Quantum theory of a one-dimensional optical cavity with output coupling. Field quantization

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The quantum theory of a one-dimensional optical cavity is developed with emphasis on the presence of output coupling. First the resonant mode is defined and calculated classically. Then the field is decomposed into a sequence of modes by introducing an imaginary boundary at a large distance. The mode functions are proved to be orthogonal with respect to an integration with the dielectric constant as a weighting factor. The Hamiltonian of the field is shown to be equivalent to that of a collection of independent harmonic oscillators, the mass of which is a function of the frequency of oscillation. This equivalent of mass appears on normalizing the mode functions and proves to carry information on the structure of the cavity. Quantization of the field is carried out, and the commutation relation for the electric fields inside and outside the cavity is derived. The commutator is composed of an infinite set of derivatives of δ functions, which discloses the effect of the presence of output coupling and that of the small size of the cavity on the radiation field. Also, the commutator is shown to have some properties common with the classical resonant mode, i.e., the property of exponential decay and its rate.

I. INTRODUCTION

Fully quantum-mechanical treatments of laser action have been published by several authors.[1] The losses of the laser cavity have been accounted for by introducing loss oscillators[2] that consisted of phonons or bound electrons. Strictly speaking, these approaches do not give information on the radiation field coupled out of the cavity. If we wish to include in the theory the radiation fields both inside and outside of the cavity coupled to each other, we cannot use the standard technique of the field decomposition into modes, which relies on the presence of homogeneous optical material in the whole space. Because most laser materials and cavity reflectors have different dielectric constants from that of the vacuum, and the size of the laser cavity is often small compared with the coherence length of typical, spontaneously emitted light, we must encounter the problems of both optical discontinuity and size effect.

In this paper we develop a quantum theory of a passive one-dimensional optical cavity with output coupling. A set of orthogonal functions are found with which we can expand the vector potential for our one-dimensional field extending from within the cavity to outside, which allows quantization of the whole field. The effect of output coupling and of the small size of the cavity appears in explicit forms in the commutator for the electric fields, one inside and the other outside the cavity.

In Sec. II we describe the model of the one-dimensional optical cavity having output coupling. The cavity is analyzed classically, and the resonant and antiresonant modes are defined in Sec. III. Resonant modes are defined as ones that have only outgoing or incoming waves outside the cavity and not both. The antiresonant modes are in a sense opposite to resonant modes. In Sec. IV the space is bounded by perfectly conducting walls and the radiation field is divided into a sequence of modes, the eigenmodes of the bounded space including the cavity. The Hamiltonian of the field is calculated and is shown to be equivalent to a sum of Hamiltonians of independent harmonic oscillators, each corresponding to the respective eigenmode. The field is quantized in Sec. V, and some discussion is made about the unique problems arising in our particular radiation field. Finally in Sec. VI, the commutation relation between the electric fields internal and external to the cavity is derived. The commutator has a rather complicated appearance because of the structure of the cavity, which forces us to depart from the usual light-cone concept applicable to a free space.

II. MODEL OF THE CAVITY

Although it is desirable to consider realistic three-dimensional cavities, in view of difficulties involved in three-dimensional analyses of optical cavities, we confine ourselves to only one-dimensional problems. We assume that the radiation field is homogeneous in the x-y plane and is a function only of the space variable z and of time t. The optical cavity is composed of a transparent dielectric extending from $z=-d$ to $z=0$. One end of the cavity at $z=-d$ is bounded by a perfectly conducting medium. The other end of the cavity at $z=0$ is not coated and has finite reflectivity allowing coupling of the fields at both sides of this boundary. The space outside of the cavity is assumed to be vacuum everywhere. The left-hand space $z<-d$ is irrelevant to later calculations.

We are considering a cavity which is infinitely large in the x-y plane. This structure is one of the simplest forms of optical cavities applicable to solid-state lasers except for the simplification to one dimension and for the assumption that one end is uncoated. Another simple form may be an empty cavity ended by a conducting wall and a slab of dielectric,[3] which is applicable to gaseous lasers. This type of cavity will be considered in subsequent papers. Symmetric cavities without conducting walls are likely to yield essentially the same results as the present model or the above alternative, except that they provide output coupling at both ends of the cavity.

Turning to our model, the dielectric is assumed to behave classically[4] and to have a simple dispersion relation $\omega=ck$, where the letters have their usual meanings. Also, the fact that the velocity of light in any dielectric is smaller than that in the vacuum is used throughout. We further assume that the magnetic permeability μ is the same for the dielectric and for the vacuum. Thus any portion of the space is characterized by only one quantity, the dielectric constant ϵ or, alternatively, by the velocity of light c in the medium. As a further simplifying assumption, we assume that the one-dimensional field is polarized in only one direction and that the vector potential has only x components. This assumption is not essential in our problem, but allows us to treat the problem with scalar quantities.

III. CLASSICAL DERIVATION OF THE RESONANT MODES OF THE CAVITY

For later comparison, we derive here the resonant modes of the cavity classically. Also, the antiresonant modes, as we shall call them, appear automatically in the calculation. These two classes of modes are, in a sense, two limiting cases of more general modes that are introduced in Sec. IV. The meanings of these modes in quantum-mechanical calculations of the radiation processes around the cavity are discussed in Secs. V and VI.

