Double-vertex contribution to the decay rate of orthopositronium*

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The recently calculated decay rate of orthopositronium differs with the experimental value by $2\frac{1}{2}$ standard deviations. This communication reports a 100-fold improvement in the numerical error estimate for the double-vertex diagram contribution to the decay rate. This improvement reduces the numerical error estimate of the previously calculated decay rate of orthopositronium by 20%. This improvement of the double-vertex diagrams alone does not bring the experimental and calculated values into good agreement; however, the discrepancy is reduced.

The decay rate of orthopositronium as calculated from the Feynman diagram in Fig. 1 is[1]

$$\Gamma_0 = (2/9\pi)(\alpha^6 mc^2/\hbar)(\pi^2 - 9)$$
$$= (0.72112 \pm 0.00001)\times 10^7\ \text{sec}^{-1}, \qquad (1)$$

where Γ_0 is the reciprocal of the orthopositronium lifetime and the error estimate is due to the uncertainty in the fundamental constants: $\alpha^{-1} = 137.03602(21)$, $m = 0.5110041(16)$ MeV, and $\hbar = 6.582183(22)\times 10^{22}$ MeV sec.[2] The decay rate of orthopositronium has been measured[3] with an accuracy that is sensitive to the lowest-order radiative corrections to Eq. (1). Recently, these lowest-order radiative corrections have been calculated.[4-8] The result is[6]

$$\Gamma' = \Gamma_0[1 + (\alpha/\pi)(1.8 \pm 0.6)]$$
$$= (0.7241 \pm 0.0010)\times 10^7\ \text{sec}^{-1}. \qquad (2)$$

This may be compared with the experimental value of Beers and Hughes,[3]

$$\Gamma_{\text{expt}} = (0.7275 \pm 0.0015)\times 10^7\ \text{sec}^{-1}. \qquad (3)$$

There is thus a discrepancy in the amount of approximately $2\frac{1}{2}$ standard deviations in the experimental error.

In view of this discrepancy there is a need to reduce the numerical estimates that lead to the error estimate in Eq. (2). This communication reports a 100-fold improvement in the numerical error estimates for the double-vertex diagram contributions depicted in Fig. 2. The double-vertex diagram contributions given in Ref. 6 contribute (in the Feynman gauge) in the amount

$$\Gamma'_{\text{dv}} = (-3.4 \pm 0.4)(\alpha/\pi)\Gamma_0. \qquad (4)$$

The calculation of the diagrams in Fig. 2 was accomplished by using the symbol-manipulating program REDUCE[9] for most of the algebraic calculations, and Gauss-Legendre integration algorithms to perform some of the necessary integrations (other integrals were done by hand). In the present work an improvement in the integration algorithms yields a new value for Eq. (4) given by (in the Feynman gauge)

$$\Gamma_{\text{dv}} = (-3.355 \pm 0.003)(\alpha/\pi)\Gamma_0. \qquad (5)$$

This improved value allows a redetermination of the decay rate given in Eq. (2).

Substituting the value in Eq. (5) for the value in Eq. (4) into Eq. (2) gives the result for the total decay rate of

$$\Gamma = \Gamma_0[1 + (\alpha/\pi)(1.86 \pm 0.45)]$$
$$= (0.7242 \pm 0.0008)\times 10^7\ \text{sec}^{-1}. \qquad (6)$$

In conclusion, it is observed that the improved value of the double-vertex diagram contributions yields a numerical error estimate in Eq. (6) which is 20% smaller than that in Eq. (2). The new result given to high accuracy in Eq. (5) is necessary for a high-precision test of quantum electrodynamics.

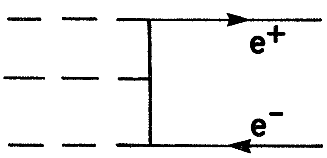

FIG. 1. Feynman diagram contributing to the lowest-order decay rate of orthopositronium. There are a total of six such diagrams, one for each of the photon permutations. Time increases from right to left.

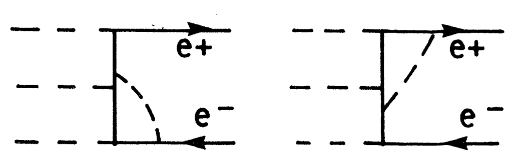

FIG. 2. Feynman diagrams contributing to the lowest-order double-vertex corrections to the decay rate of orthopositronium. There are six such diagrams for each of the diagrams above, one for each of the photon permutations. Time increases from right to left.

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[1]A. Ore and J. L. Powell, Phys. Rev. 75, 1696 (1949).

[2]B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).

[3]R. H. Beers and V. W. Hughes, Bull Am. Phys. Soc. 13, 633 (1968); also see V. W. Hughes, *Phys. 1973, Pleanarvortr. Physikertag., 37th* (Physik Verlag, Weinheim, Germany, 1973) pp. 123–155 (in English); see also P. G. Coleman, T. C. Griffith, and G. R. Heyland, J. Phys. E 5, 376 (1972); and P. G. Coleman and T. C. Griffith, J. Phys. B 6, 2155 (1973).

[4]J. M. Holt, Ph.D. thesis (Yale University, 1970) (unpublished).

[5]M. A. Stroscio, Ph.D. thesis (Yale University, 1974) (unpublished).

[6]M. A. Stroscio and J. M. Holt, Phys. Rev. A 10, 749 (1974).

[7]M. A. Stroscio, Phys. Lett. 50A, 81 (1974).

[8]M. A. Stroscio, Air Force Cambridge Research Laboratories, Technical Report 74-0248, 1974 (unpublished).

[9]A. C. Hearn, Stanford artificial intelligence project, Memo No. AIM-133, 1970 (unpublished).