{[1,2]}\cdot\frac{d\vec{u}}{dt} + b_1\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}\right)\frac{\partial}{\partial c_{\parallel}}\,\frac{\partial}{\partial c_{\perp}^2/2} \\ & - \frac{c_{\perp}^3}{8\Omega^2}\left((c_{\parallel} b_1 + b_3)\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt} \pm (c_{\parallel} b_2 + b_4)\hat{e}_{[1,2]}\cdot\frac{d\vec{u}}{dt}\right)\frac{\partial}{\partial c_{\perp}^2/2}\,\frac{\partial}{\partial c_{\perp}^2/2} , \end{aligned}$$

$$\begin{aligned} L_{[2c,2s]} = {} & -\frac{c_{\perp}^2}{4\Omega^2} b_{[2,1]}\hat{n}\cdot\nabla + \frac{c_{\perp}^2}{2\Omega^2}\left(\frac{1}{B}\frac{\partial B}{\partial y} + \hat{e}_1\cdot(\hat{e}_1\cdot\nabla)\hat{e}_2\right)\hat{e}_{[1,2]}\cdot\nabla \pm \frac{c_{\perp}^2}{2\Omega^2}\left(\frac{1}{B}\frac{\partial B}{\partial x} - \hat{e}_1\cdot(\hat{e}_2\cdot\nabla)\hat{e}_2\right)\hat{e}_{[2,1]}\cdot\nabla \\ & + \frac{c_{\perp}^2}{4\Omega^2}\left(B\frac{d}{dt}\frac{1}{B}b_{[2,1]} + (c_{\parallel} b_{[2,1]} + b_{[4,3]})\nabla\cdot\hat{n} + b_{[2,1]}\nabla\cdot\vec{u} - 2c_{[1,2]}\hat{e}_2\cdot\left[(\hat{e}_1\cdot\nabla)\hat{e}_1 + (\hat{n}\cdot\nabla)\hat{n}\right]\right. \\ & \quad \left. \pm 2c_{[2,1]}\hat{e}_1\cdot\left[(\hat{e}_2\cdot\nabla)\hat{e}_2 + (\hat{n}\cdot\nabla)\hat{n}\right] - 2B\hat{e}_{[1,2]}\cdot\nabla\frac{1}{B}c_2 \mp 2B\hat{e}_{[2,1]}\cdot\nabla\frac{1}{B}c_1\right)\frac{\partial}{\partial c_{\parallel}} \\ & - \frac{c_{\perp}^2}{4\Omega^2}\left[B\frac{d}{dt}\frac{1}{B}(c_{\parallel} b_{[2,1]} + b_{[4,3]}) + b_{[2,1]}(a + \tfrac{1}{2}\nabla\cdot\hat{n}) + 2(c_{\parallel} b_{[2,1]} + b_{[4,3]})(\nabla\cdot\vec{u} - \hat{n}\hat{n} : \nabla\vec{u})\right. \\ & \quad \mp 2(c_1 c_{[4,3]} \pm c_2 c_{[3,4]}) - 2B(\hat{e}_{[1,2]}\cdot\nabla)\left(\frac{1}{B}\hat{e}_2\cdot\frac{d\vec{u}}{dt}\right) \mp 2B(\hat{e}_{[2,1]}\cdot\nabla)\left(\frac{1}{B}\hat{e}_1\cdot\frac{d\vec{u}}{dt}\right) \end{aligned}$$

$$+2[\hat{e}_1\cdot(\hat{e}_1\cdot\nabla)\hat{e}_2]\hat{e}_{[1,2]}\cdot\frac{d\vec{u}}{dt}\mp 2[\hat{e}_1\cdot(\hat{e}_2\cdot\nabla)\hat{e}_2]\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}\Big]\frac{\partial}{\partial c_\perp^2/2}$$