In order to clarify the features of the output coupling with respect to the resonant modes of the cavity, we discard the usual definition[5] of the resonant modes—that they undergo a phase shift of integral multiples of 2π during a round trip in the cavity. Instead, we define the resonant mode as one that has only an outgoing wave in the free space, i.e., outside of the cavity.[6]

We now seek these modes in the framework of the classical electrodynamics. By assumption, our vector potential is written as

$$\vec{A}(z,t)=A(z,t)\vec{x}, \tag{1}$$

where the arrow indicates vector quantities. The wave equation for the function $A(z,t)$ in continuous media is

$$\left(\frac{\partial}{\partial z}\right)^2 A(z,t)=\frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2 A(z,t), \tag{2}$$

where $c=(\epsilon\mu)^{-1/2}$ is the velocity of light in the medium. Working in the Coulomb gauge, we have

$$\operatorname{div}\vec{A}(z,t)=0, \tag{3}$$

which is satisfied automatically by (1). At the boundaries, the field must obey the following boundary conditions.[7] At $z=-d$, where the dielectric is bounded by the perfect conductor, the tangential component of the electric vector must vanish. The tangential component of the magnetic vector should be proportional to the surface current J. Normal components are missing by assumption. At the interface of the dielectric and the vacuum, $z=0$, the tangential components of both electric and magnetic vectors must be continuous. The magnetic vector is $(1/\mu)\operatorname{curl}\vec{A}$, which reduces, in our model, to $(1/\mu)(\partial A/\partial z)$. Thus we have

$$(\partial/\partial t)A^1(z,t)=0, \quad \text{at } z=-d, \tag{4a}$$

$$(\partial/\partial t)A^1(z,t)=(\partial/\partial t)A^0(z,t), \quad \text{at } z=0, \tag{4b}$$

$$(\partial/\partial z)A^1(z,t)=(\partial/\partial z)A^0(z,t), \quad \text{at } z=0, \tag{4c}$$

$$(1/\mu)(\partial/\partial z)A^1(z,t)=J, \quad \text{at } z=-d, \tag{4d}$$

where the superscripts 1 and 0 refer to the spatial regions $-d<z<0$ and $z>0$, respectively. In the following, we adopt the convention that every physical quantity in the cavity is signified by the superscript 1 and those outside of the cavity by 0. The magnetic permeability is dropped from (4c) by the assumption that $\mu^1=\mu^0$. The last equation (4d) gives the surface current flowing perpendicular to the magnetic vector at $z=-d$ once the latter is obtained, and imposes no restriction on the solutions. Thus the problem reduces to solving the wave equation (2) under the conditions of (4a), (4b), and (4c).

According to our definition of the resonant modes of an optical cavity, we try solutions of the form

$$A^1(z,t)=u(z)e^{i\omega t}+\text{c.c.}, \quad -d<z<0 \tag{5a}$$

$$A^0(z,t)=v e^{i(\omega t-k^0 z)}+\text{c.c.}, \quad z>0 \tag{5b}$$

where v is a constant. Equation (2) then yields

$$\left(\frac{d}{dz}\right)^2 u(z)+\left(\frac{\omega}{c^1}\right)^2 u(z)=0, \quad -d<z<0 \tag{6}$$

and

$$k^0 = \pm\omega/c^0, \quad z > 0. \tag{7}$$

The solution of (6) is of the form

$$u(z) = ae^{ik^1z} + be^{-ik^1z}, \tag{8}$$

with

$$k^1 = \omega/c^1. \tag{9}$$

The boundary conditions (4a), (4b), and (4c) now yield an eigenvalue problem for the coefficients a, b, and v. After minor algebra we have the following determinantal equation giving possible values of the angular frequency ω:

$$\cos\left(\frac{\omega d}{c^1}\right) \pm i\,\frac{c^1}{c^0}\sin\left(\frac{\omega d}{c^1}\right) = 0. \tag{10}$$

This shows that ω is complex, so that we rewrite it as

$$\omega = \omega' + i\gamma_\omega, \tag{11}$$

where ω' and γ_ω are real. Equation (10) then yields

$$\cos\left(\frac{2\omega' d}{c^1}\right)\exp\left(-\frac{2\gamma_\omega d}{c^1}\right) = \frac{c^1 \mp c^0}{c^1 \pm c^0} \tag{12}$$

and

$$\sin(2\omega' d/c^1) = 0. \tag{13}$$

The light velocity c^1 in the dielectric is smaller than c^0, the light velocity in vacuum:

$$c^1 < c^0. \tag{14}$$

Under this condition we get

$$\omega_k' = (2k+1)\pi c^1/2d, \quad k = 0,1,2,\ldots, \tag{15}$$

$$\gamma_{\omega_k} = \pm\gamma = \pm\frac{c^1}{2d}\ln\left(\frac{c^0 + c^1}{c^0 - c^1}\right), \quad \gamma > 0. \tag{16}$$

When substituted into Eqs. (5b), (7), and (11), the positive value of γ_{ω_k} gives a desired outgoing wave propagated in the positive z direction:

$$A^0(z,t) = v_k \exp[i\omega_k'(t - z/c^0) - \gamma(t - z/c^0)]. \tag{17}$$

The negative value gives an incoming wave:

$$A^0(z,t) = v_k \exp[i\omega_k'(t + z/c^0) + \gamma(t + z/c^0)]. \tag{18}$$

Therefore, an alternative definition of a resonant mode may be that it should have only incoming waves outside of the cavity.

The resonant modes derived here have complex frequencies, so that they have finite bandwidths determined by γ. Accordingly, the propagation constants k^1 and k^0 have imaginary parts, which give broadenings in k space. In other words, these resonant modes are not stationary modes. These nonstationary modes are not convenient for quantization. In later sections we shall call the field components having the frequencies of Eq. (15) resonant modes.

We define antiresonant modes for later comparison. Equation (13) gives the possibility that

$$2\omega_k' d/c^1 = 2k\pi, \quad k = 1,2,3,\ldots, \tag{19}$$

which is excluded by the condition (14). This can be a solution if the inequality (14) is reversed. Also, these are frequencies of the eigenmodes of the cavity if the cavity is bounded by a perfectly conducting medium both at $z = -d$ and at $z = 0$. Let us call the field components having the frequencies of Eq. (19) antiresonant modes.

