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Parametric interaction of an arbitrary incident signal

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We have considered the process of frequency conversion when the signal mode has an arbitrary field distribution. The set of coupled partial differential equations is solved employing the angular spectrum representation for the signal mode as well as for the idler mode. Explicit expressions for the signal and the idler field are obtained in the far-field approximation. An expression for the signal mode intensity is obtained which takes into account the effect of partial coherence as well as the effect of the bounded beam aperture. As an illustration we have considered the case of a Gaussian beam in detail.

I. INTRODUCTION

With the availability of an intense light source such as a laser, it is now possible to observe several nonlinear effects. The nonlinearity of the medium allows coupling among different modes, and the energy exchange takes place from one mode to another giving rise to various nonlinear phenomena of physical interest such as parametric amplification, frequency conversion, and others.[1-3] Assuming that only three modes experience substantial interaction, these parametric processes are mathematically described by a set of three nonlinear coupled partial differential equations. These equations were first solved by Armstrong *et al.*[4] under the assumption that the three interacting waves are polarized plane waves of infinite extent. In that case, each of the partial differential equations can be exactly solved. However, in the plane-wave theory, neither the effect of the finite beam aperture nor the effect of partial coherence can be taken into account. Some numerical investigations are also carried out in the case where the field distribution across the incident beam is Gaussian.[5-8]

In the present investigation, we consider the general case when the incident beam has an arbitrary field distribution with arbitrary state of spatial coherence. In order to simplify the problem considerably, we make the usual parametric approximation; i.e., we assume that the pump field is so intense that the pump intensity does not change during interaction. Under this approximation, the three nonlinear coupled partial differential equations reduce to a set of two linear coupled partial differential equations. This set of equations has been obtained in Sec. II for a particular process of up-frequency conversion. These equations are solved in Sec. III, employing the technique of the angular-spectrum representation of the wave field.[9-12] It is assumed that initially only the signal and the pump modes are present and the idler field is generated inside the medium during interaction. We find that the signal field, as well as the idler field, can formally be expressed in terms of the incident field. In order to obtain explicit expressions for the electric fields, we evaluate the integrals in Sec. IV under the far-field approximation. We find that the signal field, as well as the idler field inside the medium, is a superposition of two angular spectra with different angular variables. In Sec. V we have used these expressions to obtain the intensity distribution of the signal mode. In Sec. VI we apply our result to a particular case where the incident-signal field has a Gaussian distribution. This case is of particular interest as the light from a laser approximately satisfies this condition (cf. Ref. 8). We have obtained the intensity expressions for the signal and the idler modes. These results are used to obtain an expression for the conversion efficiency.

We have considered in detail only the process of frequency conversion. However, it may be pointed out that the method is quite general and may be used to study other parametric processes.

II. BASIC SET OF EQUATIONS

We consider a crystalline medium filling the space $z>0$. The pump field and the signal field are incident on the plane $z=0$ from the left-hand side, and the field distribution for both the beams are known on this plane. At any point $\vec{r}$ in the medium, the frequency spectrum of the electric field $\vec{E}(\vec{r},\omega)$ and that of the nonlinear part of the polarization $\vec{P}_{NL}(\vec{r},\omega)$ satisfy the equation[2,3]

$$[\nabla^2+\epsilon(\omega)\omega^2/c^2]\vec{E}(\vec{r},\omega)=-(4\pi\omega^2/c^2)\vec{P}_{NL}(\vec{r},\omega)\,, \tag{2.1}$$

$$\vec{P}_{NL}(\vec{r},\omega)=\int \underline{\chi}^{(2)}(-\omega,\omega',\omega-\omega'):\vec{E}(\vec{r},\omega')\times\vec{E}(\vec{r},\omega-\omega')\,d\omega'\,, \tag{2.2}$$

where $\underline{\chi}^{(2)}$ is the second-order susceptibility tensor, and we have considered only the lowest-order contribution to the nonlinear polarization.

