Comments and Addenda

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Calculations of electron energy distributions for internal ionization during the alpha decay of ^{210}Po employing the binary-encounter approximation*

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(Received 5 March 1975)

The description of internal ionization during α decay, as developed by Hansen, is reexamined and extended to the calculation of the ejected-electron energy distributions. The results are compared with recent experimental data for the decay of ^{210}Po.

Hansen has applied the binary-encounter approximation (BEA) in an impact-parameter representation to the problem of internal ionization during α decay.[1] His calculations of the total probabilities for K-, L-, and M-shell ionization by this method yielded surprisingly good agreement with experimental results. Since the BEA description of the ionization process does not take into account the binding energy of the electron undergoing ionization, but rather assumes strict nonrelativistic energy and momentum conservation in a collision between a heavy charged particle and a *free* electron,[2] one would not expect it to be successful at predicting K-shell ionization probabilities for ^{210}Po where the K binding energy is relatively large compared to the α-particle energy. The recent work of Fischbeck and Freedman[3] has now provided experimental data on the energy distributions of K- and L-shell electrons ejected in this decay. As a further, more stringent, test of the BEA description, the method used by Hansen is herein reexamined and extended to the calculation of the energy distributions of the ejected electrons.

The probability of ionizing an electron as a result of a collision with an α particle emanating from the nucleus (zero impact parameter) may be written[1]

$$P=\int_0^\infty \sigma\,(v_1(r),\, v_2(r))\,\rho(r)\,dr\,, \qquad (1)$$

where $\sigma(v_1,v_2)$ is the ionization cross section as a function of α-particle velocity (v_1) and electron velocity (v_2), and ρ is the electron density given by

$$\rho(r)=(n/4\pi)R^2_{n,l}(r)\,, \qquad (2)$$

where n is the number of electrons in the shell under consideration and $R_{n,l}$ is the bound-electron radial wave function.

The differential cross section for the exchange of energy ΔE between a heavy charged particle and an electron has been derived within the framework of the binary-encounter approximation by Gerjuoy[2] and by Vriens.[4] To calculate the total ionization probability, one requires the total ionization cross section, which is obtained by integrating the differential energy-exchange cross section over all energy transfers greater than the electron binding energy. This integrated cross section has been given by Garcia[5] and is, for the range of energy exchange leading to ionization in the case under consideration (i.e., $\Delta E \geq U$, where U is the electron binding energy and $v_2 > v_1$),

$$\sigma=\int^{\Delta E}\frac{d\sigma}{d\Delta E}\,d\Delta E$$

$$=\frac{\pi}{3}\frac{Z_1^2e^4}{v_1^2v_2}\left(3\frac{v_1/m_1-v_2/m_2}{\Delta E}+\frac{(v_2'^3-v_2^3)-(v_1^3+v_1'^3)}{(\Delta E)^2}\right) \qquad (3)$$

for $U\leq\Delta E\leq a$, where

$$a=\frac{4m_1m_2}{(m_1+m_2)^2}\left[E_1-E_2+\tfrac{1}{2}v_1v_2\,(m_1-m_2)\right],$$

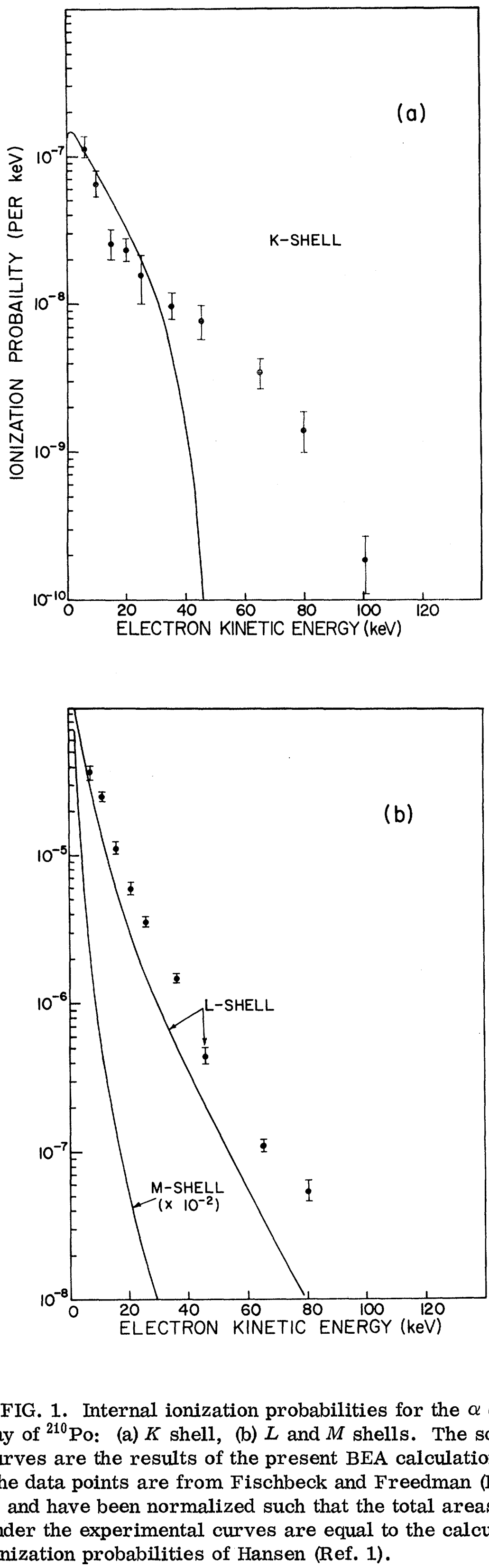

FIG. 1. Internal ionization probabilities for the α decay of ^{210}Po: (a) K shell, (b) L and M shells. The solid curves are the results of the present BEA calculations. The data points are from Fischbeck and Freedman (Ref. 3) and have been normalized such that the total areas under the experimental curves are equal to the calculated ionization probabilities of Hansen (Ref. 1).

