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Large-angle inelastic electron-helium scattering in the unrestricted Glauber approximation*

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The work of Gau and Macek on large-angle inelastic electron-hydrogen scattering in the unrestricted Glauber approximation is extended to electron-helium scattering. "Unrestricted" in this context means that we do not make the additional approximation of purely transverse linear-momentum transfer. Once again, we find at large angles for $1\,^1S_0 \rightarrow 2\,^1P_1$ collision-induced transitions the electron-nucleus interaction dominates and has a q^{-4} behavior in the differential cross section. Further, we calculate the orientation parameter $O_{1-}^{\rm col}$.

I. INTRODUCTION

In a recent article Gau and Macek[1] considered large-angle inelastic electron-hydrogen scattering in the unrestricted Glauber approximation. It was shown that the expression for the scattering amplitude contains a six-dimensional integral which can be reduced to a two-dimensional one by means of parametric differentiation and integration techniques. Further, it was shown that for high-energy collisions at a large angle, the dominant scattering mechanism is the electron-proton interaction. This electron-proton term has a Rutherford $1/q^4$ behavior in the inelastic differential cross section where $\vec{q}$ is the momentum transferred to the hydrogen atom. This behavior contrasts with the conventional Glauber approximation[2,3] which predicts a q^{-6} asymptotic dependence[4] for the $1s \rightarrow 2p$ excitation. It is of interest to see whether this $1/q^4$ behavior for the differential cross section is characteristic of the approximation or just true for hydrogen.

In this paper the work on the unrestricted Glauber approximation is extended to electron-helium scattering.[5] We confine our efforts to the $1\,^1S_0 \rightarrow 2\,^1P_1$ collision-induced excitation. Here, again, we find the differential cross section has a $1/q^4$ behavior for high-energy collisions at large angles. The coefficient of this term is evaluated for two choices of the ground-state wave function.

II. DERIVATION OF INTEGRAL EXPRESSION

The unrestricted Glauber inelastic electron-helium scattering amplitude[6,7] is given by

$$F(i\rightarrow f,\vec{q}) = -\frac{\eta K}{2\pi}\int\!\!\int\!\!\int e^{i\vec{q}\cdot\vec{r}_3}\left(\frac{1}{r_{13}}+\frac{1}{r_{23}}-\frac{2}{r_3}\right)\exp\left[-i\eta\int_{-\infty}^{z_3}\left(\frac{1}{r'_{13}}+\frac{1}{r'_{23}}-\frac{2}{r'_3}\right)dz'_3\right]u_f^* u_i\,d^3r_1\,d^3r_2\,d^3r_3\,, \qquad (1)$$

where K is the momentum of the incoming electron relative to the helium atom and $\eta = e^2/\hbar V$. V is the relative velocity of the incoming electron. Coordinates $\vec{r}_i$ refer to the position of the ith electron with respect to the helium nucleus. Note further that $r_{ij} = |\vec{r}_i - \vec{r}_j|$, $\vec{r}'_3 = (x_3, y_3, z'_3)$, and $r'_{13} = |\vec{r}_i - \vec{r}'_3|$. Electrons 1 and 2 are bound to helium, and electron 3 is the impinging electron. u_i and u_f are the initial and final states of the helium atom. $\vec{q} = \vec{K} - \vec{K}'$ is the momentum transferred by the incident electron. The coordinate system is chosen with $\hat{z}$ along $\vec{K}$, $\hat{y}$ along $\vec{K}\times\vec{K}'$, and $\hat{x}$ perpendicular to $\hat{z}$ and $\hat{y}$.

In order to evaluate Eq. (1) for states of helium, we replace $u_f^* u_i$ with the expression

$$C_{fi}\exp(-\mu_1 r_1 + i\vec{\gamma}_1\cdot\vec{r}_1)\exp(-\mu_2 r_2 + i\vec{\gamma}_2\cdot\vec{r}_2)\,. \qquad (2)$$

The product of approximate wave functions for this problem can be represented (C_{fi} is the appropriate normalization constant) by a linear combination of terms generated by differentiating Eq. (2) with respect to μ_1 and μ_2 and the components of $\vec{\gamma}_1$ and $\vec{\gamma}_2$ after which $\vec{\gamma}_1$ and $\vec{\gamma}_2$ are set equal to zero.