For the sake of simplicity, we assume that the pump and the signal modes incident at the plane $z=0$ are monochromatic, and at any point $\vec{r}$ in the medium we confine our attention to fields at three frequencies ω_1, ω_2, and ω_3 such that

$$\omega_3=\omega_1+\omega_2\,. \tag{2.3}$$

Let the modes at frequencies ω_1, ω_2, ω_3 be linearly polarized along unit vectors $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ so that we may write

$$\vec{E}(\vec{r},\omega_\lambda)=\vec{e}_\lambda E_\lambda(\vec{r}) \quad (\lambda=1,2,3;\ \text{no summation})\,. \tag{2.4}$$

From (2.1)–(2.4), we obtain the following set of coupled nonlinear equations:

$$(\nabla^2+k_1^2)E_1(\vec{r})=-(4\pi d k_1^2/\epsilon_1)E_3(\vec{r})E_2^*(\vec{r})\,, \tag{2.5}$$

$$(\nabla^2+k_2^2)E_2(\vec{r})=-(4\pi d k_2^2/\epsilon_2)E_3(\vec{r})E_1^*(\vec{r})\,, \tag{2.6}$$

$$(\nabla^2+k_3^2)E_3(\vec{r})=-(4\pi d k_3^2/\epsilon_3)E_1(\vec{r})E_2(\vec{r})\,, \tag{2.7}$$

where

$$\begin{aligned} d&=\vec{e}_1\cdot\underline{\chi}^{(2)}(-\omega_1,\omega_3,-\omega_2):\vec{e}_2\vec{e}_3\\ &=\vec{e}_2\cdot\underline{\chi}^{(2)}(-\omega_2,\omega_3,-\omega_1):\vec{e}_1\vec{e}_3\\ &=\vec{e}_3\cdot\underline{\chi}^{(2)}(-\omega_3,\omega_1,\omega_2):\vec{e}_1\vec{e}_2\,, \end{aligned} \tag{2.8}$$

and

$$k_\lambda=(\epsilon_\lambda)^{1/2}\omega_\lambda/c\,,\quad \epsilon_\lambda\equiv\epsilon(\omega_\lambda)\,. \tag{2.9}$$

The different expressions used in (2.8) are all equal on account of the symmetry properties of the susceptibility tensor $\underline{\chi}^{(2)}$. In writing Eqs. (2.5)–(2.7), it has been assumed that the interaction can be treated in terms of an effective nonlinear coefficient d (cf. Appendix 3 of Ref. 8).

Various parametric processes can be described by Eqs. (2.5)–(2.7) by suitably identifying the signal and the pump mode. For instance, if we identify the fields at ω_1 and ω_2 as the pump and the signal modes, respectively, we will be dealing with the up-frequency conversion. The special case $\omega_1=\omega_2$ corresponds to second-harmonic generation. Similarly, if we identify ω_1 as the signal frequency and ω_3 as the pump frequency, we obtain the case of parametric amplification. In what follows, we consider the process of frequency conversion and accordingly write

$$E_1(\vec{r})=E_p(\vec{r})\,,\quad E_2(\vec{r})=E_s(\vec{r})\,,\quad E_3(\vec{r})=E_i(\vec{r})\,. \tag{2.10}$$

To simplify our calculations, let us assume that the pump field is a plane wave propagating along the z axis. Further, if the pump intensity is large and remains constant throughout the interaction (parametric approximation), we may write

$$E_p(\vec{r})=A_p\exp(ik_pz)\,, \tag{2.11}$$

where A_p is a constant. From Eqs. (2.5)–(2.7) and (2.11), we find that the signal and the idler fields satisfy the coupled linear equations

$$(\nabla^2+k_s^2)E_s(\vec{r})=-4\pi d(k_s^2/\epsilon_s)A_p^*\exp(-ik_pz)E_i(\vec{r})\,, \tag{2.12}$$

$$(\nabla^2+k_i^2)E_i(\vec{r})=-4\pi d(k_i^2/\epsilon_i)A_p\exp(ik_pz)E_s(\vec{r})\,. \tag{2.13}$$

These equations are solved in Sec. III under the boundary conditions that E_s is given on the plane $z=0$ and E_i is assumed to be zero on this plane.

