Quantum electrodynamics in the presence of dielectrics and conductors. IV. General theory for spontaneous emission in finite geometries

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(Received 4 December 1974)

A quantum-electrodynamic theory of spontaneous emission in presence of dielectrics and conductors is developed. The theory makes use of the master-equation techniques and the response-function formalism of part I of this series of papers. Various observable entities such as damping coefficients (lifetimes), Lamb shifts, and frequency shifts are related to the appropriate surface-dependent response functions. The results are valid for arbitrary geometries (involving linear dielectrics) and naturally contain, as a special case, the usual results of spontaneous emission in free space. As explicit examples we consider a two-level atom (also the multilevel atom) in presence of a plane dielectric interface and between two conducting plates. Formulas for the shifts and widths are given and their asymptotic behavior for large and short distances is discussed. The behavior when the atom is embedded inside the dielectric is different than when the atom is outside the dielectric. The origin of coherence in the present model is discussed and the coherence effects in this model are contrasted with those in Dicke's model. The results are also compared with those obtained by the image method. Exact expressions for the operator radiation-reaction fields are obtained in terms of the atomic polarization operators and response functions. Approximate results for such fields are also given. The far-zone behavior of the radiation fields is obtained in terms of response functions and the polarization operators. Some of the normally ordered correlation functions are also calculated. The connection of some of the theoretical results with a recent experimental work of Carniglia, Mandel, and Drexhage is discussed. The effect of anisotropy of the dielectric function on the lifetimes as well as on the far-field correlation functions is also considered. Finally the contribution to the shifts and widths due to the excitation of surface polariton modes is computed and the results are compared with those obtained by the quantization of surface polariton field. It is found that such surface modes contribute significantly to widths.

I. INTRODUCTION

In part I of this series of papers[1] we discussed how the linear response functions can be used to study the electromagnetic field fluctuations. In parts II and III[2,3] we discussed several applications of this formalism and a number of new effects which arise as a result of the presence of dielectric and conducting surfaces. We also discussed how the formalism of I can be generalized to discuss electromagnetic fields which are not necessarily in thermal equilibrium. In the present paper we discuss how spontaneous emission can be treated in this framework and discuss a number of new coherence effects.[4]

Spontaneous emission has been the subject of a great many investigations, recently, both theoretical and experimental.[5-16] In a recent article[17] we discussed at length different concepts pertaining to spontaneous emission and also different theories which have been used to treat spontaneous emission. In that treatment it was assumed that atoms were emitting in free space. Here we discuss various aspects of spontaneous emission in presence of interfaces.

The plan of the paper is as follows. In Sec. II we obtain the master equation for the reduced density operator corresponding to the atomic system

and use it to obtain the lifetime and the shifts of the atomic states, and the relation of these to the surface-dependent response functions is discussed. In Sec. III the shifts and widths are evaluated for the case when the atom is emitting in the vicinity of a plane dielectric interface. Various limiting cases are examined. Next we study the change in the lifetimes due to the presence of two conducting mirrors. In Sec. V the radiation-reaction fields are calculated. Both positive and negative frequency parts of the field operator are obtained. The far-zone behavior of the radiation fields is discussed in Sec. VI, and the normally ordered correlation functions of the field operators are computed. The far-zone field is found to be essentially the one which one would obtain from a classical analysis.[18] In Sec. VII we examine the origin of the coherence effects in the present problem and contrast the present effect to that of Dicke's superradiance.[19] We comment on the method of images used by some authors[20,21] in the treatment of spontaneous emission and point out some of the inadequacies of that method (Sec. VIII). In Sec. IX we calculate the contributions to the shifts and widths due to the excitation of the surface polariton modes. The paper is concluded with several appendices, wherein we discuss a number of dispersion characteristics of the anticommutators of the

field operators at different space-time points and their evaluation. We also comment on the spontaneous emission from a harmonic oscillator in the presence of an interface and the effects of the anisotropy of the dielectric function on the shifts and widths of the states. As in the previous papers of this series, we will assume that nonlinear interactions inside the dielectric are not important.

II. MASTER EQUATION FOR THE REDUCED DENSITY OPERATOR OF THE ATOMIC SYSTEM AND THE RELATIONS AMONG LIFETIMES, FREQUENCY SHIFTS, AND RESPONSE FUNCTIONS

The lifetimes and the shifts of various states can be easily obtained using the first-order perturbation theory. We, however, follow a different route. We will obtain the master equation for the reduced density operator corresponding to the atomic system. This will enable us to discuss both kinematical and dynamical aspects of spontaneous emission. Consider the interaction between two quantum-mechanical systems S and R. We write the interaction Hamiltonian as

$$H = H_S + H_R + H_{RS},$$

$$H_{RS} = -\sum_\alpha \int G_\alpha^{(R)}(\vec{r}) G_\alpha^{(S)}(\vec{r})\, d^3r, \tag{2.1}$$

where H_S and H_R are the unperturbed Hamiltonians of S and R, respectively, and $G_\alpha^{(S)}$ ($G_\alpha^{(R)}$) is a system (reservoir R) operator. We will assume that the

system R is large enough so that it can be treated as a reservoir, i.e., the unperturbed spectrum of R is quasicontinuous. In our problem the radiation field acts like a reservoir. We are specifically interested in the evolution of S due to interaction with R. We assume that the initial state of the total system is such that (ρ standing for the density operator)

$$\rho(t = 0) = \rho_R(0)\rho_S(0), \tag{2.2}$$

and that $\rho_R(0)$ represents the thermal equilibrium state of R, i.e.,

$$\rho_R(0) = e^{-\beta H_R}/\mathrm{Tr}\{e^{-\beta H_R}\}. \tag{2.3}$$

Let $\rho_S(t)$ be the reduced density operator for the system S. It is obtained from the total density operator by taking trace over the reservoir variables, i.e.,

$$\rho_S(t) = \mathrm{Tr}_R \rho(t). \tag{2.4}$$

$\rho(t)$ satisfies the usual Liouville equation:

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] \equiv -i\mathcal{L}\rho, \quad \mathcal{L} \equiv [H, \;]. \tag{2.5}$$

We also assume that the initial state of the reservoir is such that

$$\mathrm{Tr}_R\{\rho_R(0) G_\alpha^{(R)}(t)\} = 0, \tag{2.6}$$

where $G_\alpha^{(R)}(t)$ is the operator $G_\alpha^{(R)}$ in the interaction picture. On using Eqs. (2.1)–(2.6) and the standard procedure,[22] we find that $\rho_S(t)$ in the Born, Markov, and long-time approximations satisfies

$$\frac{\partial \rho_S}{\partial t} = -\sum_{\alpha\beta} \frac{1}{2} \int\int d^3r_1\, d^3r_2 \int_0^\infty d\tau\, \{\langle\{G_\alpha^{(R)}(\vec{r}_1, \tau), G_\beta^{(R)}(\vec{r}_2, 0)\}\rangle [G_\alpha^{(S)}(\vec{r}_1, t), [G_\beta^{(S)}(\vec{r}_2, t - \tau), \rho_S(t)]]$$

$$+ \langle[G_\alpha^{(R)}(\vec{r}_1, \tau), G_\beta^{(R)}(\vec{r}_2, 0)]\rangle [G_\alpha^{(S)}(\vec{r}_1, t), \{G_\beta^{(S)}(\vec{r}_2, t - \tau), \rho_S(t)\}]\}, \tag{2.7}$$

where $\rho_S(t)$ is the density operator in the interaction picture. The brackets $\langle\cdots\rangle$ in (2.7) denote the ensemble average with respect to $\rho_R(0)$, i.e.,

$$\langle G_\alpha^{(R)}(\vec{r}_1, \tau) G_\beta^{(R)}(\vec{r}_2, 0)\rangle \equiv \mathrm{Tr}_R\{\rho_R(0) G_\alpha^{(R)}(\vec{r}_1, \tau) G_\beta^{(R)}(\vec{r}_2, 0)\}. \tag{2.8}$$

Equation (2.7) can be expressed in terms of the response function $\chi_{\alpha\beta}(\vec{r}_1, \vec{r}_2, \tau)$ which is the linear response of the reservoir variable $G_\alpha^{(R)}(\vec{r}_1)$ to an external perturbation of the form

$$H_{\exp} = -\sum_\alpha \int G_\alpha^{(R)}(\vec{r}, t) f_\alpha(\vec{r}, t)\, d^3r. \tag{2.9}$$

The fluctuation correlation $\langle\{G_\alpha^{(R)}(\vec{r}_1, \tau), G_\beta^{(R)}(\vec{r}_2, 0)\}\rangle$ is related to $\chi_{\alpha\beta}$ via the fluctuation-dissipation theorem. We rewrite (2.7) as

$$\frac{\partial \rho_S}{\partial t} = -\sum_{\alpha\beta} \int\int d^3r_1\, d^3r_2 \int_0^\infty d\tau\{S_{\alpha\beta}(\vec{r}_1, \vec{r}_2, \tau)[G_\alpha^{(S)}(\vec{r}_1, t), [G_\beta^{(S)}(\vec{r}_2, t - \tau), \rho_S(t)]]$$

$$+ \chi_{\alpha\beta}''(\vec{r}_1, \vec{r}_2, \tau)[G_\alpha^{(S)}(\vec{r}_1, t), \{G_\beta^{(S)}(\vec{r}_2, t - \tau), \rho_S(t)\}]\}, \tag{2.10}$$

where according to the fluctuation-dissipation theorem (I 2.10), the Fourier transforms of S and χ'' are related by

$$S_{\alpha\beta}(\vec{r}_1, \vec{r}_2, \omega) = \coth(\beta\omega/2)\chi''_{\alpha\beta}(\vec{r}_1, \vec{r}_2, \omega) .$$ (2.11)

From (2.10) it follows that the change in the system variable Q is given by

$$\frac{\partial\langle Q\rangle}{\partial t} = -\sum_{\alpha\beta} \int\int d^3r_1 d^3r_2 \int_0^\infty d\tau \{S_{\alpha\beta}(\vec{r}_1, \vec{r}_2, \tau)\langle[[Q, G_\alpha^{(s)}(\vec{r}_1, t)], G_\beta^{(s)}(\vec{r}_2, t-\tau)]\rangle$$

$$+ \chi''_{\alpha\beta}(\vec{r}_1, \vec{r}_2, \tau)\langle\{[Q, G_\alpha^{(s)}(\vec{r}_1, t)], G_\beta^{(s)}(\vec{r}_2, t-\tau)\}\rangle\} .$$ (2.12)

Equations (2.10)–(2.12) are valid generally. We now specialize to the problem of the atomic or molecular system interacting with the radiation fields. We consider only the dipole transitions. The method, of course, is also applicable to magnetic dipole as well as to higher multipole transitions. The interaction Hamiltonian in the dipole approximation has the form[23]

$$H_{RS} = -\sum_\alpha p_\alpha(t)E_\alpha(\vec{b}, t) ,$$ (2.13a)

where $p_\alpha(t)$ stands for the atomic polarization and

$\vec{b}$ denotes the position of the atom. For the two-level atom case we will write

$$p_\alpha(t) = d_\alpha(S^+e^{i\omega t} + \text{H.c.}) = d_\alpha p ,$$ (2.13b)

and for the multilevel case

$$p_\alpha(t) = \sum_{jl} d_\alpha^{jl}|j\rangle\langle l| ,$$ (2.13c)

where the d_α^{jl} are the dipole moment matrix elements. In (2.13b) the $S^\pm$ are the spin-angular-momentum operators corresponding to spin-$\frac{1}{2}$. On substituting (2.13b) in (2.10), we obtain

$$\frac{\partial\rho_S}{\partial t} = -\sum_{\alpha\beta} d_\alpha d_\beta \int_0^\infty d\tau \{S_{\alpha\beta}^{(s)}(\vec{b}, \vec{b}, \tau)[p(t), [p(t-\tau), \rho(t)]] + \chi''_{\alpha\beta EE}(\vec{b}, \vec{b}, \tau)[p(t), \{p(t-\tau), \rho(t)\}]\} ,$$ (2.14)

where we have introduced the following notation in this series of papers:

$$S_{\alpha\beta}^{(s)}(\vec{r}_1, \vec{r}_2, \tau) = \tfrac{1}{2}\langle\{E_\alpha(\vec{r}_1, \tau), E_\beta(\vec{r}_2, 0)\}\rangle ,$$

$$\chi''_{\alpha\beta EE}(\vec{r}_1, \vec{r}_2, \tau) = \tfrac{1}{2}\langle[E_\alpha(\vec{r}_1, \tau), E_\beta(\vec{r}_2, 0)]\rangle .$$ (2.15)

Using (2.12) and (2.13), we obtain a similar equation for $\langle Q\rangle$. It is clear from (2.12) and (2.13) that $\langle S^-\rangle$ obeys the equation

$$\left\langle\frac{dS^-}{dt}\right\rangle + [i(\omega + \Omega) + \gamma]\langle S^-\rangle = 0 ,$$ (2.16)

where in deriving (2.16) we made the rotating-wave approximation; i.e., we ignored the rapidly oscillating terms. γ and Ω are defined by

$$\gamma(\vec{b}, \omega) + i\Omega(\vec{b}, \omega) = 2\sum_{\alpha\beta} d_\alpha d_\beta \int_0^\infty d\tau\, e^{i\omega\tau} S_{\alpha\beta}^{(s)}(\vec{b}, \vec{b}, \tau) ,$$ (2.17)

and ω is the energy separation between two levels of the atom. We have discussed the dispersion characteristics of the one-sided Fourier transform, like the one appearing in (2.17), in Appendix A. From Eqs. (A7) and (A9) it follows that

$$\gamma(\vec{b}, \omega) = \sum_{\alpha\beta} d_\alpha d_\beta S_{\alpha\beta}^{(s)}(\vec{b}, \vec{b}, \omega) ,$$ (2.18)

$$\Omega(\vec{b}, \omega) = -\frac{1}{\pi} P \int_0^\infty d\omega_0\, \gamma(\vec{b}, \omega_0)[(\omega_0 - \omega)^{-1}$$

$$- (\omega_0 + \omega)^{-1}] .$$ (2.19)

On using (I 2.20) we can write (2.18) in terms of the response function as

$$\gamma(\vec{b}, \omega) = \sum_{\alpha\beta} d_\alpha d_\beta \coth(\beta\omega/2)\,\text{Im}\chi_{\alpha\beta EE}(\vec{b}, \vec{b}, \omega) .$$ (2.20)

It is clear from (2.16) that Ω represents the effective shift in the energy separation of the two levels. We will refer to Ω as the frequency shift of the two-level atom. Moreover γ represents the width of the excited state: It is the inverse of the lifetime of the excited state. It is interesting that γ is determined in terms of the symmetrized correlation function $S_{ij}^{(s)}$ rather than the normally ordered or antinormally ordered correlation function as was the case with the transition probabilities [cf. Eqs. (III 3.17) and (III 3.18)]. It should be noted that γ, in general, is temperature dependent (field dependent). We have thus been able to express the shifts and widths in terms of the appropriate response functions. The energy shift of the ground and excited states can also be expressed in terms of the response function. We have shown in II [Eq. (3.12)] that the energy shifts $\delta E^{(+)}$ and $\delta E^{(-)}$ of the excited and ground states are related by (restricting ourselves to the zero-temperature case)

$$\delta E^{(+)} = -\left(\delta E^{(-)} + \text{Re}\sum d_\alpha d_\beta \chi_{\alpha\beta EE}(\vec{b}, \vec{b}, \omega)\right) ,$$ (2.21)

and that by definition we have

$$\delta E^{(+)} - \delta E^{(-)} = \Omega(\vec{b}, \omega), \qquad (2.22)$$

and hence

$$\delta E^{(\mp)} = \mp \frac{1}{2}\left(\Omega(\vec{b}, \omega) \pm \mathrm{Re} \sum d_\alpha d_\beta \chi_{\alpha\beta EE}(\vec{b}, \vec{b}, \omega) \right). \qquad (2.23)$$

On substituting (2.19), (A3) in (2.23),

$$\delta E^{(\mp)} = -\frac{1}{\pi}\, \mathrm{P} \int_0^\infty d\omega_0\, \gamma(\vec{b}, \omega_0)(\omega_0 \pm \omega)^{-1}. \qquad (2.24)$$

The case of a multilevel atom can be treated similarly. For a multilevel atom whose spectrum is nondegenerate and unequidistant, one finds that the Lamb shift of the jth level is given by (restricting ourselves to zero temperatures)

$$\delta E_j = -\frac{1}{\pi} \sum_{l\alpha\beta} \mathrm{P} \int_0^\infty d\omega_0\, (\omega_0 + \omega_{lj})^{-1} d_\alpha^{jl} d_\beta^{lj}$$

$$\times \mathrm{Im}\chi_{\alpha\beta EE}(\vec{b}, \vec{b}, \omega_0), \qquad (2.25)$$

$$= -\frac{1}{2} \sum_{\alpha\beta l} d_\alpha^{jl} d_\beta^{lj}[\mathrm{Re}\chi_{\alpha\beta EE}(\vec{b}, \vec{b}, \omega_{lj})$$

$$+ \mathrm{Im}Q_{\alpha\beta EE}(\vec{b}, \vec{b}, \omega_{lj})], \qquad (2.26)$$

where Eqs. (A3) and (A11) have been used. In (2.25), d_α^{jl} is the jl element of the dipole moment operator. The result (2.25) can be obtained from the master equation (2.14) by using the same procedure which led to Eq. (15.8d) of Ref. 17.