The scattering amplitude, Eq. (1), can be written as

$$F(i\rightarrow f,\vec{q}) = -(\eta K/2\pi)C_{fi}S(1\leftrightarrow 2)D(\mu_1,\vec{\gamma}_1;\mu_2,\vec{\gamma}_2)$$
$$\times\left\{\int e^{i\vec{q}\cdot\vec{r}_3}\left(\frac{1}{r_{13}}+\frac{1}{r_{23}}-\frac{2}{r_3}\right)\exp\left[-i\eta\int_{-\infty}^{z_3}\left(\frac{1}{r'_{13}}+\frac{1}{r'_{23}}-\frac{2}{r'_3}\right)dz'_3\right]\right.$$
$$\left.\times\exp(-\mu_1 r_1 + i\vec{\gamma}_1\cdot\vec{r}_1)\exp(-\mu_2 r_2 + i\vec{\gamma}_2\cdot\vec{r}_2)\right\}d^3r_1\,d^3r_2\,d^3r_3\,. \qquad (3)$$

$S(1\to 2)D(\mu_1,\vec{\gamma}_1;\mu_2,\vec{\gamma}_2)$ is the differential operator which generates the required wave functions when operating on Eq. (2). $S(1\to 2)$ explicitly forms a sum with the proper symmetry between 1 and 2 of the derivatives in $D(\mu_1,\vec{\gamma}_1;\mu_2,\vec{\gamma}_2)$. $F(i\to f,\vec{q})$ can be written in a slightly different form:

$$F(i\to f,\vec{q})=-\frac{\eta K}{\pi}C_{fi}S(1\to 2)D(\mu_1,\vec{\gamma}_1;\mu_2,\vec{\gamma}_2)$$
$$\times\int d^3r_3\,e^{i\vec{q}\cdot\vec{r}_3}\left\{\int d^3r_2\exp\left[-i\eta\int_{-\infty}^{z_3}\left(\frac{1}{r'_{23}}-\frac{1}{r'_3}\right)dz'_3\right]\exp(-\mu_2r_2+i\vec{\gamma}_2\cdot\vec{r}_2)\right\}$$
$$\times\left\{\int d^3r_1\left(\frac{1}{r_{13}}-\frac{1}{r_3}\right)\exp\left[-i\eta\int_{-\infty}^{z_3}\left(\frac{1}{r'_{13}}-\frac{1}{r'_3}\right)dz'_3\right]\exp(-\mu_1r_1+i\vec{\gamma}_1\cdot\vec{r}_1)\right\}. \tag{4}$$

The integrals computed by Gau and Macek can be used to reduce this nine-dimensional integral to a four-dimensional one. In terms of this problem we rewrite these integrals:

$$\int d^3r_1\left(\frac{1}{r_{13}}-\frac{1}{r_3}\right)\exp\left[-i\eta\int_{-\infty}^{z_3}\left(\frac{1}{r'_{13}}-\frac{1}{r'_3}\right)dz'_3\right]\exp(-\mu_1r_1+i\vec{\gamma}_1\cdot\vec{r}_1)$$
$$=\frac{2\pi}{\Gamma(-i\eta)}(4\mu_1)\int_0^\infty d\lambda_1\lambda_1^{-i\eta}\int_0^1\frac{d\chi_1}{\chi_1}\left(\frac{d}{d\mu_1^2}\right)^2\left(\lambda_1^{-1}-\frac{1-\chi_1}{\Lambda_1}\right)\frac{\exp(-\Lambda_1r_3-i\vec{\alpha}_1\cdot\vec{r}_3)}{r_3}(r_3-z_3)^{-i\eta}, \tag{5}$$

$$\int d^3r_2\exp\left[-i\eta\int_{-\infty}^{z_3}\left(\frac{1}{r'_{23}}-\frac{1}{r'_3}\right)dz'_3\right]\exp(-\mu_2r_2+i\vec{\gamma}_2\cdot\vec{r}_2)$$
$$=\frac{2\pi}{\Gamma(-i\eta)}(4\mu_2)\int_0^\infty d\lambda_2\lambda_2^{-i\eta}\int_0^1\frac{d\chi_2}{\chi_2}\left(\frac{d}{d\mu_2^2}\right)^2\frac{(1-\chi_2)}{\Lambda_2}\exp(-\Lambda_2r_3-i\vec{\alpha}_2\cdot\vec{r}_3)(r_3-z_3)^{-i\eta}. \tag{6}$$