III. ANGULAR SPECTRUM REPRESENTATION

In order to solve Eq. (2.12) and (2.13), we first express the signal and the idler fields in angular-spectrum representations,[9]

$$E_s(\vec{r})=\int A_s(p,q;z)e^{i(px+qy)}dp\,dq\,, \tag{3.1}$$

$$E_i(\vec{r})=\int A_i(p,q;z)e^{i(px+qy)}dp\,dq\,. \tag{3.2}$$

From Eqs. (2.12), (2.13), (3.1), and (3.2), we obtain the following set of equations satisfied by the angular amplitudes A_s and A_i:

$$\left(\frac{\partial^2}{\partial z^2}+m_s^2\right)A_s(p,q;z)=-4\pi d\left(\frac{k_s^2}{\epsilon_s}\right)A_p^*\exp(-ik_pz)\times A_i(p,q;z)\,, \tag{3.3}$$

$$\left(\frac{\partial^2}{\partial z^2}+m_i^2\right)A_i(p,q;z)=-4\pi d\left(\frac{k_i^2}{\epsilon_i}\right)A_p\exp(ik_pz)\times A_s(p,q;z)\,, \tag{3.4}$$

where

$$m_\lambda=+[k_\lambda^2-(p^2+q^2)]^{1/2}\,,\quad \text{if } p^2+q^2\le k_\lambda^2 \tag{3.5a}$$

$$=i[p^2+q^2-k_\lambda^2]^{1/2}\,,\quad \text{if } p^2+q^2>k_\lambda^2\,. \tag{3.5b}$$

In the absence of the coupling ($d=0$), each of the Eqs. (3.3) and (3.4) becomes homogeneous and has the solution

$$A_\lambda(p,q;z)=B_\lambda(p,q)e^{im_\lambda z},\quad (\lambda=s,i)\,. \tag{3.6}$$

If we substitute (3.6) in (3.1) or (3.2), we find that $E_\lambda(\vec{r})$ is a superposition of two types of waves: (i) for values of m_λ given by (3.5a), it consists of homogeneous waves propagating along the direction with direction cosines proportional to (p,q,m_λ); (ii) for values of m_λ given by (3.5b), it consists of evanescent waves propagating in the xy plane and attenuating exponentially with increasing z.

In order to solve the coupled set of equations (3.3) and (3.4), we write A_λ in the following form [cf. Eq. (3.6)]:

$$A_\lambda(p,q;z)=B_\lambda(p,q;z)\exp(im_\lambda z), \quad (\lambda=s,i), \tag{3.7}$$

where B_λ is a slowly varying function of z. On neglecting the second derivative of B_λ with respect to z, we find from Eqs. (3.3), (3.4), and (3.7) that

$$\frac{\partial}{\partial z}B_s(p,q;z)=2\pi id(k_s^2/\epsilon_s m_s)A_p^*\exp(-i\Delta kz)\times B_i(p,q;z), \tag{3.8}$$

$$\frac{\partial}{\partial z}B_i(p,q;z)=2\pi id(k_i^2/\epsilon_i m_i)A_p\exp(i\Delta kz)B_s(p,q;z). \tag{3.9}$$

Here we have set

$$\Delta k=k_p-m_s-m_i. \tag{3.10}$$

As we have assumed that initially at the plane $z=0$, only the pump and signal modes are present, we wish to solve Eqs. (3.8) and (3.9) under the boundary conditions that $B_s(p,q;0)$ is known and $B_i(p,q;0)=0$. This is readily done by employing the normal modes. We find that

$$B_s(p,q;z)=B_s(p,q;0)\left(\cos gz+\frac{i\Delta k}{2g}\sin gz\right)\times\exp(-i\Delta kz/2), \tag{3.11}$$

$$B_i(p,q;z)=B_s(p,q;0)\left(\frac{2\pi ik_i^2dA_p}{\epsilon_i m_i g}\right)\sin gz\times\exp(i\Delta kz/2), \tag{3.12}$$

where

$$g=\left(\frac{2\pi k_sk_id|A_p|^2}{\epsilon_s\epsilon_i m_s m_i}+\frac{(\Delta k)^2}{4}\right)^{1/2}. \tag{3.13}$$

Finally, from Eqs. (3.1), (3.2), (3.7), (3.11), and (3.12), we obtain the following expressions for the