Having expressed all the dampings and shifts in terms of the response functions, we now proceed to study the effect of the dielectric and conducting surfaces on lifetimes, energy shifts, etc. Before we do that, we demonstrate how (2.14)–(2.26) lead to the usual results for the case of an atom emitting in the entire free space.[24] For simplicity we consider only the case of zero temperature. χ_{ijEE} is now identical to the translationally invariant response computed in Sec. IV of I [cf. (I 4.7)]

$$\chi_{ijEE}^{(0)}(\vec{r}, \vec{r}', \omega) = \left(\frac{\omega^2}{c^2} \delta_{ij} + \frac{\partial^2}{\partial x_i \partial x_j} \right) \frac{e^{i(\omega/c)|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}. \qquad (2.27)$$

Assuming that the orientation of the dipole moment is random, we obtain from (2.20) and (2.27)

$$\gamma^{(0)} = \frac{|d|^2}{3}\left(\frac{3\omega^2}{c^2} + \nabla^2 \right) \frac{\sin(\omega/c)|\vec{r}-\vec{r}'|}{|\vec{r}-\vec{r}'|}\Bigg|_{r=r'}$$

$$= \frac{2}{3} \frac{|d|^2 \omega^3}{c^3}, \qquad (2.28)$$

which is equal to $\frac{1}{2}$ the usual Einstein A coefficient. On substituting (2.28) into (2.19) and (2.24), we will obtain various energy-shift terms which are to be renormalized. Moreover on combining (2.27) and (2.14) we will obtain the usual master equation [cf. (6.75) of Ref. 17].

III. WIDTHS AND SHIFTS OF THE ATOMIC LEVELS IN THE PRESENCE OF A DIELECTRIC INTERFACE

We now use the general formalism of Sec. II to obtain the lifetime of an atom, assumed to be a two-level atom, and the frequency shift in presence of a dielectric characterized by the dielectric function $\epsilon_0(\omega)$. For the sake of simplicity we assume the following geometrical arrangement. The dielectric occupies the semi-infinite domain $-\infty \leqslant z \leqslant 0$ and the atom is located in vacuum at the point $\vec{b}$ with $b_x = b_y = 0$, $b_z = b$. The shifts and widths depend on whether the dipole transition is parallel to the interface or perpendicular to the interface. Various physical entities will carry the subscript $\parallel$ or $\perp$ depending on the orientation of the dipole moment. Moreover we assume, in the parallel case, that the dipole moment is randomly oriented in the x-y plane, and hence we will also carry out an averaging over its orientation. We will throughout this section be restricted to the case of zero temperature. From Eq. (2.20) we obtain

$$\gamma_\parallel(\vec{b}, \omega) = \frac{1}{2}|d|^2 \mathrm{Im} \sum_{i=1}^2 \chi_{iiEE}(\vec{b}, \vec{b}, \omega), \qquad (3.1)$$

$$\gamma_\perp(\vec{b}, \omega) = |d|^2 \mathrm{Im}\chi_{zzEE}(\vec{b}, \vec{b}, \omega). \qquad (3.2)$$

The translationally invariant part $\chi^{(0)}$ of the response function would lead to the usual value $\gamma^{(0)}$. We denote by $\gamma^{(1)}$ the surface-dependent contribution

$$\gamma_\parallel(\vec{b}, \omega) = \gamma^{(0)} + \gamma_\parallel^{(1)}(\vec{b}, \omega),$$
$$\gamma_\perp(\vec{b}, \omega) = \gamma^{(0)} + \gamma_\perp^{(1)}(\vec{b}, \omega), \qquad (3.3)$$

$$\gamma_\parallel^{(1)}(\vec{b}, \omega) = \frac{1}{2}|d|^2 \mathrm{Im} \sum_{i=1}^2 \chi_{iiEE}^{(1)}(\vec{b}, \vec{b}, \omega), \qquad (3.4)$$

$$\gamma_\perp^{(1)}(\vec{b}, \omega) = |d|^2 \mathrm{Im}\chi_{zzEE}^{(1)}(\vec{b}, \vec{b}, \omega). \qquad (3.5)$$

The response functions needed to evaluate (3.4) and (3.5) are given by (II 3.3), (II 3.4), viz.

$$\chi_{ijEE}^{(1)}(\vec{r}, \vec{r}', \omega) = -\frac{i}{2\pi} \int \int \frac{du\, dv}{w} \hat{\chi}_{ij}(u, v, \omega) \exp[iu(x - x') + iv(y - y') + iw(z + z')],$$

$$w^2 = k_0^2 \epsilon - k_\parallel^2, \quad w_0^2 = k_0^2 \epsilon_0 - k_\parallel^2, \quad k_\parallel^2 = u^2 + v^2, \quad k_0 = \omega/c, \qquad (3.6)$$

with square roots defined so that $\mathrm{Im}\,w \geq 0$, $\mathrm{Im}\,w_0 \geq 0$ and with

$$\hat{\chi}_{11}(u, v, \omega) = \hat{\chi}_{22}(v, u, \omega)$$

$$= k_0^2 \frac{w_0 - w}{w_0 + w} - \frac{u^2}{\epsilon}\left(1 - \frac{2w\epsilon}{\epsilon_0 w + \epsilon w_0}\right),$$

$$\hat{\chi}_{12}(u, v, \omega) = \hat{\chi}_{21}(u, v, \omega) = -\frac{uv}{\epsilon}\left(1 - \frac{2w\epsilon}{\epsilon_0 w + \epsilon w_0}\right),$$

$$\hat{\chi}_{13}(u, v, \omega) = -\hat{\chi}_{31}(u, v, \omega) = \frac{uw}{\epsilon}\frac{w\epsilon_0 - w_0\epsilon}{\epsilon_0 w + \epsilon w_0}, \quad (3.7)$$

$$\hat{\chi}_{23}(u, v, \omega) = -\hat{\chi}_{32}(u, v, \omega) = \frac{vw}{\epsilon}\frac{w\epsilon_0 - w_0\epsilon}{\epsilon_0 w + \epsilon w_0},$$

$$\hat{\chi}_{33}(u, v, \omega) = -\frac{k_{\parallel}^2}{\epsilon}\frac{w\epsilon_0 - w_0\epsilon}{\epsilon_0 w + \epsilon w_0}.$$

We have actually modified the response functions[25] given by (II 3.3) by assuming that the region $0 \leq z \leq \infty$ is occupied by a dielectric characterized by dielectric function $\epsilon(\omega)$. We will need these modified response functions at the end of this section.

On combining (3.4), (3.5), (3.6), and (3.7) with $\epsilon = 1$, and after making trivial change in variable of integration, we find that

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega) = \tfrac{3}{4}\gamma^{(0)}\,\mathrm{Re}\int_0^\infty \frac{\kappa\,d\kappa}{\mu}e^{i\mu x}$$

$$\times\left(\kappa^2 - \frac{2\mu\kappa^2}{\epsilon_0\mu + \mu_0} - 2(\mu - \mu_0)^2(\epsilon_0 - 1)^{-1}\right),$$

$$(3.8)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega) = \tfrac{3}{2}\gamma^{(0)}\,\mathrm{Re}\int_0^\infty \frac{\kappa^3\,d\kappa}{\mu}e^{i\mu x}\frac{\mu\epsilon_0 - \mu_0}{\mu\epsilon_0 + \mu_0}, \quad (3.9)$$

where

$$\mu^2 = 1 - \kappa^2, \quad \mu_0^2 = \epsilon_0 - \kappa^2, \quad x = (2\omega/c)b. \quad (3.10)$$

Equations (3.8) and (3.9) are our final expressions for the damping coefficients. Surface-dependent shifts can be obtained by substituting (3.8) and (3.9) into (2.19). We next discuss a number of special cases.

We first consider the case when the atom is emitting spontaneously in the vicinity of a perfectly conducting surface. On taking the formal limit of infinite conductivity, (3.8) and (3.9) reduce to

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega) = \tfrac{3}{4}\gamma^{(0)}\,\mathrm{Re}\int_0^\infty \frac{\kappa\,d\kappa}{\mu}(\kappa^2 - 2)e^{i\mu x}, \quad (3.11)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega) = \tfrac{3}{2}\gamma^{(0)}\,\mathrm{Re}\int_0^\infty \frac{\kappa^3\,d\kappa}{\mu}e^{i\mu x}. \quad (3.12)$$

Note that when $\kappa > 1$, then μ is pure imaginary and hence

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega) = \tfrac{3}{4}\gamma^{(0)}\int_0^1 \frac{\kappa\,d\kappa}{\mu}(\kappa^2 - 2)\cos\mu x, \quad (3.13)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega) = \tfrac{3}{2}\gamma^{(0)}\int_0^1 \frac{\kappa^3\,d\kappa}{\mu}\cos\mu x, \quad (3.14)$$

which on evaluation leads to

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega) = -\tfrac{3}{2}\gamma^{(0)}\left[\left(\frac{1}{x} - \frac{1}{x^3}\right)\sin x + \frac{\cos x}{x^2}\right], \quad (3.15)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega) = 3\gamma^{(0)}\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right). \quad (3.16)$$

There exists a close relationship between the surface-dependent damping coefficients for the case of a conductor and the free-space response functions. We have, in Appendix B, presented explicit expression for $\chi_{ij}^{(0)'}$ and $\chi_{ij}^{(0)''}$ as well as the one-sided Fourier transform of $\mathscr{E}_{ij}^{(S)}(\vec{r}, \vec{r}', t - t')$. From Eqs. (3.15), (3.16), and (B2), we find that

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega) = -\tfrac{1}{2}|d|^2\,\mathrm{Im}\sum_{i=1}^2 \chi_{iiEE}^{(0)}(\vec{b}, -\vec{b}, \omega), \quad (3.17)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega) = |d|^2\,\mathrm{Im}\chi_{zzEE}^{(0)}(\vec{b}, -\vec{b}, \omega). \quad (3.18)$$

Note that $\vec{b}$ and $-\vec{b}$ are mirror images of each other since $\vec{b}_x = \vec{b}_y = 0$. Equations (3.17) and (3.18) show that $\gamma^{(1)}$ is essentially determined from the free-space electromagnetic-field fluctuations at the points $\vec{b}$ and $-\vec{b}$. For very small distances $x \ll 1$,

$$\gamma_{\parallel}^{(1)} \sim \gamma^{(0)}(-1 + \tfrac{1}{5}x^2), \quad \gamma_{\perp}^{(1)} \sim \gamma^{(0)}(1 - \tfrac{1}{10}x^2). \quad (3.19)$$

The frequency shifts can be obtained by substituting (3.15) and (3.16) into (2.19) and calculating the resulting integrals. We can also use the results of Appendix B in view of the relationships (3.17) and (3.18). We find from (B10) and (B11) the following results for the frequency shifts:

$$\Omega_{\parallel}^{(1)}(\vec{b}, \omega) = (3\gamma^{(0)}/\pi)[1/x^2 + (\sin x\,\mathrm{Ci}x - \cos x\,\mathrm{Si}x)(1/x^3 - 1/x) - (1/x^2)(\cos x\,\mathrm{Ci}x + \sin x\,\mathrm{Si}x)], \quad (3.20)$$

$$\Omega_{\perp}^{(1)}(\vec{b}, \omega) = (6\gamma^{(0)}/\pi)[(1/x^3)(\sin x\,\mathrm{Ci}x - \cos x\,\mathrm{Si}x) - (1/x^2)(\cos x\,\mathrm{Ci}x + \sin x\,\mathrm{Si}x)]. \quad (3.21)$$

For the harmonic-oscillator model (discussed in Appendix C) the frequency shifts can be obtained by using (C5) and the response function (3.6). For the case of a perfect conductor such shifts can be eval-

uated in closed form and can also be related to the free-space response functions by using (3.17) and (3.18). The results of such a calculation are

$$\Omega_{\parallel}^{(-)(1)}(\vec{b}, \omega) = -\tfrac{1}{2}|d|^2 \, \mathrm{Re} \sum_{i=1}^{2} \chi_{iiEE}^{(1)}(\vec{b}, \vec{b}, \omega)$$

$$= \tfrac{1}{2}|d|^2 \, \mathrm{Re} \sum_{i=1}^{2} \chi_{iiEE}^{(0)}(\vec{b}, -\vec{b}, \omega)$$

$$= \frac{-3\gamma^{(0)}}{2}\left(\frac{\sin x}{x^2} + \frac{\cos x}{x^3} - \frac{\cos x}{x}\right), \quad (3.22)$$

$$\Omega_{\perp}^{(-)(1)}(\vec{b}, \omega) = -|d|^2 \, \mathrm{Re}\chi_{zzEE}^{(1)}(\vec{b}, \vec{b}, \omega)$$

$$= -|d|^2 \, \mathrm{Re}\chi_{zzEE}^{(0)}(\vec{b}, -\vec{b}, \omega)$$

$$= -3\gamma^{(0)}\left(\frac{\sin x}{x^2} + \frac{\cos x}{x^3}\right). \quad (3.23)$$

The close relation, as exhibited by (3.22) and (3.23), between the shifts for the oscillator model and the first-order dispersion forces between two atoms in free space should also be noted. For short distances (3.20)–(3.23) reduce to

$$\Omega_{\parallel}^{(1)}(\vec{b}, \omega) \sim (3\gamma^{(0)}/\pi)[-\ln x + \mathrm{const} + O(x)],$$

$$\Omega_{\perp}^{(1)}(\vec{b}, \omega) \sim \frac{-6\gamma^{(0)}}{\pi x^2}, \quad (3.24)$$

$$\Omega_{\parallel}^{(-)(1)}(\vec{b}, \omega) \sim -3\gamma^{(0)}/2x^3, \quad \Omega_{\perp}^{(-)(1)}(\vec{b}, \omega) \sim -3\gamma^{(0)}/x^3, \quad (3.25)$$

which show how different the behaviors of $\Omega^{(1)}$ and $\Omega^{(-)(1)}$ are at short distances. On introducing auxiliary functions, defined by[26]

$$f(x) = \tfrac{1}{2}\pi\cos x + (\mathrm{Ci}\,x\,\sin x - \cos x\,\mathrm{Si}\,x),$$

$$g(x) = \tfrac{1}{2}\pi\sin x - (\mathrm{Ci}\,x\,\cos x + \sin x\,\mathrm{Si}\,x), \quad (3.26)$$

we can rewrite (3.20) and (3.21) as

$$\Omega_{\parallel}^{(1)}(\vec{b}, \omega) = \Omega_{\parallel}^{(-)(1)}(\vec{b}, \omega) + \frac{3\gamma^{(0)}}{\pi}\left[\frac{1}{x^2} + \left(\frac{1}{x^3} - \frac{1}{x}\right)f + \frac{g}{x^2}\right], \quad (3.27)$$

$$\Omega_{\perp}^{(1)}(\vec{b}, \omega) = \Omega_{\perp}^{(-)(1)}(\vec{b}, \omega) + \frac{6\gamma^{(0)}}{\pi}\left(\frac{f}{x^3} + \frac{g}{x^2}\right), \quad (3.28)$$

and since for large distances[26]

$$f(x) \sim \frac{1}{x}\left(1 - \frac{2!}{x^2} + \frac{4!}{x^4}\cdots\right),$$

$$g(x) \sim \frac{1}{x^2}\left(1 - \frac{3!}{x^2} + \frac{5!}{x^4}\cdots\right), \quad (3.29)$$

hence it is clear from (3.27) and (3.28) that the large-distance behavior of Ω is dominated by the terms $\Omega_{\parallel}^{(-)(1)}$ and $\Omega_{\perp}^{(-)(1)}$, i.e.,

$$\Omega_{\parallel}^{(1)}(\vec{b}, \omega) \sim \Omega_{\parallel}^{(-)(1)}(\vec{b}, \omega) + 12\gamma^{(0)}/\pi x^4,$$

$$\Omega_{\perp}^{(1)}(\vec{b}, \omega) \sim \Omega_{\perp}^{(-)(1)}(\vec{b}, \omega) + 12\gamma^{(0)}/\pi x^4. \quad (3.30)$$

The surface-dependent contribution to the Lamb shift of the excited and ground states can be ob-

tained by substituting (3.20)–(3.23) into (2.24).

