Letter

Nonreciprocal dynamics and the non-Hermitian skin effect of repulsively bound pairs

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We study the dynamics of a Bose-Hubbard model coupled to an engineered environment which in the noninteracting limit induces an effective nonreciprocal hopping as described by the Hatano-Nelson model. At strong interactions, two bosons occupying the same site form a so-called repulsively bound pair, or doublon. Using tensor-network simulations, we clearly identify a distinct doublon light cone and show that the doublon inherits nonreciprocity from that of single particles. Applying the idea of reservoir engineering at the level of doublons, we introduce a new set of dissipators and we analytically show that then the doublon dynamics are governed by the Hatano-Nelson model. This brings about a *two-particle* non-Hermitian skin effect and nonreciprocal doublon motion. Combining features of the two models we study, we show that single particles and doublons can be made to spread with opposite directionality, opening intriguing possibilities for the study of dynamics in interacting nonreciprocal models.

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Introduction. Nonreciprocal systems appear in physics in many different forms (see, e.g., Ref. [1] and references therein). They have recently received significant attention for the interesting phenomena they host, often lacking a counterpart in reciprocal systems. These range from the dramatic sensitivity of the spectral properties on the boundary conditions, to novel topological features in the complex spectrum [2–5]. A paramount example is the celebrated Hatano-Nelson model [6] where the presence of nonreciprocal hopping leads to an exponential localization of all left and right eigenstates on the opposite boundaries of an open one-dimensional chain, the so-called non-Hermitian skin effect (NHSE) [7–11].

One physical implementation of nonreciprocity is based on reservoir engineering [12,13] where the system is coupled to a nontrivial environment resulting in an effective nonreciprocal dynamics. These strategies are widely used in the noninteracting case [14–23] where the equations of motion close and one can analytically obtain the non-Hermitian Hamiltonian giving rise to nonreciprocity. Additionally, reservoir engineering and nonreciprocity were recently realized in cold atom experiments [24–26].

The rich landscape of novel phenomena emerging in nonreciprocal systems has further stimulated the question of the interplay of nonreciprocity and interactions. Most of the literature on non-Hermitian interacting systems focuses on the no-click limit of engineered open many-body systems [27–49]. This approximation, however, completely neglects the effect of jump operators and describes a single trajectory (the one where no photon is detected, hence the name) out of the exponentially many possibilities. In certain particular cases, however, a formal relation between the eigensystem of the non-Hermitian Hamiltonian and the eigensystem of the Liouvillian can be established [50]. A different possible step towards interacting systems considers the interaction on the mean-field level, yielding tractability at the expenses of a correct quantification of quantum fluctuations [51].

To correctly take fluctuations into account, one can study the system at the level of the many-body Lindblad master equation [15,52–55]. While in this framework analytical and exact numerical methods have limited application, tensor network techniques allow the study of dynamics of large interacting systems. The use of these methods is well established in the context of open quantum dynamics [56–59], and it was recently applied to the study of universality in a nonreciprocal spin-$1/2$ XXZ chain [60].

Here, we study the dynamics of a Bose-Hubbard model in the presence of engineered dissipation. Inspired by the so-called *repulsively bound pairs* appearing in isolated optical lattices at strong interactions [61], we focus on the dynamics of a single *doublon*, i.e., a composite particle made of two bosons occupying the same site. Focusing on dynamics, our work unveils interesting features beyond the steady state which in the strong nonreciprocity regime is only weakly affected by the presence of interactions [60]. Our numerical simulations show that both single particles and doublons can move nonreciprocally, although with different velocities. The directional doublon light cone we identify is a clear indication of emergent nonreciprocity in the interacting regime.

Applying the idea of reservoir engineering [13] to the effective strong-coupling doublon Hamiltonian, we introduce a new set of dissipators and show that the resulting equations of motion within the single-doublon sector reproduce the Hatano-Nelson model. Our model is then characterized by an *interaction-induced* NHSE, where single-particle dynamics are reciprocal and *only doublons* feature directional motion. Their different behavior opens several intriguing possibilities for the study of dynamics, as we show by briefly exploring the case of opposite directionality for doublons and single particles.

