Errata

## Erratum: Electrostatic component of self-focusing electron streams [Phys. Rev. A 11, 2152 (1975)]

R. A. McCorkle

Line 23, second column, page 2153 should read

 $\sim 0.42\gamma [\beta AC(\alpha\sigma_0 p\gamma/i)^2]^{1/3}$ .

In line 18, first column, page 2154, read

 $\ldots \sim 5.6 \times 10^{11} \gamma / \delta \sigma_0^2 \alpha^2$ .

## Erratum: Effects of retardation on electromagnetic self-energy of atomic states\* [Phys. Rev. A <u>9</u>, 1794 (1974)]

C. K. Au and G. Feinberg

There are basically two errors involved. The first is in the numerical calculation, slightly affecting the answer obtained by either renormalization prescription. The second error affects only the alternative renormalization prescription in the Appendix and lies in the fact that the factor  $\frac{2}{3}$  that appears in Eq. (A1) should in fact be replaced by  $1 - \cos^2 \theta$  inside the integral, i.e.,

$$\Delta E' = \frac{\alpha}{\pi} \frac{p^2}{2m^2} \int \frac{(1 - \cos^2 \theta)k \, dk \, d \, \cos \theta}{k^2/2m + k - pk \cos \theta/m}.$$
 (1)

With this change, the difference between these two renormalization prescriptions becomes

$$P(y) = \frac{m\alpha}{3y\pi} \left[ (1 - 3y^2 - 2y^3) \ln(1 + y) - (1 - 3y^2 + 2y^3) \ln(1 - y) + \frac{16}{3}y^3 - 2y \right],$$
(2)

where y = p/m. This can be expanded, for small y, to give

$$P(y) = \frac{2\alpha m y}{\pi} \sum_{i=1}^{\infty} \frac{y^{2i+1}}{i(2i+1)(2i+3)}$$
(3)

as compared to  $F(p)p^2/2m^2$  given in Eq. (A2) in the original paper, which expands to give

$$F(p)\frac{p^2}{2m^2} = \frac{2\,\alpha m\,y}{3\pi} \sum_{i=1}^{\infty} \frac{y^{2\,i+1}}{i(2i+1)}.$$
(4)

We also wish to remark that in calculating the

renormalized energy shift according to the formula

$$W_n^{\rm ren} = \frac{2}{\pi} \int_0^\infty k \, dk \, \tilde{M}_n + \frac{\alpha^3}{6\pi} Z^2 \int_0^\infty \frac{k \, dk}{k + k^2/2m}$$
(5)

as given in Sec. III of the original paper, not only do the leading logarithmic divergences cancel, but also the terms that scale as  $1/k^2$  for large k in the integral. The integral thus converges at least as fast as  $1/k_{\text{max}}^{1.5}$  where  $k_{\text{max}}$  is the momentum cutoff in the numerical integration.

We give below amended numerical values, corrected for computational errors, corresponding to the mass-renormalization correction as given by (4) and then supplement with values corresponding to the mass renormalization given in (3).

(a) The last line in Table I should read 0.945 (3) and footnote d should be replaced by "Integral converges at least as fast as  $1/k_{\text{max}}^{1.5}$ ."

(b) In Sec. IV, the corrected values are

 $W_{2S}^{\text{ren}} = 932 \text{ MHz}$ ,  $W_{2S}^{\text{ren}} - W_{2P}^{\text{ren}} = 945 \text{ MHz}$ ,

 $\tilde{W}_{2S}^{\text{ren}} = 1078 \text{ MHz}$ ,  $\tilde{W}_{2S}^{\text{ren}} - \tilde{W}_{2P}^{\text{ren}} = 1064 \text{ MHz}$ 

using the mass-renormalization correction operator given in (4), and

 $\tilde{W}_{2S}^{\text{ren}} = 1019 \text{ MHz}$ ,  $\tilde{W}_{2S}^{\text{ren}} - \tilde{W}_{2P}^{\text{ren}} = 1016.5 \text{ MHz}$ 

using the mass-renormalization correction operator given in (3).

(c) In the Appendix, Eqs. (A4)-(A8) should read as follows:

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(A4) $\Delta E'_{a} = 87.3 \text{ MHz}$	tive P(y)
(A5) $\Delta E'_{2P}$ = 15.8 MHz	
(A6) $\Delta E'_{2S} - \Delta E'_{2P} = 71.5 \text{ MHz}$	
(A7) $\tilde{W}_{2S}^{\text{ren}} = W_{2S}^{\text{ren}} + \Delta E'_{2S} = 1019 \text{ MHz}$	
(A8) $\tilde{W}_{2S}^{\text{ren}} - \tilde{W}_{2P}^{\text{ren}} = 1016.5 \text{ MHz}$	

In addition, the following has been brought to our attention: It was suggested by Kroll<sup>1</sup> in 1964 that the em self-energy shift as calculated in a nonrelativistic theory with retardation would be finite. However, he did not give any specific values.

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<sup>1</sup>N. Kroll, Lectures delivered at Les Houches, Summer,

1964, in *Quantum Optics and Electronics*, edited by C. DeWitt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach, New York, 1965).