We next consider the case when the frequency ω happens to coincide with the frequency of one of the elementary excitations of the dielectric. These excitations are of three types[27]: (i) longitudinal, (ii) transverse, and (iii) surface excitations. In such cases it is easy to obtain the expressions for the damping coefficients. The expressions for the shifts are more involved, as one has to put in the dispersion of the dielectric function. We will also ignore the damping. When $\omega = \omega_t$ (one of the transverse excitations) then the damping coefficients are given by (3.17) and (3.18). When $\omega = \omega_l$ [longitudinal frequency $\epsilon(\omega_l) = 0$], then (3.8) and (3.9) lead to

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega_l) = \tfrac{3}{4}\gamma^{(0)} \, \mathrm{Re} \int_0^\infty \frac{\kappa\,d\kappa}{\mu} e^{i\mu x}(3\mu^2 - 1 - 2i\mu\kappa), \quad (3.31)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega_l) = -\tfrac{3}{2}\gamma^{(0)} \, \mathrm{Re} \int_0^\infty \frac{\kappa^3\,d\kappa}{\mu} e^{i\mu x}, \quad (3.32)$$

which on simplification reduce to

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega_l) = \tfrac{3}{4}\pi^{(0)}[J_1(x) + J_1''(x)]$$

$$+ \tfrac{3}{2}\gamma^{(0)}\left[3\frac{\cos x}{x^2} + \left(\frac{1}{x} - \frac{3}{x^3}\right)\sin x\right], \quad (3.33)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega_l) = -3\gamma^{(0)}\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right). \quad (3.34)$$

It is interesting to note that $\gamma_{\perp}^{(1)}(\vec{b}, \omega_l) = -\gamma_{\perp}^{(1)}(\vec{b}, \omega_t)$. For short distances one has

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega_l) \sim 0, \quad \gamma_{\perp}^{(1)}(\vec{b}, \omega_l) \sim -\gamma^{(0)}(1 - \tfrac{1}{10}x^2). \quad (3.35)$$

When $\omega = \omega_s$ [the surface plasmon frequency given by $\epsilon(\omega_s) = -1$; such surface modes exist only in dielectrics which are finite in extent], we find that (3.8) and (3.9) lead to

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega_s) = -\tfrac{3}{4}\gamma^{(0)} \int_0^1 d\mu\,[(1 - \mu^2)^2 \cos\mu x$$

$$- \mu(1 + \mu^2)(2 - \mu^2)^{1/2}\sin\mu x], \quad (3.36)$$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega_s) = -\tfrac{3}{2}\gamma^{(0)} \int_0^1 d\mu\,[(1 - \mu^2)^2 \cos\mu x$$

$$+ \mu(1 - \mu^2)(2 - \mu^2)^{1/2}\sin\mu x]. \quad (3.37)$$

It does not seem possible to do these integrals in closed form. To have some idea of the behavior of $\gamma^{(1)}(\vec{b}, \omega_s)$, we quote the limiting values for $x \to 0$

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega_s) \to -\tfrac{2}{5}\gamma^{(0)}, \quad \gamma_{\perp}^{(1)}(\vec{b}, \omega_s) \to -\tfrac{4}{5}\gamma^{(0)}. \quad (3.38)$$

In the general case, Eqs. (3.8) and (3.9) have to be integrated numerically. We defer all the numerical work to a later paper in this series. A

few numerical curves are presented in our earlier communication.[4]

We have so far considered the case when the atom was outside the dielectric. One could ask what happens if the atom were embedded inside the dielectric. This case is also covered by our formalism. We assume the following geometrical arrangement: The atom is located at $\vec{r} = \vec{b}$ inside the dielectric $\epsilon(\omega)$ which occupies the domain $0 \leq z \leq \infty$ and the region $-\infty \leq z \leq 0$ is the free space (vacuum). The translationally invariant response is now

$$\chi_{ijEE}^{(0)}(\vec{r}, \vec{r}', \omega) = \left(k_0^2 \delta_{ij} + \frac{1}{\epsilon} \frac{\partial^2}{\partial x_i \partial x_j} \right) \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r} - \vec{r}'|} . \quad (3.39)$$

The surface-dependent contribution is given by (3.7) with $\epsilon_0 = 1$. We consider a special case when $\omega = \omega_s$ $[\epsilon(\omega_s) = -1]$, then from (3.1)–(3.7) we have

$$\gamma_\perp(\vec{b}, \omega_s) = |d|^2 k_0^3 \int_0^1 \kappa^3 \, d\kappa (1 - \kappa^2)^{1/2} \exp[-x(1 + \kappa^2)^{1/2}] . \quad (3.40)$$

If we compare (3.40) with (3.37), we find that when the atom is embedded inside the dielectric then γ is a decaying function of x (no oscillatory dependence) in contrast to the case when the atom is outside whence the dependence is also damped but oscillatory. For short distances, (3.40) reduces to

$$\gamma_\perp(\vec{b}, \omega_s) = \frac{2}{15} |d|^2 k_0^3 (1 - \frac{5}{4} x + \frac{11}{7} x^2/2! \cdots)$$

$$\approx \frac{2}{15} |d|^2 k_0^3 \exp(-\frac{5}{4} x) . \quad (3.41)$$

$\gamma_\|(\vec{b}, \omega_s)$ shows a similar behavior.[28]

We now make a few remarks concerning the energy shifts and lifetimes associated with the states of a multilevel atom. We have seen [Eq. (3.24)] that in presence of a conductor $\Omega_\|^{(1)}$ has a logarithmic behavior $(\Omega_\perp^{(1)} \sim 1/x^2)$ in contrast to $\Omega^{(-)(1)}$ which has a $1/x^3$ behavior for small distances. The energy shift of each level will have a $1/x^3$ behavior as is obvious from (2.23), (3.24), and (3.25). The energy shifts for a multilevel atom will also have the $1/x^3$ behavior as is clear from (2.26). The leading term in (2.26) can be written as (considering only the surface-dependent part)

$$\delta E_j^{(1)} \sim \frac{1}{b^3} \sum_{\alpha\beta l} d_\alpha^{jl} d_\beta^{lj} = \frac{1}{b^3} \sum_{\alpha\beta} \langle j | d_\alpha d_\beta | j \rangle , \quad (3.42)$$

and hence

$$\delta E_i^{(1)} - \delta E_j^{(1)} \sim \frac{1}{b^3} \sum_{\alpha\beta} (\langle i | d_\alpha d_\beta | i \rangle - \langle j | d_\alpha d_\beta | j \rangle) , \quad (3.43)$$

showing that the frequency shift between ith and jth levels in general shows a $1/b^3$ behavior. For the two-level model the matrix elements $\langle + | d_\alpha d_\beta | + \rangle$

and $\langle - | d_\alpha d_\beta | - \rangle$ were identical and hence the $1/b^3$ term did not appear in $\Omega^{(1)}$. This is in accordance with what is well known: that the frequency shifts associated with a multilevel atom show a very different behavior from those for the two-level atom.[29,30] For large distances it is clear from (3.30), (3.22), (3.23), and (2.23) that $\delta E^{(-)} \sim 1/x^4$, whereas $\delta E^{(+)}$ will have the same asymptotic behavior as $\Omega^{(-)}$ which is different from a $1/x^4$ behavior. Similarly, the large-distance behavior of the surface-dependent energy shifts of a multilevel atom will be dominated by that of $\chi_{ijEE}'(\vec{b}, \vec{b}, \omega)$.

It is easily shown from (2.10) (cf. derivation in Ref. 17) that the diagonal matrix elements of ρ satisfy a Pauli-type equation,

$$\frac{\partial \rho_{ii}}{\partial t} = -2 \sum_k (\gamma_{ki} \rho_{ii} - \gamma_{ik} \rho_{kk}) , \quad (3.44)$$

where

$$\gamma_{ki} = \gamma_{ki}^{(0)} + \gamma_{ki}^{(1)}, \quad \gamma_{ki}^{(0)} = \frac{2}{3} (\omega_{ik}^3/c^3) |d_{ik}|^2 , \quad (3.45)$$

$$\gamma_{ki}^{(1)} = \sum_{\alpha\beta} d_\alpha^{ik} d_\beta^{ik*} \chi_{\alpha\beta EE}^{(1)''}(\vec{b}, \vec{b}, \omega_{ik}) , \quad (3.46)$$

and where allowed transitions correspond to $\omega_{ik} > 0$, $d_{ik} \neq 0$. It is clear from (3.46) and from our explicit calculations for a two-level atom that the lifetimes of different states of a multilevel atom undergo different types of changes. For instance it may happen that the lifetime of one particular state goes up whereas that of another state goes down. Relative changes depend on the relative orientations of the dipole-moment matrix elements connecting different states.

So far we have considered a dielectric medium which is nonmagnetic. We can similarly consider spontaneous emission in presence of a *magnetic body*, characterized by the permeability μ. We treat only the case of nonelectric bodies, $\epsilon = 1$, as we can use the response functions already computed in paper I. This follows from the duality of Maxwell equations (in the absence of surface charges and surface currents), i.e., Maxwell's equations are invariant under the transformations $\vec{E} \rightleftharpoons \vec{H}$, $\vec{P} \rightleftharpoons \vec{M}$, $\epsilon \rightleftharpoons \mu$, $k_0 \rightleftharpoons -k_0$. Hence the response function χ_{ijEE} for a magnetic medium is identical to χ_{ijHH} for a dielectric medium if we make the replacements $\epsilon \to \mu$, $k_0 \to -k_0$. We now have, in place of (I 5.43), (I 5.44), the equations

$$(\vec{K}_\| \times \vec{\mathscr{E}}^{(-)}) = \frac{ik_0^2}{2\pi w_0} (\vec{K}_\| \times \vec{p}_\|) \frac{w_0 \mu - w}{w_0 \mu + w} e^{-i\vec{K}_0 \cdot \vec{r}_0} , \quad (3.47)$$

$$(\vec{K}_\| \cdot \vec{\mathscr{E}}_\|^{(-)}) = \frac{-iw_0}{2\pi w_0} [k_\|^2 p_\perp - w_0 (\vec{K}_\| \cdot \vec{p}_\|)] \frac{w - w_0}{w + w_0} e^{-i\vec{K}_0 \cdot \vec{r}_0} , \quad (3.48)$$

$$w_0^2 = \omega^2/c^2 - k_{\parallel}^2, \quad w^2 = (\omega^2/c^2)\mu - k_{\parallel}^2, \tag{3.49}$$

for the geometrical arrangement: the magnetic body occupies $0 \leq z \leq \infty$, the atom is located at $\vec{r} = (0, 0, -b)$ in vacuum (the region $-\infty \leq z \leq 0$). The surface-dependent contributions to the damping coefficient of a two-level atom are obtained on combining (3.47)–(3.49), (3.4), and (3.5), the results being

$$\gamma_{\parallel}^{(1)} = \tfrac{3}{4}\gamma^{(0)} \,\mathrm{Re} \int_0^\infty \frac{\kappa \, d\kappa}{\lambda_0} \left(\frac{\lambda_0 \mu - \lambda}{\lambda_0 \mu + \lambda} + \frac{\lambda_0^2(\lambda - \lambda_0)}{\lambda + \lambda_0} \right) e^{i\lambda_0 x}, \tag{3.50}$$

$$\gamma_{\perp}^{(1)} = -\tfrac{3}{2}\gamma^{(0)} \,\mathrm{Re} \int_0^\infty \frac{\kappa^3 \, d\kappa}{\lambda_0} \frac{\lambda - \lambda_0}{\lambda + \lambda_0} e^{i\lambda_0 x}, \tag{3.51}$$

where

$$\lambda_0^2 = 1 - \kappa^2, \quad \lambda^2 = \mu - \kappa^2, \quad x = (2\omega/c)b. \tag{3.52}$$

These expressions compare very well with (3.8) and (3.9) for the case of a nonmagnetic dielectric medium. For the case of a magnetic conductor (3.50) and (3.51) reduce to

$$\gamma_{\parallel}^{(1)} = \tfrac{3}{2}\gamma^{(0)}\left[\left(\frac{1}{x} - \frac{1}{x^3}\right) \sin x + \frac{\cos x}{x^2} \right],$$
$$\gamma_{\perp}^{(1)} = -3\gamma^{(0)}\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right). \tag{3.53}$$

On comparison with (3.15) and (3.16), we find that the change for the case of a magnetic conductor is opposite in sign to that for an electric conductor. This was, of course, expected since the boundary conditions at the interface $z = 0$ are now very different. When $\epsilon \neq 1$, it is easily shown that

(3.50) and (3.51) are modified to

$$\gamma_{\parallel}^{(1)} = \tfrac{3}{4}\gamma^{(0)} \,\mathrm{Re} \int_0^\infty \frac{\kappa \, d\kappa}{\lambda_0} \left(\frac{\lambda_0 \mu - \lambda}{\lambda_0 \mu + \lambda} + \frac{\lambda_0^2(\lambda - \lambda_0 \epsilon)}{\lambda + \lambda_0 \epsilon} \right) e^{i\lambda_0 x},$$

$$\gamma_{\perp}^{(1)} = -\tfrac{3}{2}\gamma^{(0)} \,\mathrm{Re} \int_0^\infty \frac{\kappa^3 \, d\kappa}{\lambda_0} \left(\frac{\lambda - \lambda_0 \epsilon}{\lambda + \lambda_0 \epsilon} \right) e^{i\lambda_0 x},$$

$$\lambda^2 = \mu\epsilon - \kappa^2. \tag{3.54}$$

Finally, spontaneous emission in presence of an anisotropic dielectric body is discussed in Appendix D.

IV. LIFETIME OF A TWO-LEVEL ATOM IN PRESENCE OF TWO CONDUCTING MIRRORS

As another problem in spontaneous emission in presence of surfaces we consider the change in the lifetime of a two-level atom placed between two perfectly conducting mirrors. We will see how Fabry-Perot modes contribute to the lifetimes. We assume that the conductors (with plane surfaces) are placed at $z = -d$ and $z = 0$ and the atom is located at $\vec{r} = \vec{b}$ ($b_x = b_y = 0$, $b_z = b$) in the space between the two conductors, which is assumed to be free space (vacuum). We again consider the two cases separately, namely, the dipole transition is (i) parallel to the interface, and (ii) perpendicular to z axis. We again assume, as before, that in the first case the orientation of the dipole moment in x-y plane is random. The damping coefficients would be given by (3.4) and (3.5). The response functions which appear in (3.4) and (3.5) can be obtained from Eqs. (I 5.33)–(I 5.37) and Eqs. (III 2.10)–(III 2.14):

$$\chi_{zzEE}^{(1)}(\vec{b}, \vec{b}, \omega) = \frac{-i}{2\pi} \int \int \frac{du \, dv}{w_0} D_0^{-1} k_{\parallel}^2 (2 + e^{2iw_0 b} + e^{-2iw_0 b - 2iw_0 d}), \tag{4.1}$$

$$\sum_{i=1}^2 \chi_{iiEE}^{(1)}(\vec{b}, \vec{b}, \omega) = \frac{i}{2\pi} \int \int \frac{du \, dv}{w_0} D_0^{-1}(k_0^2 + w_0^2)(e^{2iw_0 b} + e^{-2iw_0 b - 2iw_0 d} - 2), \tag{4.2}$$

with

$$D_0 = (1 - e^{-2iw_0 d}). \tag{4.3}$$

We have already discussed in paper III [following Eq. (III 2.17)] the sense in which integrals of the

type (4.1) and (4.2) are to be interpreted. In the limit of both conductors receding to infinity, (4.1) and (4.2) should vanish. On combining (3.5) and (4.1) and making a change of the integration variable, we obtain for $\gamma_{\perp}^{(1)}(\vec{b}, \omega)$

$$\gamma_{\perp}^{(1)}(\vec{b}, \omega) = -\tfrac{3}{2}\gamma^{(0)} \,\mathrm{Re} \int_0^1 d\alpha\,(1 - \alpha^2)(1 - e^{-2i\alpha d k_0})^{-1}(2 + e^{2i\alpha b k_0} + e^{-2i\alpha k_0(b+d)}) \tag{4.4}$$

$$= \tfrac{3}{2}\gamma^{(0)} \int_0^1 d\alpha\,(1 - \alpha^2)(1 + \cos 2b\,\alpha k_0)\sum_{-\infty}^{+\infty} e^{2indk_0\alpha} - \gamma^{(0)}, \tag{4.5}$$

which on using Poisson's summation formula[31] leads to

$$\gamma_{\perp}(\vec{b}, \omega) = \gamma^{(0)} + \gamma_{\perp}^{(1)}(\vec{b}, \omega)$$
$$= \frac{3\pi}{2k_0 d}\gamma^{(0)} + \frac{3\pi}{k_0 d}\gamma^{(0)} \sum_1^N \left(1 - \frac{\pi^2 n^2}{k_0^2 d^2}\right)\cos^2\left(\frac{b\pi n}{d}\right), \tag{4.6}$$

where N is the largest integer less than dk_0/π.