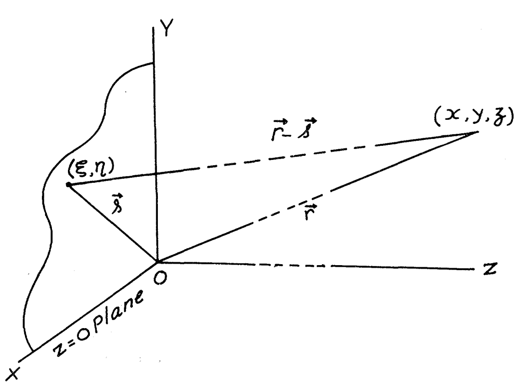

FIG. 1. Illustrating the coordinates of the source and the observation plane.

electric fields of the signal and the idler modes at any point in the medium:

$$E_s(\vec{r})=\int dp\,dq\,B_s(p,q;0)\left(\cos gz+\frac{i\Delta k}{2g}\sin gz\right)\times\exp[i(px+qy+m_sz-\tfrac{1}{2}\Delta kz)], \tag{3.14}$$

$$E_i(\vec{r})=2\pi idA_p\frac{k_i^2}{\epsilon_i}\int dp\,dq\,B_s(p,q;0)\frac{\sin gz}{m_ig}\times\exp[i(px+qy+m_iz+\tfrac{1}{2}\Delta kz)]. \tag{3.15}$$

$B_s(p,q;0)$ is obtained in terms of the boundary field $E_s(\xi,\eta)$ [where (ξ,η) denotes a point on the plane $z=0$ (cf. Fig. 1)] by setting $z=0$ in (3.1) and taking the inverse Fourier transform:

$$B_s(p,q;0)=\frac{1}{4\pi^2}\int E_s(\xi,\eta)\exp[-i(p\xi+q\eta)]\,d\xi\,d\eta. \tag{3.16}$$

IV. FAR-FIELD APPROXIMATION

Equations (3.14) and (3.15) formally represent the general solutions for the electric fields. The integrals over p and q in these equations may be carried out in the far-field approximation $k_\lambda z\gg 0$ ($\lambda=s$ or i). If it is assumed that the source and the observation planes are parallel to each other and that their linear dimensions are small compared to the asymptotic parameter $k_\lambda z$, one may readily verify that the contributions to the integrals in (3.14) and (3.15) will be significant for small values of p and q only. We may therefore write

$$m_\lambda\simeq k_\lambda[1-\tfrac{1}{2}(p^2+q^2)/k_\lambda^2], \tag{4.1}$$

and from (3.10), we may write

$$\Delta k\simeq(k_p+k_s-k_i)+\tfrac{1}{2}(k_i^{-1}-k_s^{-1})(p^2+q^2)\simeq\tfrac{1}{2}(k_i^{-1}-k_s^{-1})(p^2+q^2). \tag{4.2}$$

Keeping in view of Eqs. (2.3) and (2.9), we have here assumed that the collinear phase matching condition

$$k_p+k_s-k_i\simeq 0 \tag{4.3}$$

is satisfied. From (3.13) and (4.1), we may then also write

$$g\simeq\mu\left[1+\frac{1}{4}\left(\frac{1}{k_i^2}+\frac{1}{k_s^2}\right)(p^2+q^2)\right], \tag{4.4}$$

where μ is given by

$$\mu=2\pi d|A_p|\left(\frac{k_sk_i}{\epsilon_s\epsilon_i}\right)^{1/2}. \tag{4.5}$$

We now substitute (4.1), (4.2), and (4.4) in (3.14) and (3.15) and use the method of stationary phase to obtain the asymptotic values for large z. It is to be noted that we may set $\Delta k=0$ and $g=\mu$ everywhere except in the trigonometric- or exponential-phase terms. After straightforward evaluations, we find that

$$E_s(\vec{r})=\frac{\pi e^{ik_s z}}{iz}\left[\frac{e^{i\psi_1}}{\alpha_1}B_s\left(\frac{x}{z\alpha_1},\frac{y}{z\alpha_1};0\right)+\frac{e^{i\psi_2}}{\alpha_2}B_s\left(\frac{x}{z\alpha_2},\frac{y}{z\alpha_2};0\right)\right], \tag{4.6}$$