IV. HAMILTONIAN FORMULATION OF THE FIELD

An alternative representation of the field suitable for the field quantization should be based on stationary modes. For this purpose we set a boundary of a perfectly conducting wall at a large distance L in the positive z direction. The optical cavity is then enclosed in a large one-dimensional cavity bounded at $z = -d$ and at $z = L$. In addition to the boundary conditions (4a), (4b), and (4c), we have a new boundary condition

$$(\partial/\partial t)A^0(z,t) = 0, \quad \text{at } z = L. \tag{20}$$

We solve the wave equation (2) under these four conditions. Try solutions of the form

$$A^1(z,t) = u(z)e^{i\omega t} + \text{c.c.}, \quad -d < z < 0 \tag{21a}$$

$$A^0(z,t) = v(z)e^{i\omega t} + \text{c.c.}, \quad 0 < z < L. \tag{21b}$$

Substitution into (2) yields

$$\left(\frac{d}{dz}\right)^2 u(z) + (k^1)^2 u(z) = 0, \tag{22a}$$

$$\left(\frac{d}{dz}\right)^2 v(z) + (k^0)^2 v(z) = 0, \tag{22b}$$

with

$$k^i = \omega/c^i = \omega(\epsilon^i \mu^i)^{1/2}, \quad i = 1,0, \tag{23}$$

which have general solutions like

$$u(z) = a^1 e^{ik^1z} + b^1 e^{-ik^1z}, \tag{24a}$$

$$v(z) = a^0 e^{ik^0z} + b^0 e^{-ik^0z}. \tag{24b}$$

The boundary conditions (4a), (4b), (4c), and (20) now read

$$a^1 e^{-ik^1d} + b^1 e^{ik^1d} = 0, \tag{25}$$

$$a^1 + b^1 = a^0 + b^0, \tag{26}$$

$$a^1k^1 - b^1k^1 = a^0k^0 - b^0k^0, \tag{27}$$

$$a^0 e^{ik^0L} + b^0 e^{-ik^0L} = 0. \tag{28}$$

Again, we have an eigenvalue problem. The determinant of the coefficients of a^1, b^1, a^0, and b^0 must vanish. Using (23), we obtain for the kth eigenmode

$$c^1 \tan \frac{\omega_k d}{c^1} + c^0 \tan \frac{\omega_k L}{c^0} = 0, \tag{29}$$

or in a more convenient form for later calculations

$$\tan(k^0 L) = -(k^0/k^1)\tan(k^1 d), \tag{29$'$}$$

where k^i is an abbreviation of k_k^i, $i = 1,0$. This equation, giving allowed frequencies in our model, is the key to all the following procedure towards quantization. Now the number of independent equations for the above four coefficients reduces to three, and we can determine the ratios of these coefficients. Thus we have an explicit expression for the kth eigenfunction $A_k(z,t)$ with a single undetermined coefficient. The general solution is a superposition of these eigenfunctions. For example, we have

$$A^1(z,t) = \sum_k f_k \sin k^1(z+d)\cos(\omega_k t + \Phi_k), \quad -d < z < 0 \tag{30a}$$

$$A^0(z,t) = \sum_k f_k \frac{k^1 \cos k^1 d}{k^0 \cos k^0 L} \sin k^0(z-L)\cos(\omega_k t + \Phi_k), \quad 0 < z < L. \tag{30b}$$

Here the Φ_k are arbitrary phases and the f_k are real constants giving the amplitudes of the modes.

We understand that k^i is an abbreviation to k_k^i unless explicitly written as k_j^i, etc. Of course physical processes cannot depend on L, a quantity introduced only for mathematical convenience. L can be eliminated using (29$'$) as

$$A^0(z,t) = \sum_k f_k[(k^1/k^0)\cos k^1 d \sin k^0 z + \sin k^1 d \cos k^0 z] \times \cos(\omega_k t + \Phi_k), \quad 0 < z < L. \tag{30b$'$}$$

For complete elimination of L, it must be considered to be infinitely large and the concept of density of modes must be introduced, which will be discussed in Sec. V.

Having determined the eigenmodes of our one-dimensional field, we now proceed to calculation of the Hamiltonian H of the field. It is given by

$$H = \int_{-d}^{L} \left[\frac{\epsilon}{2}\left(\frac{\partial}{\partial t} A(z,t)\right)^2 + \frac{1}{2\mu}\left(\frac{\partial}{\partial z} A(z,t)\right)^2 \right] dz. \tag{31}$$

Writing

$$q_k = f_k \cos(\omega_k t + \Phi_k), \tag{32}$$

$$\frac{d}{dt} q_k = p_k, \tag{33}$$

we have

$$H = \int_{-d}^{0} \left[\frac{\epsilon^1}{2}\left(\sum_k p_k \sin k^1(z+d)\right)^2 + \frac{1}{2\mu}\left(\sum_k k^1 q_k \cos k^1(z+d)\right)^2 \right] dz + \int_0^L \left[\frac{\epsilon^0}{2}\left(\sum_k p_k \frac{k^1 \cos k^1 d}{k^0 \cos k^0 L} \sin k^0(z-L)\right)^2 + \frac{1}{2\mu}\left(\sum_k k^0 q_k \frac{k^1 \cos k^1 d}{k^0 \cos k^0 L}\cos k^0(z-L)\right)^2 \right] dz. \tag{34}$$

In the Appendix we show that the cross terms for any combination of two different modes vanish identically:

$$H_{i,j} = \int_{-d}^{0} \left(\frac{\epsilon^1}{2} 2p_i p_j \sin k_i^1(z+d)\sin k_j^1(z+d) + \frac{1}{2\mu} 2k_i^1 k_j^1 q_i q_j \cos k_i^1(z+d)\cos k_j^1(z+d)\right) dz + \int_0^L \left(\frac{\epsilon^0}{2} 2p_i p_j \frac{k_i^1 k_j^1 \cos k_i^1 d \cos k_j^1 d}{k_i^0 k_j^0 \cos k_i^0 L \cos k_j^0 L} \sin k_i^0(z-L)\sin k_j^0(z-L) + \frac{1}{2\mu} 2k_i^0 k_j^0 q_i q_j \frac{k_i^1 k_j^1 \cos k_i^1 d \cos k_j^1 d}{k_i^0 k_j^0 \cos k_i^0 L \cos k_j^0 L} \cos k_i^0(z-L)\cos k_j^0(z-L)\right) dz = 0, \quad i \neq j. \tag{35}$$