Model. We study bosons in a dissipative cavity array with on-site Kerr nonlinearity of strength U. The coherent part of the dynamics is encoded in the Bose-Hubbard

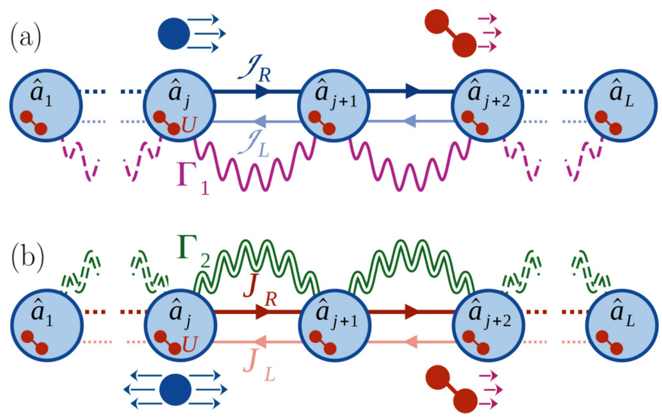

FIG. 1. (a) Neighboring cavities are coupled with amplitude J and bosons on the same site interact via the Kerr nonlinearity with strength U. In addition, the one-particle nearest-neighbor dissipation Γ_1 couples neighboring cavities [$\hat{L}_j$ in Eq. (2)], giving rise to nonreciprocal hopping $\mathcal{J}_{R(L)}$. Both single-particle and doublon dynamics are directional, although doublons propagate more slowly and decay faster. (b) As in the strong-coupling limit $U \gg J$ doublons are stable excitations, we introduce a second two-particle dissipator with rate Γ_2 [Eq. (6)]. In this second model, doublons acquire directional dynamics with nonreciprocal hopping $J_{R(L)}$, whereas single particles spread reciprocally.

Hamiltonian

$$\hat{H} = J \sum_{j=1}^{L-1} (\hat{a}_j^\dagger \hat{a}_{j+1} + \text{H.c.}) + \sum_{j=1}^{L} U (\hat{a}_j^\dagger)^2 \hat{a}_j^2, \tag{1}$$

where J is the hopping amplitude between neighboring cavities and $\hat{a}_j^\dagger$ ($\hat{a}_j$) creates (annihilates) a boson on site j. The action of the environment is represented by a set of Lindblad operators $\hat{L}_j$ which introduce a nearest-neighbor dissipation $\hat{L}_j = \sqrt{\Gamma_1}(\hat{a}_j + e^{\imath\theta_1}\hat{a}_{j+1})$. The full quantum dynamics of the system are then described by the many-body Lindblad master equation

$$\dot{\rho} = -\imath[\hat{H}, \rho] + \sum_{j=1}^{L-1} \hat{L}_j \rho \hat{L}_j^\dagger - \frac{1}{2}\{\hat{L}_j^\dagger \hat{L}_j, \rho\}, \tag{2}$$

as depicted in Fig. 1(a). This model was recently introduced in Refs. [46,55] studying the effective non-Hermitian Hamiltonian arising in its fully nonreciprocal no-click limit.

Previous studies in the noninteracting case, $U = 0$, have shown how the dynamics of the first moments $\langle \hat{a}_j \rangle$ [17] and the second moments $\langle \hat{a}_j^\dagger \hat{a}_i \rangle$ [19] are described by a non-Hermitian dynamical matrix corresponding to the Hatano-Nelson model [6]. Consequences of this effective nonreciprocity include directional exponential amplification of the cavity amplitude [16–18] and nonreciprocal dynamics of the boson densities [20].

Nonreciprocity and dynamics. To avoid the exponential growth of the Hilbert space ($\mathcal{D} \approx 10^6$ in the case we study), we use tensor-network methods which allow to obtain accurate results at a cost scaling only linearly with system size. In particular, we use the well-known time-evolving block decimation (TEBD) algorithm [62] adapted to the description of Lindblad dynamics, as detailed in the Supplemental Material [63].