and

$$E_i(\vec{r})=\frac{\pi e^{ik_i z}}{iz}\left(\frac{\epsilon_s k_i A_p}{\epsilon_i k_s A_p^*}\right)^{1/2}\left[-\frac{e^{i\psi_1}}{\alpha_1}B_s\left(\frac{x}{z\alpha_1},\frac{y}{z\alpha_1};0\right)+\frac{e^{i\psi_2}}{\alpha_2}B_s\left(\frac{x}{z\alpha_2},\frac{y}{z\alpha_2};0\right)\right], \tag{4.7}$$

where

$$\alpha_1=\frac{1}{2}\left[\frac{1}{k_s}+\frac{1}{k_i}+\mu\left(\frac{1}{k_s^2}+\frac{1}{k_i^2}\right)\right], \tag{4.8}$$

$$\alpha_2=\frac{1}{2}\left[\frac{1}{k_s}+\frac{1}{k_i}-\mu\left(\frac{1}{k_s^2}+\frac{1}{k_i^2}\right)\right], \tag{4.9}$$

$$\psi_1=-\mu z+\tfrac{1}{2}(x^2+y^2)/z\alpha_1, \tag{4.10}$$

$$\psi_2=\mu z+\tfrac{1}{2}(x^2+y^2)/z\alpha_2. \tag{4.11}$$

In the absence of the coupling, i.e., when $d=0$ (and hence also $\mu=0$), we note from Eqs. (4.8)–(4.11) that $\alpha_1=\alpha_2$ and $\psi_1=\psi_2$. In this case, the idler mode E_{i0} is absent,

$$E_{i0}(\vec{r})=0. \tag{4.12}$$

However, the expression for E_{s0} as obtained from (4.6) is not strictly valid since, in this case, the the contributions from the term containing $(\Delta k/2g)\sin gz$ to the integral in (3.14) cannot be neglected even in the asymptotic limit of large z. In fact, from (3.13) we find, on setting $d=0$, that $g=\frac{1}{2}\Delta k$, and Eq. (3.14) then reduces to

$$E_{s0}(\vec{r})=\int dp\,dq\,B_s(p,q;0)\exp[i(px+qy+m_s z)]. \tag{4.13}$$

On making use of Eq. (4.1) and applying the method of stationary phase, we then obtain the following expression for E_{s0} in the far-field approximation:

$$E_{s0}(\vec{r})=\frac{2\pi k_s}{iz}B_s\left(\frac{xk_s}{z},\frac{yk_s}{z};0\right)\exp\left[ik_s\left(z+\frac{x^2+y^2}{2z}\right)\right]. \tag{4.14}$$

It is worth noting that (4.14) may be obtained by setting $\mu=0$ and $k_i=k_s$ in (4.6).

V. INTENSITY DISTRIBUTION OF SIGNAL MODE

In Sec. IV, the expressions for the signal- and the idler-field distributions were obtained. In this section, we obtain an expression for the intensity of the signal mode at any point $\vec{r}$ of the medium. For this purpose, we rewrite Eq. (4.6) in the following form:

$$E_s(x,y,z)=\tfrac{1}{2}e^{ik_s z}[E_{s0}(x,y,\alpha_1 k_s z)e^{-iz(\alpha_1 k_s^2+\mu)}+E_{s0}(x,y,\alpha_2 k_s z)e^{-iz(\alpha_2 k_s^2-\mu)}], \tag{5.1}$$

where $E_{s0}(x,y,z)$ is the signal field in absence of any interaction and is given by Eq. (4.14).

If the signal field $E_{s0}(\vec{r})$ is assumed to be a plane wave propagating along z axis, i.e., if

$$E_{s0}(\vec{r})=C\exp(ik_s z), \tag{5.2}$$

where C is a constant, we find from (5.1) that (cf. Ref. 13)

$$E_s(\vec{r})=Ce^{ik_s z}\cos\mu z. \tag{5.3}$$

From (5.1), we may obtain an expression for the average intensity $I_s(\vec{r})$ of the signal mode. We find that

$$\begin{aligned}I_s(\vec{r})&=\langle E_s^*(\vec{r})E_s(\vec{r})\rangle\\&=\tfrac{1}{4}\{I_0(x,y,\alpha_1 k_s z)+I_0(x,y,\alpha_2 k_s z)\\&\quad+2\,|\Gamma_0(x,y,\alpha_1 k_s z;x,y,\alpha_2 k_s z)|\\&\quad\times\cos[2\mu z+(\alpha_1-\alpha_2)k_s^2 z]\}.\end{aligned} \tag{5.4}$$

