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Stimulated collective scattering from a magnetized relativistic electron beam

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We analyze stimulated scattering of a right-circularly-polarized electromagnetic pump wave, of frequency ω_0, from a cold relativistic magnetized electron beam. The pump wave is chosen to satisfy the dispersion relation associated with the magnetized beam. The scattered waves consist of collective plasma oscillations as well as right- and left-polarized electromagnetic waves, traveling parallel and antiparallel to the beam. The frequency of the forward-scattered electromagnetic wave is Doppler shifted up twice and is of order $\gamma_{0z}^2\omega_0$, where $\gamma_{0z}=(1-v_0^2/c^2)^{-1/2}$ and v_0 is the beam's speed. Enhanced stimulated scattering results if the frequency of the pump, in the beam frame, is approximately equal to the electron's cyclotron frequency. Enhanced growth rates for the scattering process are obtained for the scattered radiation. This theory describes a mechanism which is a plausible explanation of a recent experiment in which ~ 100 kW of submillimeter radiation was observed. In view of the large up-conversion in frequency and enhanced growth rates of the scattered radiation, a new high-frequency generative device based on this mechanism seems possible.

The stimulated Compton scattering of an electromagnetic wave from relativistic electrons has evoked interest[1-4] because of the very large frequency conversion which accompanies the process. In particular, Pantell *et al.*[4] have suggested that scattering of microwaves from a relativistic electron beam would result in emission of powerful radiation at submillimeter wavelengths. This suggestion was elaborated by Sukhatme and Wolf, who considered the effect both of boundaries[5] and of an external magnetic field[6]; when their analysis was applied to specific existing relativistic electron beams, the stimulated Compton scattering was found to be strong enough to be of practical importance.

However, intense electron beams[7] are often characterized by Debye lengths that would be much smaller than the scattered wavelengths. In such cases, the usual stimulated Compton scattering result is inapplicable. Rather, the dominant scattering will be a collective phenomenon in which the incident wave scatters off plasmons instead of individual electrons. The present paper presents a relativistic analysis of electromagnetic scattering from plasmons in a magnetized electron beam.

This work is related to analyses of stimulated Raman scattering off plasmons in laser-pellet plasmas and in the ionosphere.[8] However, generally, in those studies neither resonant external magnetic fields nor relativistic effects are considered. The presence of an external magnetic field has been included in the formulation of Forslund *et al.*,[9] and cyclotron resonance effects have been shown to lead to significantly enhanced growth rates when compared to the usual Raman scattering situation.[10] The extension to a relativistic formulation in the present paper makes the analysis applicable to a wider range of phenomena including, in addition to submillimeter radiation from relativistic electron beams,[11,12] astrophysical phenomena such as radiation from pulsars,[13] which have relativistic electrons streaming along magnetic fields as high as 10^{10} kG. Indeed, relativistic effects (but not magnetization) have been considered in treating wave propagation in a pulsar plasma.[14]

The frequency conversion, which occurs when an electromagnetic wave is scattered by a beam of relativistic electrons, may be appreciated by considering the following one-dimensional configuration. In the laboratory frame, an incident electromagnetic wave of frequency ω_0 and wave number k_0 propagates antiparallel to a beam of relativistic electrons with speed v_0. In general, both a backscattered wave (ω_{s_1}, k_{s_1}) and a forward scattered wave (ω_{s_2}, k_{s_2}) can result; for the time being, let us concentrate on the backscattered wave.

