L-shell x-ray production cross sections for ^{16}O ions on Ce, Pr, Sm, Eu, Dy, and Ho: 0.50 to 2.25 MeV/amu*

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L-shell and individual x-ray production cross sections for 8–36-MeV oxygen ions on thin solid targets of Ce, Pr, Sm, Eu, Dy, and Ho are measured and compared with predictions of theories of direct Coulomb ionization. In comparing theory and experiment, we use, with reasonable justification, fluorescence yields, Coster-Kronig yields, and radiative widths obtained from single-hole initial configurations. The theoretical treatment which provides the best overall fit to the data is the plane-wave Born approximation with corrections for Coulomb deflection of the projectile and increased binding of the target electrons. The effects of multiple ionization are examined through measurement of x-ray energy shifts as measured with Si(Li) detectors and by vacuum crystal-spectrometer measurements of L x-ray spectra produced by 3-MeV proton and 25-MeV oxygen-ion bombardment of a thick La target.

I. INTRODUCTION

Inner-shell ionization by light-ion ($Z < 3$) bombardment has been explored rather extensively both theoretically and experimentally. Theoretical treatments of direct Coulomb ionization by such "bare point charges" have utilized primarily three approximations: the classical binary-encounter approximation (BEA)[1]; the impact-parameter-dependent semiclassical approximation (SCA)[2]; and the quantum-mechanical plane-wave Born approximation (PWBA).[3] Improvements in the original theoretical treatments have included, among others, use of relativistic atomic-electron wave functions,[4] use of hyperbolic rather than straight-line paths for the incident projectile,[5] and inclusion of effects of increased binding energy of the inner-shell electrons due to the presence of the incident projectile.[6,7] Predictions of these theoretical treatments have been found to be in reasonable agreement with the numerous experimental investigations of light-ion-induced K-shell ionization[8] and, more recently, L- and M-shell ionization.[9]

In contrast with light-ion work, however, experimental cross-section data in the area of heavy-ion ($Z \geq 3$) ionization of inner shells has been rather sparse, especially at high energies, until quite recently.[10] In one of the most recent published works, Bissinger *et al.*[11] have made comparisons of experimental L-shell x-ray production cross sections with PWBA predictions for oxygen-ion bombardment of Ag and Au. In addition, they have presented comparisons of $L\alpha/L\beta$ and $L\alpha/L\gamma$ intensity ratios with the PWBA. The results of their work indicate that the theoretical calculations based upon the PWBA and single-hole transition probabilities vary from the experimental data, with the PWBA overestimating the observations by approximately 50%.

It is the intent of this work to extend the study of L-shell ionization in the rare-earth region ($Z = 58$–67) by oxygen-ion bombardment in the range 0.5–2.25 MeV/amu, and to investigate the effects of perturbation corrections to the PWBA for heavy ions over a range of these elements.

II. EXPERIMENTAL CONSIDERATIONS

The low-resolution experiments were performed at the T. W. Bonner Nuclear Laboratories at Rice University using the model EN tandem Van de Graaff of that facility. The experimental setup has been described in detail previously.[12] The 8–36-MeV oxygen beams were incident on thin, 20–50-μg/cm^2, targets of Ce, Pr, Sm, Eu, Dy, and Ho, which were prepared by vacuum evaporation of rare-earth fluorides on 20-μg/cm^2 carbon backings. The targets were aligned 45° with respect to the incident-beam direction, with the target side facing the beam. At 90° to the beam, a collimated KEVEX Si(Li) detector (resolution, 180 eV at 5.9 keV), mounted outside the vacuum system, was used to detect the emitted x rays, which were attenuated by the Mylar chamber window, additional Mylar absorbers, the Be detector window, air path, gold layer, and silicon dead layer. The total detector efficiency was determined by normalizing the efficiency obtained from calibrated sources (after correcting for Mylar absorption) to the relative intrinsic-efficiency curve given in Ref. 12.

The incident oxygen ions were detected, after undergoing elastic scattering from the target atoms, at angles of 40° and 135° with respect to the incident-beam direction. The ORTEC surface-barrier detectors used to detect the scattered particles subtended solid angles of 1.63×10^{-4} sr (at 40°) and 6.10×10^{-4} sr (at 135°). Solid angles were determined by comparison of actual and expected yields from a ^{244}Cm source of known intensity. Beam integration was used only as a relative monitor of beam intensity.

X-ray and charged-particle spectra were stored on disk by interfacing with an IBM 1800 computer, which also monitored dead times from x-ray and charged-particle amplifiers and from the data-collection system. Low count rates were maintained in order to minimize pileup and dead-time corrections.

Various assumptions are inherent in the reduction of raw experimental data for use in predicting x-ray production cross sections. One such assumption is that the x rays emitted by the target atoms are emitted isotropically. Experimental verification of this assumption has been accomplished for K- and L-shell x rays of Sn bombarded by deuterons and α particles at 6.25 MeV/amu,[13] and for L x rays of Au bombarded by 1.5–4.25-MeV protons.[14] Recently, Pedersen *et al.*[15] have observed polarization fractions of >20% for projectile K x rays from 33-MeV $F^{7+,9+}$ on gas targets of Ar and He. However, for target K x rays, polarization fractions were <3% for 3-MeV protons and 33-MeV F^{6+} on Ar gas targets, and <7% for fully stripped F^{9+} on the Ar targets. Whether these results affect the extension of the isotropy assumption for the characteristic target x-ray lines to different projectile-target combinations and different energy ranges awaits the proof of further experiment.

