Letter

Model-independent inference of quantum interaction from statistics

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Any physical theory aims to establish the relationship between physical systems in terms of the interaction between these systems. However, any known approach in the literature to infer this interaction is dependent on the particular modeling of the physical systems involved. Here, we propose an alternative approach where one does not need to model the systems involved but only assume that these systems behave according to quantum theory. We first propose a setup to infer a particular entangling quantum interaction between two systems from the statistics. For our purpose, we utilize the framework of Bell inequalities. We then extend this setup where an arbitrary number of quantum systems interact via some entangling interaction.

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Introduction. In physics, the exploration of interactions between two systems constitutes the foundational aspect of understanding natural phenomena. Despite significant advancements in the field, the analyses conducted thus far remain inherently model dependent. A notable example of this lies in quantum field theories, where one hypothesizes different fields to explain experimental observations. While these models have proven immensely powerful in explaining a wide array of physical phenomena, their reliance on specific theoretical constructs underscores the necessity for continued refinement and exploration. Furthermore, these models are highly dependent on the parameters and the assumptions made on the experimental setup.

Recently, the idea of device-independent (DI) certification of quantum states and measurements has gained a lot of interest as they allow one to certify the properties of an unknown quantum device by only observing the statistical data it generates and making minimal assumptions about the device. The essential resource for any DI scheme is Bell nonlocality [1–3]. For instance, any violation of a Bell inequality is a DI certification of the presence of entanglement inside the device.

The strongest form of DI certification is termed self-testing [4,5], enabling near-complete characterization of the underlying quantum state and its associated measurements by only assuming that the devices behave according to quantum theory. Consequently, a wide range of schemes has been proposed to self-test pure entangled quantum states and projective quantum measurements (see, e.g., Refs. [6–20]) as well as mixed entangled states [20,21] and nonprojective measurements [20,22]. Furthermore, schemes to self-test single unitaries [23] and the controlled-NOT gate have been proposed in Ref. [24]. Despite this progress, no scheme has been proposed that can be used to certify the interaction between two unknown systems.

Inspired by self-testing, in this Letter, we propose a model-independent approach to infer the quantum interaction between two systems from the statistics generated in the experiment. We do not delve into the physical considerations of the experimental setup but rather focus on the operational nature of the scheme, that is, we do not care about the degree of freedom in which the interaction between the systems takes place. However, the concerned degree of freedom is the one that is being measured by the detectors. We particularly focus on entangling quantum interactions, that is, quantum interactions that can generate an entangled state from a product state.

Recently, a lot of attention has been devoted to such interactions as they can be a certificate to probe the quantum nature of gravity [25,26]. Such entangling interactions have also been explored in quantum electrodynamics, for instance, two electrons that dynamically scatter get entangled either in spin or momentum degrees of freedom [27–35]. Furthermore, such entangling interactions have also been explored in the quark-quark system [36–39] and have been recently observed at the Large Hadron Collider [40]. We first propose a scenario that can be used to infer that two systems are interacting via a particular entangling quantum interaction that can generate maximally entangled states from product ones. For our purpose, we use the Bell inequalities suggested in Refs. [18,20]. Then, we generalize this result where an arbitrary number of systems interact with each other. Again, we utilize the Bell inequalities suggested in Ref. [20] to certify a particular entangling quantum interaction that can create Greenberger-Horne-Zeilinger (GHZ)-like states from product states.

Two-system interaction. Let us begin by describing the setup to infer the entangling quantum interaction in a model-independent way.