Thus H is made up of squared quantities contributed from each eigenmode. Now evaluation of H is straightforward. We interchange the sequence of integration and summation, integrate each term separately, and sum them:

$$H = \sum_k \left[\frac{\epsilon^1}{4} p_k^2 \left(d - \frac{\sin 2k^1 d}{2k^1}\right) + \frac{1}{4\mu}(k^1 q_k)^2\left(d + \frac{\sin 2k^1 d}{2k^1}\right) + \frac{\epsilon^0}{4} p_k^2 \left(\frac{k^1 \cos k^1 d}{k^0 \cos k^0 L}\right)^2 \left(L - \frac{\sin 2k^0 L}{2k^0}\right) + \frac{1}{4\mu}(k^0 q_k)^2 \left(\frac{k^1 \cos k^1 d}{k^0 \cos k^0 L}\right)^2 \left(L + \frac{\sin 2k^0 L}{2k^0}\right)\right]. \tag{36}$$

By Eq. (23) we have

$$(k^1/k^0)^2 = \epsilon^1/\epsilon^0, \quad (k^1)^2/\mu = \epsilon^1 \omega_k^2 \ . \tag{37}$$

So that the first and the third sine functions cancel by virtue of (29$'$), and so do the second and the fourth sine functions. Thus we have

$$H = \frac{\epsilon^1}{4}\sum_k \left[d + \left(\frac{\cos k^1 d}{\cos k^0 L}\right)^2 L \right] [p_k^2 + (\omega_k q_k)^2]. \tag{38}$$

This result has the familiar form of the Hamiltonian for a collection of independent harmonic oscillators except that their equivalents to mass are dependent on the frequency of oscillation. These different masses of the oscillators reflect the structure of the optical cavity and the nature of the media. They will later exhibit important consequences on the commutation relations for the field and on the radiation processes such as emission, absorption, and propagation of radiation around the cavity, which we will discuss in Secs. V and VI.

We are now in a position to quantize our one-dimensional radiation field extending over the whole space including the optical cavity and the free space outside of it.

V. QUANTIZATION

At first we normalize the mode functions appearing in (30) by introducing new variables Q_k and P_k defined by

$$Q_k=\left\{\tfrac{1}{2}\epsilon^1\left[d+\left(\frac{\cos k^1 d}{\cos k^0 L}\right)^2 L\right]\right\}^{1/2} q_k\,, \tag{39}$$

$$P_k=\left\{\tfrac{1}{2}\epsilon^1\left[d+\left(\frac{\cos k^1 d}{\cos k^0 L}\right)^2 L\right]\right\}^{1/2} p_k\,. \tag{40}$$

In terms of these variables, the vector potential (30) is rewritten as

$$A^1(z,t)=\sum_k Q_k\left(\frac{2}{\epsilon^1[d+(\cos k^1 d/\cos k^0 L)^2 L]}\right)^{1/2} \times \sin k^1(z+d),\quad -d<z<0, \tag{41a}$$

$$A^0(z,t)=\sum_k Q_k\left(\frac{2}{\epsilon^1[d+(\cos k^1 d/\cos k^0 L)^2 L]}\right)^{1/2} \times \frac{k^1\cos k^1 d}{k^0\cos k^0 L}\sin k^0(z-L),\quad 0<z<L. \tag{41b}$$

If we formally write

$$A(z,t)=\sum_k Q_k U_k(z),\quad -d<z<L \tag{42}$$

where explicit expressions for the mode functions $U_k(z)$ are given by (41), $U_k(z)$ satisfy the following normalized orthogonality relation:

$$\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz=\delta_{ij}\,, \tag{43}$$

which we prove in the Appendix. $U_k(z)$ are normalized with the weighting factor ϵ, the dielectric constant. The presence of these functions is the basis of the present quantization of our one-dimensional radiation field as is discussed in the Appendix.

Substitution of (39) and (40) into (38) yields

$$H=\tfrac{1}{2}\sum_k (P_k^2+\omega_k^2 Q_k^2). \tag{44}$$

Now we quantize the system by imposing the following commutation relations on the variables Q_k's and P_k's:

$$[Q_i,Q_j]=[P_i,P_j]=0,\quad [Q_i,P_j]=i\hbar\delta_{ij}\,. \tag{45}$$

Q_k and P_k are now operators acting on the kth mode.

Define annihilation and creation operators as

$$a_k=(2\hbar\omega_k)^{-1/2}(\omega_k Q_k+iP_k), \tag{46a}$$

$$a_k^*=(2\hbar\omega_k)^{-1/2}(\omega_k Q_k-iP_k), \tag{46b}$$

which satisfy the commutation relation

$$[a_i,a_j^*]=\delta_{ij} \tag{47}$$

as is easily derived by (45). The inverse relation to (46) is

$$Q_k=(\hbar/2\omega_k)^{1/2}(a_k+a_k^*), \tag{48a}$$

$$P_k=-i(\hbar\omega_k/2)^{1/2}(a_k-a_k^*). \tag{48b}$$

Substituting (48) into (44) and using (47), we have

$$H=\sum_k \hbar\omega_k(a_k^*a_k+\tfrac{1}{2}). \tag{49}$$

But we can subtract the zero-point energy without violation of the uncertainty,[8]

$$H=\sum_k \hbar\omega_k a_k^*a_k=\sum_k H_k\,. \tag{49$'$}$$

The state Ψ of the radiation field obeys the Schrödinger equation

$$ih\frac{\partial}{\partial t}\Psi=H\Psi. \tag{50}$$

The solution is

$$\Psi=\prod_k \psi_k=\prod_k \exp(-iE_{k,n}t/\hbar)\phi_{k,n}\,, \tag{51}$$

where $\phi_{k,n}$ and $E_{k,n}$ are the nth energy eigenstate and the corresponding eigenvalue of the kth mode. The general solution is a linear superposition of the pure states (51). $\phi_{k,n}$ and $E_{k,n}$ are given by

$$H_k\phi_{k,n}=E_{k,n}\phi_{k,n}=n\hbar\omega_k\phi_{k,n}\,. \tag{52}$$

It is easy to show that the annihilation and the creation operators a_k and a_k^* have the following effects on the energy eigenstates if $\phi_{k,n}$ are normalized to unity:

$$a_k\phi_{k,n}=\sqrt{n}\,\phi_{k,n-1}, \tag{53a}$$

$$a_k^*\phi_{k,n}=\sqrt{n+1}\,\phi_{k,n+1}. \tag{53b}$$

The nonvanishing matrix elements are thus

$$a_{k,n-1,n}=\phi_{k,n-1}^*a_k\phi_{k,n}=\sqrt{n}\,, \tag{54a}$$

$$a_{k,n+1,n}^*=\phi_{k,n+1}^*a_k^*\phi_{k,n}=\sqrt{n+1}\,. \tag{54b}$$

We next consider the density of modes $\rho(\omega)\,d\omega$

which will become necessary in later calculations. At first we note that we have no degeneracy in that two different modes have the same frequency, as is easily seen by Eqs. (23) and (29). We remember that the angular frequencies of the eigenmodes are given by Eq. (29). If $\Delta\omega$ is the frequency difference of two neighboring modes at ω_k, then

$$c^0 \tan\frac{(\omega_k+\Delta\omega)L}{c^0} + c^1 \tan\frac{(\omega_k+\Delta\omega)d}{c^1} = 0. \qquad (55)$$

