Radiation characteristics of ion waves in a weakly magnetized plasma

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The phase velocity and wave amplitude of ion waves launched by a small probe are investigated experimentally and theoretically for all directions in a weakly magnetized plasma. Ion waves in a weakly magnetized plasma are found to propagate omnidirectionally (excluding the effect of plasma flows) with respect to the phase velocity and to be launched bidirectionally along the direction of the magnetic field with respect to the amplitude. The excitation mechanism is also investigated in a fluid model in comparison with the results of a nonmagnetized plasma.

I. INTRODUCTION

Since ion waves were first observed experimentally,[1-4] many investigations have been performed both experimentally and theoretically. Gould reported the theory of the excitation of ion waves in detail.[5] The excitation of ion waves in ion beams by oscillating dipole charges has been also investigated.[6] Recently, many investigations on the excitations have been reported, mainly by Alexeff and co-workers in relation to the excitation of a pseudowave.[7] For oblique propagation, Hirose *et al.* investigated dispersions of electrostatic ion waves near the ion cyclotron frequency.[8] Shen *et al.* observed the radiation characteristics of an ion acoustic wave from a circular disk mesh.[9] Ohnuma *et al.* were able to control the radiation characteristics of ion waves with double meshes by using the reflection of ion waves near an electron-rich sheath boundary.[10] Furthermore, the excitation mechanism of ion waves by ion sheet beams has been investigated experimentally and theoretically.[11] In this paper, radiation characteristics of ion waves from a small probe are investigated experimentally and theoretically for a weakly magnetized plasma.

II. EXPERIMENT

A. Experimental setup

The experiments were performed in a vacuum chamber 32 cm in diameter and 160 cm in length. The schematic experimental setup is shown in Fig. 1. Argon gas was used at a pressure $P \approx 0.6 \times 10^{-3}$ Torr. Four oxide-coated cathodes were set at different positions near the wall of the chamber, as shown in the figure. The wall of the chamber was used as an anode. The typical plasma parameter values were $N_0 \approx 10^9$ cm^{-3} for the plasma density, $T_e = 1.5 \sim 2.0$ eV for the electron temperature, and $T_i \approx 0.3$ eV for the ion temperature. The plasma density and the electron temperature were obtained with a Langumuir probe. The ion temperature was obtained with a Faraday cup. As a property of the plasma production, a flow of plasma on the order of $V_{0r} = (3-4) \times 10^4$ cm/sec from the center of the chamber to the wall and $V_{0a} \approx 2 \times 10^4$ cm/sec to the center along the axis of the chamber was found to exist in the central region of the chamber (experimental regions). The plasma flows were obtained by using the Doppler effect for the ion acoustic wave, i.e., by measuring the phase velocity of the ion acoustic wave along and against the plasma flows.

The excitation of ion waves was performed with a small probe 1 mm in diameter and 4 mm in length. The signals were detected with a rectangular mesh 5 mm wide and 1 cm long by using an interferometer or a sampling method. The detector was movable radially and azimuthally. The resolution angle due to the finite width of the detector was $\theta \approx 4°$.

B. Experimental results

When periodic rectangular pulses are applied to the exciter, typical signals are detected at a fixed distance from the exciter, as shown in Figs. 2(a) and 2(b). The directions of $\theta = 0°$ and $\theta = 90°$ indicate those parallel and perpendicular to an applied magnetic field, respectively. When the magnetic field is not applied [the case of Fig. 2(a)], the signals are shown to propagate uniformly in all

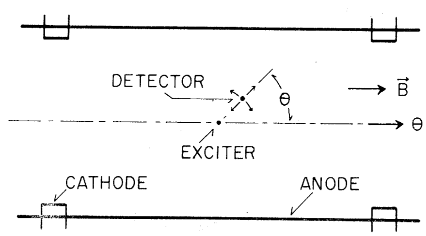

FIG. 1. Schematic experimental setup. The chamber is 32 cm in diameter and 160 cm in length.