A source P sends particles to Alice and Bob who are located in spatially separated laboratories. Alice and Bob can freely choose two inputs each denoted by $x_1, y_1 = 0, 1$, respectively, based on which local measurements are performed on their particles at time t_1. Furthermore, each measurement results in two outputs denoted by $a_1, b_1 = 0, 1$ for Alice and Bob, respectively. Then, the postmeasured particles are allowed to leave their laboratories after which they interact via some interaction $\mathcal{I}$. After the interaction, the particles come

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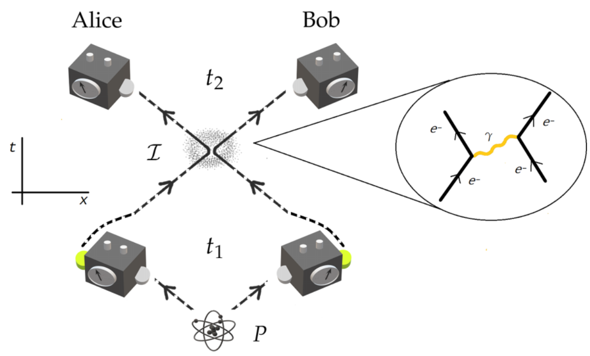

FIG. 1. Left: Setup for model-independent inference of quantum interaction. Particles from source P are sent to spatially separated laboratories where Alice and Bob reside. At time t_1, Alice and Bob choose $x_1, y_1 = 0, 1$ for local measurements, yielding $a_1, b_1 = 0, 1$. Postmeasured particles interact via $\mathcal{I}$, and return to their respective laboratories at t_2. At t_2, inputs $x_2, y_2 = 0, 1$ are chosen by Alice and Bob, respectively, resulting in $a_2, b_2 = 0, 1$. Right: An example of an entangling quantum interaction. Two electrons interacting via dynamical scattering can get entangled either in spin or momentum degrees of freedom.

back to Alice and Bob at time t_2. Again, Alice and Bob choose two inputs $x_2, y_2 = 0, 1$ based on which the incoming particles are measured and result in two outputs $a_2, b_2 = 0, 1$ (see Fig. 1).

By repeating the above-described procedure, they obtain two joint probability distributions often referred to as correlations. The first one is obtained at time t_1 given by $\vec{p}_1 = \{p(a_1, b_1|x_1, y_1, P)\}$, where $p(a_1, b_1|x_1, y_1, P)$ denotes the probability of obtaining outcomes a_1, b_1 given the inputs x_1, y_1 given the source P. The second probability distribution is obtained at time t_2 given by

$$\vec{p}_{2,a_1,b_1,x_1,y_1} = \{p(a_2, b_2|x_2, y_2, a_1, b_1, x_1, y_1, \mathcal{I}, P)\}, \quad (1)$$

where $p(a_2, b_2|x_2, y_2, a_1, b_1, x_1, y_1, \mathcal{I}, P)$ denotes the conditional probability of obtaining outcome a_2, b_2 at time t_2 given inputs x_2, y_2 when Alice and Bob at time t_1 obtained a_1, b_1 with inputs x_1, y_1 and the postmeasured states interacted via some interaction $\mathcal{I}$.

Let us now describe the above-proposed scenario (see Fig. 1) within quantum theory. The probability $p(a_1, b_1|x_1, y_1, P)$ is obtained via the Born rule as

$$p(a_1, b_1|x_1, y_1, P) = \mathrm{Tr}\big(M^A_{a_1|x_1} \otimes M^B_{b_1|y_1}\rho_{AB}\big), \quad (2)$$

where $M^A_{a_1|x_1}, M^B_{b_1|y_1}$ denote the measurement elements of Alice and Bob such that these elements are positive and $\sum_a M^A_{a_1|x_1} = \sum_b M^B_{b_1|y_1} = \mathbb{1}$. Here, ρ_{AB} is the quantum state prepared by the source P. One could similarly define the above formula (2) at time t_2 where the state ρ_{AB} will be replaced by the postinteraction states defined below in (5). It is often helpful to express the correlations in terms of the expected values of observables which are defined as

$$\langle A_m \otimes B_l\rangle = \sum_{a,b=0}^{1} (-1)^{a+b} p(a, b|m, l, \dots, P). \quad (3)$$