The efficiency of the x-ray detector was determined by comparison with sources which had been calibrated against IAEA standards. These sources were made comparable in size to the beam spot at the target, and were counted in the same environment and geometry as the target. The efficiency thus measured then accounts for Si(Li) intrinsic efficiency, detector solid-angle limitations, and absorption in the intervening media between the source and the intrinsic region of the Si(Li) detector. Use of computer fits to the efficiency measurements indicated errors of less than 10% for x-ray energies from 5–20 keV and less than 15% for energies of 4–5 keV.

The measured efficiency does not include corrections for target self-absorption of its own x rays, amplifier dead time, and pulse pileup in the amplifier due to high counting rates. As mentioned, the count rates were kept low so that the corrections for dead time and pileup were always less than 5%. As for self-absorption, the worst case considered here amounted to a maximum self-absorption of only 3%.

A number of experiments have been performed in which x rays from ionization by heavy ions have been measured with Si(Li) detectors.[16] In all cases the characteristic lines have been shifted to higher energies and broadened compared to the energies and peak shapes obtained by fluorescence excitation, by electron excitation, or by proton excitation. The interpretation of these features has been based on high-resolution measurements with crystal spectrometers both for K-shell[17] and L-shell[18] excitations. High-resolution measurements with a model ARL25600 vacuum crystal spectrometer using a LiF(200) crystal are presented in Fig. 1 for La L x rays produced by 3-MeV proton and 25-MeV oxygen-ion bombardment of a thick La target. These measurements were taken at the Center for Nuclear Studies of the University of Texas at Austin using the model EN tandem Van de Graaff accelerator to provide the beams. The characteristic lines are labeled on the spectra of Fig. 1. Also indicated are observed noncharacteristic lines. There were obvious contaminants in the La sample used to obtain the spectra in Fig. 1, and no attempt is made to examine the features of these spectra in the re-

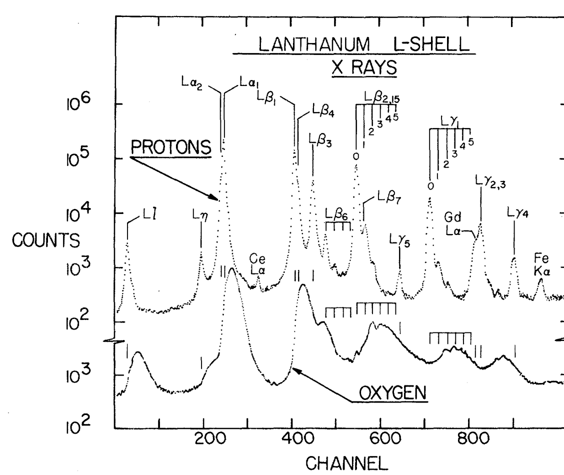

FIG. 1. High-resolution L-shell x-ray spectra for proton and ^{16}O bombardment of La. A LiF crystal (200) was employed using first-order diffraction. The single hash marks about the proton-induced spectrum indicate the unperturbed x-ray energies from Ref. 36. The HFS predictions for the LM^n configurations are indicated as for $L\beta_{2,15}$.

gions where contaminant peaks exist. There are, however, many interesting features of heavy-ion-induced L x-ray spectra which are apparent. The Ll, $L\alpha_2$, $L\alpha_1$, and $L\beta_1$ lines are readily observed to be shifted and broadened in the ^{16}O-produced spectrum. For $L\beta_6$, $L\beta_{2,15}$, and $L\gamma_1$ transitions, there are associated noncharacteristic lines which appear in the proton spectrum. In the oxygen spectrum, the $L\beta_6$ line is unresolved, but for $L\beta_{2,15}$ and $L\gamma_1$, their characteristic and noncharacteristic lines have different relative intensities than is observed for proton bombardment. The energies of the noncharacteristic lines in these spectra are interpreted in terms of a Hartree-Fock-Slater (HFS) model for atomic energy levels and x-ray transition energies[19] as due to transitions to the L shell of La which has initial M-shell vacancies in addition to the single L-shell vacancy which gives rise to a particular x-ray transition. These lines have been termed "satellite lines"—as distinguished from "hypersatellite lines" which originate from double L-shell, multiple M-shell vacancies. The lack of structure in $L\alpha$ and other LM transitions is explained by the fact that additional M-shell vacancies produce energy shifts of only 14–18 eV per vacancy, which is less than the resolution of the spectrometer (20–25 eV). On the other hand, shifts of 40–60 eV are predicted for each additional M-shell vacancy in LN transitions such as $L\beta_{2,15}$ or $L\gamma_1$. Comparisons of experimental and HFS-predicted x-ray energies for the proton spectrum in Fig. 1 are given in Table I. As inspection of the table indicates, the HFS predictions are in reasonably good agreement with experimentally observed x-ray energy shifts. Energy shifts of peak centroids in Si(Li) spectra can thus be interpreted as an indication of the *average* number of additional M-shell vacancies produced by the heavy-ion bombardment.