Throughout this Letter, we study the dynamics of the single-doublon initial state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} (\hat{a}_{L/2}^\dagger)^2 |\text{vac}\rangle, \tag{3}$$

which in the isolated scenario can form a *repulsively bound pair* [61] when strong interactions make single-particle hopping energetically unfavorable. The behavior of this composite particle at $U \gg J$ is accurately captured by the following effective Hamiltonian [64,65] which can be obtained through a Schrieffer-Wolff transformation [66,67],

$$\hat{H}_{\text{eff}} = \frac{1}{2}\frac{J^2}{U} \sum_{j=1}^{L-1} [(\hat{a}_j^\dagger)^2 (\hat{a}_{j+1})^2 + \text{H.c.}], \tag{4}$$

where doublons move coherently with a reduced hopping amplitude. The presence of engineered dissipation makes the dynamics richer as it enables nonreciprocal hopping in the noninteracting case $\mathcal{J}_{R(L)} = J - \imath e^{-(+)\imath\theta_1}\Gamma_1/2$, tuning the model from reciprocal at $\Gamma_1 = 0$ to fully nonreciprocal at $\Gamma_1 = 2J$ (for $\theta_1 = \pm\pi/2$) [13].

Considering second-order processes in the equations of motion of doublon states, one can show that doublons inherit nonreciprocity and move with an effective nonreciprocal hopping amplitude $\mathcal{J}_{R(L)}^{(d)} \approx \mathcal{J}_{R(L)}^2/(U + \imath\Gamma_1)$. We confirm this prediction numerically in Fig. 2.

At strong nonreciprocity $\Gamma_1 = 2J$ and $\theta_1 = -\pi/2$ [Figs. 2(a) and 2(b)], particles move only to the right following the single-particle *light cone* $x(t) \propto Jt$, irrespective of interaction strength U. This is a consequence of the large dissipation rate Γ_1, which quickly depletes the system thus making the effect of interactions weak [60].

In Figs. 2(c)–2(f) we decrease the dissipation rate to $\Gamma_1 = 0.1J$, hence the degree of nonreciprocity is expected to be weaker. Nevertheless, dynamics still show clear signatures of nonreciprocity in both the interacting and noninteracting cases. Importantly, the presence of interactions leads to the emergence of a *second light cone* $\tilde{x}(t) \propto (J^2/U)t$ [black dashed line in Figs. 2(e) and 2(f)]. This doublon light cone is related, to leading order, to the effective nonreciprocal doublon hopping amplitude, and it clearly highlights the slower doublon dynamics due to strong nonlinearity. We further observe a slightly larger population on the right branch of the doublon light cone, suggesting the extension of nonreciprocity also to the interacting level. In passing, comparing Figs. 2(c) and 2(e) we notice that the interference pattern at long times $Jt \gg L/2$ is washed out in the interacting case, reminiscent of many-body dephasing in isolated systems, where dynamics relax to the thermal average due to interactions and ergodicity [68–71].

To obtain a clearer picture of the doublon dynamics and distinguish it from that of single particles it is useful to define a *doublon density*

$$\hat{n}_j^d = \tfrac{1}{2} (\hat{a}_j^\dagger)^2 \hat{a}_j^2, \tag{5}$$

which is identically zero for all single-particle states and in the single-doublon sector corresponds to the doublon population on site j. In Figs. 2(d) and 2(f) we show the dynamics of