Notice that by using Eq. (2), these expectation values can be expressed as $\langle A_m \otimes B_l\rangle = \mathrm{Tr}[A_m \otimes B_l \rho_{AB}]$, where A_m, B_l are quantum operators, referred to as observables, defined via the measurement elements as $A_m = M^A_{0|m} - M^A_{1|m}$, $B_l = M^B_{0|l} - M^B_{1|l}$ for every m, l. When the measurement is projective, then the corresponding observable is unitary. Furthermore, if the measurements of Alice and Bob are projective and they observe the outputs a_1, b_1 given the inputs x_1, y_1, respectively, then the postmeasurement states are given by

$$\rho'_{a_1,b_1,x_1,y_1} = \frac{M^A_{a_1|x_1} \otimes M^B_{b_1|y_1}\rho_{AB} M^A_{a_1|x_1} \otimes M^B_{b_1|y_1}}{p(a_1, b_1|x_1, y_1)}. \quad (4)$$

Then, the particles interact via some Hamiltonian $H(t)$ for a time δt. Consequently, the state ρ'_{a_1,b_1,x_1,y_1} at time t_1 evolves via the unitary process $V(\delta t)$ [41]. For instance, when the Hamiltonian is time independent $H(t) \equiv H$ for any time t, then $V(\delta t)$ is given by $V(\delta t) = e^{-iH\delta t}$. Now, the states after the interaction, referred to as postinteraction states, are given by

$$V(\delta t)\rho'_{a_1,b_1,x_1,y_1} V(\delta t)^\dagger = \sigma_{a_1,b_1,x_1,y_1}. \quad (5)$$

For simplicity, we will further represent $V(\delta t) \equiv V$. Without loss of generality, we consider that the unitary maps quantum states from the Hilbert space $\mathcal{H}_{A(t_1)} \otimes \mathcal{H}_{B(t_1)}$ to a different Hilbert space $\mathcal{H}_{A(t_2)} \otimes \mathcal{H}_{B(t_2)}$.

Model-independent inference. Inspired by the idea of self-testing (see Ref. [42] for a review), let us introduce the idea of model-independent inference of a quantum interaction via statistics by referring back to the scenario shown in Fig. 1. First, we consider that the measurements conducted by the parties, the state prepared by the source as well as the interaction between the postmeasured states are unknown except for the fact that they obey quantum theory. The only other information that Alice and Bob have about the whole scenario is via the observed correlations $\vec{p}_1, \vec{p}_{2,a_1,b_1,x_1,y_1}$. It is worth mentioning here that we assume that dimensions of the local Hilbert spaces $\mathcal{H}_{A(t_i)}, \mathcal{H}_{B(t_i)}$ for $i = 1, 2$ is unknown but finite.

Let us then consider a reference experiment giving rise to the same correlations $\vec{p}_1$ when some known observables A'_m, B'_l are performed on a known quantum state prepared by the source $|\psi'_{AB}\rangle \in \mathcal{H}_{A'(t_1)} \otimes \mathcal{H}_{B'(t_1)}$. Then the postmeasured states interact via some known unitary V', after which they are measured using the same known observables A'_m, B'_l to obtain $\vec{p}_{2,a_1,b_1,x_1,y_1}$. The task of model-independent inference is to deduce from the observed $\vec{p}_1, \vec{p}_{2,a_1,b_1,x_1,y_1}$ that the actual experiment is equivalent to the reference one in the following sense: (i) The local Hilbert spaces admit the product form $\mathcal{H}_{A(t_i)} = \mathcal{H}_{A'(t_i)} \otimes \mathcal{H}_{A''(t_i)}$ and $\mathcal{H}_{B(t_i)} = \mathcal{H}_{B'(t_i)} \otimes \mathcal{H}_{B''(t_i)}$ for some auxiliary Hilbert spaces $\mathcal{H}_{A''(t_i)}$ and $\mathcal{H}_{B''(t_i)}$. (ii) There are local unitary operations $U_{s(t_i)} : \mathcal{H}_{s(t_i)} \to \mathcal{H}_{s'(t_i)} \otimes \mathcal{H}_{s''(t_i)}$ for $s = A, B$ and $i = 1, 2$ such that