Similarly, on combining (3.4) and (4.2) we find that

$$\gamma_{\parallel}^{(1)}(\vec{b}, \omega) = \tfrac{3}{4}\gamma^{(0)} \, \mathrm{Re} \int_0^1 d\alpha \, (1 + \alpha^2)(1 - e^{-2i\alpha dk_0})^{-1} \, (e^{2i\alpha bk_0} + e^{-2i\alpha k_0(b+d)} - 2) \tag{4.7}$$

$$= \tfrac{3}{2}\gamma^{(0)} \int_0^1 d\alpha \, (1 + \alpha^2) \sin^2(b\,\alpha k_0) \sum_{-\infty}^{+\infty} \frac{\pi}{k_0 d} \, \delta\left(\alpha - \frac{\pi n}{dk_0}\right) - \gamma^{(0)}, \tag{4.8}$$

and hence

$$\gamma_{\parallel}(\vec{b}, \omega) = \gamma^{(0)} + \gamma_{\parallel}^{(1)}(\vec{b}, \omega)$$

$$= \frac{3\pi}{2k_0 d} \gamma^{(0)} \sum_1^N \left(1 + \frac{\pi^2 n^2}{k_0^2 d^2}\right) \sin^2\left(\frac{b\,\pi n}{d}\right). \tag{4.9}$$

If the orientation of the dipole moment were completely random, then

$$\gamma^{(1)}(\vec{b}, \omega) = \tfrac{1}{3}|d|^2 \, \mathrm{Im} \sum_{i=1}^3 \chi_{iiEE}(\vec{b}, \vec{b}, \omega)$$

$$= \tfrac{1}{3}\gamma_{\perp}^{(1)}(\vec{b}, \omega) + \tfrac{2}{3}\gamma_{\parallel}^{(1)}(\vec{b}, \omega). \tag{4.10}$$

Hence on combining (4.6), (4.9), and (4.10) we have

$$\gamma(\vec{b}, \omega) = \frac{\pi}{2k_0 d} \gamma^{(0)} + \frac{\pi}{k_0 d} \gamma^{(0)} \sum_1^N \left[1 - \frac{\pi^2 n^2}{d^2 k_0^2} \cos\left(\frac{2b\,\pi n}{d}\right)\right]. \tag{4.11}$$

Equations (4.6), (4.9), and (4.11) are our final expressions for the damping coefficients[32] (inverse of the lifetimes) for the case of a two-level atom placed between two mirrors. It is easily verified that in the limit $d \to \infty$, (4.6) and (4.9) reduce to the results obtained in Sec. III. One has for example from (4.9) in the limit $d \to \infty$

$$\gamma_{\parallel}(\vec{b}, \omega) = \tfrac{3}{2}\gamma^{(0)} \int_0^1 d\alpha \, (1 + \alpha^2) \sin^2(b\,\alpha k_0)$$

$$= \gamma^{(0)} - \tfrac{3}{4}\gamma^{(0)} \int_0^1 d\alpha \, (1 + \alpha^2) \cos(\alpha x),$$

$$x = 2bk_0, \tag{4.12}$$

which is identical to (3.13). For $b \to 0$ (or $-d$), $\gamma_{\parallel} \to 0$, and

$$\gamma_{\perp}(\vec{b}, \omega)$$

$$\to \frac{3\pi}{2k_0 d} \gamma^{(0)} + \frac{3\pi}{k_0 d} \gamma^{(0)} N \left(1 - \frac{\pi^2}{6k_0^2 d^2}(N+1)(2N+1)\right). \tag{4.13}$$

Equation (4.13) shows that for $k_0 d \sim \pi$, $\gamma_{\perp} \sim \tfrac{3}{2}\gamma^{(0)}$, whereas for $k_0 d \gg \pi$, $\gamma_{\perp} \sim 2\gamma^{(0)}$.

To obtain the damping coefficients for the case of a magnetic conductor, we use the duality of Maxwell's equations and Eqs. (I 5.38)–(I 5.41). We now have for the electric-field response functions the relations

$$(\vec{K}_{\parallel} \cdot \vec{\mathcal{E}}_{\parallel}^{(-)}) = \frac{iw_0}{2\pi w_0} D_0^{-1}\{[k_{\parallel}^2 p_{\perp} - w_0(\vec{K}_{\parallel} \cdot \vec{p}_{\parallel})] e^{-i\vec{K}_0 \cdot \vec{r}_0 - 2iw_0 d} - [k_{\parallel}^2 p_{\perp} + w_0(\vec{K}_{\parallel} \cdot \vec{p}_{\parallel})]e^{-i\vec{K}_0' \cdot \vec{r}_0}\} = w_0 \mathcal{E}_z^{(-)}, \tag{4.14}$$

$$(\vec{K}_{\parallel} \cdot \vec{\mathcal{E}}_{\parallel}^{(+)}) = -w_0 \mathcal{E}_z^{(+)} = \frac{iw_0}{2\pi w_0} D_0^{-1}\{[k_{\parallel}^2 p_{\perp} - w_0(\vec{K}_{\parallel} \cdot \vec{p}_{\parallel})] e^{-i\vec{K}_0 \cdot \vec{r}_0} - [k_{\parallel}^2 p_{\perp} + w_0(\vec{K}_{\parallel} \cdot \vec{p}_{\parallel})]e^{-i\vec{K}_0' \cdot \vec{r}_0}\}, \tag{4.15}$$

$$(\vec{K}_{\parallel} \times \vec{\mathcal{E}}_{\parallel}^{(-)}) = \frac{-ik_0^2}{2\pi w_0} D_0^{-1}(\vec{K}_{\parallel} \times \vec{p}_{\parallel})(e^{-i\vec{K}_0' \cdot \vec{r}_0} + e^{-i\vec{K}_0 \cdot \vec{r}_0 - 2iw_0 d}), \tag{4.16}$$

$$(\vec{K}_{\parallel} \times \vec{\mathcal{E}}_{\parallel}^{(+)}) = \frac{-ik_0^2}{2\pi w_0} D_0^{-1}(\vec{K}_{\parallel} \times \vec{p}_{\parallel})(e^{-i\vec{K}_0 \cdot \vec{r}_0} + e^{-i\vec{K}_0' \cdot \vec{r}_0}), \tag{4.17}$$

and hence

$$\chi_{zzEE}^{(1)}(\vec{b}, \vec{b}, \omega) = \frac{-i}{2\pi} \int \int \frac{du\,dv}{w_0 D_0} k_{\parallel}^2 (2 - e^{+2iw_0 b} - e^{-2iw_0 b - 2iw_0 d}), \tag{4.18}$$

$$\sum_{i=1}^2 \chi_{iiEE}^{(1)}(\vec{b}, \vec{b}, \omega) = \frac{-i}{2\pi} \int \int \frac{du\,dv}{w_0 D_0} (k_0^2 + w_0^2)(2 + e^{2iw_0 b} + e^{-2iw_0 b - 2iw_0 d}). \tag{4.19}$$

On simplifying (4.18) and (4.19), we obtain for the damping coefficients

$$\gamma_{\parallel} = \frac{3\pi}{4k_0 d} \gamma^{(0)} + \frac{3\pi}{2k_0 d} \gamma^{(0)} \sum_1^N \left(1 + \frac{\pi^2 n^2}{k_0^2 d^2}\right)\cos^2\left(\frac{b\,\pi n}{d}\right), \tag{4.20}$$

$$\gamma_\perp = \frac{3\pi}{k_0 d} \gamma^{(0)} \sum_1^N \left(1 - \frac{\pi^2 n^2}{k_0^2 d^2}\right) \sin^2\left(\frac{b\pi n}{d}\right). \qquad (4.21)$$

For $b \to 0$ $(-d)$, we have, from (4.20) and (4.21), $\gamma_\perp \to 0$ and

$$\gamma_\parallel \to \frac{3\pi}{4k_0 d}\gamma^{(0)} + \frac{3\pi}{2k_0 d}\gamma^{(0)}N\left(1 + \frac{\pi^2(N+1)(2N+1)}{6k_0^2 d^2}\right).$$
$$(4.22)$$

V. RADIATION REACTION FIELDS

A very interesting concept in connection with spontaneous emission is that of radiation-reaction fields which have a c-number nature[5] or an operator nature[33-37] depending on the theory under consideration. It is the field seen by the atom. Such fields have a long history and have been computed by a number of authors[5, 33-37] for the case of an atom emitting spontaneously in free space (i.e., in the absence of dielectrics and conductors). The spontaneous emission has been considered to be totally due to the radiation-reaction fields or the

free fields or some combination of the two.[36] We have elsewhere[37] discussed the interrelations between free fields and the radiation-reaction fields. In the present section we discuss the form of such fields when the atom is emitting in presence of dielectrics.

The positive and the negative frequency parts of the electric field operator $\vec{E}$ are, in the Heisenberg picture, given by

$$\vec{E}^{(\pm)}(\vec{r}, t) = \exp(i \mathcal{L} t)\vec{E}^{\pm}(\vec{r}, 0), \qquad (5.1)$$

where the Liouville operator $\mathcal{L}$ is given by

$$\mathcal{L} = [H, \ ] = [H_0, \ ] - \sum_\alpha [p_\alpha E_\alpha(\vec{b}, 0), \ ] \qquad (5.2)$$

$$= \mathcal{L}_0 + \mathcal{L}_1. \qquad (5.3)$$

We have assumed that the atom is located at $\vec{b}$ and have denoted by $\vec{p}$ the atomic polarization. Or using the identity[38]

$$e^{i\mathcal{L}t} = e^{i\mathcal{L}_0 t} + \int_0^t d\tau\, e^{i\mathcal{L}\tau}(i\mathcal{L}_1)e^{i\mathcal{L}_0(t-\tau)}, \qquad (5.4)$$

we obtain

$$E_\beta^{(+)}(\vec{r}, t) = E_{0\beta}^{(+)}(\vec{r}, t) - i \sum_\alpha \int_0^t d\tau\, e^{i\mathcal{L}\tau}[p_\alpha(0)E_{0\alpha}(\vec{b}, 0), E_{0\beta}^{(+)}(\vec{r}, t - \tau)], \qquad (5.5)$$

where we have replaced $\vec{E}(b, 0)$ by $\vec{E}_0(b, 0)$ as the two are identical in the Schrödinger picture. In (5.5), E_0 is the "free-field operator," i.e., the electric field operator evolving according to the unperturbed Hamiltonian

$$\vec{E}_0^{(\pm)}(\vec{r}, t) = e^{i\mathcal{L}_0 t}\vec{E}_0^{(\pm)}(\vec{r}, 0). \qquad (5.6)$$

We note that $p_\alpha(0)$ commutes with $\vec{E}_0(\vec{r}, t)$ and hence can be taken outside the commutator. Moreover the commutator $[\vec{E}_0(\vec{b},0), \vec{E}_0^{(+)}(\vec{r}, t)]$ is a c number and is related to the response function in the following manner [cf. (III 5.15)]

$$\int [E_{0i}^{(+)}(\vec{R}, t), E_{0j}(\vec{r}, \tau)]e^{i\omega(t-\tau)}d(t-\tau)$$
$$= 2\chi_{ijEE}''(\vec{R}, \vec{r}, \omega)\eta(\omega), \qquad (5.7)$$

and hence

$$[E_{0i}^{(+)}(\vec{R}, t - \tau), E_{0j}(\vec{r}, 0)]$$
$$= \frac{1}{\pi}\int_0^\infty d\omega\, \chi_{ijEE}''(\vec{R}, \vec{r}, \omega)e^{-i\omega(t-\tau)}. \qquad (5.8)$$

In view of the above remarks, $e^{i\mathcal{L}\tau}$ in (5.5) acts only on $p_\alpha(0)$ giving rise to $p_\alpha(\tau)$ (atomic polarization in the Heisenberg picture) and hence

$$E_\beta^{(+)}(\vec{r}, t) = E_{0\beta}^{(+)}(\vec{r}, t) - i \int_0^t d\tau \sum_\alpha p_\alpha(\tau)[E_{0\alpha}(\vec{b}, 0), E_{0\beta}^{(+)}(\vec{r}, t - \tau)]. \qquad (5.9)$$

Finally on combining (5.8) and (5.9) we have

$$E_\beta^{(+)}(\vec{r}, t) = E_{0\beta}^{(+)}(\vec{r}, t) + \frac{i}{\pi}\int_0^t d\tau \sum_\alpha p_\alpha(\tau)\int_0^\infty d\omega \chi_{\beta\alpha EE}''(\vec{r}, \vec{b}, \omega)e^{-i\omega(t-\tau)}. \qquad (5.10)$$

From (5.10) it follows that the field at the position of the atom is given by

$$E_\beta^{(+)}(\vec{b}, t) = E_{0\beta}^{(+)}(\vec{b}, t) + E_{RR\,\beta}^{(+)}(\vec{b}, t), \qquad (5.11)$$

where $\vec{E}_{RR}$ is the radiation-reaction field operator defined by

$$E_{RR\,\beta}^{(+)}(\vec{b}, t) = \frac{i}{\pi}\sum_\alpha \int_0^t d\tau p_\alpha(t - \tau)$$
$$\times \int_0^\infty \chi_{\beta\alpha EE}''(\vec{b}, \vec{b}, \omega)e^{-i\omega\tau} d\omega. \qquad (5.12)$$

In view of the fact that the response function is

usually a sum of two contributions: the translationally invariant part and the surface-dependent part, we would have

$$E_{RR\,\beta}^{(+)}(\vec{b}, t) = E_{RR\,\beta}^{(0)(+)}(\vec{b}, t) + E_{RR\,\beta}^{(1)(+)}(\vec{b}, t) , \qquad (5.13)$$

where $\vec{E}_{RR\,\beta}^{(0)}$ is the usual free-space contribution whose value can be found from Refs. 33–35. The surface-dependent part is given by

$$E_{RR\,\beta}^{(1)(+)}(\vec{b}, t) = \frac{i}{\pi} \int_0^t d\tau \sum_\alpha p_\alpha(t - \tau)$$

$$\times \int_0^\infty d\omega\, \chi_{\beta\alpha EE}^{(1)\,\prime\prime}(\vec{b}, \vec{b}, \omega)e^{-i\omega\tau} . \qquad (5.14)$$

The total radiation-reaction field will be

$$E_{RR\,\beta}^{(1)}(\vec{b}, t) = \frac{i}{\pi} \int_0^t d\tau \sum_\alpha p_\alpha(t - \tau)$$

$$\times \int_0^\infty d\omega\, [\chi_{\beta\alpha EE}^{(1)\,\prime\prime}(\vec{b}, \vec{b}, \omega)e^{-i\omega\tau}$$

$$- \chi_{\beta\alpha EE}^{(1)\,\prime\prime}(\vec{b}, \vec{b}, \omega)e^{i\omega\tau}] ,$$

which on using the symmetry property (A1) reduces to

$$E_{RR\,\beta}^{(1)}(\vec{b}, t) = \frac{i}{\pi} \int_0^t d\tau \sum_\alpha p_\alpha(t - \tau)$$

$$\times \int_{-\infty}^{+\infty} d\omega\, \chi_{\beta\alpha EE}^{(1)\,\prime\prime}(\vec{b}, \vec{b}, \omega)e^{-i\omega\tau} \qquad (5.15)$$

$$= 2i \int_0^t d\tau \sum_\alpha p_\alpha(t - \tau)\chi_{\beta\alpha EE}^{(1)\,\prime\prime}(\vec{b}, \vec{b}, \tau) . \qquad (5.16)$$

The expressions (5.14)–(5.16) are exact, i.e., no approximation has been made as to the strength of the interaction. We have expressed radiation-reaction fields as a linear functional of the atomic polarization. All the above results are valid for any geometrical arrangement—all one has to do is to feed in the right response function. It is clear from (5.16) that the expression for the total radiation-reaction field is relatively simple. It should be noted that the radiation-reaction fields are *independent* of the initial state of the radiation field because these are determined in terms of χ'' which for the radiation problem does not depend on the state of the field. The free-field terms lead to the

state-dependent contribution. A simple argument also enables us to rewrite (5.14) as

$$E_{RR\,\beta}^{(+)}(\vec{b}, t) = \frac{i}{2\pi} \int_0^t d\tau \sum_\alpha p_\alpha(t - \tau)\{\chi_{\beta\alpha EE}^{\prime\prime}(\vec{b}, \vec{b}, \tau)$$

$$+ Q_{\beta\alpha EE0}(\vec{b}, \vec{b}, \tau)\} . \qquad (5.17)$$

The exact expressions are quite involved, and as usual we assume that the interaction is weak and that the retardation effects can be ignored; then (5.17) leads to a manageable expression. For the case of a two-level atom $[p_\alpha(t) = d_\alpha(S^+ e^{i\omega t} + \text{H.c.})]$, (5.17) reduces to

$$\vec{d} \cdot \vec{E}_{RR}^{(1)(+)}(\vec{b}, t) = -\tfrac{1}{2}(\Omega^{(1)} + \Omega^{(-)(1)})S^-$$

$$+ \tfrac{1}{2}(\Omega^{(1)} - \Omega^{(-)(1)})S^+ - i\gamma^{(1)} S^{(-)} \qquad (5.18)$$

and (5.16) leads to

$$\vec{d} \cdot \vec{E}_{RR}^{(1)}(\vec{b}, t) = i\gamma^{(1)}(S^- - S^+) - \Omega^{(-)(1)}(S^+ + S^-) , \qquad (5.19)$$

where $\Omega^{(-)}$ is defined by (C1), Ω by (2.19) and γ by (2.18), with $T = 0$ ($\beta = \infty$). The usual QED results can be obtained by using (5.18). The neoclassical results[5,17] for the case of atom emitting in presence of a dielectric can be obtained by using

$$\langle \vec{d} \cdot \vec{E}_{RR}^{(1)}(\vec{b}, t)\rangle = i\gamma^{(1)}\langle S^- - S^+\rangle - \Omega^{(-)(1)}\langle S^+ + S^-\rangle , \qquad (5.20)$$

as the reaction field.