$$\mp\frac{c_\perp^2}{2\Omega^2}[(\hat{e}_1\cdot\nabla)(\hat{e}_{[2,1]}\cdot\nabla)\pm(\hat{e}_2\cdot\nabla)(\hat{e}_{[1,2]}\cdot\nabla)]-\frac{c_\perp^2}{\Omega^2}c_{[1,2]}(\hat{e}_2\cdot\nabla)\frac{\partial}{\partial c_\parallel}\mp\frac{c_\perp^2}{\Omega^2}c_{[2,1]}(\hat{e}_1\cdot\nabla)\frac{\partial}{\partial c_\parallel}$$

$$+\frac{c_\perp^2}{\Omega^2}\left(\hat{e}_{[1,2]}\cdot\frac{d\vec{u}}{dt}(\hat{e}_2\cdot\nabla)\pm\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}(\hat{e}_1\cdot\nabla)\right)\frac{\partial}{\partial c_\perp^2/2}\mp\frac{c_\perp^2}{2\Omega^2}[2c_1c_2,c_1^2-c_2^2]\frac{\partial}{\partial c_\parallel}\frac{\partial}{\partial c_\parallel}$$

$$+\frac{c_\perp^2}{\Omega^2}\left(c_{[1,2]}\hat{e}_2\cdot\frac{d\vec{u}}{dt}\pm c_{[2,1]}\hat{e}_1\cdot\frac{d\vec{u}}{dt}\right)\frac{\partial}{\partial c_\parallel}\frac{\partial}{\frac{1}{2}\partial c_\perp^2}\mp\frac{c_\perp^2}{2\Omega^2}\left[2\left(\hat{e}_1\cdot\frac{d\vec{u}}{dt}\right)\left(\hat{e}_2\cdot\frac{d\vec{u}}{dt}\right),\left(\hat{e}_1\cdot\frac{d\vec{u}}{dt}\right)^2\right.$$

$$\left.-\left(\hat{e}_2\cdot\frac{d\vec{u}}{dt}\right)^2\right]\frac{\partial}{\frac{1}{2}\partial c_\perp^2}\frac{\partial}{\frac{1}{2}\partial c_\perp^2},$$

$$L_{[3c,3s]}=\frac{c_\perp}{\Omega^2}\left(\tfrac{1}{4}c_\perp^2 b_{[2,1]}\hat{e}_1\cdot[(\hat{e}_2\cdot\nabla)\hat{e}_2+(\hat{n}\cdot\nabla)\hat{n}]\mp\tfrac{1}{4}c_\perp^2 b_{[1,2]}\hat{e}_2\cdot[(\hat{e}_1\cdot\nabla)\hat{e}_1+(\hat{n}\cdot\nabla)\hat{n}]\right.$$

$$\left.+\frac{c_\perp^2 B}{8}\hat{e}_{[1,2]}\cdot\nabla\frac{1}{B}b_2\mp\frac{c_\perp^2 B}{8}\hat{e}_{[2,1]}\cdot\nabla\frac{1}{B}b_1\right)\frac{\partial}{\partial c_\parallel}$$

$$+\frac{c_\perp}{\Omega^2}\Big[\tfrac{1}{4}c_\perp^2(c_\parallel b_2+b_4)\hat{e}_1\cdot(\hat{e}_{[2,1]}\cdot\nabla)\hat{e}_2$$

$$-\tfrac{1}{4}c_\perp^2(c_\parallel b_1+b_3)\hat{e}_1\cdot(\hat{e}_{[1,2]}\cdot\nabla)\hat{e}_2+\tfrac{1}{4}c_\perp^2 b_{[1,2]}(\tfrac{1}{2}c_2+c_4)\mp\tfrac{1}{4}c_\perp^2 b_{[2,1]}(\tfrac{1}{2}c_1+c_3)$$

$$+\frac{c_\perp^2 B}{8}\left(\hat{e}_{[2,1]}\cdot\nabla\frac{1}{B}(c_\parallel b_1+b_3)\mp\hat{e}_{[1,2]}\cdot\nabla\frac{1}{B}(c_\parallel b_2+b_4)\right)\Big]\frac{\partial}{\partial c_\perp^2/2}$$