Using (29), (29′), and (55) we have

$$\tan\frac{\Delta\omega L}{c^0}\left\{\left[\tan^2\left(\frac{\omega_k L}{c^0}\right)+1\right]\tan\frac{\omega_k d}{c^1} + \left[\tan^2\left(\frac{\omega_k L}{c^0}\right) - \tan^2\left(\frac{\omega_k d}{c^1}\right)\right]\tan\frac{\Delta\omega d}{c^1}\right\} - \tan\frac{\Delta\omega d}{c^1}\tan\frac{\omega_k L}{c^0}\left[1+\tan^2\left(\frac{\omega_k d}{c^1}\right)\right] = 0. \qquad (56)$$

Provided that

$$L/c^0 \gg d/c^1, \qquad (57)$$

terms of $\tan(\Delta\omega d/c^1)$ may be ignored, thus

$$\tan(\Delta\omega L/c^0) = 0,$$

and we have

$$\Delta\omega = (c^0/L)\pi. \qquad (58)$$

Therefore, the inequality (57) is equivalent to

$$\Delta\omega \ll c^1/d. \qquad (57')$$

That is, we require that $\Delta\omega$ is much smaller than the separation of the resonant modes of the cavity defined by (15). If L is finite, we may add corrections to (58) in powers of d/L. To the first order in d/L, we obtain, by repeated use of (29),

$$\Delta\omega = \frac{c^0\pi}{L}\left[1 - \left(\frac{\cos k^0 L}{\cos k^1 d}\right)^2 \frac{d}{L}\right]. \qquad (58')$$

Thus we have

$$\rho(\omega) = L/c^0\pi, \quad L \to \infty, \qquad (59a)$$

$$\rho(\omega) = \frac{L}{c^0\pi}\left[1 + \left(\frac{\cos k^0 L}{\cos k^1 d}\right)^2 \frac{d}{L}\right], \quad L \text{ finite}. \qquad (59b)$$

We note that the factor before d/L in (58′) and (59b) is always finite as is easily seen by (29′). The density of modes (59a) will be used in later calculations rather than (59b).

Now that we have quantized the field and obtained the density of modes, we prove here a very useful and important equation concerning the normalization factor in (41).

Since the coefficient of L in this factor is never zero as is easily verified by (29′), we can ignore d compared to the term of L if L is large. Thus the square of the normalization factor, which appears in various calculations, has the quantity $(\cos k^0 L/\cos k^1 d)^2$, which can be rewritten as

$$\left(\frac{\cos k^0 L}{\cos k^1 d}\right)^2 = \frac{2(k^1)^2/[(k^1)^2+(k^0)^2]}{1+\{[(k^1)^2-(k^0)^2]/[(k^1)^2+(k^0)^2]\}\cos 2k^1 d} \qquad (60a)$$

$$= \frac{2k^1}{k^0}\sum_{n=0}^{\infty}\frac{1}{1+\delta_{0,n}}\left(-\frac{k^1-k^0}{k^1+k^0}\right)^n \cos 2nk^1 d, \qquad (60b)$$

where $\delta_{0,n}$ is the Kronecker delta. The first equality is easily verified by (29). To prove the second is rather tedious but not difficult. We shall only outline the proof.

Consider an expansion of the form

$$1/[1+B\cos(y)] = b_0 + b_1\cos(y) + b_2\cos(2y) + b_3\cos(3y) + \cdots.$$

Multiply both sides by $1+B\cos(y)$ and compare the coefficient of $\cos(ny)$. Then we have

$$b_0 + \tfrac{1}{2}Bb_1 = 1, \quad Bb_0 + b_1 + \tfrac{1}{2}Bb_2 = 0,$$

$$\tfrac{1}{2}Bb_n + b_{n+1} + \tfrac{1}{2}Bb_{n+2} = 0, \quad n \geq 1.$$

Rewrite the last equation as

$$b_{n+2} - \theta b_{n+1} = \xi(b_{n+1} - \theta b_n),$$

where

$$\theta + \xi = -2/B, \quad \theta\xi = 1.$$

Let θ_1, ξ_1, and θ_2, ξ_2 be two alternative solutions to this. Then we have

$$b_{n+2} = \frac{(\xi_1)^n(b_2 - \theta_1 b_1)\theta_2 - (\xi_2)^n(b_2 - \theta_2 b_1)\theta_1}{\theta_2 - \theta_1}.$$

If $|B|$ is smaller than unity, $|\xi_1| > 1$ and $|\xi_2| < 1$, say. To obtain a mathematically tractable series, b_n should converge to zero. Thus the coefficient of ξ_1 must vanish. This requirement determines b_0 when applied to the first two equations for b's. Hence all the b_n are determined uniquely. Substitution of B by $[(k^1)^2-(k^0)^2]/[(k^1)^2+(k^0)^2]$ completes the proof.