Here $\Gamma_0(\vec{r}_1,\vec{r}_2)$ is the mutual coherence function at the two points $\vec{r}_1,\vec{r}_2$,

$$\Gamma_0(\vec{r}_1,\vec{r}_2)=\langle E_{s0}^*(\vec{r}_1)E_{s0}(\vec{r}_2)\rangle, \tag{5.5}$$

and $I_0(\vec{r})$ is the intensity at the point $\vec{r}$ in absence of any nonlinearities. The expressions for I_0 and Γ_0 which correspond to a linear medium have been obtained earlier.[14,15]

VI. GAUSSIAN BEAM

In this section, we consider the special case when the incident-signal field has a Gaussian distribution and obtain the intensity distribution of the signal mode as well as of the idler mode at any point inside the nonlinear medium. The boundary-field distribution $E_s(\xi,\eta)$ at the plane $z=0$ is given by

$$E_s(\xi,\eta)=C\exp[-(\xi^2+\eta^2)/\Omega_0^2], \tag{6.1}$$

where Ω_0 is the spot size and C is a constant. Substituting (6.1) in (3.16) and integrating over ξ and η, we find that

$$B_s(p,q;0)=(C\Omega_0^2/4\pi)\exp[-\tfrac{1}{4}\Omega_0^2(p^2+q^2)]. \tag{6.2}$$

On substituting (6.2) in (4.6) and (4.7), we obtain the following expressions for the signal and the idler fields:

$$E_s(\vec{r}) = \frac{Ce^{ik_s z}\Omega_0^2}{4zi}\left\{\frac{e^{i\psi_1}}{\alpha_1}\exp\left[-\left(\frac{\Omega_0\rho}{2\alpha_1 z}\right)^2\right] + \frac{e^{i\psi_2}}{\alpha_2}\exp\left[-\left(\frac{\Omega_0\rho}{2\alpha_2 z}\right)^2\right]\right\}, \tag{6.3}$$

$$E_i(\vec{r}) = \frac{Ce^{ik_i z}\Omega_0^2}{4zi}\left(\frac{\epsilon_s k_i A_p}{\epsilon_i k_s A_p^*}\right)^{1/2}\left\{-\frac{e^{i\psi_1}}{\alpha_1}\exp\left[-\left(\frac{\Omega_0\rho}{2\alpha_1 z}\right)^2\right] + \frac{e^{i\psi_2}}{\alpha_2}\exp\left[-\left(\frac{\Omega_0\rho}{2\alpha_2 z}\right)^2\right]\right\}, \tag{6.4}$$

where

$$\rho^2 = x^2 + y^2 . \tag{6.5}$$

We may further simplify the above expressions for the electric fields since the parameter μ/k_s is usually very small ($\sim 10^{-5}$). The quantities $1/\alpha_1$ and $1/\alpha_2$ may therefore be expanded in a binomial series in powers of $\mu(k_i^2+k_s^2)/[k_i k_s(k_i+k_s)]$. Whenever these quantities occur in the exponent (as phase terms), one may, to a very good approximation, retain terms up to the first order, whereas when these terms occur alone in the denominator, one may retain only the zeroth-order term. Under this approximation, we obtain, after some calculations, the following expressions for the signal and the idler fields:

$$E_s(\vec{r}) = e^{ik_s z}E_0(\vec{r})\cos\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right) + i\mu\delta\frac{\Omega_0^2\rho^2}{z^2}\frac{k_i k_s}{k_i+k_s}\right], \tag{6.6}$$

$$E_i(\vec{r}) = ie^{ik_i z}\left(\frac{\epsilon_s k_i A_p}{\epsilon_i k_s A_p^*}\right)^{1/2}E_0(\vec{r}) \times\sin\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right) + i\mu\delta\frac{\Omega_0^2\rho^2}{z^2}\frac{k_i k_s}{k_i+k_s}\right], \tag{6.7}$$

where

$$\delta = 2(k_s^2+k_i^2)/(k_s+k_i)^2 \tag{6.8}$$

and

$$E_0(\vec{r}) = \frac{k_i k_s}{k_i+k_s}\frac{C\Omega_0^2}{iz}\exp\left[i\frac{k_i k_s}{k_i+k_s}\frac{\rho^2}{z}\right]\exp\left[-\left(\frac{k_i k_s}{k_i+k_s}\frac{\Omega_0\rho}{z}\right)^2\right]. \tag{6.9}$$