$$v_1' = (v_1^2 - 2\Delta E/m_1)^{1/2}, \quad v_2' = (v_2^2 + 2\Delta E/m_2)^{1/2}.$$

In order to compute the cross section needed in Eq. (1), one must relate the velocity of the bound electron and the α particle to their distances from the nucleus. Hansen obtained such a relationship for the electron using the condition[1,6]

$$\int_{v'}^{\infty} R_{n,l}^2(v)\,dv = \int_0^{r'} R_{n,l}^2(r)\,dr\,, \tag{4}$$

where $R_{n,l}(v)$ and $R_{n,l}(r)$ are the bound-electron radial wave functions in momentum space and configuration space, respectively. The relationship between the α-particle velocity and its distance from the nucleus is obtained by equating the Q value for α decay to the sum of the total kinetic energy (α particle + recoil) and potential energy of the system. The result is

$$v_1 = \left(\frac{2Q - 4(Z-2)e^2/r}{M_1[M_1/(M-4)+1]}\right)^{1/2}, \tag{5}$$

where Z and M are the atomic number and mass of the decaying atom.

The same general procedure used by Hansen is employed in the present calculations of the electron energy distributions. In accordance with Eq. (1) the ionization probability as a function of energy transfer is expressed as

$$P_{\Delta E} = \int_0^{\infty} \sigma_{\Delta E}(v_1(r), v_2(r))\,\rho(r)\,dr\,. \tag{6}$$

The cross section, $\sigma_{\Delta E}$, is obtained from Eq. (3) for intervals of energy transfer $\Delta E_i - \delta$ to $\Delta E_i + \delta$.

In Fig. 1, the experimental K- and L-shell ionization probabilities of Fischbeck and Freedman[3] are compared with the calculated electron energy distributions. The experimental values have been normalized such that the areas under the experimental curves are equal to the total ionization probabilities calculated previously by Hansen.[1] With regard to the comparison for the K shell, it is seen that at low energies there is good agreement between theory and experiment but that as the electron kinetic energy increases, the BEA prediction drops off much too rapidly. A similar trend is observed in the L-shell comparison, although the divergence between theory and experiment with increasing electron kinetic energy is not nearly as large as for the K shell.

One additional point to be made concerns the way in which relativistic velocity and mass corrections have been incorporated into the calculations. Hansen uses a procedure whereby the electron velocity obtained from Eq. (4) is used to calculate the electron kinetic energy nonrelativistically. This kinetic energy is then transformed

TABLE I. Total ionization probabilities for the α decay of ^{210}Po.

Shell	Present "nonrelativistic" calculation	Hansen's "relativistic" calculation
L	4.00×10^{-4}	5.92×10^{-4}
$2s$	1.36×10^{-4}	2.30×10^{-4}
$2p$	2.64×10^{-4}	3.62×10^{-4}
M	1.67×10^{-2}	1.90×10^{-2}
$3s$	1.28×10^{-3}	1.74×10^{-3}
$3p$	4.95×10^{-3}	5.66×10^{-3}
$3d$	1.05×10^{-2}	1.16×10^{-2}

back to obtain a relativistic electron velocity and mass which are subsequently used in the calculation of the energy-transfer cross section [Eq. (3)].[6] This substitution of relativistic quantities into a nonrelativistic expression is not theoretically justifiable and it causes the calculation to yield different results depending upon the form in which the energy-transfer cross section is written. For example, the correspondence between the cross-section equation given by Rudd *et al.*[7] and Eq. (3) relies upon the identity $E_2=\frac{1}{2}m_2v_2^2$.[8] When one employs the relativistic correction procedure of Hansen, this equality no longer holds.

The questionable validity of the relativistic correction procedure causes one particular concern when it is realized that within the framework of the Hansen BEA treatment, K-shell ionization in the α decay of ^{210}Po becomes possible *only* as a direct result of its utilization. Calculations carried out without the use of relativistic electron velocities and masses (unmodified BEA) yielded a maximum possible energy transfer of 84 keV—which is less than the K-electron binding energy. Thus, the unmodified BEA calculation predicts a total K-shell ionization probability of zero for this case. This result inherently stems from the classical nature of the model. In particular, Eqs. (4) and (5) require that the bound-electron velocity be highest in the small-r region where the α-particle velocity is lowest. Quantally, of course, one cannot associate a specific position for the bound electron with a given velocity, and it is this association which causes the K-shell ionization probability to be zero in the unmodified calculation and to drop off so rapidly with ejected electron energy in the Hansen calculation.

The results of unmodified BEA calculations for the L and M shells are compared with Hansen's previous results in Table I.

I thank T. Kishimoto for many helpful discussions concerning this work.

*Supported in part by the U. S. Energy Research and Development Administration and the Robert A. Welch Foundation.

[1]J. S. Hansen, Phys. Rev. A 9, 40 (1974).
[2]E. Gerjuoy, Phys. Rev. 148, 54 (1966).
[3]H. J. Fischbeck and M. S. Freedman, Phys. Rev. Lett. 34, 173 (1975).
[4]L. Vriens, Proc. Phys. Soc. Lond. 90, 935 (1967).
[5]J. D. Garcia, Phys. Rev. A 1, 280 (1970).
[6]J. S. Hansen, Phys. Rev. A 8, 822 (1973).
[7]M. E. Rudd, D. Gregoire, and J. B. Crooks, Phys. Rev. A 3, 1635 (1971).
[8]In addition, it requires that $m_1 \gg m_2$, but even at the highest electron velocities encountered in the ^{210}Po K-shell calculation, this is still a valid approximation.