In Eqs. (5) and (6) we use the following notation:

$$\Lambda_i=[\lambda_i^2(1-\chi_i)^2+2i\lambda_i(1-\chi_i)\gamma_{iz}\chi_i+\mu_1^2\chi_i+\gamma_i^2(1-\chi_i)\chi_i]^{1/2},$$
$$\vec{\alpha}_i=i\lambda_i(1-\chi_i)\hat{z}-\vec{\gamma}_i\chi_i. \tag{7}$$

Using Eqs. (5) and (6) in Eq. (4), we get a form for $F(i\to f,\vec{q})$ with seven integrals in it:

$$F(i\to f,\vec{q})=-\frac{4\pi}{[\Gamma(-i\eta)]^2}(\eta K)C_{fi}S(1\to 2)D(\mu_1,\vec{\gamma}_1;\mu_2,\vec{\gamma}_2)(4\mu_1)(4\mu_2)$$
$$\times\int_0^\infty d\lambda_1\lambda_1^{-i\eta}\int_0^1\frac{d\chi_1}{\chi_1}\int_0^\infty d\lambda_2\lambda_2^{-i\eta}\int_0^1\frac{d\chi_2}{\chi_2}(1-\chi_2)\left(\frac{d}{d\mu_1^2}\right)^2\left(\frac{d}{d\mu_2^2}\right)^2\left(\lambda_1^{-1}-\frac{1-\chi_1}{\Lambda_2}\right)\frac{1}{\Lambda_2}$$
$$\times\int\frac{d^3r_3}{r_3}\exp[-(\Lambda_1+\Lambda_2)r_3+i\vec{q}''\cdot\vec{r}_3](r_3-z_3)^{-2i\eta}. \tag{8}$$

Here $\vec{q}''=\vec{q}-\vec{\alpha}_1-\vec{\alpha}_2$.

The integration over $\vec{r}_3$ is performed using the following integral also from the work of Gau and Macek.

$$\int\frac{d^3r_3}{r_3}\exp[-(\Lambda_1+\Lambda_2)r_3+i\vec{q}''\cdot\vec{r}_3](r_3-z_3)^{-2i\eta}=2^{2-2i\eta}\pi\Gamma(1-2i\eta)[(\Lambda_1+\Lambda_2)^2+q''^2]^{2i\eta-1}(\Lambda_1+\Lambda_2-iq''_z)^{-2i\eta}. \tag{9}$$

Equation (9) is inserted in Eq. (8) to obtain $F(i\to f,\vec{q})$ in terms of four integrals over λ_1, λ_2, χ_1, and χ_2:

$$F(i\to f,\vec{q})=-\frac{2^{4-2i\eta}}{[\Gamma(-i\eta)]^2}C_{fi}(\eta K)\pi^2\Gamma(1-2i\eta)S(1\to 2)D(\mu_1,\vec{\gamma}_1;\mu_2,\vec{\gamma}_2)(4\mu_1)(4\mu_2)$$
$$\times\int_0^\infty d\lambda_1\lambda_1^{-i\eta}\int_0^1\frac{d\chi_1}{\chi_1}\int_0^\infty d\lambda_2\lambda_2^{-i\eta}\int_0^1\frac{d\chi_2}{\chi_2}(1-\chi_2)\left(\frac{d}{d\mu_1^2}\right)^2\left(\frac{d}{d\mu_2^2}\right)^2\left(\lambda_1^{-1}-\frac{1-\chi_1}{\Lambda_1}\right)\frac{1}{\Lambda_2}$$
$$\times[(\Lambda_1+\Lambda_2)^2+q''^2]^{2i\eta-1}(\Lambda_1+\Lambda_2-iq''_z)^{-2i\eta}. \tag{10}$$

It is remarkable that $F(i\to f,\vec{q})$ can be reduced from nine to four integrals in such a straightforward fashion. No further simplification without approximations or numerical methods appears possible.