$$\begin{aligned}(U_{A(t_1)} \otimes U_{B(t_1)})\rho_{AB}(U_{A(t_1)} \otimes U_{B(t_1)})^\dagger \\ = |\psi'\rangle\langle\psi'|_{A'(t_1)B'(t_1)} \otimes \xi_{A''(t_1)B''(t_1)},\end{aligned} \quad (6)$$

where $\xi_{A''_i E''_i}$ acting on $\mathcal{H}_{A''(t_i)} \otimes \mathcal{H}_{B''(t_i)}$ is some auxiliary quantum state, and

$$\begin{aligned}U_{A(t_i)}\overline{A}_{m(t_i)}U^\dagger_{A(t_i)} &= A'_m \otimes \mathbb{1}_{A''(t_i)}, \\ U_{B(t_i)}\overline{B}_{l(t_i)}U^\dagger_{B(t_i)} &= B'_l \otimes \mathbb{1}_{B''(t_i)},\end{aligned} \quad (7)$$

where $\mathbb{1}_{s''(t_i)}$ is the identity acting on the parties auxiliary system and $\overline{A}_{m(t_i)} = \Pi_i^A A_m \Pi_i^A$ and $\overline{B}_{l(t_i)} = \Pi_i^B B_l \Pi_i^B$ such that Π_i^A, Π_i^B denotes the projection of the observables A_m, B_l onto the Hilbert space $\mathcal{H}_{A(t_i)}, \mathcal{H}_{B(t_i)}$ respectively. (iii) The interaction V is certified as

$$(U_{A(t_2)} \otimes U_{B(t_2)})V(U_{A(t_1)} \otimes U_{B(t_1)})^\dagger = V' \otimes V_0, \tag{8}$$

where V_0 is a unitary matrix mapping $\mathcal{H}_{A''(t_1)} \otimes \mathcal{H}_{B''(t_1)}$ to $\mathcal{H}_{A''(t_2)} \otimes \mathcal{H}_{B''(t_2)}$. For a note, if the above conditions (i) and (ii) are met, one may say that the reference state and measurements are self-tested in the actual experiment from the observed correlations. Then if condition (iii) is met, the interaction between the systems is certified in a model-independent way.

Consider now the following Bell inequalities for $a_1, b_1 = 0, 1$,

$$\mathcal{B}_{a_1,b_1} = (-1)^{a_1}\langle \tilde{A}_1 \otimes B_1 + (-1)^{b_1}\tilde{A}_0 \otimes B_0\rangle \leqslant \beta_C, \tag{9}$$

where

$$\tilde{A}_0 = \frac{A_0 - A_1}{\sqrt{2}}, \quad \tilde{A}_1 = \frac{A_0 + A_1}{\sqrt{2}}. \tag{10}$$

The classical bound of the above Bell inequalities is $\beta_C = \sqrt{2}$ for any a_1, b_1. Consider now the following states,

$$|\phi_{a_1,b_1}\rangle = \frac{1}{\sqrt{2}}\big[|a_1 b_1\rangle + (-1)^{a_1}\big|a_1^\perp b_1^\perp\big\rangle\big], \tag{11}$$

where $a_1^\perp = 1 - a_1$, $b_1^\perp = 1 - b_1$, and the following observables,

$$A_0 = \frac{X+Z}{\sqrt{2}}, \quad A_1 = \frac{X-Z}{\sqrt{2}}, \quad B_0 = Z, \quad B_1 = X. \tag{12}$$

As shown in Fact 1 of Sec. A of the Supplemental Material [43], using these states (11) and observables (12), one can attain the value $\mathcal{B}_{a_1,b_1} = 2$. This is the quantum bound β_Q of $\mathcal{B}_{a_1,b_1}$, that is, the maximal value of $\mathcal{B}_{a_1,b_1}$ that can be attained within quantum theory.