$$+\frac{3c_\perp^3}{8\Omega^2}b_{[2,1]}(\hat{e}_1\cdot\nabla)\frac{\partial}{\partial c_\parallel}\mp\frac{3c_\perp^3}{8\Omega^2}b_{[1,2]}(\hat{e}_2\cdot\nabla)\frac{\partial}{\partial c_\parallel}$$

$$-\frac{3c_\perp^3}{8\Omega^2}\left((c_\parallel b_2+b_4)(\hat{e}_{[1,2]}\cdot\nabla)\pm(c_\parallel b_1+b_3)(\hat{e}_{[2,1]}\cdot\nabla)\right)\frac{\partial}{\partial c_\perp^2/2}+\frac{3c_\perp^3}{8\Omega^2}(b_{[2,1]}c_1\mp b_{[1,2]}c_2)\frac{\partial}{\partial c_\parallel}\frac{\partial}{\partial c_\parallel}$$

$$-\frac{3c_\perp^3}{8\Omega^2}\left((c_\parallel b_2+b_4)c_{[1,2]}\mp(c_\parallel b_1+b_3)c_{[2,1]}+b_{[2,1]}\hat{e}_1\cdot\frac{d\vec{u}}{dt}\mp b_{[1,2]}\hat{e}_2\cdot\frac{d\vec{u}}{dt}\right)\frac{\partial}{\partial c_\parallel}\frac{\partial}{\partial c_\perp^2/2}$$

$$+\frac{3c_\perp^3}{8\Omega^2}\left((c_\parallel b_2+b_4)\hat{e}_{[1,2]}\cdot\frac{d\vec{u}}{dt}\mp(c_\parallel b_1+b_3)\hat{e}_{[2,1]}\cdot\frac{d\vec{u}}{dt}\right)\frac{\partial}{\partial c_\perp^2/2}\frac{\partial}{\partial c_\perp^2/2},$$

$$L_{[4c,4s]}=\frac{c_\perp^4}{16\Omega^2}[2b_1b_2,\ b_1^2-b_2^2]\left(\frac{\partial}{\partial c_\parallel}\frac{\partial}{\partial c_\parallel}-\frac{\partial}{\partial c_\perp^2/2}\right)-\frac{c_\perp^4}{8\Omega^2}[(c_\parallel b_1+b_3)b_{[2,1]}\pm(c_\parallel b_2+b_4)b_{[1,2]}]\frac{\partial}{\partial c_\parallel}\frac{\partial}{\partial c_\perp^2/2}$$

$$+\frac{c_\perp^4}{16\Omega^2}[2(c_\parallel b_1+b_3)(c_\parallel b_2+b_4),\ (c_\parallel b_1+b_3)^2-(c_\parallel b_2+b_4)^2]\frac{\partial}{\partial c_\perp^2/2}\frac{\partial}{\partial c_\perp^2/2}, \quad (21)$$

in which

$$a=\frac{e}{m}E_\parallel+\tfrac{1}{2}c_\perp^2\nabla\cdot\hat{n}-\hat{n}\cdot\frac{d\vec{V}}{dt},$$

$$b_{[1,3]}=\hat{e}_1\cdot(\hat{e}_1\cdot\nabla)[\hat{n},\vec{V}]-\hat{e}_2\cdot(\hat{e}_2\cdot\nabla)[\hat{n},\vec{V}], \qquad b_{[2,4]}=-\hat{e}_2\cdot(\hat{e}_1\cdot\nabla)[\hat{n},\vec{V}]-\hat{e}_1\cdot(\hat{e}_2\cdot\nabla)[\hat{n},\vec{V}],$$

$$c_{[1,2]}=\hat{e}_{[1,2]}\cdot\left(\frac{d\hat{n}}{dt}-\nabla\vec{V}\cdot\hat{n}\right), \qquad c_{[3,4]}=\hat{e}_{[1,2]}\cdot\left(\frac{d\hat{n}}{dt}+c_\parallel(\hat{n}\cdot\nabla)\hat{n}\right).$$