Theories which predict the relative intensities of the various satellites using a simple binomial distribution of multiple-ionization probabilities have met with some success in their predictions for light-ion-induced single K-, multiple L-shell excitation, but generally fail for heavy-ion, higher-shell excitation.[20] The expermental results shown in Fig. 1 then give the best indication of how to proceed in attempting to extract individual x-ray yields from low-resolution Si(Li) spectra produced by heavy-ion bombardment. Although each resolved x-ray "peak" in the low-resolution spectra is composed of a multiplicity of satellite peaks, the resolved experimental peaks may be fitted and appropriate theoretical parameters chosen in order to describe the actual composition of the peaks.

Inspection of the spectra in Fig. 1 indicates that an appropriate function for fitting the resolved low-resolution peaks would be a series of Gaussian peaks with tailing parameters to account for the asymmetry due to multiple ionization. The computer code SAMPO,[21] which used Gaussian peaks with exponential tails, has been utilized to perform this task. Having only a limited knowledge of the relative satellite intensities, this procedure appears to be adequate.

III. RESULTS AND DISCUSSION

The measured total L-shell x-ray production cross sections (σ_{LX}) for ^{16}O bombardment over the energy range 8.0–36.0 MeV of the elements Ce, Pr, Sm, Eu, Dy, and Ho, are presented in Fig. 2 and Table II. Also presented in Table II are the partial x-ray production cross sections for the Ll, $L\alpha$, $L\beta_1$, $L\beta_2$, $L\gamma_1$, and $L\gamma_{2,3}$ transitions over the same energy range. The data shown in Fig. 2 for the total x-ray production cross sections are systematic as a function of E_1 and show the expected increase of σ_{LX}

TABLE I. Energy shifts in La L x-ray transitions due to multiple ionization.[a]

X-ray	M vacancies	Energy or energy shift (eV) Bearden[b]	HFS[c]	Observed[d]
Ll	0	4124	4149	4129
	1	···	18	28
	2	···	36	43
$L\eta$	0	4525	4530	4528
	1	···	18	20
	2	···	36	44
$L\alpha_2$	0	4634	4623	4632
$L\alpha_1$	0	4651	4641	4653
$L\beta_1$	0	5042	5004	5043
$L\beta_4$	0	5061	4999	5064
$L\beta_3$	0	5143	5073	5142
	1	···	17	21
	2	···	34	40
$L\beta_6$	0	5211	5207	5212
	1	···	42	48
	2	···	85	94
$L\beta_{2,15}$	0	5383	5370	5383
	1	···	45	49
	2	···	90	95
$L\beta_7$	0	5450	5437	5452
$L\gamma_5$	0	5620	5588	5618
	1	···	43	45
$L\gamma_1$	0	5788	5748	5787
	1	···	45	46
	2	···	90	87
$L\gamma_4$	0	6251	6160	6248

[a] Only peak energies determined unambiguously from computer fits to the proton spectrum in Fig. 1 are included in the table.

[b] Reference 36.

[c] Reference 19.

[d] Uncertainties in computer fits are 2–4 eV.

with increasing projectile energy and decreasing atomic number of the target, as predicted both by BEA and PWBA theories in the ion-energy range of this work.

The applicability of the theories of direct Coulomb ionization by heavy ions is predicated on being able to treat the incident ion as a bare nuclear charge, ignoring whatever electronic structure the projectile may possess. Although the validity conditions for this assumption are somewhat vague,[6,7] one fairly definitive criterion for L-shell ionization is that the mean K-shell radius of the projectile electrons be greater than the mean L-shell radius of the target electrons ($r_{1K} > r_{2L}$). For the worst case examined in this work (^{16}O on Ce), $r_{1K} \simeq 2r_{2L}$.

In order to make specific comparisons between theory and experiment, it is necessary to calculate the theoretical x-ray production cross sections for the individual lines or groups of lines which can be resolved experimentally. As an example, the theoretical x-ray production cross section for the $L\alpha_{1,2}$ transitions is given by

$$\sigma_{L\alpha_{1,2}} = [\sigma_{L_{III}} + f_{23}\sigma_{L_{II}} + (f_{13} + f_{12}f_{23})\sigma_{L_I}] \times \omega_3(\Gamma_{3\alpha_1} + \Gamma_{3\alpha_2})/\Gamma_3, \quad (1)$$

where σ_{L_i} are the theoretical ionization cross sections for the ith subshell, f_{ij} are the Coster-Kronig yields, ω_i are the ith-subshell fluorescence yields, Γ_{ij} are partial radiative widths for the ith subshell, and Γ_i are the total radiative widths for the ith subshell.