Let us now suppose that correlations $\vec{p}_1$ achieve the quantum bound of $\mathcal{B}_{0,0}$, that is, the quantum state ρ_{AB} maximally violates the Bell inequality $\mathcal{B}_{0,0}$. Furthermore, the correlations $\vec{p}_{2,a_1,b_1,0,0}$ achieve the quantum bound of $\mathcal{B}_{a_1,b_1}$ for each a_1, b_1, that is, the postinteraction quantum states $\sigma_{a_1,b_1,0,0}$ maximally violate the Bell inequalities $\mathcal{B}_{a_1,b_1}$. Along with them, one also needs to observe that when $a_1 = b_1 = 0$ and $x_1 = 1, y_1 = 1$, the postinteraction states $\sigma_{0,0,1,1}$ satisfy

$$\langle \mathbb{1} \otimes B_1\rangle = -\langle \tilde{A}_0 \otimes \mathbb{1}\rangle = 1. \tag{13}$$

Consider now the following unitary,

$$\mathcal{U} = \sum_{a_1,b_1=0,1} |\phi_{a_1,b_1}\rangle\langle \overline{a}_1 b_1|, \tag{14}$$

where $\{|\overline{0}\rangle, |\overline{1}\rangle\}$ are the eigenvectors of $(X+Z)/\sqrt{2}$. It is straightforward to verify that if the state $|\phi_{0,0}\rangle$ (11), after being measured by the observables (12) and obtaining outcome a_1, b_1 with inputs $x_1 = y_1 = 0$, evolves via the unitary (14), then one obtains the postinteraction state as $|\phi_{a_1,b_1}\rangle$. Consequently, all these states and observables satisfy the above-mentioned statistics and thus we take them as the reference quantum states, observables, and interaction.

Let us now state the main result.

Theorem 1. Assume that the Bell inequality $\mathcal{B}_{0,0}$ (9) is maximally violated at t_1. Furthermore, when Alice and Bob obtain the outcome a_1, b_1 with inputs $x_1 = y_1 = 0$, then the Bell inequalities $\mathcal{B}_{a_1,b_1}$ (9) for any a_1, b_1 are maximally violated at t_2. Along with it, when Alice and Bob observe the outcomes $a_1 = b_1 = 0$ with inputs $x_1 = 1, y_1 = 1$, then the condition (13) is also satisfied. Then, the quantum state prepared by the source and the observables of both parties are certified as in (6) and (7), with the reference strategies given below Eq. (14). Importantly, the unitary V is certified as defined in (8)

$$U_{A(t_2)} \otimes U_{B(t_2)}\, V\, U_{A(t_1)}^\dagger \otimes U_{B(t_1)}^\dagger = \mathcal{U} \otimes V_0, \tag{15}$$

where $\mathcal{U}$ is given in Eq. (14) and V_0 is unitary.

The proof of the above theorem is presented in Sec. A of the Supplemental Material [43]. Here, we provide a short description of the proof. The proof is mainly divided into two parts. In the first part, we self-test the state prepared by the source and the measurements acting on the local support of this state. This further allows one to certify the postmeasurement states of the parties. We then self-test the postinteraction states along with the measurements acting on their local supports. Both of these self-tests are based on the sum of squares (SOS) decomposition of the Bell operator corresponding to the inequality (9) for which we follow the techniques introduced in Ref. [20]. Importantly, we do not assume any state to be pure or measurements to be projective in our proof. In the second part, utilizing the certified postmeasured and postinteraction states and the condition (13), we then conclude that the only quantum interaction that can reproduce all the statistics has to be of the form (15).

Let us now generalize the above result to the scenario where an arbitrary number of particles interact with each other via some entangling quantum interaction.