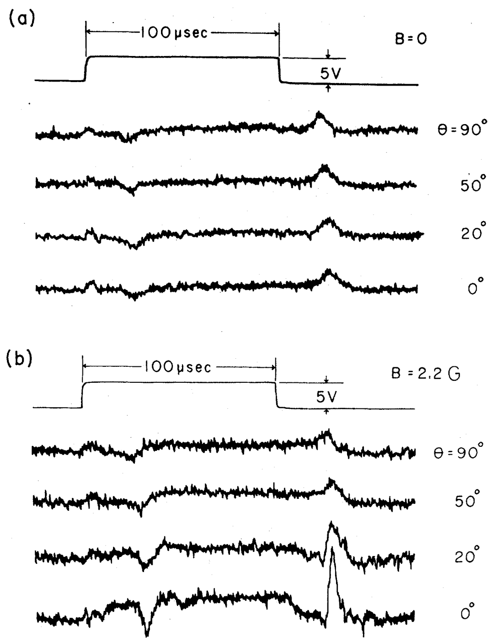

FIG. 2. (a) Typical ion wave signals detected by a sampling method for several directions are indicated at a fixed distance ($d = 4$ cm) from the exciter probe. (b) Typical ion wave signals for several propagation angles in a weakly magnetized plasma ($d = 4$ cm).

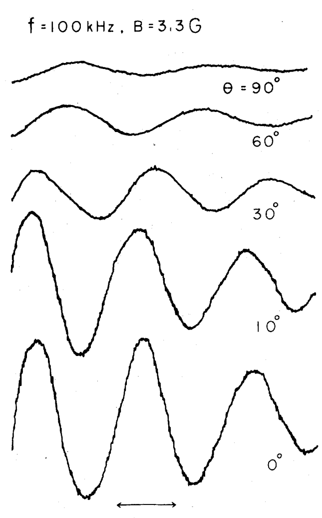

FIG. 3. Typical ion wave signals detected by interferometer method when a continuous sinusoidal signal with 1 V peak to peak is applied to a probe in weakly magnetized plasma. The directions $\theta = 0°$ and $90°$ are, respectively, parallel and perpendicular to the magnetic field.

directions with respect to both velocity and amplitude, although strictly speaking the velocity for $\theta = 90°$ is faster than that for $\theta = 0°$ as a result of the effects of plasma flows. Figure 2(b) indicates the signals when a weak magnetic field is applied. The signal for $\theta = 0°$ with the magnetic field on is greatly enhanced compared to that with the magnetic field off, although the signals at $\theta = 90°$ propagate similarly for both cases. Typical signals in a weak magnetic field, obtained by a lock-in amplifier, are shown for several fixed directions in Fig. 3. The signal for $\theta = 0°$ is clearly much greater than that for $\theta = 90°$. The signals are consistent with those from pulse excitations in a magnetic field, as shown in Fig. 2(b).

Figure 4 indicates the phase velocity and wave

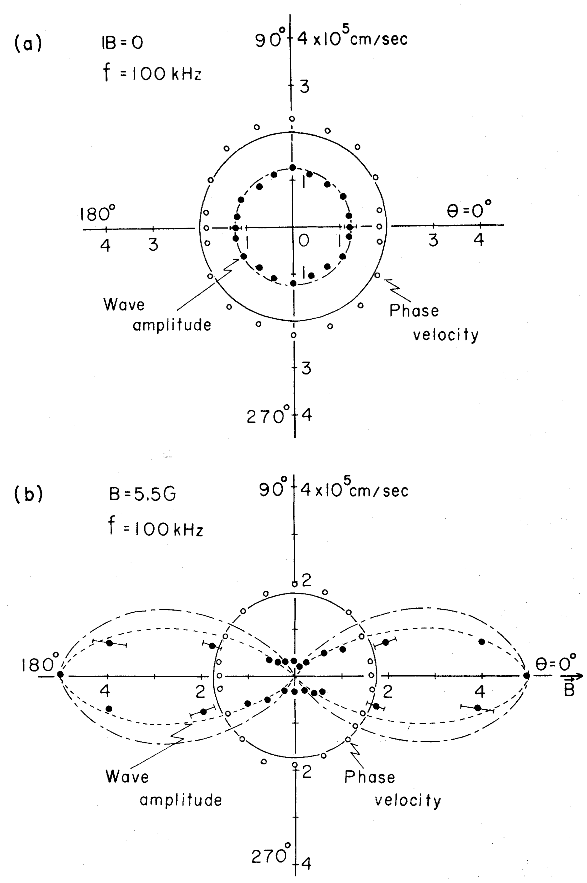

FIG. 4. Typical radiation characteristics of ion waves from a small probe. The open and solid circles indicate the experimental phase velocity and wave amplitude, respectively. The solid and dot-dashed (or dotted) lines are the theoretical phase velocity and wave amplitude excluding the effect of plasma flows, respectively. $f_{pe} \approx 1.8 \times 10^9$ sec^{-1}, $f_{pi} \approx 1.1 \times 10^6$ sec^{-1}. (a) In an unmagnetized plasma; (b) in a weakly magnetized plasma; $f_{ce} = 15.4 \times 10^6$ Hz, $f_{ci} = 2.1 \times 10^2$ Hz. For the amplitude of (a), three times the experimental values are plotted for comparison with the amplitudes of (b).