In order to compare the individual x-ray production cross sections for various theoretical models, the computer program XCODE was written.[22] This code performs (through a table-look-up procedure) calculations of K-, L_I-, L_{II}-, and L_{III}-ionization

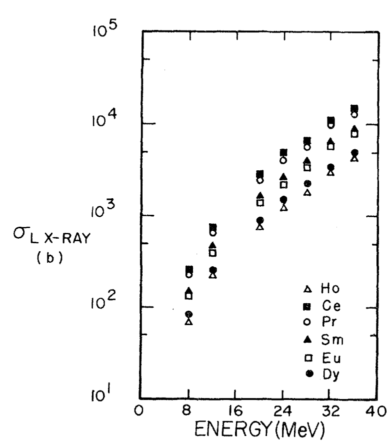

FIG. 2. Experimental total L x-ray production cross sections for ^{16}O ions on Ce, Pr, Sm, Eu, Dy, and Ho.

cross sections using the BEA,[1] the PWBA,[3] and the PWBA including both binding-energy and Coulomb-deflection corrections (PWBABC).[6,7] Theoretical x-ray production cross sections are also calculated through the use of transition probabilities [cf. Eq. (1)] taken from McGuire[23] and from Crasemann *et al.*[24] as given in the review article by Bambynek *et al.*[25] The radiative widths are taken from Scofield,[26] and binding energies used in the theories are from Bearden and Burr.[27] In making theoretical comparisons to the experimental data of this work, it is emphasized that fluorescence yields, Coster-Kronig yields, and radiative widths employed result from initial configurations containing only a single L-shell vacancy. As has already been shown in Fig. 1, multiple ionization is a dominant factor in the observed x-ray energy spectra arising in moderate-energy oxygen-ion bombardment. Therefore, some justification for the use of single-hole rates is required.

The degree of multiple ionization observed in the Si(Li) low-resolution data for this work can be estimated, as mentioned previously, from a comparison of the observed energy shifts in the x-ray energy peaks with HFS calculations[19] assuming various initial- and final-state configurations. Shown in Fig. 3 are energy-shift data for the $L\gamma_1$, $L\alpha_{1,2}$ and Ll lines of Ce and the $L\gamma_1$ lines of Ce, Sm, and Ho, under oxygen-ion bombardment, as a function of ion energy. The number of additional M-shell holes required to produce the observed shifts is of order 3 to 4, on the average, for the Ce data. The energy shifts increase as expected because the peak in the M-shell ionization cross section is not attained in the energy range of this work. It is emphasized that the data of Fig. 3 indicate the *average* number of additional M-shell vacancies. It should be pointed out further that neither low- nor high-resolution spectra are capable of discerning unambiguously the number of additional M-shell vacancies, since peak shifts resulting from successive removal of N-shell electrons are on the order of 1–2 eV or less per N-shell vacancy. The presence of additional N-shell vacancies thus results in severe spectral broadening under presently available instrumental resolution. This is typified by the disappearance of observable structure in individual noncharacteristic lines for ^{16}O on Sn,[28] and the similar poor resolution in the oxygen-induced spectrum of Fig. 1.

The effects of multiple ionization on x-ray and Auger transition probabilities have been calculated for a limited number of cases either by the use of statistical scaling for a single 1s vacancy based on the number of electrons in each subshell[29] or by explicit relativistic HFS calculations.[30] In low-Z elements, outer-shell ionization can cause dra-

TABLE II. X-ray production cross sections for ^{16}O ions.[a]

E_1 (MeV)	Ll	$L\alpha$	σ_{LX} (barns) $L\beta_1$	$L\beta_2$	$L\gamma_1$	$L\gamma_{2,3}$	L_{total}
			Cerium (Ce)				
8.00	8.04	150	69.0	17.8	7.19	2.65	255
12.00	23.5	444	191	58.5	20.6	7.76	746
20.00	101	1660	688	254	84.4	38.7	2830
24.00	172	2790	1220	455	149	86.1	4880
28.00	217	3650	1740	610	208	136	6560
32.00	347	5980	3000	990	349	250	10 900
36.00	476	7950	4170	1320	498	345	14 800
			Praseodymium (Pr)				
8.00	6.47	134	61.5	16.1	6.57	2.14	226
12.00	19.9	389	170	51.3	18.9	6.63	655
20.00	83.4	1440	589	222	71.5	33.3	2440
24.00	122	2350	997	378	126	62.1	4040
28.00	171	3180	1460	535	189	107	5640
32.00	282	5350	2620	896	314	207	9670
36.00	377	6880	3580	1160	420	303	12 700
			Samarium (Sm)				
8.00	4.34	84.4	39.2	11.6	4.83	1.22	146
12.00	14.0	271	117	40.1	14.8	3.78	461
20.00	51.1	950	355	164	52.4	15.4	1590
24.00	89.2	1530	576	277	87.2	29.3	2580
28.00	129	2230	894	419	134	54.2	3860
32.00	199	3680	1540	684	212	91.9	6410
36.00	275	4920	2170	918	302	146	8730
			Europium (Eu)				
8.00	3.83	76.5	35.4	10.2	4.45	1.25	132
12.00	11.4	229	98.6	33.1	12.9	3.83	389
20.00	46.2	816	295	141	44.1	15.7	1360
24.00	74.0	1310	471	233	69.6	28.2	2180
28.00	112	1970	731	367	110	49.9	3340
32.00	190	3330	1290	601	176	92.1	5680
36.00	251	4490	1830	830	251	136	7790
			Dysprosium (Dy)				
8.00	2.23	46.2	22.7	6.64	3.36	0.708	81.8
12.00	7.17	145	67.3	22.2	9.64	2.25	254
20.00	27.4	513	207	92.1	32.4	9.75	882
24.00	47.4	855	346	166	55.6	17.1	1490
28.00	69.5	1280	524	254	84.4	28.8	2240
32.00	107	1970	794	374	115	45.4	3400
36.00	149	2770	1180	529	171	73.1	4870
			Holmium (Ho)				
8.00	1.93	38.4	19.1	5.80	2.76	0.568	68.5
12.00	6.68	129	59.6	20.4	8.65	1.92	226
20.00	24.6	443	183	82.3	29.5	7.60	771
24.00	39.7	726	289	140	47.5	13.5	1260
28.00	56.0	1030	412	206	66.6	22.7	1790
32.00	95.5	1750	685	334	104	39.4	3010
36.00	133	2470	1000	482	151	62.8	4310