N-system interaction. We begin by generalizing the scenario depicted in Fig. 1. A source P_N sends particles to N Alices who are located in spatially separated laboratories. All the parties freely choose two inputs, each denoted by $x_{n,1} = 0, 1$ for $n = 1, \ldots, N$ based on which local measurements are performed on their particles at time t_1, respectively. Furthermore, each measurement results in two outputs denoted by $a_{n,1} = 0, 1$ for the nth Alice. Then, the postmeasured particles are allowed to leave their laboratories after which they interact via some interaction $\mathcal{I}_N$. After the interaction, the particles come back to their respective starting laboratories at time t_2. Again, all the parties choose two inputs $x_{n,2} = 0, 1$ based on which the incoming particles are measured and result in two outputs $a_{n,2} = 0, 1$. We further denote $\mathbf{x_i} = x_{1,i}, \ldots, x_{N,i}$ and $\mathbf{a_i} = a_{1,i}, \ldots, a_{N,i}$ for $i = 1, 2$.

By repeating the above-described procedure, they obtain two joint probability distributions or correlations. The first one is obtained at time t_1 given by $\vec{p}_{N,1} = \{p(\mathbf{a_1}|\mathbf{x_1}, P_N)\}$ where $p(\mathbf{a_1}|\mathbf{x_1}, P_N)$ denotes the probability of obtaining outcome $\mathbf{a_1}$ given the input $\mathbf{x_1}$ and source P_N. The second probability distribution is obtained at time t_2 given by

$$\vec{p}_{N,2,\mathbf{a_1},\mathbf{x_1}} = \{p(\mathbf{a_2}|\mathbf{x_2}, \mathbf{a_1}, \mathbf{x_1}, \mathcal{I}_N, P_N)\}, \tag{16}$$

where $p(\mathbf{a_2}|\mathbf{x_2}, \mathbf{a_1}, \mathbf{x_1}, \mathcal{I}_N, P_N)$ denotes the conditional probability of obtaining outcome $\mathbf{a_2}$ at time t_2 given inputs $\mathbf{x_2}$ when

Alice and Bob at time t_1 obtained $\mathbf{a_1}$ with input $\mathbf{x_1}$ and the postmeasured states interacted via some interaction $\mathcal{I}_N$.

Within quantum theory, the probability $p(\mathbf{a_1}|\mathbf{x_1}, P_N)$ is obtained via the Born rule as

$$p(\mathbf{a_1}|\mathbf{x_1}, P_N) = \mathrm{Tr}\left(\bigotimes_{n=1}^{N} M^n_{a_{n,1}|x_{n,1}}\rho_N\right), \tag{17}$$

where $M^n_{a_{n,1}|x_{n,1}}$ denote the measurement elements of nth Alice with ρ_N denoting the quantum state prepared by P_N. If the measurements of all parties are projective and they observe the output $\mathbf{a_1}$ given the input $\mathbf{x_1}$, then the postmeasurement states are given by

$$\rho'_{\mathbf{a_1},\mathbf{x_1}} = \frac{\left(\bigotimes_{n=1}^{N} M^n_{a_{n,1}|x_{n,1}}\right)\rho_N\left(\bigotimes_{n=1}^{N} M^n_{a_{n,1}|x_{n,1}}\right)}{p(\mathbf{a_1}|\mathbf{x_1}, P_N)}. \tag{18}$$

Then, the particles interact via some unitary V_N to evolve to the postinteraction states $\sigma_{\mathbf{a_1},\mathbf{x_1}}$. We consider that the unitary V_N maps quantum states from the Hilbert space $\bigotimes_n \mathcal{H}_{A_n(t_1)}$ to some other Hilbert space $\bigotimes_n \mathcal{H}_{A_n(t_2)}$.

Model-independent inference for N systems. One can define the model-independent inference for N systems in the same way as done for two systems, that is, the state, measurements, and interaction are certified to be equivalent to the reference strategies up to the action of local unitaries and the presence of the junk part.