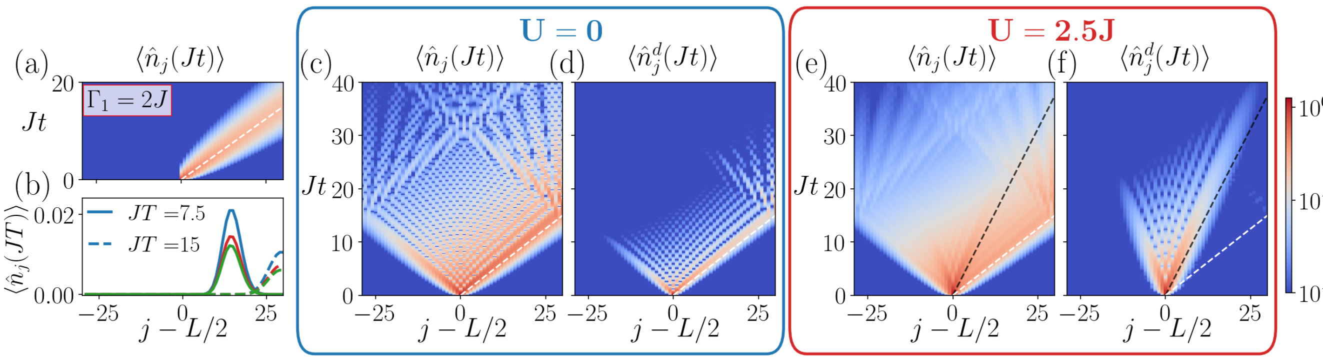

FIG. 2. (a) At $\Gamma_1 = 2J$, dynamics are restricted to the right half of the system and follow the single-particle light cone (white dashed line) for all U ($U = 0$ is shown here). (b) Remarkably, the full nonreciprocity expected in the noninteracting case is extended to $U \neq 0$ substantially unchanged (red and green lines), as shown by the snapshots at different times. (c)–(f) At smaller $\Gamma_1 = 0.1J$ the system is effectively less nonreciprocal, the left half of the system becomes slightly populated, and novel interaction-driven phenomena emerge. (e), (f) In particular, at $U = 2.5J$ we observe the appearance of a *light cone* corresponding to stable doublons moving nonreciprocally (black dashed line) and distinct from the single-particle light cone observed in (c) and (d). This feature is more evident in the doublon density dynamics for $U = 2.5J$ (f). The single-particle light cone is highly suppressed, and the density propagation follows the black dashed line corresponding to J^2/U. These data were obtained for a system of $L = 60$ sites, using an vectorized matrix product operator (MPO) of bond dimension $\chi = 128$ and $\theta_1 = -\pi/2$.

$\langle \hat{n}^d_j\rangle$ for a small dissipation rate $\Gamma_1 = 0.1J$ and for $U = 0$ and $U = 2.5J$, respectively. In the noninteracting case, the doublon density follows the single-particle light cone, indicating the absence of coherent doublon motion. On the other hand, at $U = 2.5J$, the single-particle light cone is strongly suppressed and the dominant contribution comes from the slower and nonreciprocal doublon motion. Hence, the interacting system inherits the nonreciprocity characterizing single-particle dynamics. The emergence of metastable nonreciprocal doublons is a genuine consequence of interactions, clearly distinguishable from the single-particle case, and is one of the main results of this Letter.

Stabilizing doublon directional motion. Inspired by the structure of the effective Hamiltonian Eq. (4), we introduce a different set of dissipators which *stabilize* doublon nonreciprocity. Following the approach of Ref. [13], we replace the one-particle nearest-neighbor dissipator with its doublon version

$$\Gamma_2 \mathcal{D}\big[\hat{a}_j^2 + e^{\imath\theta_2}\hat{a}_{j+1}^2\big], \tag{6}$$

where only pairs of bosons (i.e., doublons) are lost to the environment, as sketched in Fig. 1(b).

Using the effective Hamiltonian (4) and the dissipator above we obtain the equation of motion for the doublon amplitudes $\langle \hat{a}_\ell^2\rangle$ and correlations $\langle(\hat{a}_\ell^\dagger)^2\hat{a}_m^2\rangle$ [63]. In the single-doublon sector these simplify and can be written in terms of a non-Hermitian dynamical matrix $\mathbb{H}$ acting nontrivially on the *single-doublon* space only,

$$\imath\frac{\partial\big\langle(\hat{a}_\ell^\dagger)^2\hat{a}_m^2\big\rangle}{\partial t} = \sum_j \mathbb{H}_{m,j}\big\langle(\hat{a}_\ell^\dagger)^2\hat{a}_j^2\big\rangle - \mathbb{H}^\dagger_{j,\ell}\big\langle(\hat{a}_j^\dagger)^2\hat{a}_m^2\big\rangle, \tag{7}$$

with $\mathbb{H}$ the Hatano-Nelson matrix

$$\mathbb{H} = \sum_j J_R|j+1\rangle_2\langle j|_2 + J_L|j-1\rangle_2\langle j|_2 - 2\imath\Gamma_2|j\rangle_2\langle j|_2. \tag{8}$$

Similarly to the noninteracting case [17,19], the interference of the coherent nearest-neighbor coupling with the doublon dissipation causes the emergence of different left and right hopping amplitudes $J_L = \frac{J^2}{U} - \imath e^{\imath\theta_2}\Gamma_2$ and $J_R = \frac{J^2}{U} - \imath e^{-\imath\theta_2}\Gamma_2$. The dissipation rate required for full nonreciprocity is then $\Gamma_2 = J^2/U$ and is of the same order of the doublon motion timescale. This results in more stable nonreciprocal doublon dynamics, as compared to the one-particle dissipator case where full nonreciprocity is achieved at $\Gamma_1 = 2J \gg J^2/U$.