[a]Errors in the tabulated values are ±15%.

matic changes in Auger rates, whereas in high-Z elements, the influence of outer-shell vacancies is less pronounced. Although statistical scaling tends to underestimate the x-ray and Auger rates in comparison with HFS predictions, the procedure does indicate that for Cu ($Z = 29$), the ratio of L-shell x-ray to Auger transition strengths is approximately constant for up to as many as six additional M-shell vacancies. For even higher-Z elements (rare earths), the results should be similar to those for Cu. It is further assumed that the Coster-Kronig rates are also not affected by additional vacancies outside the L-shell. Since these various transition probabilities have not been calculated using multiple-vacancy configurations for the L shells of the rare earths, and since there have been no experimental measurements of these rates, the single-hole assumptions will have to be considered a possible source of error in comparisons of theory and experiment. However, the approach to the calculations employing single-hole rates does offer a consistent basis for the comparison of theory to experiment.

The effect of target thickness on the measured cross sections of this work has also been investigated. For the targets used, charge-state equilibrium is expected to be obtained. This places the measurements of x-ray production cross section in the asymptotic region for dynamic screening effects.[31] Variation of x-ray yield with target thickness was checked experimentally, with the resulting cross-section measurements agreeing to within the assigned experimental error of ± 15%. The use of $^{16}O^{3+,4+,5+}$ ions at a fixed bombarding energy yielded similar results. In view of these results, the theoretical calculations were performed using the charge of the projectile as $Z_1 = 8$, the bare nuclear charge.

The total L-shell x-ray production cross sections for ^{16}O on Sm are shown in Fig. 4. Comparison of the PWBA, BEA, and PWBABC calculations to the data shows that the BEA and PWBA overestimate the magnitude of the cross section, with the deviation between the PWBA and the data increasing as the ion energy is decreased. The BEA calculations overestimate the measured cross sections by ~120%. The PWBABC calculations fall ~40%–50% below the data. These results are typical for all of the elements studied in this work.

A plot of the ratio of theoretical to experimental L-shell x-ray production cross sections is given in Fig. 5 for the PWBA and the PWBABC for ^{16}O and

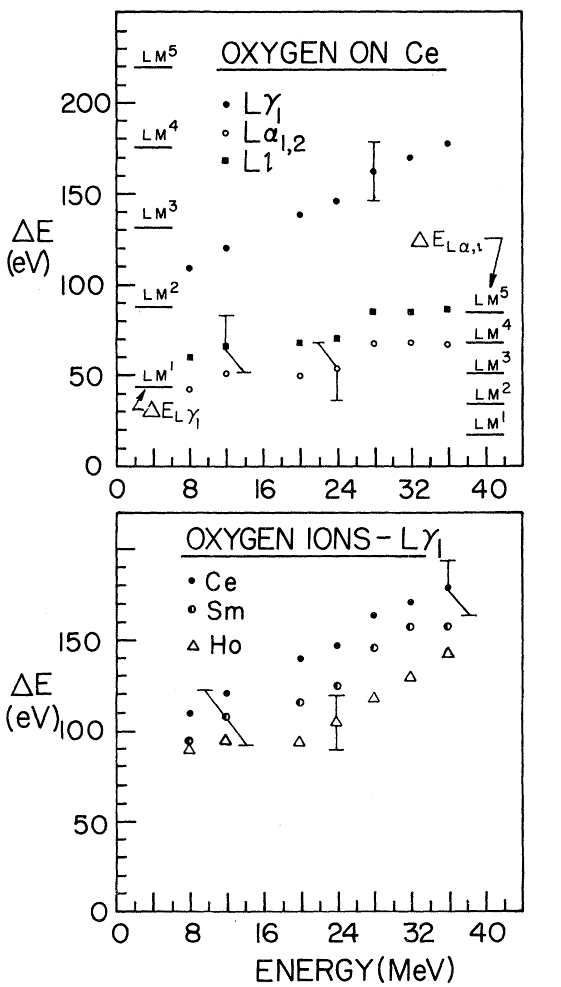

FIG. 3. Typical average energy shifts for ^{16}O bombardment as determined by the x-ray peak centroid positions for Si(Li) measurements (low-resolution data). The quantities ΔE_{Li} are HFS predictions for the configurations LM^n.