VI. FAR-ZONE BEHAVIOR OF THE RADIATION FIELDS

In this section we discuss how the fields behave in the far zone and calculate some of the normally ordered correlation functions of the field operators, as these are the entities measured by a photodetector placed in the far zone. We have already obtained in Sec. V an exact expression, Eq. (5.10), for the positive-frequency part of the field operator at any point. We now specialize to the case of a two-level atom, ignore the retardation effects, make the Born approximation, and take the long-time limit. Then (5.10) reduces to

$$E_\beta^{(+)}(\vec{r}, t) \sim E_{0\,\beta}^{(+)}(\vec{r}, t) + i \sum d_\alpha \chi_{\beta\alpha EE}^{\prime\prime}(\vec{r}, \vec{b}, \omega)S^- + \frac{1}{\pi} \sum d_\alpha \int_0^\infty d\omega_0 \chi_{\beta\alpha EE}^{\prime\prime}(\vec{r}, \vec{b}, \omega_0)[(\omega_0 + \omega)^{-1}S^+ + (\omega_0 - \omega)^{-1}S^-] , \qquad (6.1)$$

which on using the relation (A3) can be written as

$$E_\beta^{(+)}(\vec{r}, t) \approx E_{0\,\beta}^{(+)}(\vec{r}, t) + \sum d_\alpha \chi_{\beta\alpha EE}(\vec{r}, \vec{b}, \omega)S^- + (S^+ - S^-)\frac{1}{\pi} \sum d_\alpha \int_0^\infty d\omega_0 \chi_{\beta\alpha EE}^{\prime\prime}(\vec{r}, \vec{b}, \omega_0)(\omega_0 + \omega)^{-1} . \qquad (6.2)$$

Next we use a relation which can be proved by using an analysis similar to that which led to (II 1.3):

$$\int_0^\infty d\omega_0 \, \chi_{\beta\alpha EE}''(\vec{r}, \vec{b}, \omega_0)(\omega_0 + \omega)^{-1} = \omega \int_0^\infty dx \, \chi_{\beta\alpha EE}(\vec{r}, \vec{b}, ix)(x^2 + \omega^2)^{-1} , \tag{6.3}$$

to express (6.2) as

$$E_\beta^{(+)}(\vec{r}, t) = E_{0\beta}^{(+)}(\vec{r}, t) + \sum_\alpha d_\alpha \chi_{\beta\alpha EE}(\vec{r}, \vec{b}, \omega) S^- + (S^+ - S^-) \sum_\alpha \omega d_\alpha \int_0^\infty dx \, \chi_{\beta\alpha EE}(\vec{r}, \vec{b}, ix)(x^2 + \omega^2)^{-1} . \tag{6.4}$$

The structure of the last term in (6.4) is similar to what one encounters in the theory of dispersion forces [cf. Eq. (II 3.5)], and it is known from the work of Casimir and Polder[39] that this term will make negligible contribution in the far zone. Hence in the far zone (6.4) can be approximated by

$$E_\beta^{(+)}(\vec{r}, t) \sim E_{0\beta}^{(+)}(\vec{r}, t) + \sum d_\alpha \tilde{\chi}_{\beta\alpha EE}(\vec{r}, \vec{b}, \omega) S^- , \tag{6.5}$$

where $\tilde{\chi}$ denotes the asymptotic value of χ. A similar formula holds for the magnetic-field operator:

$$H_\beta^{(+)}(\vec{r}, t) \sim H_{0\beta}^{(+)}(\vec{r}, t) + \sum d_\alpha \tilde{\chi}_{\beta\alpha HE}(\vec{r}, \vec{b}, \omega) S^- , \tag{6.6}$$

where $\chi_{\beta\alpha HE}$ denotes the response of the variable H_β to an applied polarization.

For the case of emission in free space, we have as is seen from (2.27) that the asymptotic expansion of $\chi^{(0)}$, for fixed b, is given by

$$\tilde{\chi}_{\beta\alpha EE}^{(0)}(\vec{r}, \vec{b}, \omega) \approx \frac{\omega^2}{c^2}(\delta_{\alpha\beta} - \hat{r}_\beta \hat{r}_\alpha)\frac{e^{i(\omega/c)(r - b\cos\theta)}}{r} , \tag{6.7}$$

where $\hat{r}$ is the unit vector in the direction $\vec{r}$. On substituting (6.7) in (6.5), we find that (letting $b = 0$)

$$E_\beta^{(+)}(\vec{r}, t) \sim E_{0\beta}^{(+)}(\vec{r}, t) + \frac{\omega^2}{c^2}[d_\beta - \hat{r}_\beta(\vec{d} \cdot \hat{r})]\frac{e^{i(\omega/c)r}}{r} S^- , \tag{6.8}$$

which is the standard result obtained in Ref. 17 by a different method. In that reference $E_\beta^{(+)}$ as defined by (6.4) was also evaluated and found to be[40]

$$\vec{d} \cdot \vec{E}^{(+)} = \vec{d} \cdot \vec{E}_0^{(+)} + i(\gamma_{ij} + i\Omega_{ij})S^-$$
$$+ [(S^+ - S^-)/\pi]\vec{d} \cdot \vec{\nabla} \times \vec{\nabla} \times [(\vec{d}/r)f(k_0 r)] , \tag{6.9}$$

where f is the auxiliary function defined by (3.26) and γ_{ij}, Ω_{ij} are defined by (8.2) with $\vec{r}_{ij}$ replaced by $\vec{r}$. Again since in the far zone $f \sim 1/x$ [Eq. (3.29)], the last term does not contribute and (6.9) reduces to (6.8).

For the dielectric problem (Sec. III) the response function has been expressed in the form of angular spectrum (3.6) (assume $\epsilon = 1$)

$$\chi_{ijEE}^{(1)}(\vec{r}, \vec{b}, \omega) = \frac{-i}{2\pi} \int \int \frac{du\, dv}{w} \hat{\chi}_{ij}(u, v, \omega)$$

$$\times \exp\{iux + ivy + iw(z + b)\},$$

$$z > 0 . \tag{6.10}$$

The asymptotic expansion of the angular spectrum is well known.[41] We keep b fixed and let r go to infinity in a fixed direction. Then

$$\tilde{\chi}_{ijEE}^{(1)}(\vec{r}, \vec{b}, \omega) \to -\frac{e^{ik_0(r + b\cos\theta)}}{r}$$

$$\times \hat{\chi}_{ij}(k_0\sin\theta\cos\varphi, k_0\sin\theta\sin\varphi, \omega) ,$$

$$\tag{6.11}$$

where φ, θ are the polar coordinates associated with the vector $\vec{r}$. It is clear from (6.7) and (6.11) that the power radiated in the far zone will show an interference pattern with respect to the distance of the atom from the interface.

In a recent experiment[18] the fields were measured in the left half-space. So we need to calculate $\chi^{(1)}$ for $z < 0$. This calculation is done using Eqs. (I 5.4), (I 5.9), and (I 5.42)–(I 5.44), and we quote the result:

$$\chi_{ijEE}(\vec{r}, \vec{b}, \omega) = \frac{i}{2\pi} \int \int \frac{du\, dv}{w_0} \hat{\chi}_{ij<}(u, v, \omega)$$

$$\times \exp(iux + ivy - iw_0 z + iwb),$$

$$z < 0 , \tag{6.12}$$

where

$$\hat{\chi}_{11<}(u, v, \omega) = \hat{\chi}_{22<}(v, u, \omega)$$

$$= 2w_0\left(\frac{k_0^2}{w + w_0} - \frac{u^2}{\epsilon_0 w + w_0 \epsilon}\right),$$

$$\hat{\chi}_{12<}(u, v, \omega) = \hat{\chi}_{21<}(v, u, \omega) = \frac{-2uvw_0}{\epsilon_0 w + w_0 \epsilon} ,$$

$$\hat{\chi}_{13<}(u,v,\omega) = \hat{\chi}_{23<}(v,u,\omega) = \frac{2uw_0^2}{\epsilon_0 w + w_0 \epsilon}$$

$$= \frac{w_0}{w}\,\hat{\chi}_{31<}(u,v,\omega) = \frac{w_0}{w}\,\hat{\chi}_{32<}(v,u,\omega),$$

$$\hat{\chi}_{33<}(u,v,\omega) = \frac{2k_\parallel^2 w_0}{\epsilon_0 w + \epsilon w_0}\;. \tag{6.13}$$

The asymptotic expansion of (6.12) will be $(z<0)$

$$\tilde{\chi}_{ijEE}(\vec{r},\vec{b},\omega) \cong \frac{\exp[ik r + ik_0 b(1-\epsilon_0 \sin^2\theta)^{1/2}]}{r}$$

$$\times \hat{\chi}_{ij<}\!\left(\frac{kx}{r},\frac{ky}{r},\omega\right),$$

$$k^2 = k_0^2 \epsilon_0,\quad \cos\theta = z/r \tag{6.14}$$

where we have now assumed that ϵ_0 is real and put $\epsilon = 1$. The last exponential in (6.14) is oscillatory

or decaying depending on whether $\theta < \theta_c$ or $\theta > \theta_c$ where θ_c is the critical angle defined by

$$\sin^2\theta_c = 1/\epsilon_0\;. \tag{6.15}$$

On combining (6.5) and (6.14) we find that the positive-frequency part of the electric-field operator in the far zone $(z<0)$ is given by

$$E_i^{(+)} \sim E_{0i}^{(+)} + \sum_j d_j S^- \hat{\chi}_{ij<}\!\left(\frac{kx}{r},\frac{ky}{r},\omega\right)\frac{1}{r}$$

$$\times \exp[ik r + ik_0 b(1-\epsilon_0 \sin^2\theta)^{1/2}]\;. \tag{6.16}$$

We now discuss the connection with the classical calculations of Carniglia, Mandel, and Drexhage.[18] In that work a polarizer was placed so that only the y component of the field was detected, and it was assumed that $\vec{r}$ lies in the x-z plane so that $v = 0$ $(y=0)$. Moreover since the vector $\vec{d}$ itself lies in x-y plane, Eqs. (6.13) and (6.16) lead to

$$E_2^{(+)} \sim E_{02}^{(+)} + S^- d_2 \frac{e^{ikr}}{r}\exp[ik_0 b(1-\epsilon_0\sin^2\theta)^{1/2}]\frac{2w_0 k_0^2}{w+w_0}\;. \tag{6.17}$$

Hence the normally ordered correlation function will be given by

$$\langle E_2^{(-)}(\vec{r},t)E_2^{(+)}(\vec{r},0)\rangle \approx \frac{1}{2}\frac{|d|^2}{r^2}\langle S^+(t)S^-(0)\rangle (4k_0^4)\left|\frac{w_0}{w+w_0}\right|^2,\quad (\theta<\theta_c)$$

$$\approx \frac{1}{2}\frac{|d|^2}{r^2}\langle S^+(t)S^-(0)\rangle (4k_0^4)\left|\frac{w_0}{w+w_0}\right|^2 \exp[-2bk_0(\epsilon_0\sin^2\theta - 1)^{1/2}],\quad (\theta>\theta_c). \tag{6.18}$$

The free-field terms do not contribute to (6.18) as the field is initially, in vacuum state. We have also carried out an averaging over the orientation of the dipole moment. The atomic correlation function can be obtained from (2.10) (cf. with our proof for the case of emission in free space[17]):

$$\langle S^+(t)S^-(0)\rangle = \exp(i\omega t - \gamma|t|)\;, \tag{6.19}$$

where ω now represents the renormalized frequency. The intensity will be given by

$$I(\theta,t) \approx \frac{2|d|^2 k_0^4}{r^2}e^{-2\gamma|t|}\frac{w_0^2}{(w+w_0)^2},\quad \theta<\theta_c$$

$$\approx \frac{2|d|^2 k_0^4}{r^2}\exp[-2\gamma|t| - 2k_0 b(\epsilon_0\sin^2\theta - 1)^{1/2}]$$

$$\times \frac{w_0^2}{w_0^2 + |w|^2}\,,\quad \theta>\theta_c. \tag{6.20}$$

The result (6.20) for the intensity is identical to the one obtained in Ref. 18 using a classical analysis, except for the factor $e^{-2\gamma|t|}$. The radiation damping effects were ignored in that work. We also comment briefly about the nature of the radiation emitted by the atom. It is known from the work of Lalor and Wolf[42] concerning the reflection

and transmission of electromagnetic waves on a medium whose dielectric function exceeds unity that (i) in any direction $\theta<\theta_c$ the transmitted wave is homogeneous and the incident wave is also homogeneous, (ii) in any direction $\theta_c<\theta<\pi/2$ the transmitted wave is homogeneous whereas the incident wave must be an evanescent wave with $1 \leqslant \sin^2\theta_{\rm inc} \leqslant \epsilon_0$, (iii) for incident angles such that $\sin^2\theta_{\rm inc} > \epsilon_0$ the transmitted waves are evanescent in nature. Hence it is clear that for $\theta<\theta_c$, I is entirely due to the homogeneous waves emitted by the atom whereas for $\pi/2 \geqslant \theta \geqslant \theta_c$ the contribution is due to the evanescent waves emitted by the atom such that $1 \leqslant \sin^2\theta_{\rm inc} \leqslant \epsilon_0$.

For the case of the atom embedded inside the dielectric (which we assume to be occupying right half-space), we have from (6.12) (for $\epsilon_0 = 1$)

$$\tilde{\chi}_{ijEE}(\vec{r},\vec{b},\omega) = \frac{1}{r}\exp[ik_0 r + ibk_0(\epsilon - \sin^2\theta)^{1/2}]$$

$$\times \hat{\chi}_{ij<}\!\left(\frac{k_0 x}{r},\frac{k_0 y}{r},\omega\right),\quad z<0\,, \tag{6.21}$$

and hence if the frequency of the atom is in the range in which ϵ is negative, then

$$\tilde{\chi}_{ijEE}(\vec{r}, \vec{b}, \omega) = \frac{1}{r} \exp[ik_0 r - bk_0(|\epsilon| + \sin^2\theta)^{1/2}]$$

$$\times \tilde{\chi}_{ij} \cdot \left(\frac{k_0 x}{r}, \frac{k_0 y}{r}, \omega\right), \qquad z < 0,$$

$$(6.22)$$

which show a decaying amplitude with respect to b for all directions in the left half-space.

VII. COHERENCE IN THE PRESENT MODEL VERSUS THE DICKE MODEL

We have seen how the presence of a dielectric or conductor leads to an increase or decrease in the lifetime of the excited states depending upon the dielectric function and the distance of the atom from the interface. This might be compared with Dicke's superradiance.[19] Dicke found that the radiation rate from a collection of identical atoms or molecules confined to a region whose dimensions are small compared to a wavelength, can be proportional to N^2 (N being the number of atoms) depending upon the initial excitation. We wish to point out that the coherence in the present model is due to the *coherence properties of the radiation field* in the new environment whereas in Dicke model it is due to *atomic coherence*.