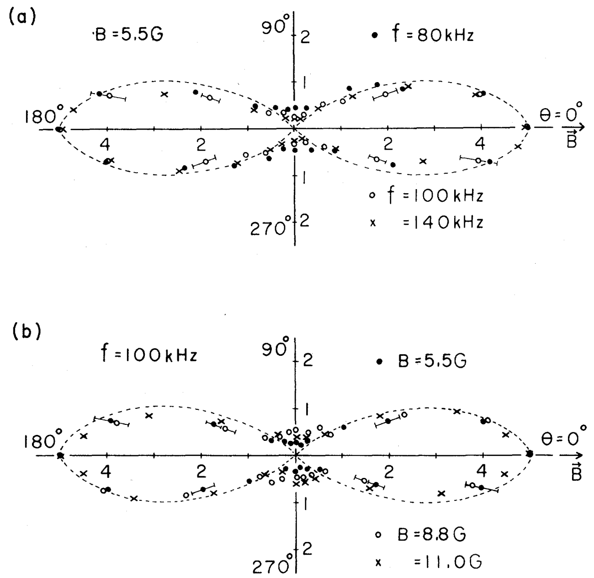

FIG. 5. θ dependence of an ion wave amplitude launched by a small probe in a weakly magnetized plasma for several frequencies at a fixed strength of the magnetic field (a), and for several strengths of the magnetic field at a fixed frequency (b). Those amplitudes are normalized at $\theta=0°$.

amplitude for all directions at a fixed frequency and a fixed magnetic field. The plotted wave amplitudes are those extrapolated at the exciter position with a consideration of the wave damping. The case with the magnetic field off is also indicated. When the magnetic field is not applied, ion waves propagate roughly omnidirectionally in phase velocity with equal amplitudes. When a weak magnetic field is applied, the ion waves also propagate roughly omnidirectionally in the phase velocity. However, the dependence of the wave amplitude of the excited ion wave on θ is much different from that with the magnetic field off.

Figure 5 indicates that the θ dependence of the amplitude of the launched ion waves in a weakly magnetic field is roughly independent of the strength of the magnetic field and the frequency of the ion acoustic wave. In Fig. 5, the strength of the magnetic field is chosen as the electron Larmor radius, which is less than 1 mm. Under present experimental conditions, the frequency of ion waves ($f \sim 10^5$ Hz) is much higher than the ion cyclotron frequency ($f_{ci} \sim 10^2$ Hz) and much lower than the electron cyclotron frequency ($f_{ce} \sim 10^6$ Hz) and the ion plasma frequency ($f_{pi} \sim 10^6$ Hz). Furthermore, the results of Figs. 4 and 5 are found to be independent of the exciting voltage V_{ex} ($0.1\,\mathrm{V} \lesssim V_{ex} \lesssim 5\,\mathrm{V}$ peak to peak). We took great care in confirming that the signals detected in the experiments are ion acoustic waves and not pseudowaves.[7]

III. THEORY

In this section we treat the radiation of ion waves in a weakly magnetized plasma in a three-dimensional problem. Considering the experimental conditions, we use the fundamental equations in a fluid model as follows:

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j \vec{v}_j) = 0, \quad j = e, i \tag{1}$$

$$\frac{\partial \vec{v}_i}{\partial t} = \frac{e}{m_i}\vec{E}, \tag{2}$$

$$\frac{\partial \vec{v}_e}{\partial t} = -\frac{e}{m_e}\left(\vec{E} + \frac{1}{c}\vec{v}_e \times \vec{B}\right) - C_e^2 \frac{\nabla n_e}{n_e} - \nu_{en}\vec{v}_e, \tag{3}$$

$$\nabla\cdot\vec{E} = 4\pi e(n_i - n_e - n_s), \quad \vec{E} = -\nabla\phi \tag{4}$$

where n_j, $\vec{v}_j$, $\vec{E}$, $\vec{B}$, and ϕ indicate the density, the velocity of jth component, the electric field, the magnetic field, and the potential, respectively. The symbols C_e^2 ($\equiv \kappa T_e/m_e$) and ν_{en} denote the electron thermal velocity and the electron-neutral collision frequency, respectively. The source term of the plasma waves is included as n_s in the Poisson equation. A "weak" magnetic field means the approximation that the magnetic field affects only the motions of electrons in the experimental regions. The approximation is fairly good for the present experimental conditions because the electron and ion Larmor radii, for example, at $B=5$ G are 0.7 mm and 200 cm, respectively, and the wavelength of ion waves is nearly 1–2 cm.