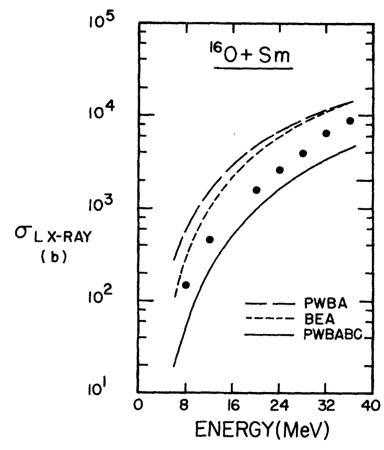

FIG. 4. Comparison of the experimental total L-shell x-ray production cross sections to the PWBA, PWBABC, and BEA predictions for ^{16}O on Sm. The BEA calculations are taken from Ref. 1 (J. S. Hansen) using $2s$ and $2p$ electron velocity distributions.

for protons[32] on Pr, Sm, and Dy. The deviations between the PWBA and the data are observed to be consistent and systematic. Clearly, the PWBA is having serious difficulty in predicting the observed energy dependence for the heavy-ion data. The PWBABC, however, gives much better agreement with the heavy-ion data and surprisingly similar results for the proton data. This similaritv in the predictions of the PWBABC for both the proton and oxygen data tends to support the sssumption that multiple ionization is not causing a major change in the fluorescence parameters for oxygen bombardment. These same trends have been observed for ^{12}C ions on the rare earths,[33] and for ^{4}He and ^{7}Li ions on Sm, Yb, and Pb.[34]

The individual x-ray production cross sections for the Ll, $L\alpha$, $L\beta_1$, $L\beta_2$, and $L\gamma_{2,3}$ peaks for ^{16}O on Ho are shown in Fig. 6. The PWBA and PWBABC predictions are compared to the data. For all but the $L\gamma_{2,3}$ cross section, the PWBA overestimates the data, while the PWBABC falls below the data, as expected from the total cross section comparisons. In the case of the $L\gamma_{2,3}$ cross section, however, a distinct change in structure is noted. This is consistent with observations of ^{1}H, ^{4}He, ^{7}Li, and ^{12}C on Yb,[35] where similar changes in structure are observed in the $L\gamma_{2,3}$ x-ray production cross section. This structure is associated with the nodal character of the 2*s* electron wave functions. The change in structure of the $L\gamma_{2,3}$ cross section can also be observed through examination of the $\sigma_{L\alpha}/\sigma_{L\gamma_{2,3}}$ ratio from the data presented in Table II. This ratio is almost constant over the energy range of this work. This is in contrast to the peaked structure predicted by theory and observed in protons on rare-earth targets in the same E/M range.[32] This change in the structure of the $L\gamma_{2,3}$ cross section for heavy-ion bombardment tends to indicate that a calculational approach which is more complex than a simple shift in the binding energy of the target electron may be necessary. The present PWBABC calculations preserve the structure of the $L\gamma_{2,3}$ cross section for ^{16}O bombardment (cf. Fig. 6), with the most noticeable difference being a shift in that structure to higher ion energies.

IV. CONCLUSION

For the elements studied in this work, the PWBABC (the PWBA with binding-energy and Coulomb-deflection corrections) does give a better representation of the measured total *L*-shell x-ray production cross sections for ^{16}O ions in the energy range 0.5–2.25 MeV/amu than other comparable theories. The PWBABC is typically ~50% below the oxygen data. This can be considered an excellent fit considering the various approximations used, but especially considering the fact that the PWBABC

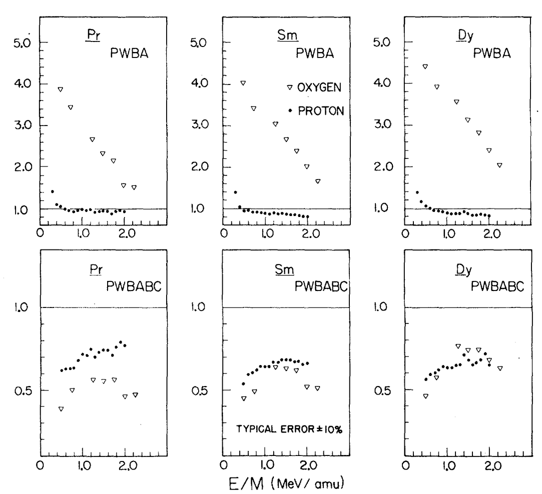

FIG. 5. Ratios of $\sigma_{\rm theory}/\sigma_{\rm exp}$ for the total *L*-shell x-ray production cross sections for ^{16}O-ion bombardment, based upon the PWBA and PWBABC theories. The proton data is from Ref. 32.

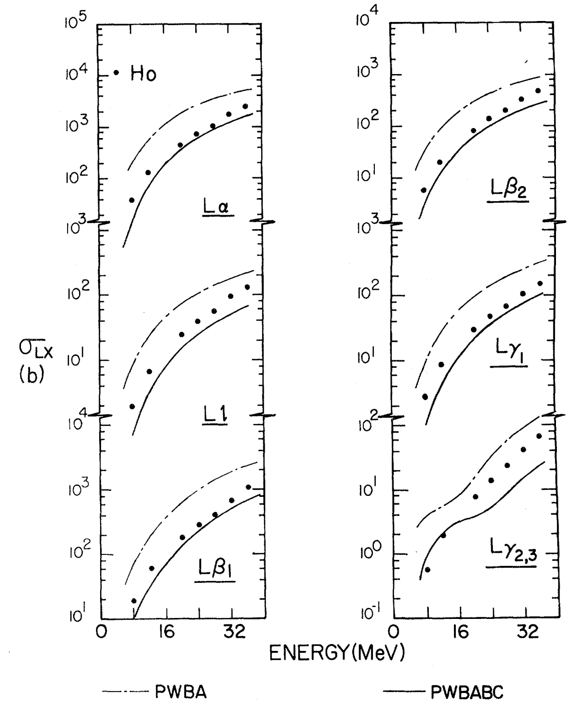

FIG. 6. X-ray production cross sections for the individual x-ray lines Ll, $L\alpha$, $L\beta_1$, $L\beta_2$, $L\gamma_1$, and $L\gamma_{2,3}$ for ^{16}O bombardment of Ho. Comparisons are made to the PWBA and PWBABC theoretical predictions. Single-hole values are taken for the radiative and nonradiative parameters required for the theoretical x-ray production cross sections [cf. Eq. (1)].

is ~40% below proton cross sections in the same E/M range.