In the Dicke model the radiation rate is given by

$$I = 2\omega\gamma^{(0)} \sum_{ij} \langle S_i^+ S_j^- \rangle$$

$$= 2\gamma^{(0)} \omega \left(\sum_i \langle S_i^+ S_i^- \rangle + \sum_{i \neq j} \langle S_i^+ S_j^- \rangle \right), \qquad (7.1)$$

where $\gamma^{(0)}$ is the free-space value of the damping. The radiation rate now depends on the mean value $\sum_{i \neq j} \langle S_i^+ S_j^- \rangle$. Thus enhancement or suppression depends on the atomic correlations. In the present model the effective damping (which is proportional to the radiation rate) is given by

$$\gamma(\vec{b}, \omega) = \sum_{ij} d_i d_j \mathcal{E}_{ij}^{(S)}(\vec{b}, \vec{b}, \omega)$$

$$= \gamma^{(0)} + \sum_{ij} d_i d_j \mathcal{E}_{ij}^{(S)(1)}(\vec{b}, \vec{b}, \omega), \qquad (7.2)$$

where $\mathcal{E}^{(S)(1)}$ is the extra contribution to the field coherence due to the presence of the dielectric. Such a contribution has already been computed in I. Thus there is a basic difference in the origin of coherence in the two models, though it is true of both models that the change could be positive or negative. From the point of view of radiation-reaction fields, it is true that in both models the atom sees a different field. In one case the change in the field is due to the dipole fields radiated by the other atoms. In the other case the change is due to the fact that the field radiated by a given atom is reflected from the dielectric interface and acts back on the atom. However, the operator character of the radiation-reaction fields is very different in two cases. Similar remarks can be made in connection with the far-zone behavior of the fields. We first note that the intensity of emission in the free space for the Dicke problem[19] (small sample) using perturbation theory will be

$$I(\theta, t) \approx I_0(\tfrac{1}{2}N + m)(\tfrac{1}{2}N - m + 1), \qquad I_0 = |d|^2 k_0^4 / r^2,$$

$$(7.3)$$

where it is assumed that the atomic system was initially excited to the Dicke state $|\tfrac{1}{2}N, m\rangle$. I_0 denotes the intensity due to a single atom. The enhancement property is a property of the initial state of the atomic system. For the dielectric problem the corresponding expression is given by (6.20), the enhancement property in the dielectric case being the property of the initial state of the radiation field. In general, as we have shown in III [Eq. (III 3.16)], the transition probability per unit time is given by

$$\gamma_{fi} = \sum_{mn} \int\int d^3r_1 \, d^3r_2 \langle \psi_i | P_{Am}(\vec{r}_1, 0) | \psi_f \rangle \langle \psi_f | P_{An}(\vec{r}_2, 0) | \psi_i \rangle [\chi''_{mn\,EE}(\vec{r}_1, \vec{r}_2, \omega_{if}) + \mathcal{E}_{mn}^{(S)}(\vec{r}_1, \vec{r}_2, \omega_{if})]. \qquad (7.4)$$

Here the terms in the square brackets depend on the coherence properties of the electromagnetic field and the rest of the terms on the atomic coherence and thus, in general, both the atomic correlations and field correlations affect the transition probabilities. One can also discuss Dicke type of superradiance in presence of dielectrics, and it is clear from (7.4) that one will have large changes in transition probabilities. The combined effects of atomic and field correlations on γ_{fi} will be discussed elsewhere.

VIII. METHOD OF IMAGES IN THE CALCULATION OF SHIFTS AND WIDTHS

In this section we discuss how the method of images[20, 21] has been used in the computation of lifetimes and Lamb shifts in presence of conductors and also discuss some of the inadequacies of this method. We first recall a few results concerning the spontaneous emission from a collection of N two-level atoms.[17] The reduced density operator for the atomic system satisfies the following master equation[43]:

$$\frac{\partial\rho}{\partial t} = -i\omega\sum_i [S_i^z, \rho] - i\sum_{i\neq j} \Omega_{ij}[S_i^+ S_j^-, \rho] - \sum_{ij}\gamma_{ij}(S_i^+ S_j^-\rho - 2S_j^-\rho S_i^+ + \rho S_i^+ S_j^-), \qquad (8.1)$$

where each two-level atom is described by the appropriate spin angular momentum operators. The parameters γ_{ij} and Ω_{ij} are given by[43]

$$\gamma_{ij} + i\Omega_{ij} = -\sum_{\alpha\beta} d_\alpha d_\beta \chi^{(0)}_{\alpha\beta EE}(\vec{r}_i, \vec{r}_j, \omega), \quad i \neq j, \tag{8.2}$$

where $\chi^{(0)}$ is the translationally invariant response function. γ_{ii} is nothing but $\gamma^{(0)}$. The frequency which appears in (8.1) is already renormalized. It is clear from (8.1) that the rate at which the system dissipates energy is given by

$$-\sum_i \frac{\partial}{\partial t} \langle S_i^z \rangle = \sum_{ij} 2\gamma_{ij} \langle S_i^+ S_j^- \rangle \equiv 2\gamma, \tag{8.3}$$

which of course depends on the initial state of the system.

We first consider how the image method has been used for a single two-level atom in presence of a conductor. One replaces the effect of the conductor by an image atom located at $(0, 0, -b)$. The state of the combined system is taken to depend on the orientation of the dipole moment. The initial state of the two atom system is taken to be

$$\psi_{\parallel}(0) = (1/\sqrt{2})(|+-\rangle - |-+\rangle), \tag{8.4}$$

if the dipole moment is parallel to the surface $z = 0$ and is taken to be

$$\psi_{\perp}(0) = (1/\sqrt{2})(|+-\rangle + |-+\rangle), \tag{8.5}$$

if the dipole moment is perpendicular to the surface $z = 0$. Thus the emission from a two-level atom in presence of a mirror is taken to be equivalent to that from a system of two atoms with appropriate initial state. On substituting (8.4) and (8.5) in (8.3), we easily obtain

$$\gamma_{\parallel} = \gamma^{(0)} - \frac{1}{2}|d|^2 \sum_{i=1}^{2} \mathrm{Im}\chi^{(0)}_{iiEE}(\vec{b}, -\vec{b}, \omega)$$

$$= \gamma^{(0)} + \gamma_{\parallel}^{(1)}, \tag{8.6}$$

$$\gamma_{\perp} = \gamma^{(0)} + |d|^2 \mathrm{Im}\chi^{(0)}_{zzEE}(\vec{b}, -\vec{b}, \omega)$$

$$= \gamma^{(0)} + \gamma_{\perp}^{(1)}, \tag{8.7}$$

where (8.2) has been used. The results so obtained coincide with the results (3.17) and (3.18) obtained by using QED. Thus the above *ad hoc* assumptions (8.4) and (8.5) are good enough to compute the damping coefficients. The Lamb shift is, however, an entirely different matter. We see from the master equation (8.1) that the term

$$\Delta = \sum_{i \neq j} \Omega_{ij} S_i^+ S_j^- \tag{8.8}$$

corresponds to the energy-shift terms. The mean values of Δ, in the states (8.4) and (8.5), are given by

$$\Delta_{\parallel} = \frac{1}{2}|d|^2 \mathrm{Re} \sum_{i=1}^{2} \chi^{(0)}_{iiEE}(\vec{b}, -\vec{b}, \omega), \tag{8.9}$$

$$\Delta_{\perp} = -|d|^2 \mathrm{Re}\chi^{(0)}_{zzEE}(\vec{b}, -\vec{b}, \omega), \tag{8.10}$$

which on comparing with (3.22) and (3.23) lead to

$$\Delta_{\parallel} = \Omega_{\parallel}^{(-)(1)}, \quad \Delta_{\perp} = \Omega_{\perp}^{(-)(1)}, \tag{8.11}$$

which are very different from the frequency shift terms (3.27) and (3.28). We thus see that though the image method leads to the right value for the damping coefficients, it yields incorrect values for the frequency shifts. Hence the image argument fails for the shift terms which are due to the intrinsic nature of the spin-$\frac{1}{2}$ operators ($S^- S^+ + S^+ S^- = 1$ in the present case). One would have expected right from the start that the image argument will fail since the master equation involves the parameters (for $i \neq j$) which appear in the total reaction field whereas the Lamb shift is something which is determined[44] from the positive and negative frequency parts of the reaction field. There is, of course, as such no *a priori* justification for taking (8.4) and (8.5) as the initial state of the equivalent system. Moreover, it is not clear how the image method could be generalized to the case of dielectrics. Quantum electrodynamic calculations in a sense partially justify (8.4) and (8.5) as far as the calculations of the lifetimes are concerned. It is clear that the equivalence cannot be complete. We have already seen the problems which arise in connection with the Lamb shift. What one really hopes to achieve in the image method is to replace the coherence characteristics of the field in the changed environment with the coherence characteristics in the old environment but with the atomic correlations changed. From Eqs. (8.6)–(8.11) it is clear that the above task can only be partially accomplished.

Similar remarks apply to the case of an atom emitting spontaneously between two conducting mirrors. Again our quantum electrodynamic results (4.6) and (4.9) justify the results obtained in Ref. 21 by the image method.

IX. SURFACE POLARITON CONTRIBUTION TO THE WIDTH OF THE EXCITED STATES

We have calculated in Sec. IV of the lifetime of the excited state of a two-level atom in presence of two conducting mirrors. We found that the radiation damping was essentially due to the Fabry-Perot modes. The response functions (4.1) and (4.2) contain the resonant denominator, the vanish-

ing of which gives the dispersion relations for the Fabry-Perot modes:

$$D_0 = 0 \Rightarrow w_0 = n\pi/d = (k_0^2 - k_{\parallel}^2)^{1/2} , \qquad (9.1)$$

where n is an integer. Only modes with $n < k_0 d/\pi$ contributed to the radiation dampings. One has a similar situation in dielectrics. In a dielectric, which is finite in extent, both surface modes as well as bulk modes could be excited. The modes of a dielectric are usually referred to as the polariton modes[27] (plasmon modes in the context of a metal). As discussed in Ref. 2, the response functions contain a resonant denominator, the vanishing of which gives the dispersion relations for bulk and surface modes.[46,47] The dispersion relations for the surface modes depend on the geometry under consideration. For the geometry considered in Sec. III, the surface polariton modes are given by

$$\mu\epsilon_0 + \mu_0 = 0 \Rightarrow \kappa^2 + \mu\mu_0 = 0 \Rightarrow \kappa^2 = \epsilon_0/(\epsilon_0 + 1) . \quad (9.2)$$

It is clear from (9.2) that such modes occur at frequencies for which $\epsilon_0 \leq -1$ and μ, μ_0 are pure imaginary ($\kappa > 1$, ϵ_0 assumed to be real). Hence for the excitation of surface polariton modes to take place, the energy separation ω between the two levels of the atom should be such that $\epsilon_0(\omega)$ ≤ -1. From now on we assume that $\epsilon_0(\omega) \leq -1$. The expressions (3.8) and (3.9) for the dampings involve integrations over κ from zero to infinity. We break the integration region in two parts, $0 \leq \kappa \leq 1$, $\infty \geq \kappa \geq 1$. It is clear from (9.2) that the surface polariton contributions, which we will denote by Γ^{sp}, come from the part $1 \leq \kappa \leq \infty$. We will focus our attention only on this part. From (3.8) and (3.9) we obtain

$$\Gamma_{\parallel}^{sp} = -\tfrac{3}{2}\gamma^{(0)} \int_1^{\infty} \frac{\kappa^3 d\kappa}{|\mu|} e^{-|\mu||x|} \mathrm{Im}\left(\frac{\mu}{\epsilon_0\mu + \mu_0}\right) ,$$
$$(9.3)$$

$$\Gamma_{\perp}^{sp} = \tfrac{3}{2}\gamma^{(0)} \int_1^{\infty} \frac{\kappa^3 d\kappa}{|\mu|} e^{-|\mu||x|} \mathrm{Im}\left(\frac{\mu\epsilon_0 - \mu_0}{\mu\epsilon_0 + \mu_0}\right) . \quad (9.4)$$

In view of the resonant character of the denominator $\mu\epsilon_0 + \mu_0$, one can show that

$$\mathrm{Im}\left(\frac{\mu\epsilon_0 - \mu_0}{\mu\epsilon_0 + \mu_0}\right) = \delta(\kappa - \kappa_0)\left(\frac{-2\pi\epsilon_0^{3/2}}{(\epsilon_0 - 1)(\epsilon_0 + 1)^{3/2}}\right),$$
$$(9.5)$$

where κ_0 is given by (9.2). On combining (9.3)–(9.5) we obtain

$$\Gamma_{\parallel}^{sp} = (1/2|\epsilon_0|)\Gamma_{\perp}^{sp}$$
$$= \tfrac{3}{2}\pi\gamma^{(0)} \kappa_0^4 e^{-\sigma_0 x} (|\epsilon_0| + 1)^{-1}(|\epsilon_0| - 1)^{-1/2},$$
$$\sigma_0^2 = \kappa_0^2 - 1 . \quad (9.6)$$

It is clear that $2\Gamma^{sp}$ is identical to the transition

probability per unit time for the atom to decay to the ground state with the excitation of a surface polariton.

To have some idea of the order of magnitude of the surface polariton contribution to the widths, consider $\Gamma_{\parallel}^{sp}$ for the case of the free electron model of a metal, $\epsilon_0(\omega) = 1 - \omega_p^2/\omega^2$, and for ω_p/ω $= 1.5$. In this case $\Gamma_{\parallel}^{sp}/\gamma^{(0)} \sim 100 \exp(-\tfrac{8}{3}b\omega_p/c)$ ≈ 90 for $2bw_p/c = 0.075$. Let us denote by $R_{\parallel}$ $(R_{\perp})$ the integrated probability $R_{\parallel} = \int_0^{\infty} db \, \Gamma_{\parallel}^{sp}(b)$; then

$$R_{\parallel} = \frac{1}{2|\epsilon_0|} R_{\perp} = \frac{3\pi}{4} \frac{\gamma^{(0)}}{k_0} \kappa_0^4 (|\epsilon_0| + 1)^{-1} . \quad (9.7)$$

This integrated probability can be obtained from an experiment of the type done by White and Tolk[48] as it is closely connected with the probability that the atom escapes to infinity without radiationless deexcitation. We have checked by computer calculations that the contribution of the integral $\int_0^1$ in (3.8) for $\epsilon_0(\omega) \leq -1$ is negligible compared to Γ^{sp}.

One can similarly consider the surface polariton contribution to the damping in presence of a uniaxial crystal. We have obtained in Appendix D the full expressions for the damping coefficients. We have, for example, from (D12) the following expression for the probability that the atom decays with the emission of a surface polariton,

$$\Gamma_{\perp}^{sp} = \tfrac{3}{2}\gamma^{(0)} \mathrm{Re} \int_1^{\infty} \frac{\kappa^3 d\kappa}{\lambda_0} e^{i\lambda_0 x}\left(\frac{\lambda_0\epsilon_1 - \lambda_3}{\lambda_0\epsilon_1 + \lambda_3}\right) . \quad (9.8)$$

The surface polaritons occur at frequencies and wave vectors such that[47]

$$\lambda_0\epsilon_1 + \lambda_3 = 0 \Rightarrow \kappa^2 = \frac{\epsilon_3(\epsilon_1 - 1)}{\epsilon_1\epsilon_3 - 1} . \quad (9.9)$$

We ignore the damping from the dielectric function and then it is clear that surface polaritons occur at frequencies for which ϵ_1 is negative. There are two types of surface polaritons depending on the sign of ϵ_3: (i) If ϵ_3 is negative, then (9.9) can be written as

$$\kappa^2 = \frac{|\epsilon_3|(|\epsilon_1| + 1)}{|\epsilon_1||\epsilon_3| - 1} ; \quad (9.10)$$

(ii) and if ϵ_3 is positive, then

$$\kappa^2 = \frac{\epsilon_3(|\epsilon_1| + 1)}{|\epsilon_1|\epsilon_3 + 1} . \quad (9.11)$$

In case (ii) one should have $\epsilon_3 > 1$ and $1 \leq \kappa^2 \leq \epsilon_3$. A simple analysis shows that

$$\mathrm{Im}\left(\frac{\lambda_0\epsilon_1 - \lambda_3}{\lambda_0\epsilon_1 + \lambda_3}\right) = \delta(\kappa - \kappa_0)\frac{2\pi|\epsilon_1||\epsilon_3|(|\epsilon_3| + 1)}{\kappa_0(|\epsilon_1||\epsilon_3| - 1)^2} , \quad \text{case (i)},$$

$$= \delta(\kappa - \kappa_0)\frac{2\pi|\epsilon_1|\epsilon_3(\epsilon_3 - 1)}{\kappa_0(|\epsilon_1|\epsilon_3 + 1)^2} , \quad \text{case (ii)} ,$$

$$(9.12)$$

where κ_0 is the root of (9.10) and (9.11). On sub-
stituting (9.12) in (9.8) we obtain, for the surface
polariton contribution, the following expression:

$$\Gamma_{\perp}^{sp} = 3\pi\gamma^{(0)}\kappa_0^4 e^{-\lambda_0 x}\, \alpha, \quad \lambda_0^2 = \kappa_0^2 - 1 \ , \qquad (9.13)$$

where

$$\alpha = \frac{|\epsilon_1|\,(|\epsilon_3| + 1)^{1/2}}{(|\epsilon_1| + 1)(|\epsilon_1||\epsilon_3| - 1)^{1/2}} \ , \quad \text{case (i)},$$

$$= \frac{|\epsilon_1|\,(\epsilon_3 - 1)^{1/2}}{(|\epsilon_1| + 1)(|\epsilon_1|\,\epsilon_3 + 1)^{1/2}} \ , \quad \text{case (ii)} . \qquad (9.14)$$

