Optical coherence transfer in atom arrays

P. R. Berman and A. Kuzmich
Physics Department, University of Michigan, Ann Arbor, Michigan 48109-1040, USA

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A $J = 0$ to $J = 1$ to $J = 0$ ladder scheme can be used as a prototype for probing optical coherence transfer in an atom array. Two incident σ_- polarized fields drive transitions from the ground state to the intermediate state and the intermediate state to the uppermost state in the atoms. The uppermost state is taken to be a long-lived Rydberg level. In the absence of any atom-atom interactions, excitation to the Rydberg level is not allowed, owing to the selection rules. However, dipole-dipole interactions can provide a transfer of the state amplitude associated with the $m = -1$ sublevel in one atom to the $m = +1$ sublevel in another atom. The overall process of field excitation and optical coherence transfer can then lead to an excitation of the Rydberg level. One can then apply a readout pulse to obtain phase-matched emission from the array. For a linear array of atoms, the phase-matched signal can increase more rapidly than N^2, where N is the number of atoms in the array. For arrays that are invariant under a rotation $\delta\phi < \pi$ about the z axis, however, it turns out that the total field intensity vanishes identically in the phase-matched direction. For such arrays, the Rydberg state population can serve as a measure of the optical coherence transfer.

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I. INTRODUCTION

Optical coherence transfer (OCT) is a process in which a dipole coherence between states in an atom that are separated by an optical frequency is transferred to another atom, owing to atom-atom interactions. The quantum information "explosion" has rekindled an interest in collective decay and cooperative Lamb shifts [1–7] in atomic ensembles and arrays, both of which involve OCT between the atoms. In most theories of the collective decay and cooperative Lamb shifts [8–13], the atoms are modeled as two-level systems, with the magnetic structure of the levels not taken into account. There are some papers that include the magnetic sublevel structure [14–16], but these papers do not focus on the fundamental underlying process of OCT. Moreover, we are unaware of any experiments in arrays that directly probe this coherence transfer [17,18].

Actually, we are interested in the somewhat more esoteric OCT illustrated schematically in Fig. 1. Imagine that we use a σ_- polarized field $\mathbf{E}_1$ to prepare a state in which each atom is in a coherent superposition of its ground state $|g\rangle$ and its $m = -1$ intermediate state $|-1\rangle$ in an array of N atoms. In other words, atom j is prepared so it has nonvanishing density matrix elements $\rho_{gg}^{(j)}$, $\rho_{g,-1}^{(j)}$, $\rho_{-1,g,}^{(j)}$, and $\rho_{-1,-1}^{(j)}$. Dipole-dipole interactions between this atom and atom j' can result in OCT and population transfer, creating density matrix elements $\rho_{gg}^{(j')}$, $\rho_{g1}^{(j')}$, $\rho_{1,g,}^{(j')}$, and $\rho_{11}^{(j')}$ in atom j'. We want to stress that we are concerned here with transfer of optical coherence with a change in magnetic quantum number (OCT-M) and *not* in transfer of population. In all the cases to be considered below, the contributions to the measured signals arising from populations $\rho_{11}^{(j)}$ are negligibly small and can be safely ignored. A second σ_- polarized field $\mathbf{E}_2$ then completes the transition to the Rydberg state $|e\rangle$. In the absence of interactions, there is no Rydberg excitation, owing to selection rules. The Rydberg state population then serves as a measure of the OCT-M. Alternatively, one can apply a readout pulse on the Rydberg to upper transition to produce phase-matched emission on the lower transition.

It is not all that surprising that OCT-M has not received a great deal of attention, since effects related to OCT-M often average to zero in disordered atomic samples, such as those encountered in hot or cold atomic vapors. However for particular atom arrays in one, two, or three dimensions, the OCT-M can provided the dominant contribution to certain spectroscopic signals and also significantly modify the collective decays and/or cooperative Lamb shifts, especially for atom separations less than a wavelength. Specifically, we show that in a one-dimensional atom array, the OCT-M can lead to phase-matched signals that are dramatically enhanced in certain directions, even when the atoms are separated by greater than a wavelength. On the other hand, for arrays possessing a certain type of axial symmetry, we will show that the total signal radiated by the array in the phase-matched direction vanishes identically [19]. For such geometries, one must rely on the Rydberg state population or non-phase-matched emission to provide evidence for the OCT-M.

II. ATOM-FIELD GEOMETRY

We consider an array of N fixed atoms, each having the level scheme shown in Fig. 1. The ground state is a $J = 0$ state denoted by the ket $|g\rangle$ and the uppermost excited state is a long-lived $J = 0$ Rydberg state denoted by the ket $|e\rangle$. The intermediate state is a $J = 1$ state whose sublevels are denoted by the kets $|m\rangle$ ($m = -1$, 0, and 1). The frequency differences between the intermediate states and the ground state are denoted by ω_{mg} and those between the uppermost excited state and the intermediate states are denoted by ω_{em}. A static magnetic field is applied in the z direction, giving rise to

