Comment on charged-particle scattering in dilute and dense plasmas in the presence of intense electromagnetic radiation

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It is shown that the quantum theory in the Born approximation gives the classical result when scattering of electrons in the presence of a radiation field occurs in a time short with respect to the period of the radiation. It is argued that the result of Bunkin and Fedorov may be used only when the collision frequency of the electrons in a plasma is much smaller than the frequency of the electromagnetic radiation.

In recent years, there has been much interest in the heating of a plasma by a laser. In particular, a quantum-mechanical expression for the rate of energy transfer from a laser to plasma electrons has been obtained by Bunkin and Fedorov,[1] Pert,[2] Kroll and Watson,[3] Geltman and Teague,[4] Brehme,[5] and Rahman.[6] The case of circularly polarized radiation has been studied by Seely and Harris.[7] All of the above authors discuss quantum effects in what is often called the inverse-bremsstrahlung process. In this note, we point out that the classical as well as the quantum results follow from quantum mechanics. The important point is that the result depends upon whether the scattering occurs over a time short with respect to a cycle of the electromagnetic radiation or over very many cycles. This is shown quite simply in the following consideration.

It is convenient to take advantage of the fact that the effect of the radiation field in the dipole approximation, together with the Coulomb potential (perhaps modified by screening), may be combined to form a time-dependent effective potential.[8] Thus, the Schrödinger equation can be written

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{r},t)+V(\vec{r}+\vec{\alpha}(t))\psi(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t), \tag{1}$$

where

$$\vec{\alpha}(t)=-\int_{-\infty}^{t}\frac{e}{mc}\vec{A}(t)\,dt\,. \tag{2}$$

The quantity $\vec{\alpha}(t)$ is the classical displacement of a free electron in the vector potential $\vec{A}(t)$. We consider the initial and final states, $(1/\sqrt{\mathcal{V}})e^{i\vec{k}_0\cdot\vec{r}}$ and $(1/\sqrt{\mathcal{V}})e^{i\vec{k}\cdot\vec{r}}$. We may consider $\mathcal{V}$ to be the interaction volume of the electron with the field. The S-matrix element is given by

$$S_{fi}=-\frac{i}{\hbar}\langle f|\int_{-\infty}^{+\infty}dt\,V(\vec{r}+\vec{\alpha}(t))|i\rangle\,, \tag{3}$$

where we have adopted the interaction representation of quantum mechanics. It is convenient to introduce the Fourier transform of the scattering potential,

$$V(\vec{r})=\int d^3q\,\tilde{V}(\vec{q})\,e^{i\vec{q}\cdot\vec{r}}\,. \tag{4}$$

Then $V(\vec{r}+\vec{\alpha}(t))$ is given by

$$V(\vec{r}+\vec{\alpha}(t))=\int d^3q\,\tilde{V}(q)\exp\{i[\vec{q}\cdot\vec{r}+\vec{q}\cdot\vec{\alpha}(t)]\}\,. \tag{5}$$

At this point, we emphasize that either the classical result, or the result of Bunkin and Fedorov can be obtained by inserting Eq. (5) into Eq. (3). The result depends entirely upon the treatment of the quantity $\vec{\alpha}(t)$.

In order to obtain the result of Bunkin and Fedorov, we assume a sinusoidal radiation field having plane polarization. We assume further, that the scattering occurs over very many oscillations of the electromagnetic wave. We are thus assuming that the wave functions are spread over a large volume and that the collision frequency of the electrons with each other, as well as with ions, is very small compared to the frequency of the electromagnetic wave. This condition does not generally hold in a cold, dense plasma. In the framework of these assumptions, we now assume the time dependence of $\vec{\alpha}(t)$ to be

$$\vec{\alpha}(t)=\vec{\alpha}_0 \sin\omega t . \tag{6}$$

The assumptions listed above allow us to expand the exponential of Eq. (5) in a Fourier series in Bessel functions,

$$e^{i\vec{q}\cdot\vec{\alpha}(t)}=e^{i\vec{q}\cdot\vec{\alpha}_0 \sin\omega t}=\sum_{n=-\infty}^{\infty} J_{-n}(\vec{\alpha}_0\cdot\vec{q})e^{-in\omega t} . \tag{7}$$

Thus, Eq. (3) involves a series of integrals of the form

$$\int_{-\infty}^{+\infty} dt \exp\left[i\left(\frac{E_f-E_i}{\hbar}-n\omega\right)t\right]=2\pi\delta\left(\frac{E_f-E_i}{\hbar}-n\omega\right). \tag{8}$$

Equation (8) explicitly demonstrates the necessity of the condition that the scattering time be long with respect to $1/\omega$. Hence, we assume reasonably long wave trains and infrequent collisions of the electrons with any other objects. The nth term then corresponds to the absorption of n photons. We then obtain

$$\begin{aligned} S_{fi} &= -\frac{i}{\hbar\upsilon}\int\int d^3r\, d^3q\, e^{-i\vec{k}\cdot\vec{r}}\,\tilde{V}(q)e^{i\vec{q}\cdot\vec{r}}\sum_{n=-\infty}^{\infty} J_{-n}(\vec{\alpha}_0\cdot\vec{q})\,e^{i\vec{k}_0\cdot\vec{r}}\,2\pi\delta\left(\frac{E_f-E_i}{\hbar}-n\omega\right) \\ &= -[i(2\pi)^4/\upsilon]\,\tilde{V}(\vec{k}-\vec{k}_0)J_{-n}(\vec{\alpha}_0\cdot(\vec{k}-\vec{k}_0))\,\delta(E_f-E_i-n\hbar\omega). \end{aligned} \tag{9}$$