Assuming the temporal term is proportional to $e^{i\omega t}$ in Eqs. (1)–(4), we can obtain the following equation after some lengthy calculations:

$$\left[1 - C_e^2\left(\frac{(1+\nu_{en}/i\omega)(\nabla_x^2+\nabla_y^2)}{(\nu_{en}+i\omega)^2+\omega_{ce}^2} + \frac{\nabla_z^2}{i\omega(\nu_{en}+i\omega)}\right)\right]\left[\left(1-\frac{\omega_{pi}^2}{\omega^2}\right)\nabla^2\phi + 4\pi e n_s\right] + \omega_{pe}^2\left(\frac{(1+\nu_{en}/i\omega)(\nabla_x^2+\nabla_y^2)}{(\nu_{en}+i\omega)^2+\omega_{ce}^2} + \frac{\nabla_z^2}{i\omega(\nu_{en}+i\omega)}\right)\phi = 0, \tag{5}$$

where ω_{pe}, ω_{pi}, and ω_{ce} indicate the electron plasma, ion plasma, and electron cyclotron frequencies, respectively. The symbols ∇_x, ∇_y, and ∇_z denote partial derivatives with respect to x, y, and z, respectively. By imposing on Eq. (5) the approximations met by the experimental conditions, viz.,

$$\nu_{en}/\omega \gg 1, \quad k^2 C_e^2 \gg \omega\nu_{en}, \quad \omega_{pi}^2 \gg \omega^2, \tag{6}$$

we can obtain the following equation for the potential:

$$\left(\frac{\nabla_x^2+\nabla_y^2}{1+(\omega_{ce}/\nu_{en})^2}+\nabla_z^2\right)\left(\nabla^2\phi+k^2\phi-4\pi e n_s\frac{\omega^2}{\omega_{pi}^2}\right)=0\,, \tag{7}$$

$$k^2\equiv\omega^2/C_p^2,\quad C_p^2\equiv kT_e/m_i\,. \tag{8}$$

C_p is the ion acoustic velocity. When n_s is a point source, the solution of Eq. (7) can be obtained as

$$\phi=-en_s\frac{\omega^2}{\omega_{pi}^2}\frac{e^{-ikR}}{R}\,, \tag{9}$$

where R is the distance from an exciter to a detector. The assumption that n_s is a point source is a good one since the dimensions of the exciter are much smaller than a wavelength. Equation (9) indicates the propagation of a spherical ion wave from a point source. The phase velocity is given as the ion acoustic velocity $\omega/k=C_p$ and is independent of θ. Furthermore, the wave amplitude is proportional to n_s and is independent of θ. It must be noted that Eq. (9) is the same solution for both cases, with magnetic field on or off, under the present experimental conditions.

IV. DISCUSSION

First, the experimental results will be compared with the theoretical results of Eq. (9). When a magnetic field is not applied, ion waves propagate roughly omnidirectionally in phase velocity and are excited omnidirectionally in amplitude, as shown in Fig. 4(a). The theoretical phase velocity, in which the electron temperature was measured experimentally, is also plotted by solid line in Fig. 4(a). Except for the Doppler-shift effect of the plasma flows, the theoretical results are in fair agreement with the experimental results. The wave amplitude is nearly in accord with the theoretical results.