The question of the effects of multiple ionization on the theoretical calculations is not resolved, but comparisons of the oxygen data with available proton data tend to support the use of single-hole parameters for the radiative and nonradiative parameters involved in calculating L-shell x-ray production cross sections for ^{16}O bombardment in this E/M range. Since high-resolution instruments are currently incapable of resolving unambiguously the separation and intensities of the noncharacteristic L-shell x-ray lines in the rare-earths region, further clarification of multiple-ionization effects will probably have to come from theory.

In summary, L-shell ionization by oxygen ions on the rare earths can be reasonably and consistently described by assuming direct Coulomb ionization and using single-hole parameters for the theoretical predictions of x-ray production cross sections. There may be difficulties, however, with the method of calculating the binding-energy corrections, as comparisons of data and theory for the $L\gamma_{2,3}$ or the L_I -shell cross sections have revealed. Certainly some clarification of these difficulties seems in order.

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[1]J. S. Hansen, Phys. Rev. A 8, 822 (1973); J. H. McGuire and P. Richard, Phys. Rev. A 8, 1374 (1973); J. H. McGuire and K. Omidvar, Phys. Rev. A 10, 182 (1974).

[2]J. Bang and J. M. Hansteen, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 31, No. 13 (1959); J. M. Hansteen and O. P. Mosebekk, Z. Phys. 234, 281 (1970).

[3]G. S. Khandelwal, B.-H. Choi, and E. Merzbacher, At. Data 1, 103 (1969); B.-H. Choi, E. Merzbacher, and G. S. Khandelwal, At. Data 5, 291 (1973).

[4]D. Jamnik and Ĉ. Zupanĉiĉ, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 31, No. 2 (1957); B.-H. Choi, Phys. Rev. A 4, 1002 (1971).

[5]W. Brandt, R. Laubert, and I. Sellin, Phys. Rev. 151, 56 (1966).

[6]G. Basbas, W. Brandt, and R. Laubert, Phys. Rev. A 7, 983 (1973).

[7]W. Brandt and G. Lapicki, Phys. Rev. A 10, 474 (1974).

[8]J. D. Garcia, R. J. Fortner, and T. M. Kavanagh, Rev. Mod. Phys. 45, 111 (1973); C. H. Rutledge and R. L. Watson, At. Data Nucl. Data Tables 12, 195 (1973); T. L. Hardt and R. L. Watson, Phys. Rev. A 7, 1917 (1973); R. Lear and Tom J. Gray, Phys. Rev. A 8, 2469 (1973); T. L. Criswell and Tom J. Gray, Phys. Rev. A 10, 1145 (1974); N. Khelil and Tom J. Gray, Phys. Rev. A 11, 893 (1975).

[9]Garcia *et al.*, Ref. 8; S. Datz, J. L. Duggan, L. C. Feldman, E. Laegsgaard, and J. U. Andersen, Phys. Rev. A 9, 192 (1974); D. H. Madison, A. B. Baskin, C. E. Busch, and S. M. Shafroth, Phys. Rev. A 9, 675 (1974); F. Abrath and Tom J. Gray, Phys. Rev. A 9, 682 (1974); 10, 1157 (1974); H. Tawara, K. Ishii, S. Morita, H. Kaji, C. N. Hsu, and T. Shiokawa, Phys. Rev. A 9, 1617 (1974); S. T. Thornton, R. H. McKnight, and R. R. Karlowicz, Phys. Rev. A 10, 219 (1974); K. Ishii, S. Morita, H. Kaji, and T. Shiokawa, Phys. Rev. A 10, 774 (1974); 11, 119 (1975); C. N. Chang, J. F. Morgan, and S. L. Blatt, Phys. Rev. A 11, 607 (1975).