Consider now the following Bell inequalities for any $\mathbf{a_1}$ introduced in Ref. [20],

$$\mathcal{B}_{\mathbf{a_1}} = (-1)^{a_{1,1}}\left\langle (N-1)\tilde{A}_{1,1}\otimes\bigotimes_{n=2}^{N} A_{n,1} + \sum_{i=2}^{N}(-1)^{a_{n,1}}\tilde{A}_{1,0}\otimes A_{n,0}\right\rangle \leqslant \beta_C, \tag{19}$$

where $A_{n,1}$ are quantum observables defined via the measurement operators of the nth party as in (3) and

$$\tilde{A}_{1,0} = \frac{A_0 - A_1}{\sqrt{2}}, \quad \tilde{A}_{1,1} = \frac{A_0 + A_1}{\sqrt{2}}. \tag{20}$$

The classical bound of the above Bell inequalities is $\beta_C = \sqrt{2}(N-1)$ for any $\mathbf{a_1}$. Consider now the following states,

$$|\phi_{\mathbf{a_1}}\rangle = \frac{1}{\sqrt{2}}\left[|a_{1,1}\cdots a_{N,1}\rangle + (-1)^{a_{1,1}}\left|a^{\perp}_{1,1}\cdots a^{\perp}_{N,1}\right\rangle\right], \tag{21}$$

where $a^{\perp}_{n,1} = 1 - a_{n,1}$, and the following observables for $n = 2,\dots,N$,

$$A_{1,0} = \frac{X+Z}{\sqrt{2}}, \quad A_{1,1} = \frac{X-Z}{\sqrt{2}}, \quad A_{n,0} = Z, \quad B_{n,1} = X. \tag{22}$$

As shown in Fact 2 of Sec. B of the Supplemental Material [43], using these states (21) and observables (22), one can attain the quantum bound $\beta_Q = 2(N-1)$ of all the Bell inequalities (19).

Let us now suppose that correlations $\vec{p}_{N,1}$ achieves the quantum bound of $\mathcal{B}_{0,\dots,0}$. Furthermore, the correlations $\vec{p}_{N,2,\mathbf{a_1},0,0,1,\dots,1}$ achieve the quantum bound of $\mathcal{B}_{\mathbf{a_1}}$ for every $\mathbf{a_1}$. Along with them, suppose one also observes that when $a_{n,1} = 0$ for all n and $x_{1,1} = x_{2,1} = 1, x_{n,1} = 0\ (n = 3,\dots,N)$ that the postinteraction state satisfies

$$-\langle\tilde{A}_{1,0}\otimes\mathbb{1}\rangle = \langle\mathbb{1}\otimes A_{n,1}\rangle = 1, \quad n = 2,\dots,N. \tag{23}$$

Consider now the following unitary,

$$\mathcal{U}_{\mathcal{N}} = \sum_{\mathbf{a_1}=0,1} |\phi_{\mathbf{a_1}}\rangle\langle\bar{a}_{1,1}a_{2,1}a^x_{3,1}\cdots a^x_{N,1}|, \tag{24}$$

where $\{|\bar{0}\rangle, |\bar{1}\rangle\}$ and $\{|0^x\rangle, |1^x\rangle\}$ are the eigenvectors of $(X+Z)/\sqrt{2}$ and X, respectively. It is straightforward to verify that if the state $|\phi_{0,\dots,0}\rangle$ (21) after being measured by the observables (22) and obtaining outcome $\mathbf{a_1}$ with inputs $x_{n,1} = 0$ evolves via the unitary (24), then one obtains the postinteraction state as $|\phi_{\mathbf{a_1}}\rangle$. Consequently, all these states and observables satisfy the above-mentioned statistics and thus we take them as the reference quantum states, observables, and interaction.

Let us now state the result.