We perform an evaluation of the transport equation similar to the execution of Eq. (14) in using the definition of pressure tensor given in Eq. (15), grouping mathematical terms into two classes. Therefore we obtain

$$mN\left(C_{\parallel}\frac{d\hat{n}}{dt}+\frac{d\vec{V}}{dt}\right)_{\perp}-\hat{n}\times mN\frac{d}{dt}\hat{n}\times(\vec{U}_{\perp}-\vec{V})$$

$$+(C_{\parallel}^{(0)}\hat{n}\cdot\nabla\times\hat{n}+\hat{n}\cdot\nabla\times\vec{V})\hat{n}\times mN(\vec{U}_{\perp}-\vec{V})+mN(\vec{U}_{\perp}-\vec{V})(C_{\parallel}^{(0)}\nabla\cdot\hat{n}+\nabla\cdot\vec{V}-\hat{n}\hat{n}:\nabla\vec{V})$$

$$+mN(\vec{U}_{\perp}-\vec{V})\cdot(C_{\parallel}\nabla\hat{n}+\nabla\vec{V})-Ne\vec{E}_{\perp}+(\nabla\cdot\vec{P}^{(0)})_{\perp}+(\nabla\cdot\vec{P}^{(1)})_{\perp}-\frac{1}{c}\vec{j}\times\vec{B}=0\ . \quad (22)$$

Thus Eqs. (18) and (22) combine as an equation of motion, namely *Madeleine* Eq. III (the continuity equation (17) is named as *Madeleine* Eq. II), corresponding to the Navier-Stokes equation for an anisotropic inhomogeneous media with general magnetic field configuration from magnetofluid mechanics; they give the full aspects of magnetohydrodynamics of an axisymmetric plasma. The theory has now been developed and a rigorous kinetic foundation of the phenomenological Navier-Stokes equation is stated.

In conclusion, one should notice that in the present article the obtained drift-kinetic equation is not equivalent to that of FDL's work.[16] The problem arises starting from different expressions of our operator D and FDL's Eq. (24) or (50). In the lowest-order case and for the calculation of $f^{(1)}$, the terms that carry the gyrophase derivative $\partial/\partial\zeta$ do not play a role and thus the same result is reached. But in the higher orders, these terms play a dominant role and cause the difference from the first-order result.

It appears likely that our handling of the problem of magnetohydrodynamics differs from that given by Ref. 16. Frieman *et al.* deal with the simple case of a two-dimensional plasma in a straight magnetic field but we study a generalized case of a three-dimensional plasma in an axisymmetric magnetic configuration. The continuity equation (17) of our present work simply reduces to FDL's result (76) when the simplified conditions $E_{\parallel}=0$, $\hat{n}\cdot\nabla=0$, and $\nabla\hat{n}=0$ are applied. Also, the second term in parathenses of FDL's Eq. (76) is in error because the important effect of the magnetic momentum on the particle flux is not taken into account. Our equations of motion (18) and (22) reduces simply to their Eq. (72) with the application of the above simplified conditions. As we pointed out earlier in Sec. III, the pressure tensor, Eq. (15), and the heat flux tensor, Eq. (16), are generalizations of FDL's special case, Eq. (75). In particular, these transport coefficients play the dominant role in the study of transport theory and magnetohydrodynamics. Furthermore, in our paper the first-order transport equation (22) is obtained from the never-calculated expression of $f^{(2)}$, which could lead the problem to a further step in the studies of instabilities of $k_{\perp}\rho_i \gtrsim 1$. As we see in FDL's paper [p. 1477, Eqs. (40), (56), and (72)], their first-order transport equation (72) is, in fact, a result obtained as usual from the Vlasov equation (1) but not from the basis of the drift equations (62) and (63). It appears to be a speculation and does not give proof of the drift-kinetic equation.

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