Remarkably, the non-Hermitian skin effect arising from the Hatano-Nelson matrix (8) affects only doublon states $|j\rangle_2 = |0\ldots 2_j \ldots 0\rangle$. Binding particles together, interactions have a dramatic effect and enable the exponential localization of doublons at the boundaries of the system, whereas single particles behave reciprocally. The nonreciprocity and non-Hermitian skin effect resulting from the quadratic dissipator (6) combined with the interaction-induced doublon stability represent the second central result of our work.

In our numerical analysis, we go beyond the approximate picture of the effective Hamiltonian and simulate the full dynamics of the system using the interacting Hamiltonian (1) and the quadratic dissipator (6). As we want to separate single-particle from doublon contributions to the dynamics, the density $\hat{n}_j = \hat{a}_j^\dagger\hat{a}_j$ is not a convenient quantity, as it is affected by both. We then define the single-particle population $\hat{P}_j^{(1)} = \hat{n}_j - 2\hat{n}_j^d$ which accounts for the weight of single-particle states, when the total number of bosons is $N = 2$.

In Fig. 3(a), we show the single-particle and doublon profiles at time $JT = 15$ in the noninteracting case. Due to the absence of stable doublons at $U = 0$, dynamics are reciprocal and particles spread in both directions equally, irrespective of the value of Γ_2.

A dramatic difference is observed when $U \gg J$ [Figs. 3(b)–3(e)], where the doublon forms a stable excitation and the quadratic dissipative coupling (6) leads to strong nonreciprocity. In particular, at $\Gamma_2 = J^2/U$ and $\theta_2 = -\pi/2$, where $J_L = 0$, the doublon density propagates exclusively towards the right boundary as predicted by the equations of motion (7). As a consequence of the finite interaction U,

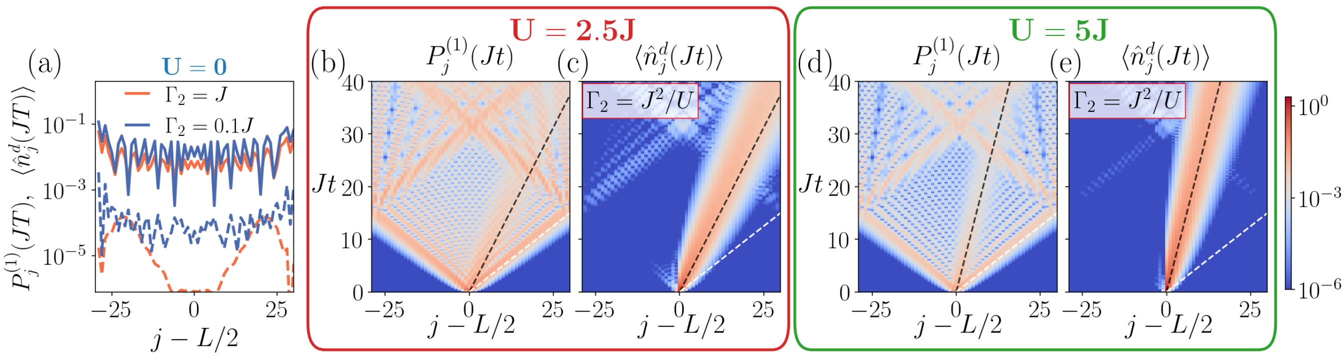

FIG. 3. (a) In the absence of interactions, $U = 0$, dynamics are reciprocal, irrespective of dissipation rate Γ_2. Both single-particle (solid lines) and doublon (dashed lines) population profiles at time $JT = 15$ are equally spread in the two halves of the system. As expected, the value of Γ_2 affects significantly the doublon density only. (b)–(e) As interactions are turned on, $U \gg J$, the system shows clear signatures of directional motion. (c), (e) Due to the destructive interference induced by the quadratic dissipator, a highly nonreciprocal doublon light cone $\tilde{x} \propto J^2/U$ (black dashed line) appears, affecting the doublon density. (b), (d) The single-particle population, however, spreads reciprocally and is only slightly affected by the doublon light cone due to its decay into single-particle states. (e), (d) As the interaction strength is increased, the nonreciprocal doublon spreading becomes slower, in agreement with the smaller effective hopping amplitude. Additionally, we notice a smaller population of single-particle states due to the increased stability of the doublon. These data were obtained for a system of $L = 60$ sites, using an MPDO of bond dimension $\chi = 128$ and $\theta_2 = -\pi/2$.

single-particle hopping processes are allowed, and the initial doublon can decay into single-particle states. These are free to propagate in both directions, as they are not affected by the dissipator (6).