[10]Several articles on heavy-ion-induced ionization are to be found in *Proceedings of the International Conference on Inner Shell Ionization Phenomena and Future Applications, Atlanta, Georgia, 1972*, edited by R. W. Fink, S. T. Manson, J. M. Palms, and P. V. Rao, CONF-720 404 (U. S. Atomic Energy Commission, Oak Ridge, Tenn., 1973). More recent L-shell works include: T. M. Kavanagh, R. C. Der, R. J. Fortner, and M. E. Cunningham, Phys. Rev. A 8, 2322 (1974); F. Folkmann, J. Borggreen, and A. Kjeldgaard, Nucl. Instrum. Methods 119, 117 (1974); R. J. Fortner, Phys. Rev. A 10, 2118 (1974); L. Winters, M. D. Brown, L. D. Ellsworth, T. Chiao, E. W. Pettus, and J. R. Macdonald, Phys. Rev. A 11, 174 (1975); F. Hopkins, R. Brenn, A. R. Whittemore, J. Karp, and S. K. Bhattacherjee, Phys. Rev. A 11, 916 (1975). Recent K-shell works include: W. E. Meyerhof, Phys. Rev. A 10, 1005 (1974); D. Burch, W. B. Ingalls, H. Wieman, and R. Vandenbosch, Phys. Rev. A 10, 1245 (1974); M. D. Brown, L. D. Ellsworth, J. A. Guffey, T. Chiao, E. W. Petters, L. M. Winters, and J. R. Macdonald, Phys. Rev. A 10, 1255 (1974); R. Laubert *et al.*, Ref. 16; L. Winters *et al.* (see L-shell above); F. Hopkins *et al.* (see L-shell above); W. E. Meyerhof, R. Anholt, T. K. Saylor, and P. D. Bond, Phys. Rev. A 11, 1083 (1975); W. Brandt and R. Laubert, Phys. Rev. A 11, 1233 (1975); R. Laubert, H. H. Haselton, J. R. Mowat, R. S. Peterson, and I. A. Sellin, Phys. Rev. A 11, 1468 (1975); F. Hopkins, R. Brenn, A. R. Whittemore, N. Cue, and V. Dutkiewicz, Phys. Rev.

A 11, 1482 (1975).
[11]G. Bissinger, P. H. Nettles, S. M. Shafroth, and A. W. Waltner, Phys. Rev. A 10, 1932 (1974).
[12]R. B. Liebert, T. Zabel, D. Miljanić, H. Larson, V. Valković, and G. C. Phillips, Phys. Rev. A 8, 2336 (1973).
[13]C. W. Lewis, R. L. Watson, and J. B. Natowitz, Phys. Rev. A 5, 1773 (1972).
[14]E. M. Bernstein and H. W. Lewis, Phys. Rev. 95, 83 (1954).
[15]E. H. Pedersen, S. J. Czuchlewski, M. D. Brown, L. D. Ellsworth, and J. R. Macdonald, Phys. Rev. A 11, 1267 (1975).
[16]See, for example, R. Laubert, H. Haselton, J. R. Mowat, R. S. Peterson, and I. A. Sellin, Phys. Rev. A 11, 135 (1975).
[17]See, for example, A. R. Knudson, P.G. Burkhalter, and D. J. Nagel, Phys. Rev. A 10, 2118 (1974); K. A. Jamison, C. W. Woods, R. L. Kauffman, and P. Richard, Phys. Rev. A 11, 505 (1975).
[18]P. G. Burkhalter, A. R. Knudson, and D. J. Nagel, Phys. Rev. A 7, 1936 (1973); D. K. Olsen, C. Fred Moore, and P. Richard, Phys. Rev. A 7, 1244 (1973).
[19]F. Herman and S. Skillman, *Atomic Structure Calculations* (Prentice-Hall, Englewood Cliffs, N.J., 1963).
[20]R. L. Kauffman, J. H. McGuire, and P. Richard, Phys. Rev. A 8, 1233 (1973); J. H. McGuire (private communication).
[21]J. T. Routti and S. G. Prussin, Nucl. Instrum. Meth. 72, 125 (1969).
[22]G. Pepper, "XCODE—A Computer Code for X-Ray Analysis" (unpublished).
[23]E. J. McGuire, Phys. Rev. A 3, 587 (1971).
[24]B. Crasemann, M. H. Chen, and V. O. Kostroun, Phys. Rev. A 4, 2161 (1971).
[25]W. Bambynek, B. Crasemann, R. W. Fink, H. U. Freund, H. Mark, C. D. Swift, R. E. Price, and P. V. Rao, Rev. Mod. Phys. 44, 716 (1972).
[26]J. H. Scofield, Phys. Rev. 179, 9 (1969); Phys. Rev. A 9, 1041 (1974); 10, 1507 (1974).
[27]J. A. Bearden and A. F. Burr, Rev. Mod. Phys. 39, 125 (1967).
[28]Olsen *et al.*, Ref. 18.
[29]F. P. Larkins, J. Phys. B 4, L29 (1971); T. M. Kavanagh, R. C. Der, R. J. Fortner, and M. E. Cunningham, Phys. Rev. A 8, 2322 (1973).
[30]C. P. Bhalla, N. O. Folland, and M. A. Hein, Phys. Rev. A 8, 649 (1973); C. P. Bhalla and M. Hein, Phys. Rev. Lett. 30, 39 (1973).
[31]W. Brandt, R. Laubert, M. Mourino, and A. Schwarzschild, Phys. Rev. Lett. 30, 358 (1973).
[32]F. Abrath and Tom J. Gray, Ref. 9.
[33]G. Pepper and Tom J. Gray (unpublished).
[34]Tom J. Gray, F. D. McDaniel, G. M. Light, J. L. Duggan, J. Tricomi, R. M. Wheeler, A. R. Zander, and S. J. Cipolla, Bull. Am. Phys. Soc. 19, 1107 (1974).
[35]Tom J. Gray, *Proceedings of the Third Conference on Application of Small Accelerators*, North Texas State University, Oct. 1974, edited by J. L. Duggan and I. L. Morgan (unpublished).
[36]J. A. Bearden, Rev. Mod. Phys. 39, 78 (1967).