Here we briefly discuss the meanings of the squared normalization factor (60). The first form (60a) gives some insight into the nature of the resonant and antiresonant modes defined in Sec. III. This factor has maximum values at the frequencies of the resonant modes, whereas minimum values are at the antiresonant modes. Because this factor appears in the interaction Hamiltonian with electrons, it may be said that the resonant modes are most active when coupled to electrons. The antiresonant modes are the reverse.[9] The second meaning of the factor is revealed in (60b). For each mode, the vector potential is in fact made up of an infinite number of terms with phase differences equal to that occurring during the time of a round trip in the cavity and with decaying amplitudes with increasing phase. This is not surprising, but can be expected by inspection of the wave nature of the radiation field and the finite size and the coupling loss of the cavity. For example, if an atom emits in the cavity and if the coherence length of the emitted light is larger than the size of the cavity, reaction of the emitted light onto the atom occurs many times during the emission process, which causes the atomic decay time to vary as a function of the relative separation of the atomic frequency and cavity resonances, as will be shown in a future paper.

Before closing this section, a remark should be in order with regard to the completeness of our mode functions in (42). In order that we can include in our theory an arbitrary function in our one-dimensional space, $-d<z<L$, the basis functions must be complete. Or, equivalently, they must satisfy the closure relation

$$\sum_k \epsilon(z')U_k(z')U_k(z)=\int_0^\infty \epsilon(z')U_k(z')U_k(z)\rho(\omega_k)\,d\omega_k =\delta(z'-z),$$
$$-d<z<L,\quad -d<z'<L,\quad \text{except } z=z'=0. \qquad (61)$$

The proof can be easily obtained using the density of modes (59a), the expansion (60b), and the definition of the δ function (68′) in Sec. VI, as well as the relation (37). The calculation is similar to that for the commutator for the electric field carried out in Sec. VI. The exception of $z=z'=0$ is reasonable, since at $z=0$ the dielectric constant cannot be specified. Also, the boundary conditions (4b) and (4c) demand that the fields should be continuous across this boundary, so that a δ function at $z=0$ is not permissible.

Thus we are surely including in our discussion an arbitrary field in the one-dimensional space, $-d<z<L$, except for discontinuities of the field at $z=0$.

VI. COMMUTATION RELATION FOR THE ELECTRIC FIELD

Finally, we derive the commutation relation for the electric fields at two different space-time points. Since we are interested in the relations between the fields inside and outside the cavity, we take one of the space point z_A from inside of the cavity and the other z_B from outside. The corresponding time variables will be written as t_A and t_B, respectively. In order to include the time variables in the commutation relation, we go to the Heisenberg picture. It can easily be verified that the nonvanishing matrix elements are related to those in (54) in the Schrödinger picture as

$$a_{kH,n-1,n}=a_{k,n-1,n}\,e^{-i\omega_k t}, \qquad (62a)$$
$$a^*_{kH,n+1,n}=a^*_{k,n+1,n}\,e^{i\omega_k t}, \qquad (62b)$$

where the subscript H indicates the operators in the Heisenberg picture. The commutator for the annihilation and the creation operators is unchanged. Regarding the vector potential (41) as an operator and substituting (48a) and (62), we have

$$E_H(z,t)=-\frac{\partial}{\partial t}A_H(z,t)=i\sum_k\left[\frac{\hbar\omega_k}{\epsilon^1 L}\left(\frac{\cos k^0L}{\cos k^1 d}\right)^2\right]^{1/2}\sin k^1(z+d)(a_k e^{-i\omega_k t}-a_k^* e^{i\omega_k t}),\quad -d<z<0 \qquad (63a)$$
$$=i\sum_k\left[\frac{\hbar\omega_k}{\epsilon^1 L}\left(\frac{\cos k^0L}{\cos k^1 d}\right)^2\right]^{1/2}\frac{k^1\cos k^1 d}{k^0\cos k^0 L}\sin k^0(z-L)(a_k e^{-i\omega_k t}-a_k^* e^{i\omega_k t}),\quad 0<z<L, \qquad (63b)$$

where we ignored d in the normalizing factors assuming that L is much larger than d. This assumption is natural since we are interested only in the effects of the cavity on the radiation field and not of the mathematical boundary, as stated earlier.

After minor algebra, using Eqs. (29′), (47), and (63), we get

$$[E_H(z_A,t_A),E_H(z_B,t_B)]=2i\sum_k\frac{\hbar\omega_k}{\epsilon^1 L}\left(\frac{\cos k^0L}{\cos k^1 d}\right)^2\sin k^1(z_A+d)\left(\frac{k^1}{k^0}\cos k^1 d\sin k^0 z_B+\sin k^1 d\cos k^0 z_B\right)\sin\omega_k(t_B-t_A). \qquad (64)$$

We replace the sum by an integration using the density of modes (59a). Also, we use the series expansion (60b):

$$[E_H(z_A,t_A),E_H(z_B,t_B)]=2i\int_0^\infty \frac{\hbar\omega_k}{\pi c^0\epsilon^1}\frac{2k^1}{k^0}\left[\sum_{n=0}^{\infty}\frac{1}{1+\delta_{0,n}}\left(-\frac{k^1-k^0}{k^1+k^0}\right)^n\cos 2nk^1d\right]$$
$$\times\sin k^1(z_A+d)\left(\frac{k^1}{k^0}\cos k^1d\sin k^0z_B+\sin k^1d\cos k^0z_B\right)\sin\omega_k(t_B-t_A)\,d\omega_k\,, \tag{65}$$

where Eq. (23) should be taken into account, that is, $k^i=\omega_k/c^i$ $(i=1,0)$. After a rather exhaustive manipulation of the sinusoidal functions and rearrangement of terms, we have

$$[E_H(z_A,t_A),E_H(z_B,t_B)]=\frac{i\hbar}{\pi c^0\epsilon^1}\frac{(k^1)^2}{k^0(k^1+k^0)}\int_0^\infty \omega_k\sum_{n=0}^{\infty}\left(-\frac{k^1-k^0}{k^1+k^0}\right)^n$$
$$\times\left[\sin\omega_k\left(\frac{z_B}{c^0}-\frac{z_A}{c^1}+\frac{2nd}{c^1}+t_B-t_A\right)-\sin\omega_k\left(\frac{z_B}{c^0}+\frac{z_A+2d}{c^1}+\frac{2nd}{c^1}+t_B-t_A\right)\right.$$
$$\left.-\sin\omega_k\left(\frac{z_B}{c^0}-\frac{z_A}{c^1}+\frac{2nd}{c^1}-t_B+t_A\right)+\sin\omega_k\left(\frac{z_B}{c^0}+\frac{z_A+2d}{c^1}+\frac{2nd}{c^1}-t_B+t_A\right)\right]d\omega_k\,. \tag{66}$$