It is a simple matter to show that the wave which is backscattered off plasma oscillations has a frequency given by

$$\omega_{s_1}=\gamma_{0z}(1+\beta_0)[\gamma_{0z}(1+v_0/v_{\rm ph})\omega_0-\omega_p], \tag{1}$$

where $\beta_0=v_0/c$, $\gamma_{0z}=(1-\beta_0^2)^{-1/2}$, $v_{\rm ph}=\omega_0/k_0$, $\omega_{s_1}\approx ck_{s_1}$, and ω_p is the invariant plasma frequency. Clearly, from Eq. (1) for a highly relativistic beam with $\omega_p \ll \gamma_{0z}(1+v_0/v_{\rm ph})\omega_0$, the scattered frequency $\omega_{s_1}\approx\gamma_{0z}^2(1+\beta_0)(1+v_0/v_{\rm ph})\omega_0$ and the up conversion of frequency in the laboratory frame can be impressively large. It should also be noted that from conservation of wave action, the energy of the backscattered wave is also enhanced by the factor $\gamma_{0z}^2(1+\beta_0)(1+v_0/v_{\rm ph})$. In what follows, we carry out a detailed analysis of the relativistic scattering process.

We take for our model an infinite, homogeneous, neutralized, electron stream which is drifting parallel to an external magnetic field $B_0\hat{e}_z$. The electrons are illuminated by a supraluminous right-handed circularly polarized (RHCP) pump electric field. We choose to perform the analysis in a reference frame in which the pump's wave number along the z axis vanishes. For supraluminous waves, such a transformation is always possible.[15]

A subluminous RHCP pump could have been chosen,[16-19] since for a magnetized beam such a wave is a normal mode of the system. In this case a transformation to a reference frame in which the frequency of the pump vanishes would be possible. Choosing a luminous pump, on the other hands, leads to inconsistencies, since for a magnetized beam there are no waves traveling at phase velocity c, and hence the pump would not be supported by the system.

For the supraluminous pump wave of the present analysis and in the reference frame where $k_0=0$ (laboratory frame),

$$\vec{E}_0(t)=E_0(\cos\omega_0 t\,\hat{e}_x-\sin\omega_0 t\,\hat{e}_y), \tag{2}$$

where E_0 and ω_0 are constants. The electron orbits in the presence of $B_0\hat{e}_z$ and $\vec{E}_0(t)$ are given by

$$\vec{V}_0=V_{0s}(\sin\omega_0 t\,\hat{e}_x+\cos\omega_0 t\,\hat{e}_y)+v_0\hat{e}_z, \tag{3}$$

where

$$V_{0s}=\frac{qE_0}{\gamma_0 m_0(\omega_0-\Omega_0/\gamma_0)},$$

$$\Omega_0=\frac{qB_0}{m_0c},\quad \gamma_0=\left(1-\beta_0^2-\frac{V_{0s}^2}{c^2}\right)^{-1/2},$$

and m_0 is the electron's rest mass. In obtaining Eq. (3) it was assumed that the pump field had been turned on adiabatically and that the beam is cold. In what follows it will be assumed that $\beta_{0s}=V_{0s}/c$ is small. Since the infinite-wavelength pump field is assumed to be supported by the magnetized beam, the pump frequency ω_0 must satisfy the linear dispersion relation for the system, viz.,

$$\omega_0=\frac{[1+(1+4\omega_p^2\gamma_0^2/\Omega_0^2)^{1/2}]\Omega_0}{2\gamma_0}, \tag{4}$$

where the invariant plasma frequency $\omega_p=(4\pi q^2 n_0/\gamma_0 m_0)^{1/2}$, and n_0 is the beam density measured in the laboratory frame.

To obtain the dispersion characteristics of the scattered waves, we assume first that these waves propagate solely along the magnetic field, hence we neglect electrostatic cyclotron waves. Also, the scattered waves are assumed sufficiently weak as to be considered perturbations on the time-dependent equilibrium outlined in Eqs. (2), (3), and (4). The scattered electric field $\delta\vec{E}(z,t)$ satisfies the wave equation

$$\nabla^2\delta\vec{E}-\nabla(\nabla\cdot\delta\vec{E})-\frac{1}{c^2}\frac{\partial^2\delta\vec{E}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial\delta\vec{J}}{\partial t}, \tag{5}$$