When a weak magnetic field is applied, the phase velocity of ion waves is roughly omnidirectional as shown in Fig. 4(b), which is fairly similar to the case without the applied magnetic field. The solid line is also the theoretical phase velocity, and the difference between the experimental and theoretical phase velocities is due to the Doppler-shift effect of plasma flows. It is due to the change of the electron temperature that the phase velocity in the weak magnetic field is a little smaller than it is with the magnetic field off. Concerning the wave amplitude, the theoretical results due to a point source indicate omnidirectional excitation, as shown in Eq. (9). However, the experimental results show that the waves are greatly enhanced in the magnetic field direction. That is, the excitation mechanism due to a point-charge source cannot explain the θ dependence of the wave amplitude for the experimental results in a magnetized plasma. Hence, we now consider another excitation mechanism contributing to n_s in Eq. (7). For the present experimental conditions (see Fig. 5), the electron Larmor radius is smaller than 1 mm and the exciter is 1 mm in diameter. The electrons are assumed to move roughly along the B axis. That is, the application of the voltage to the exciting probe may result in current flows along the magnetic field. We hypothesize, as the currents flow from both sides of the probe, "quater-dipole currents" along the magnetic field as one model resulting from an application of voltage to the probe. By "quater" we mean that double current dipoles are assumed for both sides of the probe. The details of double current dipoles are explained as follows. If one applies, for example, a positive potential to a wire probe, an excess of electrons gathers near the probe along the magnetic field. The gathering of the excess charge near the probe may result in the existence of a place along the magnetic field which is lacking in charge. Half of the excess charge and the external charges may form one current dipole. Furthermore, another dipole current may be formed between the position of the rest of the excess charge and that lacking in charge. That is, an application of a potential to a wire probe in a magnetic field may form two current dipoles for one side. As is known from the above-mentioned explanation, the two current dipoles are directed opposite to each other. It may thus be said that there exists a dipole of the dipole current for one side. In all, there may exist two oppositely directed dipoles of the dipole currents.[12]

One can obtain an identical relation to Eq. (7) even for the quater dipole currents, in which n_s has, however, a spatial variation. That is, for that case, the potential is given as a spatial integral of n_s near the currents, as follows:

$$\phi(\vec{r})=-e\frac{\omega^2}{\omega_{pi}^2}\int_{V'}n_s(\vec{r}\,')\frac{e^{-i\vec{k}\cdot(\vec{r}-\vec{r}\,')}}{|\vec{r}-\vec{r}\,'|}\,dV'\,, \tag{10}$$

where the primed coordinate refers to the sources. The integral is performed in a region of quater dipole currents. Noting that currents correspond to a time derivative of charges and that the wave amplitude due to a unit dipole is proportional to $ikl\cos\theta$ (l is the dipole length) for a far-field approximation, one can obtain the following approximate solution for quater dipoles by assuming $kl\ll 1$ as follows:

$$\phi\propto\frac{\omega^2}{\omega_{pi}^2}(kl\,\cos\theta)^4\frac{e^{-ikR}}{R}\,. \tag{11}$$

Equation (11) indicates that the ion wave propagates omnidirectionally in the phase velocity, which agrees with experiment. The θ dependence of the wave amplitude is proportional to $\cos^4\theta$, which is plotted as the dot-dashed line in Fig. 4. The effect on the damping due to the quater dipole currents is shown to be independent of angle. When we assume quater dipole currents for both sides of the probe, the wave amplitude is proportional to $(\cos^4\theta)^2$, which is also plotted for reference as a dotted line in Fig. 4. It must be noted that those curves are more appropriate for explaining the experimental results than the results of Eq. (9). Furthermore, the fact that the θ dependence of the wave amplitude is independent of the frequency and the strength of the magnetic field agrees with the experimental results in Fig. 5. In Fig. 5 the curve of $(\cos^4\theta)^2$ is also plotted for reference as a dotted line.

Next, we compare our experimental results with those of Hirose *et al.*[8] Hirose *et al.* observed that the phase velocity is dependent on angle near the ion cyclotron frequency ω_{ci}. In our experimental results, the phase velocity is independent of angle, which is not in contradiction with the observation of Hirose *et al.*, because $\omega \gg \omega_{ci}$ is satisfied in our experiments. Under the condition $\omega \gg \omega_{ci}$, the phase velocity is independent of angle. Furthermore, Hirose *et al.* observed the damping due to electron Landau damping. Our observations in Figs. 4 and 5 are different from the effect due to Landau damping for the following reasons. The wave amplitudes in Figs. 4 and 5 indicate those extrapolated at the exciter position with wave damping taken into account. The wave amplitudes in Figs. 4 and 5 show the intensity near the exciter, excluding the effect of the damping. That is, the bidirectional patterns in Figs. 4 and 5 occur near the exciter. This fact is a fundamental point for comparing the experimental results with Eq. (11).

V. CONCLUSIONS

Ion waves with no magnetic field are launched and propagate omnidirectionally (excluding the effect of plasma flows) in the phase velocity and the amplitude, which are explained by an oscillating point-charge excitation. Ion waves in a weakly magnetized plasma are found to propagate omnidirectionally with uniform phase velocity and are launched along the magnetic field with a greatly enchanced amplitude. The results can be explained more appropriately by the excitation mechanism due to oscillating-current flows to the probe along the magnetic field than by that due to oscillating point charges.

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