Theorem 2. Assume that the Bell inequality $\mathcal{B}_{0,\dots,0}$ (19) is maximally violated at t_1. Furthermore, when the parties obtain the outcome $\mathbf{a_1}$ with inputs $x_{1,1} = x_{2,1} = 0$ and $x_{n,1} = 1$ $(n = 3,\dots,N)$, then the Bell inequalities $\mathcal{B}_{\mathbf{a_1}}$ (19) for any $\mathbf{a_1}$ are maximally violated at t_2. Along with it, when the parties observe the outcomes $a_{n,1} = 0$ for all n and $x_{1,1} = x_{2,1} = 1$, $x_{n,1} = 0$ $(n = 3,\dots,N)$, then the condition (23) is also satisfied. Then, the quantum state prepared by the source and the observables of both parties are certified as in the straightaway generalization of (6) and (7) to N parties, with the reference strategies given below Eq. (24). Importantly, the unitary V_N is certified as

$$\left(\bigotimes_n U_{n(t_2)}\right) V_N \left(\bigotimes_n U_{n(t_1)}\right)^{\dagger} = \mathcal{U}_{\mathcal{N}}\otimes V_{\text{aux}}, \tag{25}$$

where $\mathcal{U}_{\mathcal{N}}$ is given in Eq. (24) and V_{aux} is unitary.

The proof of the above theorem follows similar lines as the two-system interaction case and is presented in Sec. B of the Supplemental Material [43].

Discussions. For completeness, let us describe the protocol to certify entangling interactions among particles. In step one, the source P generates a state and distributes it to all the parties. At time t_1, on each of their respective subsystems the parties perform two binary measurements and record their outcomes. From the joint statistics, they will be able to certify that the incoming state is a particular entangled state along with their measurements to be projective and of a particular form as stated above. In step two, in half the rounds of the experiment (which are randomly chosen by the parties) they allow the individual postmeasured states to interact with each other and then return to their respective laboratories. In the other half of rounds of the experiment, which we will call checking rounds, they again measure their postmeasured states before they interact with any other system and ensure that they obtain the same outcome as at time t_1. This step is important as it could well be that the measurement device might cheating by not sending the actual postmeasured state and sending some auxiliary ones. However, if the checking rounds are chosen randomly, the device would not know beforehand when to cheat and thus would be caught by the parties. Moreover, as the measurement is certified to be projective, if the device is not cheating, then performing the same measurement as at time t_1 in the checking rounds will always give the same outcome. Then at time t_2, in the rounds when the postmeasured states interacted, the parties again measure their respective subsystems and obtain the joint statistics using that which they can evaluate the final states after the interaction. Using the initial and final certified state along with some additional statistics one can then certify the interaction between the particles in a model-independent way. For a remark, we require nondestructive measurements in the above-described protocol which might be experimentally challenging to implement (see Ref. [44] for a review).

The controlled-NOT gate, which is an entangling quantum operation, has been self-tested in Ref. [24]. However, their scheme required not applying the gate at half the rounds of the experiment, thus making it not applicable to certify the interaction between two systems as one would be required to assume a particular model of the interaction to impose it, or put simply, one cannot switch off the interaction between two systems without assuming the nature of the interaction. Additionally, the scheme is not entirely device independent, as it relies on certified measurements performed on the support prior to the interaction. To certify any operation based on these measurements, one must assume that the operation does not alter the local supports. Then, the scheme of Ref. [24] also needed two independent sources which is again an additional assumption that cannot be verified. Here, we do not make any such assumptions, thus making our scheme model independent. Although our proposed scheme currently has limited applicability, it can be considered as an alternative to infer quantum interactions rather than the standard approach to observe scattering amplitudes. Furthermore, if one assumes that the interaction does not change the local supports of the postmeasured states, then the source, instead of preparing a maximally entangled state, can even send local white noise to both the detectors and the scheme would still allow certification of the interaction between the postmeasured states.

Several follow-up problems arise from our work. The first problem would be to compute the robustness of the proposed scheme towards experimental errors. Then, one could explore the other classes of interaction that could be certified in a model-independent way. Furthermore, one might extend this approach where one could also certify time-dependent evolutions.

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