Opposite directionality. Combining the one-particle and doublon nearest-neighbor dissipators as in Fig. 4(a), one can separately control doublons and single particles. In Fig. 4, we show dynamics of both single-particle population [Fig. 4(b)] and doublon density [Fig. 4(c)]. Choosing $\theta_1 = -\theta_2$ generates opposite interference of the nearest-neighbor hopping with the two dissipators and results in the different directionality of single particles [Fig. 4(b)] and doublons [Fig. 4(c)]. This simple example points out how introducing the dissipator (6) causing *doublon nonreciprocity* opens several interesting directions for the investigation of interacting nonreciprocal dynamics.

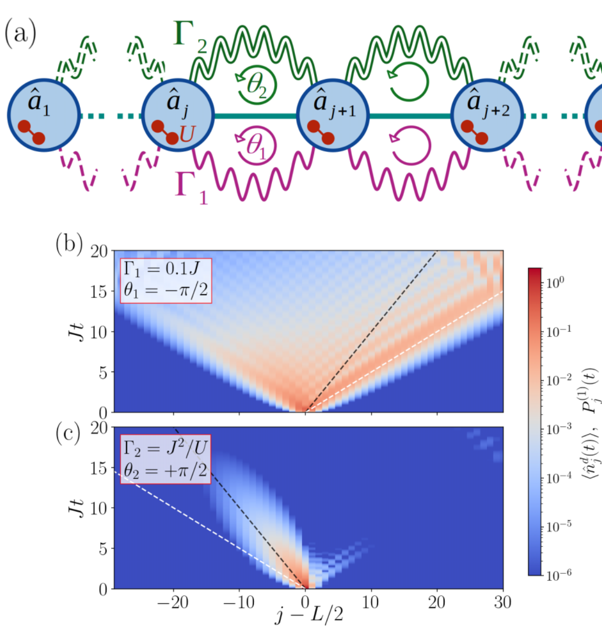

FIG. 4. (a) Combining the one- and two-particle nearest-neighbor dissipators gives rise to fascinating effects on particle dynamics. (b) At $\theta_1 = -\pi/2$ and $\Gamma_1 = 0.1J$ the single-particle density is slightly nonreciprocal towards the *right*. (c) However, choosing $\theta_2 = +\pi/2$ and $\Gamma_2 = J^2/U$ leads to almost full nonreciprocity of doublons to the *left*. These data were obtained for a system of $L = 60$ sites, using an MPDO of bond dimension $\chi = 128$ and $U = 2J$.

Conclusions. In this Letter, we investigated the dynamics of a Bose-Hubbard model coupled to engineered dissipators. Showing the emergence of a nonreciprocal doublon light cone, we highlighted how the study of time evolution can unveil genuinely interacting effects, which would be hidden in the study of steady states alone [60].

We introduced a different type of dissipator, based on the structure of the strongly interacting effective Hamiltonian, and showed how it gives rise to a doublon non-Hermitian skin effect. This arises at the level of the Lindblad master equation, going beyond the no-click limit studied in previous works [47,48]. The quadratic dissipator leads to the two-particle nonreciprocal dynamics observed in our numerical simulations, and opens new possibilities for the study of dynamics in nonreciprocal systems.

Strictly related to the study of dynamics we presented is the issue of how relaxation and thermalization are affected by interactions in nonreciprocal models [20,52]. Our approach can be easily generalized to other systems which support stable excitations such as one-dimensional spin-1/2 chains. This setup allows for the study of strongly correlated systems, raising some intriguing questions regarding non-Hermitian topology of many-body systems [33] as well as the nature of transport in nonreciprocal bosonic and fermionic models [22,23].

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