The diffracted wave fields of a Gaussian beam at frequency ω_λ in the medium in absence of nonlinear interaction may be obtained by setting $\mu=0$ and $k_i=k_s$ in (6.6) and (6.7). It is evident from (6.6)–(6.9) that the signal mode as well as the idler mode is Gaussian with spot size

$$\Omega = \frac{k_i+k_s}{k_i k_s}\frac{z}{\Omega_0} . \tag{6.10}$$

The intensities of the signal mode and of the idler mode are given by

$$I_s(\vec{r}) = I_0(\vec{r})\left\{\cos^2\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right)\right]\cosh^2\left(\frac{\mu\delta\rho^2\Omega_0^2}{z^2}\frac{k_i k_s}{k_i+k_s}\right) + \sin^2\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right)\right]\sinh^2\left(\frac{\mu\delta\rho^2\Omega_0^2}{z^2}\frac{k_i k_s}{k_i+k_s}\right)\right\}, \tag{6.11}$$

$$I_i(\vec{r}) = \frac{\epsilon_s k_i}{\epsilon_i k_s}I_0(\vec{r})\left\{\sin^2\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right)\right]\cosh^2\left(\frac{\mu\delta\rho^2\Omega_0^2}{z^2}\frac{k_i k_s}{k_i+k_s}\right) + \cos^2\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right)\right]\sinh^2\left(\frac{\mu\delta\rho^2\Omega_0^2}{z^2}\frac{k_i k_s}{k_i+k_s}\right)\right\}, \tag{6.12}$$

where

$$I_0(\vec{r}) = \langle E_0^*(\vec{r})E_0(\vec{r})\rangle = |C|^2(\Omega_0/\Omega)^2\exp(-2\rho^2/\Omega^2) . \tag{6.13}$$

It may easily be verified by setting $\rho=0$ in (6.11) or (6.12) that in the plane-wave limit, the signal and the idler intensities are proportional to $\cos^2\mu z$ and $\sin^2\mu z$, respectively.

Equation (6.12) can be used to obtain an expression for the conversion efficiency. The conversion efficiency η at any point $\vec{r}$ is defined as

$$\eta(\vec{r}) = \frac{P_i(\vec{r})}{P_s(x,y,0)} = \left(\frac{\epsilon_i}{\epsilon_s}\right)^{1/2}\frac{I_i(\vec{r})}{I_s(x,y,0)} , \tag{6.14}$$

where $P(\vec{r})$ represents the power at the point $\vec{r}$, and we have used the fact that $P \propto \epsilon^{1/2}I$. From (6.1), (6.12), (6.14), and (2.9), we find that

$$\eta(\vec{r})=\frac{\omega_i}{\omega_s}\left(\frac{\Omega_0}{\Omega}\right)^2 \exp\left[-2\left(\frac{1}{\Omega^2}-\frac{1}{\Omega_0^2}\right)\rho^2\right]\left\{\sin^2\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right)\right]\cosh^2\left(\frac{\mu\delta\rho^2\Omega_0^2}{z^2}\frac{k_ik_s}{k_i+k_s}\right)\right.$$
$$\left.+\cos^2\left[\mu\left(z+\frac{1}{2}\frac{\delta\rho^2}{z}\right)\right]\sinh^2\left(\frac{\mu\delta\rho^2\Omega_0^2}{z^2}\frac{k_ik_s}{k_i+k_s}\right)\right\} . \qquad (6.15)$$

We find from (6.15) that the conversion efficiency $\eta(\vec{r})$ has an angular dependence as well. On the axis ($\rho=0$), it has the value

$$\frac{\omega_i}{\omega_s}\left(\frac{\Omega_0}{\Omega}\right)^2 \sin^2\mu z .$$

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