Here we resort to some mathematical tools.[8] Evidently,

$$\sin\omega_k(\text{const}+t_B-t_A)=\left(\frac{1}{\omega_k}\right)^2\frac{\partial^2}{\partial t_A\partial t_B}\sin\omega_k(\text{const}+t_B-t_A),\quad\text{etc.} \tag{67}$$

Consequently, we are left with integrals of the form

$$I(X)=\int_0^\infty\frac{\sin\omega_kX}{\omega_k}\,d\omega_k\,, \tag{68}$$

which is a step function, an integral of the Dirac δ function:

$$\delta(t)=(1/\pi)\lim_{K\to\infty}\int_0^K\cos\omega t\,d\omega,\qquad I(X)=\pi\int_0^X\delta(t)\,dt=\tfrac{1}{2}\pi e(X),\qquad e(x)=\begin{cases}+1, & x>0\\ -1, & x<0\end{cases}. \tag{68$'$}$$

The second derivative of this function is

$$\left(\frac{\partial}{\partial x}\right)^2 e(x)=2\frac{\partial}{\partial x}\delta(x)=2\delta'(x). \tag{68$''$}$$

Using Eqs. (67), (68), (68′), (68″), and (23), we finally obtain

$$[E_H(z_A,t_A),E_H(z_B,t_B)]=i\hbar\frac{\mu c^0c^1}{c^0+c^1}\sum_{n=0}^{\infty}\left(-\frac{c^0-c^1}{c^0+c^1}\right)^n$$
$$\times\left[-\delta'\left(\frac{z_B}{c^0}-\frac{z_A}{c^1}+\frac{2nd}{c^1}+t_B-t_A\right)+\delta'\left(\frac{z_B}{c^0}+\frac{z_A+2d}{c^1}+\frac{2nd}{c^1}+t_B-t_A\right)\right.$$
$$\left.+\delta'\left(\frac{z_B}{c^0}-\frac{z_A}{c^1}+\frac{2nd}{c^1}-t_B+t_A\right)-\delta'\left(\frac{z_B}{c^0}+\frac{z_A+2d}{c^1}+\frac{2nd}{c^1}-t_B+t_A\right)\right],$$
$$-d<z_A<0,\quad 0<z_B. \tag{69}$$

Thus the resultant expression of the commutator for the electric field at (z_A,t_A) and that at (z_B,t_B), where z_A lies inside the one-dimensional cavity and z_B is outside, consists of an infinite number of terms. All the terms are in the form of derivatives of Dirac δ functions, so that they have nonzero values only when their arguments are zero. The physical meanings of these terms are obvious. For example, the third class of terms in the infinite sum gives the intensity and the time of arrival at z_B of the disturbances when a flash of light is emitted instantaneously at z_A at time t_A. The first of these terms ($n=0$) is obviously the directly transmitted disturbance from z_A to z_B, and the subsequent terms are those reflected at the output interface, at $z=0$, n times, made to undergo n round trips in the cavity, and then propagated to z_B. The coefficients of these terms correctly give the relative intensities of successive disturbances, because they are in powers of the well-known reflectivity for the electric vector[7] at the output surface. The minus sign in the coefficients is representative of the phase shift at $z=-d$. The fourth class of terms corresponds to those disturbances emitted at z_A which at the outset started toward the coated end of the cavity, that is, toward the negative z direction. The first and the second classes of terms represent the inverse situation, where light is emitted at z_B and transmitted to z_A. Thus we must discard the simple light-cone concept applicable in a free space.[8] Instead, we are left with infinite number of light cones fading monotonically with increasing order. The measurements of the electric fields at two space-time points belonging to any of these cones have uncertainties. The degree of the uncertainty depends on the order of the cone.

It is interesting to rewrite the coefficients in (69) using the property of the δ function. The power index n in (69) is given, for instance, for the third and the first classes of terms as

$$n=\frac{c^1}{2d}\left(t_B-t_A-\frac{z_B}{c^0}+\frac{z_A}{c^1}\right), \tag{70a}$$

$$n=-\frac{c^1}{2d}\left(t_B-t_A+\frac{z_B}{c^0}-\frac{z_A}{c^1}\right), \tag{70b}$$

so that, respectively, for the third and first classes of terms

$$\left(-\frac{c^0-c^1}{c^0+c^1}\right)^n=(-1)^n\exp\left\{-\gamma\left[(t_B-t_A)-\left(\frac{z_B}{c^0}-\frac{z_A}{c^1}\right)\right]\right\}, \tag{71a}$$

$$\left(-\frac{c^0-c^1}{c^0+c^1}\right)^n=(-1)^n\exp\left\{\gamma\left[(t_B-t_A)+\left(\frac{z_B}{c}-\frac{z_A}{c}\right)\right]\right\}, \tag{71b}$$

where

$$\gamma=\frac{c^1}{2d}\ln\left(\frac{c^0+c^1}{c^0-c^1}\right),$$

which is exactly the same as that appeared in the classical resonant modes, (17) and (18). The resemblance of the commutator (69) to the classical waves (17) and (18) is noticeable when (71) is substituted into it.

In this paper, we have developed a theory of a one-dimensional optical cavity having output coupling and have shown that the consequences of the structure of the cavity are mathematically represented in terms of the normalization factor for the mode functions. Basic radiation processes in the one-dimensional space including the cavity with output coupling will be considered in a future paper. A laser theory based on the present formalism will be published later. The unique feature of such a theory is that it gives a direct expression for the field coupled outside the cavity and that it does not necessarily require the presence of loss oscillators because the cavity loss is automatically included in the theory and no difficulty will arise as to conservation of the uncertainty in the radiation field, so long as the laser-active atoms are treated fully quantum mechanically. Also, various problems concerning spatial variation of the field, such as mode-locked oscillation or build up of the laser oscillation from arbitrary spatial distribution, can be treated systematically without any assumption on the spatial mode.

