# Comment on the decay curves of hydrogenlike atoms observed at long times following excitation

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Explicit calculations demonstrate that cascade repopulation effects are a possible explanation of the $t^{-1.5}$ dependence in the decay of Ly-$\alpha$ radiation at long times after excitation observed recently by Braithwaite, Matthews, and Moore. Good agreement with all available experimental data is obtained with reasonable population models; excitation proportional to $n^{-m}$ with $m = 2.2$–$4$ is indicated.

Recently, Braithwaite *et al.*[1] observed that the Ly-$\alpha$ decay curve for hydrogenlike oxygen in the time range of 1–10 nsec following excitation was approximately proportional to $t^{-1.5}$ rather than exponential. Similar decay curves approximately proportional to $t^{-1.5}$ for Ly-$\beta$ and a composite decay curve which is that of a superposition of a large number of Lyman lines near the series limit were also presented in this work. These authors suggest cascades from high-lying levels as the likely explanation of the nonexponential decay observed; while no particular cascades are specifically suggested there as being dominant, their discussion does imply that $2p$-$ns$ or $2p$-$nd$ replenishment from very high-lying states is intended. A similar measurement for hydrogenic fluorine was published earlier by Richard[2] who also observed decay approximately proportional to $t^{-1.5}$. Richard speculated that such an effect might arise from cascade repopulation of the $2p$ level due to atoms originally excited to states having rather high $n$ and maximum orbital angular momentum $l=n-1$. Since Braithwaite *et al.* report in their paper that their attempts to calculate the $t^{-1.5}$ behavior were unsuccessful, it is the purpose of this note to present the results of some explicit calculations of cascade effects which demonstrate that (i) the mechanisms suggested above can explain their experimental observations, and that (ii) this then can provide information concerning the primary beam-foil excitation process.

An expression for the population of a level due to cascading from higher-lying levels has been presented in a convenient form by Curtis.[3] This algorithm has previously been coded as a Fortran function CASC($K$, $T$) which computes the contribution to the population of a level at time $T$ after excitation due to the $K$th order cascade specified, where $K$ may be arbitrarily large. Of course, to carry out such a calculation, transition probabilities between all levels involved must be calculated and relative initial level populations must be assumed. This function CASC was used first to investigate the Ly-$\alpha$ decay curve expected at times between 1 and 10 nsec after excitation due to the superposition of cascades from states of varying $n$ and maximum $l$, beginning with the $2p$-$3d$ first-order cascade and continuing to successively higher-order transitions until no further significant contributions to the decay resulted. Transition probabilities between maximum $l$ states were computed using the equation of Garcia[4] and initial populations of the states with principal quantum number $n$ and orbital quantum number $l=n-1$ were assumed to decrease as $n^{-m}$, where $m$ was an adjustable parameter. Typical results of such calculations are shown as the solid curves in Fig. 1, where it is clear that these assumptions, indeed, do lead to power-law decay curves and that the slope obtained here depends sensitively upon the model assumed for the initial populations; the $t^{-1.5}$ decay can be reproduced by a level population proportional to $n^{-4}$. In this case, the most significant cascades 1 nsec after excitation are from $6h$ level, while 10 nsec later it is the $10l$ level which dominates the cascade repopulation.

Next, CASC($K$, $T$) was used to investigate the effect on the population of the $2p$ level of the $ns$ and $nd$ levels, again superposing cascades (here only first-order cascades are required) until no significant contribution resulted from considering still higher $n$ values. Needed transition probabilities were computed from the formula of Gordon and the asymptotic formulas of Bethe and Salpeter,[5] and again initial populations proportional to $n^{-m}$ were studied. Figure 1 also shows the curves which result for $m=4$ if it is assumed that the $ns$, $nd$, and the $(n, n-1)$ states are equally populated. It should be observed that again power-law decays result, falling faster than the measured decay, and which quickly become much smaller than the result for the cascades previously considered. Contributions to the $2p$ level population proportional to $t^{-1.5}$ are obtained for $m=2.2$ and 2.4, respectively. Here, significantly higher-ly-