where

$$\nabla=\frac{\partial}{\partial z}\hat{e}_z$$

and $\delta\vec{J}(z,t)=qn_0\delta\vec{v}(z,t)+q\delta n(z,t)\vec{V}_0$ is the perturbed nonlinear response current density which drives the scattered fields. In the expression for $\delta\vec{J}$, δn is the perturbed number density of the beam and δv is the perturbed velocity which obeys the linearized relativistic equation of motion

$$\frac{d\delta\vec{v}}{dt}=\frac{q}{\gamma_0 m_0}\left(\delta\vec{E}+\vec{V}_0\times\frac{\delta\vec{B}}{c}+\delta\vec{v}\times\frac{\vec{B}_0}{c}-\vec{V}_0\frac{\delta\vec{v}\cdot\vec{E}_0+\vec{V}_0\cdot\delta\vec{E}}{c^2}\right)-\beta_0\gamma_0^2\frac{d\vec{V}_0}{dt}\frac{\delta\vec{v}\cdot\hat{e}_z}{c}, \tag{6}$$

where terms of order β_{0s}^2 times a perturbed quantity have been neglected.

Defining $\delta E_{R,L}=\delta E_x\pm i\delta E_y$, $\delta v_{R,L}=\delta v_x\pm i\delta v_y$, and $V_{0\perp}^{\pm}=V_{0x}\pm iV_{0y}$ (where V_{0x} and V_{0y} are the components of $\vec{V}_0$), the wave equation (5) and orbit equation (6) take the form

$$\frac{\partial^2\delta E_{R,L}}{\partial t^2}+c^2k^2\delta E_{R,L}=-4\pi qn_0\frac{\partial\delta}{\partial t}v_{R,L}-ik\frac{\partial}{\partial t}(V_{0\perp}^{\pm}\delta E_z), \tag{7a}$$

$$\frac{\partial^2\delta E_z}{\partial t^2}=-4\pi qn_0\frac{\partial\delta v_z}{\partial t}-ikv_0\frac{\partial\delta E_z}{\partial t}, \tag{7b}$$

and

$$\frac{\partial\delta v_{R,L}}{\partial t}+i\frac{\xi_{R,L}}{c^2k^2}\delta v_{R,L}=\frac{q}{\gamma_0 m_0}\left(1-\frac{i\beta_0}{ck}\frac{\partial}{\partial t}\right)\delta E_{R,L}\pm\frac{i\beta_0}{4\pi qn_0}\gamma_0^2\omega_0\frac{V_{0\perp}^{\pm}}{c}\left(ikv_0+\frac{\partial}{\partial t}\right)\delta E_z, \tag{8a}$$

$$\frac{\partial\delta v_z}{\partial t}=\frac{ikv_0}{4\pi qn_0}\left(ikv_0+\frac{\partial}{\partial t}\right)\delta E_z+\frac{q}{\gamma_0 m_0}\left[\frac{\delta E_z}{\gamma_{0z}^2}-\frac{1}{2}\frac{V_{0\perp}^-}{c}\left(\beta_0-\frac{i}{ck}\frac{\partial}{\partial t}\right)\delta E_R-\frac{1}{2}\frac{V_{0\perp}^+}{c}\left(\beta_0-\frac{i}{ck}\frac{\partial}{\partial t}\right)\delta E_L\right]+\frac{i}{2}\frac{\phi_R\xi_R}{c^2k^2}\frac{V_{0\perp}^-}{c}\delta v_R+\frac{i}{2}\frac{\phi_L\xi_L}{c^2k^2}\frac{V_{0\perp}^+}{c}\delta v_L, \tag{8b}$$

where

$$\xi_{R,L} = c^2k^2(kv_0 \pm \Omega_0/\gamma_0 + \beta_0\omega_p^2/ck) \text{ and } \phi_{R,L} = c^2k^2[\omega_p^2/ck \mp \beta_0(\omega_0 - \Omega_0/\gamma_0)]/\xi_{R,L}.$$