We will now consider the surface polariton con-
tribution with the spatial dispersion of the dielec-
tric function included. For simplicity we will
ignore the retardation effects for the reason that
the dispersion relations for the surface polaritons
with k-dependent dielectric function are extremely
cumbersome[49] when retardation is included. For
the case of the spatially dispersive dielectric oc-
cupying $-\infty \leqslant z \leqslant 0$ and the atom in the free space
$0 \leqslant z \leqslant \infty$, the response function was found to be
[cf. Eq. (I 6.30)]

$$\chi_{ijEE}^{(1)}(\vec{r}, \vec{r}_0, \omega) = \frac{i}{2\pi}\frac{\partial^2}{\partial x_i \partial x_{0j}}\int\int \frac{du\,dv}{w_1}\frac{\psi - 1}{\psi + 1}\exp[iu(x - x_0) + iv(y - y_0) + iw_1(z + z_0)] . \qquad (9.15)$$

The form of ψ depends on the model of the dielec-
tric function and the assumed behavior at $z = 0$.
For the electron gas in the hydrodynamic approxi-
mation and diffuse type of boundary condition ψ is
given by Eq. (II 5.13). We depart from our earlier
treatment and assume specular reflection at $z = 0$.
The expression for ψ in this case is simple and
can be obtained following the work of Ritchie and
Marusak.[50] It is found to be

$$\psi^{-1} = \frac{\kappa}{\pi}\int_{-\infty}^{+\infty}\frac{dk_3}{(k_3^2 + \kappa^2)\epsilon((k_3^2 + \kappa^2)^{1/2}, \omega)} \ . \qquad (9.16)$$

The surface polariton dispersion relation is given
by

$$\psi + 1 = 0 \ . \qquad (9.17)$$

It follows from (9.16) and the dispersion relations
for $\epsilon(\vec{K}, \omega)$ that

$$\psi(\kappa, i\omega) \geqslant 1 \ , \qquad (9.18)$$

which is of importance in the considerations of the
nature of dispersion forces. In paper II we found
the following expression for the dispersion force
between a spatially dispersive and a spatially non-
dispersive body [Eq. (II 5.12)]

$$F = \frac{\hbar}{2\pi^2}\int_0^\infty d\omega \int_0^\infty \kappa^2\,d\kappa\left(\frac{\epsilon_3 + 1}{\epsilon_3 - 1}\frac{\psi + 1}{\psi - 1}e^{2\kappa d} - 1\right)_{i\omega}^{-1} . \qquad (9.19)$$

The dispersion force, as given by (9.19), is al-
ways attractive in view of (9.18) and the inequality
$\epsilon_3(i\omega) \geqslant 1$, though the first term in the expansion of
F in powers of (effective mass)$^{-1/2}$ is repulsive
in nature.

The probability that the atom decays with the
emission of a surface polariton will now be given
by

$$\Gamma_{\perp}^{sp} = 2\Gamma_{\parallel}^{sp} = \int_0^\infty \kappa^2\,d\kappa\,e^{-2\kappa b}\,\mathrm{Im}\left(\frac{\psi - 1}{\psi + 1}\right) . \qquad (9.20)$$

We now approximate $(\psi - 1)/(\psi + 1)$ by assuming that

there exists only one branch of the dispersion
curve:

$$\frac{\psi - 1}{\psi + 1} \approx \frac{\alpha_\kappa}{\omega_\kappa^2 - \omega^2 - i\omega\Gamma_\kappa} \ . \qquad (9.21)$$

From (9.20) one has the relations (cf. Ref. 51)

$$\frac{\alpha_\kappa}{\omega_\kappa^2} = \lim_{\omega \to 0}\mathrm{Re}\left(\frac{\psi - 1}{\psi + 1}\right),$$

$$\frac{\alpha_\kappa}{\omega^2} = -\lim_{\omega \to \infty}\mathrm{Re}\left(\frac{\psi - 1}{\psi + 1}\right) ,$$

$$\mathrm{Re}\,\psi(\kappa, \omega_\kappa) + 1 \approx 0 \ , \qquad (9.22)$$

$$\Gamma_\kappa \approx \left.\frac{\mathrm{Im}\,\psi(\kappa, \omega)}{(\partial/\partial\omega)\mathrm{Re}\,\psi(\kappa, \omega)}\right|_{\omega_\kappa} ,$$

and hence

$$\Gamma_{\perp}^{sp} = \int_0^\infty d\kappa\,\kappa^2\,e^{-2\kappa b}\frac{\alpha_\kappa\omega\Gamma_\kappa}{\omega^2\Gamma_\kappa^2 + (\omega^2 - \omega_\kappa^2)^2} \qquad (9.23)$$

$$\approx \frac{\alpha_{\kappa_0}\kappa_0^2 e^{-2\kappa_0 b}}{\omega}\int_0^\infty \frac{d\kappa}{d\omega_\kappa}\,d\omega_\kappa\frac{\Gamma_{\kappa_0}/4}{(\omega - \omega_\kappa)^2 + \Gamma_{\kappa_0}^2/4}$$

$$\approx \frac{2\pi\alpha_{\kappa_0}\kappa_0^2 e^{-2\kappa_0 b}}{2\omega}\frac{d\kappa_0}{d\omega_{\kappa_0}} . \qquad (9.24)$$

On assuming a dielectric function of the form

$$\epsilon(k, \omega) = 1 + \frac{\chi}{k^2 - \mu^2} \ ,$$

$$\mu^2 = \frac{\omega^2}{\beta^2} \ , \quad \chi = \frac{\omega_p^2}{\beta^2} \ , \quad \beta^2 = \tfrac{3}{5}v_F^2 \ , \qquad (9.25)$$

(v_F is the Fermi velocity) and specular reflection
at $z = 0$, we find that

$$\kappa_0^2 = \tfrac{1}{4}\mu^2(1 + \epsilon_1)^2 \ , \quad \epsilon_1 = 1 - \chi/\mu^2 = 1 - \omega_p^2/\omega^2 \qquad (9.26)$$

and

$$\Gamma_{\perp}^{sp} \approx \frac{\pi\chi^2}{4\mu^3}\kappa_0^2 e^{-2\kappa_0 b} \ . \qquad (9.27)$$

In obtaining (9.26) and (9.27) we have ignored the damping of the dielectric function and thereby the damping of the surface polariton mode. The experimentally observable number will be

$$R_\perp = \frac{\pi\chi}{16}(1 - \epsilon_1^2) = \frac{\pi}{16}\frac{\omega_p^4}{\beta^2\omega^2}\left(2 - \frac{\omega_p^2}{\omega^2}\right). \quad (9.28)$$

The result (9.24) is similar to the one obtained by Tzoar and Gersten,[51] who used the quantization of the surface polariton field. The results (9.6) and (9.13), for the case when retardation is included, can also be obtained from an explicit quantization of the surface polariton field. We would present this elsewhere[52] as in this series we have avoided the explicit quantization of the electromagnetic field.

Finally the contributions to the Lamb shifts due to the excitation of surface polaritons can be obtained by substituting (9.6), (9.13), (9.24) in (2.25).

APPENDIX A: DIGRESSION ON SOME PROPERTIES OF THE ANTICOMMUTATORS OF THE FIELD OPERATORS AT TWO DIFFERENT SPACE-TIME POINTS

Here some properties of the anticommutators of the field operators at different space-time points are presented. Such properties are useful in the text [for example in the derivation of Eqs. (2.18) and (2.19)]. Recall that since $\vec{E}$ is an even variable under time reversal, χ'' satisfies the symmetry relation

$$\chi''_{ijEE}(\vec{r}, \vec{r}', \omega) = -\chi''_{ijEE}(\vec{r}, \vec{r}', -\omega)$$

$$= \chi''_{jiEE}(\vec{r}', \vec{r}, \omega). \quad (A1)$$

The real part of χ is related to χ'' by

$$\chi'_{ijEE}(\vec{r}, \vec{r}', \omega) = \frac{P}{\pi}\int_{-\infty}^{\infty} d\omega'(\omega' - \omega)^{-1}\chi''_{ijEE}(\vec{r}, \vec{r}', \omega'), \quad (A2)$$

which on using (A1) can be written as

$$\chi'_{ijEE}(\vec{r}, \vec{r}', \omega) = \frac{P}{\pi}\int_0^{\infty} d\omega'\,\mathrm{Im}\chi_{ijEE}(\vec{r}, \vec{r}', \omega')$$

$$\times [(\omega' - \omega)^{-1} + (\omega' + \omega)^{-1}]. \quad (A3)$$

The symmetrized correlation function $\mathcal{S}_{ij}^{(S)}$ defined by (2.15), is real and hence

$$\mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', \omega) = [\mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', -\omega)]^*. \quad (A4)$$

On combining (A4) with the relation

$$\mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', \omega) = \mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', -\omega), \quad (A5)$$

obtained from time-reversal invariance, we see that $\mathcal{S}_{ij}^{(S)}(\omega)$ is a real function. Let $Q_{ijEE}(\vec{r}, \vec{r}', \omega)$

be the one-sided Fourier transform of $2\mathcal{S}_{ij}^{(S)}$, i.e.,

$$Q_{ijEE}(\vec{r}, \vec{r}', \omega) = 2\int_0^{\infty} d\tau\, e^{i\omega\tau}\mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', \tau)$$

$$= Q'_{ijEE}(\vec{r}, \vec{r}', \omega) + iQ''_{ijEE}(\vec{r}, \vec{r}', \omega), \quad (A6)$$

where Q' and Q'' are, respectively, the real and imaginary parts of Q. It is clear from (A5) and (A6) that

$$Q'_{ijEE}(\vec{r}, \vec{r}', \omega) = \mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', \omega) \quad (A7)$$

and

$$Q''_{ijEE}(\vec{r}, \vec{r}', \omega) = -\frac{P}{\pi}\int_{-\infty}^{\infty} d\omega'\,(\omega' - \omega)^{-1}$$

$$\times Q'_{ijEE}(\vec{r}, \vec{r}', \omega'). \quad (A8)$$

On combining (A5), (A7), and (A8), we obtain

$$Q''_{ijEE}(\vec{r}, \vec{r}', \omega) = -\frac{P}{\pi}\int_0^{\infty} d\omega'\,\mathcal{S}_{ij}^{(S)}(\vec{r}, \vec{r}', \omega')$$

$$\times [(\omega' - \omega)^{-1} - (\omega' + \omega)^{-1}]. \quad (A9)$$

The relations presented above are valid for arbitrary stationary fields described by time-reversal-invariant ensembles. For thermal fields, (A9) becomes on using (I 2.20)

$$Q''_{ijEE}(\vec{r}, \vec{r}', \omega)$$

$$= -\frac{P}{\pi}\int_0^{+\infty} d\omega'\,\coth(\tfrac{1}{2}\beta\omega')\,\mathrm{Im}\chi_{ijEE}(\vec{r}, \vec{r}', \omega')$$

$$\times [(\omega' - \omega)^{-1} - (\omega' + \omega)^{-1}]. \quad (A10)$$

At zero temperature (A10) reduces to

$$Q''_{ijEE0}(\vec{r}, \vec{r}', \omega) = -\frac{P}{\pi}\int_0^{\infty} d\omega'\,\mathrm{Im}\chi_{ijEE}(\vec{r}, \vec{r}', \omega')$$

$$\times [(\omega' - \omega)^{-1} - (\omega' + \omega)^{-1}], \quad (A11)$$

where the additional subscript indicates the temperature. In view of the interpretation of Q'', as the frequency shift, we will write (A10) as

$$Q''_{ijEE}(\vec{r}, \vec{r}', \omega) = Q''_{ijEE0}(\vec{r}, \vec{r}', \omega) + Q''_{ijEET}(\vec{r}, \vec{r}', \omega), \quad (A12)$$

where Q''_{ijEET} is the temperature-dependent part of Q given by

$$Q''_{ijEET}(\vec{r}, \vec{r}', \omega)$$

$$= -\frac{2P}{\pi} \int_0^\infty d\omega' (e^{\beta\omega'} - 1)^{-1}$$

$$\times \operatorname{Im}\chi_{ijEE}(\vec{r}, \vec{r}', \omega')[(\omega' - \omega)^{-1} - (\omega' + \omega)^{-1}].$$

(A13)

It should be noted that in our problem of electro-magnetic field fluctuations χ', χ'' are independent of

the temperature. Furthermore Q''_{ijEE0} will in general show a very different behavior than χ'_{ijEE}.

APPENDIX B: COMPUTATION OF $Q_{ijEE}(\vec{r}, \vec{r}', \omega)$ FOR THE CASE OF ENTIRE FREE SPACE

Here we calculate the real and imaginary parts of Q for electromagnetic fields in the entire free space. The response function $\chi^{(0)}_{ijEE}(\vec{r}, \vec{r}', \omega)$ is given by Eq. (2.27) and $\mathcal{S}^{(S)}_{ij}(\vec{r}, \vec{r}', \omega)$ is obtained from (2.11) and (2.15). On carrying out all the differentiations in (2.27), we obtain for the response function[53]

$$\chi^{(0)\prime}_{ijEE}(\vec{r}, \vec{r}', \omega) = -k_0^3 \left[(\delta_{ij} - 3\hat{R}_i\hat{R}_j)\left(\frac{\sin k_0 R}{k_0^2 R^2} + \frac{\cos k_0 R}{k_0^3 R^3}\right) - (\delta_{ij} - \hat{R}_i\hat{R}_j)\frac{\cos k_0 R}{k_0 R} \right],$$

(B1)

$$\chi^{(0)\prime\prime}_{ijEE}(\vec{r}, \vec{r}', \omega) = k_0^3 \left[(\delta_{ij} - \hat{R}_i\hat{R}_j)\frac{\sin k_0 R}{k_0 R} + (\delta_{ij} - 3\hat{R}_i\hat{R}_j)\left(\frac{\cos k_0 R}{k_0^2 R^2} - \frac{\sin k_0 R}{k_0^3 R^3}\right) \right]$$

(B2)

and the symmetrized correlation function

$$\mathcal{S}^{(S)}_{ij}(\vec{r}, \vec{r}', \omega) = k_0^3 \coth(\tfrac{1}{2}\beta\omega)\left[(\delta_{ij} - \hat{R}_i\hat{R}_j)\frac{\sin k_0 R}{k_0 R} + (\delta_{ij} - 3\hat{R}_i\hat{R}_j)\left(\frac{\cos k_0 R}{k_0^2 R^2} - \frac{\sin k_0 R}{k_0^3 R^3}\right) \right],$$

(B3)

where $\hat{R}_i$ denotes the ith component of the unit vector $\vec{R}/R = (\vec{r} - \vec{r}')/|\vec{r} - \vec{r}'|$. For $R \to 0$, (B2) and (B3) reduce to

$$\chi^{(0)\prime\prime}_{ijEE}(\vec{r}, \vec{r}, \omega) = \tfrac{2}{3}k_0^3\delta_{ij}, \quad \mathcal{S}^{(S)}_{ij}(\vec{r}, \vec{r}, \omega) = \tfrac{2}{3}k_0^3\coth\tfrac{1}{2}\beta\omega\delta_{ij}.$$

(B4)

$\chi^{(0)\prime}$ diverges as $R \to 0$, the meaning of it is discussed in Appendix C. We first calculate $Q''_{ijEE0}(\vec{r}, \vec{r}', \omega)$. On substituting (B2) in (A11) we obtain

$$Q''_{ijEE0}(\vec{r}, \vec{r}', \omega) = -\frac{2k_0}{\pi} P \int_0^\infty \frac{k^3\, dk}{k^2 - k_0^2}\left[(\delta_{ij} - \hat{R}_i\hat{R}_j)\frac{\sin kR}{kR} + (\delta_{ij} - 3\hat{R}_i\hat{R}_j)\left(\frac{\cos kR}{k^2 R^2} - \frac{\sin kR}{k^3 R^3}\right) \right]$$

$$= -\frac{2k_0}{\pi}(\delta_{ij} - \hat{R}_i\hat{R}_j)\frac{1}{R}\frac{\partial^2}{\partial R^2}\frac{1}{k_0}[\sin(k_0 R)\operatorname{Ci}(k_0 R) - \cos(k_0 R)\operatorname{Si}(k_0 R)]$$

$$+ \frac{2k_0}{\pi}(\delta_{ij} - 3\hat{R}_i\hat{R}_j)\frac{1}{R^2}[\cos(k_0 R)\operatorname{Ci}(k_0 R) + \sin(k_0 R)\operatorname{Si}(k_0 R)]$$

$$- \frac{2k_0}{\pi}(\delta_{ij} - 3\hat{R}_i\hat{R}_j)\frac{1}{k_0 R^3}[\sin(k_0 R)\operatorname{Ci}(k_0 R) - \cos(k_0 R)\operatorname{Si}(k_0 R)],$$