The integrals in Eq. (10) may be rewritten in the following convenient form:

$$L(\mu_1,\mu_2;\vec{\gamma}_1,\vec{\gamma}_2;\eta)=\int_0^\infty d\lambda_1\,\lambda_1^{-i\eta-1}\int_0^\infty d\lambda_2\,\lambda_2^{-i\eta-1}\int_0^1\frac{d\chi_1}{\chi_1}\int_0^1\frac{d\chi_2}{\chi_2}\left(\frac{d}{d\mu_1^2}\right)^2\left(\frac{d}{d\mu_2^2}\right)^2 \times[F(0,0,0;1,1,1;1,0)-F(1,1,1;1,1,1;1,0)]\,, \tag{11}$$

where

$$F(a,b,c;d,e,f;g,h)=\lambda_1^a(1-\chi_1)^b\Lambda_1^{-c}\lambda_2^d(1-\chi_2)^e\Lambda_2^{-f}[(\Lambda_1+\Lambda_2)^2+q''^2]^{2i\eta-g}(\Lambda_1+\Lambda_2-q''_z)^{-2i\eta-h}\,. \tag{12}$$

III. LARGE-ANGLE APPROXIMATION

Up to this point no approximations have been performed except that of the unrestricted Glauber approximation. Next we investigate high-energy scattering at large angles. We expand parts of the integrand of Eq. (10) and keep the lowest-order terms in $1/q$:

$$[(\Lambda_1+\Lambda_2)^2+q''^2]^{2i\eta-g}=q^{2(2i\eta-g)}\left[\left(1-2\vec{q}\cdot\frac{(\vec{\alpha}_1+\vec{\alpha}_2)}{q^2}\right)^{2i\eta-g}+O(1/q^2)\right]\,, \tag{13}$$

$$[\Lambda_1+\Lambda_2-iq''_z]^{-2i\eta-h}=(-iq_z)^{-2i\eta-h}\left(1+\frac{-2i\eta-h}{-iq_z}(\Lambda_1+\Lambda_2+i\alpha_{1z}+i\alpha_{2z})+O(1/q^2)\right)\,. \tag{14}$$

The approximate form of $F(a,\ b,\ c;\ d,\ e,\ f;\ g,\ h)$ in Eq. (12) is $F'(a,\ b,\ c;\ d,\ e,\ f;\ g,\ h)$ where

$$F'(a,\ b,\ c;\ d,\ e,\ f;\ g,\ h)=\lambda_1^a(1-\chi_1)^b\Lambda_1^{-c}\lambda_2^d(1-\chi_2)^e\Lambda_2^{-f}q^{2(2i\eta-g)}(-iq_z)^{-2i\eta-h} \times\left(1-2\vec{q}\cdot\frac{(\vec{\alpha}_1+\vec{\alpha}_2)}{q^2}\right)^{2i\eta-g}\left(1+\frac{-2i\eta-h}{-iq_z}(\Lambda_1+\Lambda_2+i\alpha_{1z}+i\alpha_{2z})\right)\,. \tag{15}$$

The second derivatives of $F'(a,\ b,\ c;\ d,\ e,\ f;\ g,\ h)$ with respect to μ_1^2 and μ_2^2 can be performed:

$$\left(\frac{d}{d\mu_1^2}\right)^2\left(\frac{d}{d\mu_2^2}\right)^2F'(a,\ b,\ c;\ d,\ e,\ f;\ g,\ h)=\frac{\chi_1^2\chi_2^2}{16}\lambda_1^a(1-\chi_1)^b\lambda_2^d(1-\chi_2)^e\left(1-2\vec{q}\cdot\frac{(\vec{\alpha}_1+\vec{\alpha}_2)}{q^2}\right)^{2i\eta-g}q^{2(2i\eta-g)}(-iq_z)^{-2i\eta-h}$$
$$\times\left[c(c+2)f(f+2)\Lambda_1^{-c-4}\Lambda_2^{-f-4}\left(1+\frac{-2i\eta-h}{-iq_z}(i\alpha_{1z}+i\alpha_{2z})\right)\right.$$
$$+c(c+2)(f-1)(f+1)\Lambda_1^{-c-4}\Lambda_2^{-f-3}\frac{-2i\eta-h}{-iq_z}$$
$$\left.+(c-1)(c+1)f(f+2)\Lambda_1^{-c-3}\Lambda_2^{-f-4}\frac{-2i\eta-h}{-iq_z}\right]\,. \tag{16}$$