In writing Eqs. (8), the relation $\delta v_z = -(ikv_0 + \partial/\partial t)\delta E_z/4\pi q n_0$, obtained from the continuity equation and Gauss's law, was used. The scattered quantities in Eqs. (7) and (8) were assumed to have a spatial dependence of the form e^{ikz}. Equations (7) and (8) can be combined to give coupled equations for the scattered RHCP and LHCP waves $\delta E_{R,L}$ and electrostatic wave δE_z:

$$\left(\frac{\partial^3}{\partial t^3} + i\mu_{R,L}\frac{\partial^2}{\partial t^2} + \nu\frac{\partial}{\partial t} + i\xi_{R,L}\right)\delta E_{R,L} = \pm\beta_{0s}\frac{\partial}{\partial t}\left[e^{\mp\omega_0 t}\left(\alpha_{R,L}\frac{\partial}{\partial t} + i\beta_{R,L}\right)\delta E_z\right], \tag{9a}$$

$$\left[\left(\frac{\partial}{\partial t} + iv_0k\right)^2 + \frac{\omega_p^2}{\gamma_{0z}^2}\right]\delta E_z = \frac{i}{2}\beta_{0s}e^{i\omega_0 t}\left(\phi_R\frac{\partial^2}{\partial t^2} + i\chi_R\frac{\partial}{\partial t} - \Psi_R\right)\delta E_R - \frac{i}{2}\beta_{0s}e^{-i\omega_0 t}\left(\phi_L\frac{\partial^2}{\partial t^2} + i\chi_L\frac{\partial}{\partial t} - \Psi_L\right)\delta E_L, \tag{9b}$$

where

$$\mu_{R,L} = kv_0 \pm \Omega_0/\gamma_0, \quad \nu = c^2k^2 + \omega_p^2,$$

$$\alpha_{R,L} = ck \mp \gamma_0^2\omega_0\beta_0,$$

$$\beta_{R,L} = ck(kv_0 \pm \Omega_0/\gamma_0 + \beta_0\omega_p^2/ck \mp \gamma_0^2\omega_0),$$

$$\chi_{R,L} = (\omega_p^2/ck)(1 - \beta_0\phi_{R,L}),$$

and

$$\Psi_{R,L} = (\omega_p^2\beta_0 - \nu\phi_{R,L}).$$

Since β_{0s} is small, the solutions of Eqs. (9a) and (9b) can be expressed as a product of the homogeneous solutions with a slowly varying amplitude, i.e.,

$$\delta E_{R,L}(k,t) = \sum_{m=1}^{3} a(m,t)_{R,L}\exp[-i\omega(m)_{R,L}t], \tag{10a}$$

$$\delta E_z(k,t) = \sum_{n=1}^{2} b(n,t)\exp[-i\omega(n)t], \tag{10b}$$

where

$$\frac{d\ln a(m,t)_{R,L}}{dt} \ll \omega(m)_{R,L} \quad\text{and}\quad \frac{d\ln b(n,t)}{dt} \ll \omega(n).$$

The frequencies $\omega(m)_{R,L}$ and $\omega(n)$ are solutions of the linear dispersion relation for the RHCP and LHCP waves, and electrostatic waves in the absence of the pump, $\beta_{0s}=0$, viz.,

$$[\omega^2(m)_{R,L} - c^2k^2][\omega(m)_{R,L} - v_0k \mp \Omega_0/\gamma_0] - [\omega(m)_{R,L} - v_0k]\omega_p^2 = 0, \tag{11a}$$

$$[\omega(n) - v_0k]^2 - \omega_p^2/\gamma_{0z}^2 = 0. \tag{11b}$$

Assuming a solution for the mode amplitudes $a(m,t)_{R,L}$ and $b(n,t)$ of the form $e^{\Gamma_{R,L}t}$ and keeping terms to lowest order in the small parameters $\Gamma_{R,L}/\omega(m)_{R,L}$, $\Gamma_{R,L}/\omega(n)$, and β_{0s}, we arrive at the following expression for the growth rates:

$$\Gamma_{R,L} = \frac{\beta_{0s}}{2}\left(\frac{\omega(m)_{R,L}[\omega(n)\alpha_{R,L} - \beta_{R,L}]}{\omega(n) - kv_0}\right)^{1/2} \times\left(\frac{\phi_{R,L}\omega^2(m)_{R,L} - \chi_{R,L}\omega(m)_{R,L} + \Psi_{R,L}}{3\omega^2(m)_{R,L} - 2\mu_{R,L}\omega(m)_{R,L} - \nu}\right)^{1/2}, \tag{12}$$

where $\omega(n) \pm \omega_0 - \omega(m)_{R,L} = 0$.

An explicit dispersion relation governing the scattered electromagnetic waves can also be obtained. To accomplish this the fields in Eqs. (9) are assumed to have the form

$$\delta\vec{G} = \sum_n \delta\vec{G}_n e^{-i\omega_n t},$$

where $n = 0, \pm 1, \pm 2, \ldots$, and $\delta\vec{G}_n$ and $\omega_n = \omega + n\omega_0$ represent the Fourier coefficient and frequency of the nth mode, respectively. Substituting the above expression for the fields into Eqs. (9) leads to the following dispersion relations, valid to order β_{0s}^2, for the RHCP and LHCP waves:

$$(\omega^3 - \mu_{R,L}\omega^2 - \nu\omega + \xi_{R,L})[(\omega \mp \omega_0 - v_0k)^2 - \omega_p/\gamma_{0z}^2] = -\tfrac{1}{2}\beta_{0s}^2\omega[\alpha_{R,L}(\omega \mp \omega_0) - \beta_{R,L}] \times(\phi_{R,L}\omega^2 - \chi_{R,L}\omega + \Psi_{R,L}). \tag{13}$$

A pictorial representation of the decay of the pump wave into supraluminous LHCP waves and plasma waves is given in Fig. 1 in the reference frame of the beam. We see that the pump wave, located at point P on the dispersion curve of the fast RHCP cyclotron wave, decays into LHCP fast waves S_1, S_2, and two corresponding plasma waves C_1 and C_2. The pump wave together with the two scattered LHCP waves are supraluminous. Transforming from the beam frame back to the laboratory will increase the frequency of the backscattered wave at S_1, while the frequency of the forward scattered wave S_2 is de-

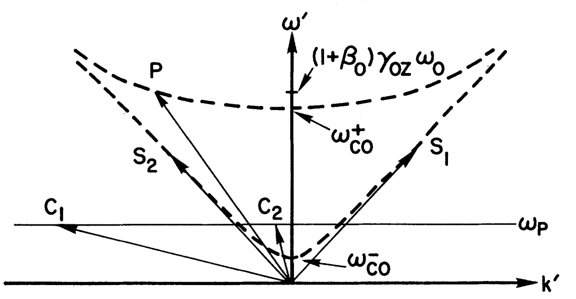

FIG. 1. Dispersion curves showing decay of pump wave into LHCP electromagnetic waves and plasma waves, $\omega_{c0}^{\pm} = (1 + 4\omega_p^2\gamma_0^2/\Omega_0^2)^{1/2} \pm 1$ (in reference frame of beam).

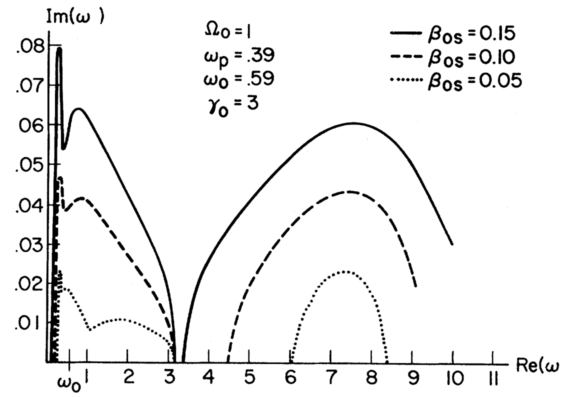

FIG. 2. Growth rate versus frequency for the scattered RHCP electromagnetic wave.

creased. Hence, in the laboratory frame, S_1 corresponds to anti-Stokes scattering, while S_2 represents Stokes scattering. A diagram similar to Fig. 1 can be drawn for scattering into the RHCP modes.