This leads immediately to the result of Refs. 1–6, which is

$$w_{fi}=[(2\pi)^7/\hbar\upsilon^2]\,|\tilde{V}(\vec{k}-\vec{k}_0)J_n(\vec{\alpha}_0\cdot(\vec{k}-\vec{k}_0))|^2 \times\delta(E_f-E_i-n\hbar\omega). \tag{10}$$

A more frequent condition in plasmas is the case in which the electron-electron collision frequency is larger than $\omega/2\pi$. In this case, the previous treatment makes little sense. Accordingly, assuming the collision to take place at $t=0$, we set

$$\vec{\alpha}(t)\simeq\vec{\alpha}(0)+\vec{\alpha}\,'(0)t+(\text{negligible terms}), \tag{11}$$

and insert this into Eq. (5). This leads to the matrix element

$$S_{fi}=-\frac{i}{\hbar\upsilon}\int_{-\infty}^{+\infty}\int\int d^3r\, d^3q\, dt\, e^{-i\vec{k}\cdot\vec{r}}\,\tilde{V}(\vec{q}) \times e^{i\vec{q}\cdot\vec{r}}e^{i\vec{q}\cdot[\vec{\alpha}(0)+\vec{\alpha}\,'(0)t+\cdots]} \times\exp\left[i\left(\frac{E_f-E_i}{\hbar}\right)t\right]e^{i\vec{k}_0\cdot\vec{r}} . \tag{12}$$

The integrations lead to the result

$$S_{fi}=-[i(2\pi)^4/\hbar\upsilon]\,\tilde{V}(\vec{k}-\vec{k}_0)e^{i(\vec{k}-\vec{k}_0)\cdot\vec{\alpha}(0)} \times\delta[(E_f-E_i)/\hbar+(\vec{k}-\vec{k}_0)\cdot\vec{\alpha}\,'(0)] . \tag{13}$$

As we shall see, this is the amplitude that gives the classical result. We note that the term involving $\vec{\alpha}(0)$ is merely a phase factor. It would be surprising if the scattering depended directly upon the amplitude of the electron oscillations. Large values of α_0 can be obtained by simply going to low frequencies. Making use of the relation

$$\vec{\alpha}\,'(0)=-(e/mc)\vec{A}(0), \tag{14}$$

we obtain the transition rate

$$w_{fi}=[(2\pi)^7 2m/\hbar^3\upsilon^2]\,|\tilde{V}(\vec{k}-\vec{k}_0)|^2 \times\delta(k^2-k_0^2-(2e/\hbar c)(\vec{k}-\vec{k}_0)\cdot\vec{A}(0)). \tag{15}$$

This gives a scattering rate

$$R_\Omega(\vec{k},\vec{A})\,dk=[(2\pi)^4 2m/\hbar^3\upsilon]\,|\tilde{V}(\vec{k}-\vec{k}_0)|^2k^2\,dk\,d\Omega_k \times\delta(k^2-k_0^2-(2e/\hbar c)(\vec{k}-\vec{k}_0)\cdot\vec{A}(0)). \tag{16}$$

For a sinusoidal wave, we have

$$\vec{A}(0)=\vec{A}_0\cos\delta_s , \tag{17}$$

where δ_s is the scattering phase. This must be averaged over all values of $\cos\delta_s$.

The scattering rate is therefore given by

$$R_\Omega(k)=\frac{1}{\pi}\int_0^\pi R_\Omega(k,\delta_s)\,d\delta_s . \tag{18}$$

Averaging the delta function over the scattering phase yields

$$\frac{1}{\pi}\int_0^\pi \delta(k^2-k_0^2-(2e/\hbar c)(\vec{k}-\vec{k}_0)\cdot\vec{A}_0\cos\delta_s)\,d\delta_s , \tag{19}$$

where δ_s is now the particular value of the scattering phase defined by the delta function. Making use of the relation $k^2dk=\frac{1}{2}k\,d(k^2)$ and the incident electron flux, which is $\hbar k_0/m\upsilon$, we arrive at the differential cross section

$$\frac{d\sigma}{d\Omega}=\frac{k}{k_0}\left(\frac{d\sigma}{d\Omega}\right)_0\frac{d(k^2)}{|2\pi e/(\hbar c)(\vec{k}-\vec{k}_0)\cdot\vec{A}\sin\delta_s|} . \tag{20}$$

This is the classical result in the form quoted by Kroll and Watson.[3] It is not surprising that the classical limit corresponds to the case of transfer of a large number of photons, as shown in Ref. 3. One can always find a sufficiently large

n such that $n\omega/2\pi$ is much greater than any collision frequency of the electrons. It is precisely the higher Fourier components of the perturbation that determine its behavior over a very short period of time. Thus the classical limit must correspond to the simultaneous absorption of a large, but undetermined number of photons. However, as seen in the derivation of Eq. (20), the photon concept does not shed much light on our understanding of energy transfer to electrons by a light beam in a dense plasma. On the other hand, observation of effects predicted by Eq. (10) would require a tenuous plasma, in order that the electron mean free path be of the order of the dimensions of the experimental apparatus. Thus, quantum effects are unlikely to play a role in most plasma applications.

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