In order to calculate the $1\,^1S_0$-$2\,^1P_1$ transition amplitude, we will need the derivatives of L' with respect to γ_{1x}, γ_{1y}, γ_{1z}, γ_{2x}, γ_{2y}, and γ_{2z} evaluated at $\vec{\gamma}_1=\vec{\gamma}_2=0$ where L' is defined by

$$L'=\int_0^\infty d\lambda_1\,\lambda_1^{-i\eta-1}\int_0^\infty d\lambda_2\,\lambda_2^{-i\eta-1}\int_0^1\frac{d\chi_1}{\chi_1}\int_0^1\frac{d\chi_2}{\chi_2}\left(\frac{d}{d\mu_1^2}\right)^2\left(\frac{d}{d\mu_2^2}\right)^2F'(a,\ b,\ c;\ d,\ e,\ f;\ g,\ h)\,. \tag{17}$$

Fortunately, Eq. (17) is symmetric in electron 1 and electron 2. Further, Eq. (17) treats γ_{1x} in the same way as γ_{1y}. These symmetries mean that only the derivatives with respect to γ_{1x} and γ_{1z} need be performed. The derivatives with respect to γ_{1y}, γ_{2x}, γ_{2y}, and γ_{2z} can be obtained from these two. We exhibit the derivatives of Eq. (17) with respect to γ_{1x} and γ_{1z} and set $\vec{\gamma}_1=\vec{\gamma}_2=0$:

$$\left.\frac{dL'}{d\gamma_{1x}}\right|_{\vec{\gamma}_1=\vec{\gamma}_2=0}\approx\frac{c(c+2)f(f+2)}{2^3}q_xq^{4i\eta-2g-2}(-iq_z)^{-2i\eta-h}(2i\eta-g)$$
$$\times\int_0^\infty d\lambda_1\,\lambda_1^{-i\eta-1}\int_0^\infty d\lambda_2\,\lambda_2^{-i\eta-1}\int_0^1 d\chi_1\,\chi_1^2\int_0^1 d\chi_2\,\chi_2\,\lambda_2^d(1-\chi_2)^e(1-\chi_1)^b\lambda_1^a\left(1-2\vec{q}\cdot\frac{(\vec{\alpha}_1+\vec{\alpha}_2)}{q^2}\right)^{2i\eta-g-1}\Lambda_1^{-c-4}\Lambda_2^{-f-4}\,, \tag{18}$$

$$\left.\frac{dL'}{d\gamma_{1z}}\right|_{\vec{\gamma}_1=\vec{\gamma}_2=0} \approx -\frac{ic(c+2)(c+4)f(f+2)}{2^4}(q^{4i\eta-2g})(-iq_z)^{-2i\eta-h}$$
$$\times\int_0^\infty d\lambda_1\,\lambda_1^{-i\eta}\int_0^\infty d\lambda_2\,\lambda_2^{-i\eta-1}\int_0^1 d\chi_1\,\chi_1^2\int_0^1 d\chi_2\,\chi_2\,\lambda_2^d(1-\chi_2)^e(1-\chi_1)^{b+1}\lambda_1^a\left(1-2\vec{q}\cdot\frac{(\vec{\alpha}_1+\vec{\alpha}_2)}{q^2}\right)^{2i\eta-g}\Lambda_1^{-c-6}\Lambda_2^{-f-4}\,. \tag{19}$$

In Eqs. (18) and (19) the expressions for Λ_1, $\vec{\alpha}_1$, Λ_2, and $\vec{\alpha}_2$ are evaluated at $\vec{\gamma}_1=\vec{\gamma}_2=0$.