(B5)

where Ci and Si are cosine and sine integrals defined by[26]

$$\operatorname{Si}(\alpha) = \int_0^\alpha \frac{\sin k}{k}\, dR, \quad \operatorname{Ci}(\alpha) = -\int_\alpha^\infty \frac{\cos k}{k}\, dk.$$

(B6)

On using the relations

$$\frac{\partial}{\partial\alpha}\operatorname{Si}(\alpha) = \frac{\sin\alpha}{\alpha}, \quad \frac{\partial^2}{\partial\alpha^2}\operatorname{Si}(\alpha) = \frac{\cos\alpha}{\alpha} - \frac{1}{\alpha^2}\sin\alpha, \quad \frac{\partial}{\partial\alpha}\operatorname{Ci}(\alpha) = \frac{\cos\alpha}{\alpha}, \quad \frac{\partial^2}{\partial\alpha^2}\operatorname{Ci}(\alpha) = -\frac{\sin\alpha}{\alpha} - \frac{1}{\alpha^2}\cos\alpha,$$

(B7)

(B5) reduces to

$$Q''_{ijEE0}(\vec{r}, \vec{r}', \omega) = -\frac{2k_0^3}{\pi x^2}(\delta_{ij} - \hat{R}_i\hat{R}_j) - \frac{2k_0^3}{\pi}\left(\frac{1}{x^3}(\delta_{ij} - 3\hat{R}_i\hat{R}_j) - \frac{1}{x}(\delta_{ij} - \hat{R}_i\hat{R}_j)\right)(\sin x\operatorname{Ci}x - \cos x\operatorname{Si}x)$$

$$+ \frac{2k_0^3}{\pi x^2}(\delta_{ij} - 3\hat{R}_i\hat{R}_j)(\cos x\operatorname{Ci}x + \sin x\operatorname{Si}x), \quad x = k_0 R.$$

(B8)

Let us now consider two points $\tilde{r}$ and $\tilde{r}'$ which are mirror images of each other with respect to $z = 0$, i.e.,

$$\tilde{r} = x, y, z, \quad \tilde{r}' = x, y, -z, \quad \tilde{R} = 0, 0, 2z .$$

<div align="right">(B9)</div>

For such points we have from (B8) for diagonal elements

$$Q''_{xxEE0}(\tilde{r}, \tilde{r}', \omega) = Q''_{yyEE0}(\tilde{r}, \tilde{r}', \omega) = -\frac{2k_0^3}{\pi}\left[\frac{1}{x^2} + (\sin x \, \mathrm{Ci}x - \cos x \, \mathrm{Si}x)\left(\frac{1}{x^3} - \frac{1}{x}\right) - \frac{1}{x^2}(\cos x \, \mathrm{Ci}x + \sin x \, \mathrm{Si}x)\right] ,$$

<div align="right">(B10)</div>

$$Q''_{zzEE0}(\tilde{r}, \tilde{r}', \omega) = \frac{4k_0^3}{\pi x^3}(\sin x \, \mathrm{Ci}x - \cos x \, \mathrm{Si}x) - \frac{4k_0^3}{\pi x^2}(\cos x \, \mathrm{Ci}x + \sin x \, \mathrm{Si}x), \quad x = 2k_0 z .$$

<div align="right">(B11)</div>

Also for such points (B1) leads to

$$\chi^{(0)}_{xxEE}(\tilde{r}, \tilde{r}', \omega) = \chi^{(0)'}_{yyEE} \quad (\tilde{r}, \tilde{r}', \omega)$$

$$= -k_0^3\left(\frac{\sin x}{x^2} + \frac{\cos x}{x^3} - \frac{\cos x}{x}\right) ,$$

<div align="right">(B12)</div>

$$\chi^{(0)'}_{zzEE}(\tilde{r}, \tilde{r}', \omega) = 2k_0^3\left(\frac{\sin x}{x^2} + \frac{\cos x}{x^3}\right) . \qquad (B13)$$

The expressions (B8)–(B13) will be useful in our discussion of Lamb shifts and frequency shifts in presence of conductors in Sec. III. We emphasize once again that all the above results have been derived using the *translationally invariant* response functions. Any other response function would yield different results for Q. The evaluation of Q''_{ijEET} is much more involved and will not be discussed.

APPENDIX C: HARMONIC OSCILLATOR VERSUS A TWO-LEVEL ATOM

Spontaneous emission from a system of harmonic oscillators has been discussed in detail in Ref. 45. Here we discuss some aspects of harmonic-oscillator emission in presence of a dielectric. There are fundamental differences between the emission from a harmonic oscillator and a two-level atom. By following similar analysis as that of Sec. II and using the boson commutation relations, it is easily shown that the damping $\gamma^{(-)}(\tilde{b}, \omega)$ and the frequency shift $\Omega^{(-)}(\tilde{b}, \omega)$ (change in the energy separation between any two adjacent levels) are given by

$$\gamma^{(-)}(\tilde{b}, \omega) + i\Omega^{(-)}(\tilde{b}, \omega) = -i\sum d_\alpha d_\beta \chi_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega) ,$$

<div align="right">(C1)</div>

which shows that the damping and the shift are determined from the response function itself. The response function $\chi_{\alpha\beta EE}$ is essentially the mean value of the commutator of E at two space-time points, and since this commutator is a c number, $\chi_{\alpha\beta EE}$ does *not* depend on the initial state of the field. Therefore the oscillator does not possess any *field-dependent* shifts and widths. The corresponding shifts and widths for the two-level atom problem were determined from the anticommutator of the field operators and hence are field dependent,

$$\gamma(\tilde{b}, \omega) + i\Omega(\tilde{b}, \omega) = \sum d_\alpha d_\beta Q_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega) . \quad (C2)$$

From (C1) and (C2) and the relations (A6) and (A7), we have

$$\gamma^{(-)}(\tilde{b}, \omega) = \mathrm{Im}\sum d_\alpha d_\beta \chi_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega) , \qquad (C3)$$

$$\gamma(\tilde{b}, \omega) = \mathrm{Im}\,\coth(\tfrac{1}{2}\beta\omega)\sum d_\alpha d_\beta \chi_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega) ,$$

<div align="right">(C4)</div>

which shows explicitly the dependence of the damping on the temperature. At zero temperature $\gamma^{(-)} = \gamma$. For the frequency shifts we have from (A3) and (A10)

$$\Omega^{(-)}(\tilde{b}, \omega) = -\sum d_\alpha d_\beta \chi'_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega) = -\frac{P}{\pi}\sum\int_0^\infty d\omega' \, d_\alpha d_\beta \chi''_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega')[(\omega' - \omega)^{-1} + (\omega' + \omega)^{-1}] , \qquad (C5)$$

$$\Omega(\tilde{b}, \omega) = -\frac{P}{\pi}\sum\int_0^\infty d\omega' \, d_\alpha d_\beta \coth(\tfrac{1}{2}\beta\omega')\chi''_{\alpha\beta EE}(\tilde{b}, \tilde{b}, \omega)[(\omega' - \omega)^{-1} - (\omega' + \omega)^{-1}] . \qquad (C6)$$

Relations (C5) and (C6) show that the frequency shift $\Omega^{(-)}(\tilde{b}, \omega)$ is very different from that $\Omega(\tilde{b}, \omega)$ [cf. our explicit expression for the case of a conductor Eqs. (3.22), (3.23), and (3.25)], whereas dampings as given by (C3) and (C4) are simply related $\gamma/\gamma^{(-)} = \coth(\tfrac{1}{2}\beta\omega)$. The harmonic oscillator is closest to a classical system. The differences in the two models are again reflections of the different nature of the commutation relations for bosons and the atomic systems.

APPENDIX D: DAMPING COEFFICIENT OF A TWO-LEVEL ATOM IN PRESENCE OF AN ANISOTROPIC, NONMAGNETIC MEDIUM

We now discuss how the anisotropy of the medium affects the damping coefficients. We assume that the anisotropic medium, occupying the domain $0 \leq z \leq \infty$, is characterized by a dielectric tensor of the form

$$\epsilon_{ij} = \epsilon_i \delta_{ij}, \quad \epsilon_1 = \epsilon_2 \neq \epsilon_3 . \tag{D1}$$

As shown by Lalor,[54] the general solution of the Maxwell equations, subject to outgoing boundary conditions at infinity, can be expressed as

$$\vec{E}(\vec{r}, \omega) = \iint_{-\infty}^{+\infty} du\, dv\, e^{iux + ivy}$$

$$\times [\, \vec{e}_1(u, v) e^{iw_1 z} + \vec{e}_3(u, v) e^{iw_3 z} \,] ,$$

$$z \geq 0 , \tag{D2}$$

where

$$w_1^2 = k_0^2 \epsilon_1 - k_\|^2, \quad w_3^2 = k_0^2 \epsilon_1 - (\epsilon_1/\epsilon_3) k_\|^2 ,$$

$$e_{1z} = 0, \quad (\vec{K}_\| \cdot \vec{e}_1) = 0, \quad \vec{e}_1 = e_1(1, -u/v, 0) , \tag{D3}$$

$$\vec{e}_3 = e_3 \left(-\frac{uw_3}{k_\|^2} \frac{\epsilon_3}{\epsilon_1}, -\frac{vw_3}{k_\|^2} \frac{\epsilon_3}{\epsilon_1}, 1 \right) .$$

The angular spectrum representation essentially involves two unknowns, e_1, and e_3, which are to be fixed by the boundary conditions at $z = 0$. The asymptotic expansion of (D2), for ϵ_1 and ϵ_3 real and positive, is given by[54]

$$\vec{E} \sim -2\pi i k_0 \epsilon_1^{1/2} \frac{z}{r} \left[\vec{e}_1 \left(k_0 \sqrt{\epsilon_1} \frac{x}{r}, k_0 \sqrt{\epsilon_1} \frac{y}{r} \right) e^{ik_0 \sqrt{\epsilon_1} r} \right.$$

$$\left. + \frac{\vec{e}_3}{\xi^2} \left(k_0 \sqrt{\epsilon_3} \frac{x}{r} \xi, k_0 \sqrt{\epsilon_3} \frac{y}{r} \xi \right) e^{ik_0 \sqrt{\epsilon_3} \xi r} \right] , \tag{D4}$$

where

$$\xi = \left[1 - \left(1 - \frac{\epsilon_1}{\epsilon_3} \right) \left(\frac{z}{r} \right)^2 \right]^{1/2} . \tag{D5}$$

In order to obtain the response functions we solve the boundary value problem as we did in paper I. The result of such a calculation is that the equations (I 5.43) and (I 5.44) should be replaced by

$$(\vec{K}_\| \times \vec{\mathcal{E}}_\|^{(-)}) = \frac{ik_0^2}{2\pi w_0} (\vec{K}_\| \times \vec{p}_\|) \frac{w_0 - w_1}{w_0 + w_1} e^{-i\vec{K}_0 \cdot \vec{r}_0} , \tag{D6}$$

$$(\vec{K}_\| \cdot \vec{\mathcal{E}}_\|^{(-)}) = \frac{iw_0}{2\pi w_0} [k_\|^2 p_\perp - w_0 (\vec{K}_\| \cdot \vec{p}_\|)] \frac{w_0 \epsilon_1 - w_3}{w_0 \epsilon_1 + w_3} e^{-i\vec{K}_0 \cdot \vec{r}_0}$$

$$= w_0 \mathcal{E}_z^{(-)}, \quad w_0^2 = k_0^2 - k_\|^2 . \tag{D7}$$

In the limit $\epsilon_1 = \epsilon_3$, (D6) and (D7) reduce, respectively, to (I 5.43) and (I 5.44). The amplitudes e_1 and e_3 are found to be

$$e_1 = -\frac{2iw_0 k_0^2 v}{2\pi w_0 k_\|^2} \frac{(\vec{K}_\| \times \vec{p}_\|) e^{-i\vec{K}_0 \cdot \vec{r}_0}}{w_0 + w_1} , \tag{D8}$$

$$e_3 = \frac{2iw_0 \epsilon_1}{2\pi w_0 \epsilon_3} \frac{[k_\|^2 p_\perp - w_0 (\vec{K}_\| \cdot \vec{p}_\|)] e^{-i\vec{K}_0 \cdot \vec{r}_0}}{w_0 \epsilon_1 + w_3} . \tag{D9}$$

In the absence of an applied probe only $\vec{e}_3$ exists provided the frequency ω and the propagation vector are such that

$$w_0 \epsilon_1 + w_3 = 0 , \tag{D10}$$

which is the well-known surface polariton dispersion relation[47] for an anisotropic nonmagnetic medium.

Using (D6), (D7) and (3.4), (3.5) the surface-dependent contributions to the damping coefficients are found to be (the atom is now assumed to be located at $z = -b$)

$$\gamma_\|^{(1)}(b, \omega) = \frac{3}{4} \gamma^{(0)} \text{Re} \int_0^\infty \frac{\kappa\, d\kappa}{\lambda_0} e^{i\lambda_0 x}$$

$$\times \left(\frac{\lambda_0 - \lambda_1}{\lambda_0 + \lambda_1} - \lambda_0^2 \frac{\lambda_0 \epsilon_1 - \lambda_3}{\lambda_0 \epsilon_1 + \lambda_3} \right) , \tag{D11}$$

$$\gamma_\perp^{(1)}(b, \omega) = \frac{3}{2} \gamma^{(0)} \text{Re} \int_0^\infty \frac{\kappa^3 d\kappa}{\lambda_0} e^{i\lambda_0 x} \frac{\lambda_0 \epsilon_1 - \lambda_3}{\lambda_0 \epsilon_1 + \lambda_3} , \tag{D12}$$

where

$$\lambda_0^2 = 1 - \kappa^2, \quad \lambda_1^2 = \epsilon_1 - \kappa^2,$$

$$\lambda_3^2 = \epsilon_1 - (\epsilon_1/\epsilon_3)\kappa^2, \quad x = 2k_0 b . \tag{D13}$$

We can now use (D11) and (D12) to obtain the results in special cases, i.e., when ω corresponds to one of the natural modes of the medium. The relations (D8), (D9), and (65) can be used to calculate the normally ordered correlation functions for the field operators in the far zone. In an experiment of the type carried out by Carniglia, Mandel, and Drexhage,[18] where y component of the field was detected and $\vec{r}$ was in xz plane, the intensity in the far field will again be given by (6.20) with ϵ_0 replaced by ϵ_1. The wave $\vec{e}_3$ makes no contribution. However, if observations are made in y-z plane, then only $\vec{e}_3$ contributes. The result analogous to (6.17) will now be

$$E_2^{(+)} \sim E_{02}^{(+)} + \frac{S^- d_2 k_0^2}{r}$$

$$\times \exp[i k_0 \sqrt{\epsilon_3}\, \xi r + i b k_0 (1 - \xi^2 \epsilon_3 \sin^2 \theta)^{1/2}] .$$

$$\times \frac{\epsilon_1^{1/2} \cos\theta}{\xi^2} \frac{2 w_0 w_3}{w_0 \epsilon_1 + w_3} . \tag{D14}$$

On comparison of (D14) and (6.17) we find that the

results of observation, say of intensity vs θ, in the x-z plane and in the y-z plane will be very different. This is also true of an isotropic medium; however the critical angles are different. The critical angle θ_c (in the sense that the amplitude is an oscillating or decaying function of b) for observations in the y-z plane depends on the relative magnitudes of ϵ_1 and ϵ_3 and is given by

$$\sin^2\theta_c = \frac{\epsilon_1 - [\epsilon_1^2 - 4(\epsilon_1 - \epsilon_3)]^{1/2}}{2(\epsilon_1 - \epsilon_3)}, \quad \epsilon_1 > \epsilon_3$$

$$= \frac{[\epsilon_1^2 + 4(\epsilon_3 - \epsilon_1)]^{1/2} - \epsilon_1}{2(\epsilon_3 - \epsilon_1)}, \quad \epsilon_3 > \epsilon_1 \qquad \text{(D15)}$$

subject to the restriction that $\sin^2\theta$ so defined corresponds to a physical angle.

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