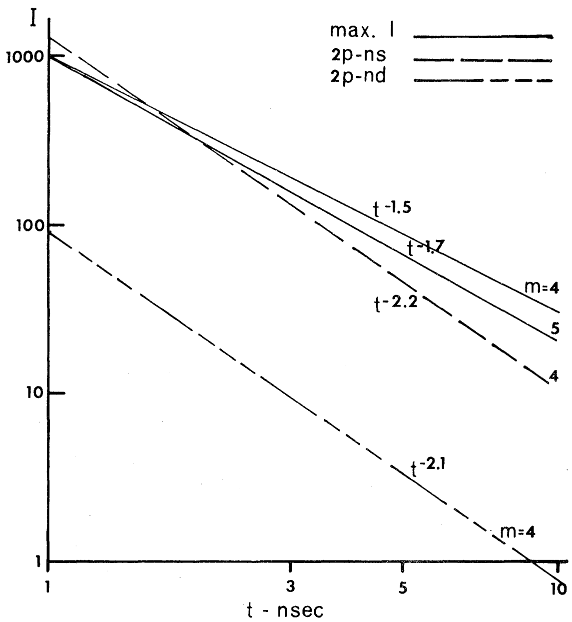


FIG. 1. Computed decay curves for the Ly-$\alpha$ transition in F IX at long times after excitation. The two solid curves show the result of cascades from states with maximum orbital angular momentum for population varying as $n^{-m}$; the value $m=4$ agrees with the experimental measurements of Richard (Ref. 2). The dashed curves show the contribution of $2p-ns$ and $2p-nd$ cascades for initial $ns$ and $nd$ populations equal to that of the $n$, $n-1$ state population, again varying as $n^{-4}$.

ing levels are important with the maximum contribution 10 nsec after excitation coming from the $26s$ and the $35d$ levels.

It is thus established that cascades from high-lying states can in fact give decay curves at long times after excitation which are indistinguishable from power-law decays and that the data of Refs. 1 and 2 may be explained with reasonable assumptions concerning the excitation. Since the cascades described above are limiting cases, initial populations proportional to $n^{-m}$ with $m$ between 2.2 and 4 are indicated. Should the states of a given $n$ be nearly statistically populated—as suggested by the measurement of Rydberg states of B III of Bromander[6]—the cascades from maximum $l$ levels will be dominant and a value of $m$ close to 4 would result. This is roughly consistent with, but represents perhaps a somewhat faster fall off with $n$ than the rather qualitative observations of Lennard *et al.*[7] of Rydberg transitions in highly ionized Fe and Ni. It is also faster than the $n^{-3}$ dependence for free-electron recombination.[5]

Finally, one must consider the variety of experimental circumstances under which essentially identical decay curves have been obtained. We have carried out calculations for both $Z=8$ and $Z=9$ and observe that the resulting decay curves for maximum $l$-state cascades are essentially identical; the predictions based upon cascades from the high-lying $s$ and $d$-states do differ in slope, but only by a few percent. Thus, either model is in good agreement with the experimental results that the same shape decay curve is observed for both fluorine[2] and oxygen.[1] Experimental data for a wider range of atomic numbers at similar ion velocities would therefore be of interest. If the excitation model does not depend strongly upon $Z$—more definite information can thereby be obtained concerning whether it is the high or low $l$ states which dominate the cascade repopulation. The experimental result that a similar $t^{-1.5}$ decay curve was observed for Ly-$\beta$ was also addressed. Ly-$\beta$ decay curves due to cascade repopulation of the $3p$ level from the $ns$ levels were constructed. The result was again a power-law decay with the power differing from $-1.5$ by only 6%.

We thus conclude that simple-cascade-repopulation calculations can reproduce all of the data available at present. A second computer code, POP($N$, $L$, $T$), which utilizes CASC($K$, $T$) to compute the population of the level with quantum numbers $N$ and $L$ at time $T$ including cascades of all allowed orders from levels up to any specified maximum $n$ value, is also available and can be used to generate predictions of more elaborate population models when warranted by more extensive experimental data.

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