The $[1-2\vec{q}\cdot(\vec{\alpha}_1+\vec{\alpha}_2)/q^2]^{2i\eta-g}$ parts can now be expanded keeping only the first term without fear of introducing spurious divergences in λ_1 or λ_2. The integrands are well behaved for the choices of $(a, b, c; d, e, f; g, h)$ prescribed by the $F(a, b, c; d, e, f; g, h)$ in Eq. (11). The integrations produce beta functions which depend on η.

We will write out the derivatives of L of Eq. (11) with respect to γ_{1x} and γ_{1z} keeping only the lowest-order term. It should be noted that both for $dL/d\gamma_{1x}$ and $dL/d\gamma_{1z}$ the lowest-order term arises from the $F(1,1,1;1,1,1;1,0)$ part of Eq. (11). This is the incoming electron-nuclear part of the Glauber scattering amplitude.

$$\left.\frac{dL}{d\gamma_{1x}}\right|_{\vec{\gamma}_1=\vec{\gamma}_2=0} \approx -\frac{3^2}{2^5}\frac{q_x}{q^2}(2i\eta-1)q^{4i\eta-2}(-iq_z)^{-2i\eta}(\mu_1)^{-i\eta-4}(\mu_2)^{-i\eta-4}B((-i\eta+1)/2,2+i\eta/2)$$
$$\times B(1-i\eta/2,1+i\eta)B((-i\eta+1)/2,2+i\eta/2)B(-i\eta/2,1+i\eta)\,, \tag{20}$$

$$\left.\frac{dL}{d\gamma_{1z}}\right|_{\vec{\gamma}_1=\vec{\gamma}_2=0} \approx \frac{i3^2 5}{2^6}(-iq_z)^{-2i\eta}q^{4i\eta-2}(\mu_1)^{-i\eta-5}(\mu_2)^{-i\eta-4}B(-\tfrac{1}{2}i\eta+1,\tfrac{5}{2}+\tfrac{1}{2}i\eta)B(\tfrac{1}{2}-\tfrac{1}{2}i\eta,i\eta+1)$$
$$\times B((-i\eta+1)/2,2+i\eta/2)B(-\tfrac{1}{2}i\eta,i\eta+1)\,. \tag{21}$$

Before presenting our helium wave functions, let us review the coordinate system we have been employing. The $\hat{z}$ axis is chosen in the direction $\vec{K}$ of the impinging electron. The $\hat{y}$ axis is along $\vec{K}\times\vec{K}'$ and $\hat{x}$ is perpendicular to $\hat{z}$ and $\hat{y}$.

The approximate wave functions for the $2\,{}^1P_1$ state of helium are those given by Eckart[8] in the following orthonormal basis:

$$u_{2\,{}^1P_{1j}}(\vec{r}_1,\vec{r}_2) = (Z_a^{5/2}Z_i^{3/2}/2^3\pi)S(1\leftrightarrow 2)r_{1j}\exp(-\tfrac{1}{2}Z_a r_1 - Z_i r_2)\,. \tag{22}$$

Here $Z_a = 0.97/a_0$ and $Z_i = 2.0/a_0$. (a_0 is the Bohr radius of hydrogen.) Further, j refers to either x, y, or z. $S(1\leftrightarrow 2)$ ensures the symmetry between electron 1 and electron 2.

First we use the hydrogenlike approximation with screening in the wave function for the $1\,{}^1S_0$ ground state

$$u_{1\,{}^1S_0}(\vec{r}_1,\vec{r}_2) = (Z_G^3/\pi)\exp(-Z_G r_1 - Z_G r_2)\,, \tag{23}$$

where $Z_G = 27/16a_0$.