Although the present discussion is limited to one dimension, we hope that the present consideration can be extended to two or three dimensions in the near future. In this respect, we should like to point out here that we can prove the orthogonality of three-dimensional mode functions in a space including a cavity. Consider a block of dielectric, which we will regard as an optical cavity, and a boundary of a perfectly conducting medium at some distance from the surface of the cavity. The latter is a mathematical boundary. Assuming the presence of the solutions to the wave equation for the vector potential inside the boundary, we decompose the field into modes which are labeled by the frequency of oscillation. Then, it is easy to show that the volume integral, within the larger boundary, of the cross terms like $\epsilon(\vec{r})\vec{A}_i(\vec{r})\cdot\vec{A}_j(\vec{r})$ and $(1/\mu)\,\mathrm{curl}\vec{A}_i(\vec{r})\cdot\mathrm{curl}\vec{A}_j(\vec{r})$ vanishes for different ω_i and ω_j. Here $\vec{A}_i(\vec{r})$ is the spatial portion of the ith-mode function. The proof requires only a few vector identities and the Green's theorem as well as the boundary conditions at the cavity surface and at the conducting wall. The calculation is similar to that presented in the Appendix for the one-dimensional case. Thus we can obtain the Hamiltonian of the field in the form of that of uncoupled harmonic oscillators. However, we cannot in general obtain the equivalents to their masses unless explicit expressions for the mode functions are given. As in the present calculation, this factor will contain all the consequences of the structure of the cavity. Therefore, to proceed to three dimensions, the first task is to find correct mode functions for a given model.

APPENDIX

In this appendix, we prove Eqs. (35) and (43) of the text. Although we can prove them by direct integrations and repeated use of (29′) and (23), we will give the proof by a more general method which may be applicable to more complicated models and be extended to two or three dimensions.

Since the mode functions appearing in (35) are the same as those in (41) and (42), we may use the latter in proving Eq. (35). At first, we show that the electric part of the cross term $H_{i,j}$ vanishes. Using the property of the mode functions that they obey equations like (22), we have

$$\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz=\int_{-d}^{0}\epsilon^1U_i(z)U_j(z)\,dz+\int_{0}^{L}\epsilon^0U_i(z)U_j(z)\,dz$$
$$=\int_{-d}^{0}\epsilon^1\left(-\frac{1}{(k_i^1)^2}\right)\left[\left(\frac{\partial}{\partial z}\right)^2U_i(z)\right]U_j(z)dz+\int_{0}^{L}\epsilon^0\left(-\frac{1}{(k_i^0)^2}\right)\left[\left(\frac{\partial}{\partial z}\right)^2U_i(z)\right]U_j(z)\,dz, \tag{A1}$$

where P_iP_j is omitted for simplicity. Integrating by parts twice and using Eq. (22) again, we have

$$\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz=-\frac{\epsilon^1}{(k_i^1)^2}\left\{\left[\left(\frac{\partial}{\partial z}U_i(z)\right)U_j(z)\right]_{z=-0}-\left[\left(\frac{\partial}{\partial z}U_i(z)\right)U_j(z)\right]_{z=-d}\right.$$
$$\left.-\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=-0}+\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=-d}\right\}$$
$$-\frac{\epsilon^0}{(k_i^0)^2}\left\{\left[\left(\frac{\partial}{\partial z}U_i(z)\right)U_j(z)\right]_{z=L}-\left[\left(\frac{\partial}{\partial z}U_i(z)\right)U_j(z)\right]_{z=+0}\right.$$
$$\left.-\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=L}+\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=+0}\right\}$$
$$+\left(\frac{k_j^1}{k_i^1}\right)^2\int_{-d}^{0}\epsilon^1U_i(z)U_j(z)\,dz+\left(\frac{k_j^0}{k_i^0}\right)^2\int_{0}^{L}\epsilon^0U_i(z)U_j(z)\,dz. \tag{A2}$$

The second, fourth, fifth, and seventh terms vanish by the boundary conditions (4a) and (20). The first and the sixth terms and the third and the eighth terms cancel by boundary conditions (4b) and (4c) by virtue of (23). Adding the last two terms yields

$$\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz=\left(\frac{\omega_j}{\omega_i}\right)^2\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz, \tag{A3}$$

also by (23). Since we have no degenerate modes,

$$\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz=0,\quad i\neq j. \tag{A4}$$

For the magnetic part of the cross term $H_{i,j}$, we have, omitting Q_iQ_j,

$$\int_{-d}^{L}\frac{1}{\mu}\frac{\partial}{\partial z}U_i(z)\frac{\partial}{\partial z}U_j(z)\,dz=\frac{1}{\mu}\left[\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=-0}-\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=-d}+\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=L}\right.$$
$$\left.-\left(U_i(z)\frac{\partial}{\partial z}U_j(z)\right)_{z=+0}-\int_{-d}^{0}U_i(z)\left(\frac{\partial}{\partial z}\right)^2U_j(z)\,dz-\int_{0}^{L}U_i(z)\left(\frac{\partial}{\partial z}\right)^2U_j(z)\,dz\right]. \tag{A5}$$

The second and the third terms vanish and the first and the fourth terms cancel by the boundary conditions (4a), (4b), (4c), and (20). The last two terms are rewritten, by (22), as

$$\frac{1}{\mu}(k_j^1)^2\int_{-d}^{0}U_i(z)U_j(z)\,dz+\frac{1}{\mu}(k_j^0)^2\int_{0}^{L}U_i(z)U_j(z)\,dz$$
$$=\omega_j^2\int_{-d}^{L}\epsilon(z)U_i(z)U_j(z)\,dz, \tag{A6}$$

which vanishes by (A4). The last expression is allowed by (23). Thus we have proved Eq. (35).

The orthogonality of U's in (43) is proved in (A4). The normality of U's is easily seen by substituting (41) into (31) and following the procedure to obtain (38). Thus (43) can be considered to be proved. The above calculation shows that the vanishing of the cross term (35) is the consequence of the orthogonality of U's, the normalized mode functions. Thus the orthonormality relation (43) is a basic equation in our formulation.

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[9]In fact these effects arise in the emission and absorption of a photon in the cavity and in the absorption of a photon outside the cavity which is emitted inside, as shown in a future paper.