For the backscattered LHCP wave S_1, the beam temperature can be neglected if the thermal velocity along the z axis in the beam frame v_{th} is much less than $|\omega_p/k_p'|$. Transforming to the laboratory frame, this restriction becomes $\Delta\gamma_{0z}/\gamma_{0z} < \omega_p/\gamma_{0z}\omega_0$, where $\Delta\gamma_{0z}$ is the thermal spread in γ_{0z}.

An example of the growth-rate spectrum in the laboratory frame for the RHCP electromagnetic waves is shown in Fig. 2. The two peaks in the growth rate correspond to the wave S_1, which is scattered parallel to the beam, $\omega_{s_1} \approx (1+\beta_0)\gamma_0^2\omega_0$, and the wave S_2, which is scattered antiparallel to the beam, $\omega_{s_2} \approx \omega_0$. It is noteworthy that the growth rates in Fig. 2 for S_1 and S_2 are approximately equal. The growth-rate spectrum for the LHCP wave is simply the mirror image (about the $\omega = 0$ axis) of the RHCP waves. Note that, if a change of reference frames were made such that the pump were traveling antiparallel to the beam ($k_0 < 0$), the scattered RHCP wave would be supraluminous. Hence, by a simple change of inertial frames, both scattered electromagnetic waves can be made supraluminous.

The parameters chosen in Fig. 2 pertain to a regime presently realizable in the laboratory. For example, if one were to generate the pump wave with a klystron amplifier of the type used on the Stanford linear accelerator (peak power = 24 MW; $\omega_0/2\pi = 2.856$ GHz), then $\beta_{0s} \approx 0.1$ is achievable. The corresponding beam parameters [viz., $\Omega_0/2\pi = 4.8$ GHz ($B_0 = 1.7$ kG), $\omega_p/2\pi = 3.2$ GHz, $\gamma_0 = 3$] are compatible with modest examples of present-day intense relativistic electron-beam accelerators. For this choice of parameters, the beam can be considered cold if $\Delta\gamma_{0z}/\gamma_{0z} \ll 20\%$. The high-frequency scattered wave would be at a wavelength of 6 mm and the e-folding length of the scattering instability would be $(\Gamma/c)^{-1} \approx 20$ cm.

In a recent experiment using a relativistic electron beam with $\gamma \approx 5.3$ and $\omega_p/2\pi \approx 3.5$ GHz, submillimeter radiation (390–540 μm) having a total power of $\approx 10^5$ W was observed.[12] The submillimeter radiation varied with the uniform guiding magnetic field and was maximum at $B_0 \approx 16$ kG. By a mechanism described elsewhere,[15] a pump field $E_0 \approx 8 \times 10^4$ V/cm and frequency $\omega_0/2\pi \approx 14$ GHz was generated, making $\beta_{0s} \approx 0.15$ and $\omega_0/(\omega_0 - \Omega_0/\gamma_0) \approx 2.8$. Applying these parameters to the present theory, we find that the high-frequency scattered wave has a growth rate $\Gamma \approx 10^{10}$ sec^{-1} and wavelength $\lambda = 2\pi c/\omega \approx 440$ μm. Hence, the scattered submillimeter radiation e-folds in ~3.0 cm, whereas the experimental scattering region was ~30 cm. It should also be noted that in the experiment the output window was ~50% reflecting to submillimeter radiation and hence multiple reflections were possible during the 50-nsec lifetime of the beam. Hence the present paper describes a mechanism which is a plausible explanation of the observed submillimeter radiation.

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