The scattering amplitudes are given by

$$F(1\,{}^1S_0\to 2\,{}^1P_{1x},\vec{q}) = -i2^{4-2i\eta}3^2(\eta K)C_{fi}\pi^2[\Gamma(-i\eta)]^{-2}\Gamma(1-2i\eta)\mu_1^{-3-i\eta}\mu_2^{-3-i\eta}q_x(2i\eta-1)q^{4i\eta-4}(-iq_z)^{-2i\eta}$$
$$\times B((-i\eta+1)/2,2+i\eta/2)B(1-i\eta/2,1+i\eta)B((-i\eta+1)/2,2+i\eta/2)B(-i\eta/2,1+i\eta)\,, \tag{24}$$

$$F(1\,{}^1S_0\to 2\,{}^1P_{1z},\vec{q}) = -3^2\times 5\times 2^{2-2i\eta}C_{fi}(\eta K)\pi^2[\Gamma(-i\eta)]^{-2}\Gamma(1-2i\eta)(-iq_z)^{-2i\eta}q^{4i\eta-2}(\mu_1^{-i\eta-4}\mu_2^{-i\eta-3}+\mu_1^{-i\eta-3}\mu_2^{-i\eta-4})$$
$$\times B(-\tfrac{1}{2}i\eta+1,\tfrac{5}{2}+\tfrac{1}{2}i\eta)B((1-i\eta)/2,i\eta+1)B((-i\eta+1)/2,2+i\eta/2)B(-i\eta/2,i\eta+1)\,, \tag{25}$$

$$C_{fi} = \frac{Z_a^{5/2}Z_i^{3/2}Z_G^3}{2^3\pi^2}\,. \tag{26}$$

Note $F(1\,{}^1S_0\to 2\,{}^1P_{1y},\vec{q})$ is identically equal to zero because from the way our coordinate system is arranged $q_y=0$. Thus, the only dependence on γ_{1y} or γ_{2y} in Eq. (10) comes from terms dependent on γ_{1y}^2 or γ_{2y}^2. In the process of taking derivatives with respect to γ_{1y} and γ_{2y} and setting $\vec{\gamma}_1=\vec{\gamma}_2=0$ these terms vanish. Thus, $F(1\,{}^1S_0\to 2\,{}^1P_{1y},\vec{q})=0$.

The differential cross section for this large-angle collision is determined by the $1\,{}^1S_0\to 2\,{}^1P_{1z}$ part of the scattering amplitude. For K large, which means $\eta = me^2/\hbar^2 K$ is small, we find the asymptotic differential cross section for this process:

$$\frac{d\sigma}{d\Omega}(1\,{}^1S_0\to 2\,{}^1P_1) \simeq 0.74\frac{(K\eta^2)^2}{q^4}\,. \tag{27}$$

Using the ground-state wave function given by Mott and Massey,[9] we find

$$\frac{d\sigma}{d\Omega}(1\,{}^1S_0 \rightarrow 2\,{}^1P_1) \simeq 0.90\,\frac{(K\eta^2)^2}{q^4}. \tag{28}$$

As can be seen, these differential cross sections are dependent upon the choice of approximate wave functions.

IV. CONCLUSION

In this paper the work of Gau and Macek on the unrestricted Glauber approximation is extended to electron-helium collisions. We reduce the original nine-integral expression for the unrestricted Glauber amplitude to one with four integrals. For the $1\,{}^1S_0 \rightarrow 2\,{}^1P_1$ collision-induced transition we calculate the inelastic differential cross section for large angles at high fixed energy. The electron-nucleus term dominates and gives a q^{-4} behavior to the differential cross section. From using two wave functions for the ground state, we find that this asymptotic differential cross section depends somewhat on the choice of approximate helium wave functions.

As in the case of hydrogen there is a 90° phase difference between $F(1\,{}^1S_0 \rightarrow 2\,{}^1P_{1x}, \vec{q})$ and $F(1\,{}^1S_0 \rightarrow 2\,{}^1P_{1z}, \vec{q})$ in the backward direction giving rise to a large O_{1-}^{col}, the orientation parameter.[1] This is a characteristic of both hydrogen and helium in the unrestricted Glauber approach. We give the expression for O_{1-}^{col} in the high-fixed-energy large-angle approximation:

$$O_{1-}^{\text{col}} \approx -1.8 q_x / q^2 a_0 . \tag{29}$$

Inasmuch as q_x behaves as $-K\sin\theta$ and q as $2K \times \sin(\theta/2)$ at large K, O_{1-}^{col} given in Eq. (29) goes linearly to zero with $(180° - \theta)$ in the backward direction.

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