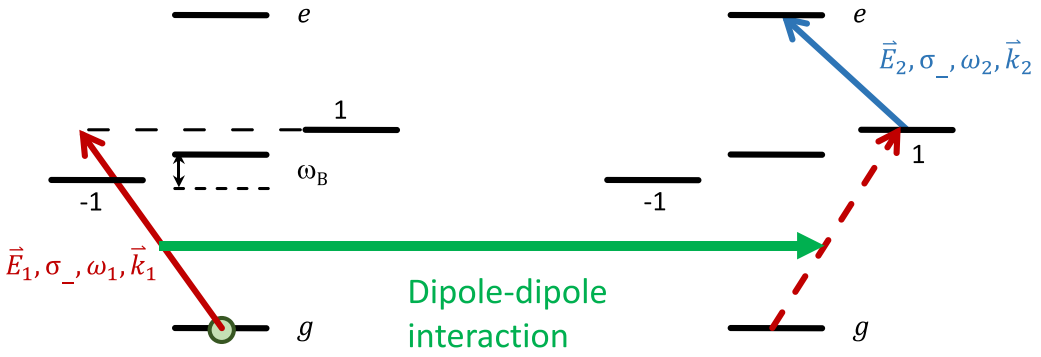

FIG. 1. The atom-field geometry proposed in this work. The atoms are subjected to two optical field pulses incident in the z direction. The first field drives the g to $m = -1$ transition in one atom and the second field completes the two-photon excitation process in another atom, with the transfer between the $m = -1$ and $m = +1$ sublevels accomplished via the dipole-dipole interaction. There is a magnetic field present (Zeeman splitting equal to ω_B) which allows for a resonant transfer, when the first field is detuned by $2\omega_B$ from the g to $m = -1$ transition.

a frequency splitting of ω_B between adjacent intermediate sublevels. The atoms are located at positions $\mathbf{R}_j$ $(j = 1, \ldots, N)$. Two atoms are shown in the figure. The atoms are subjected to two optical field pulses incident in the z direction.

The first pulse has frequency ω_1 and propagation vector $\mathbf{k}_1 = k_1\hat{\mathbf{z}}$, is σ_- polarized, and is detuned from the ground to $m = -1$ transition by

$$\delta_1 = \omega_1 - \omega_{-1g} = 2\omega_B. \tag{1}$$

In other words, the first field is resonant with the ground to $m = +1$ transition, but cannot drive this transition owing to the selection rules. Choosing the frequency in this manner will allow for a *resonant* interaction-induced transition between the $m = -1$ level in one atom and the $m = +1$ level in another atom. The Rabi frequency of the first field is denoted by

$$\Omega_1(t) = 2\chi_1(t) = 2\chi_1 e^{-t^2/T^2}. \tag{2}$$

The second pulse has frequency ω_2 and propagation vector $\mathbf{k}_2 = k_2\hat{\mathbf{z}}$, is σ_- polarized, and is detuned from the excited to $m = +1$ transition by

$$\delta_2 = \omega_2 - \omega_{e1}. \tag{3}$$

This field completes the "two-photon" transition from the ground to Rydberg level. The Rabi frequency of the second field is denoted by

$$\Omega_2(t) = 2\chi_2(t) = 2\chi_2 e^{-t^2/T^2}. \tag{4}$$

It is assumed that the Rabi frequencies are approximately constant over the transverse directions of the sample—that is, they are independent of X_j and Y_j.

In the absence of any atom-atom interactions, there will be no excitation of the Rydberg level in any atom since the second field has the wrong polarization to complete the excitation from the $m = -1$ sublevel. For the second field to complete the excitation, dipole-dipole interactions must induce transitions between the $m = -1$ and $m = +1$ sublevels in *different* atoms. The primary focus of our investigation is to see if such transfers are possible and to examine the experimental consequences associated with these transfers.

A simple measure of the OCT-M is provided by the Rydberg state population. In general the dependence of the OCT-M population signal on N will be greater than linear owing to the fact that the number of OCT-M transfers increases with increasing N. The actual dependence depends on the array geometry. An enhanced OCT-M signal, varying at least as N^2, can be obtained if a σ_--polarized readout pulse is applied on the upper transition at some time $t \gg T$ following the excitation pulses. The readout pulse can then lead to a phase-matched signal on the $m = 1$ to g transition. We shall see that this is possible, in general, but not for arrays that are invariant under a rotation $\delta\phi < \pi$ about the z axis. For such arrays, the total signal radiated in the phase-matched direction vanishes identically.

From source-field theory, the field intensity radiated on the $m = 1$ to g transition is proportional to [20]

$$I(\theta, \phi; t) = \frac{3\gamma_2}{8\pi}\frac{1+\cos^2\theta}{2}\sum_{j=1}^{N}\Bigg[\langle\sigma_{11}^{(j)}(t_r)\rangle + \sum_{j'\neq j=1}^{N}\langle\sigma_{1g}^{(j)}(t_r)\sigma_{g1}^{(j')}(t_r)\rangle e^{-i\mathbf{k}\cdot\mathbf{R}_{jj'}}\Bigg], \tag{5}$$

where

$$\mathbf{k} = \big(\omega_{1g}/c\big)(\sin\theta\cos\phi\hat{\mathbf{x}} + \sin\theta\sin\phi\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}}), \tag{6}$$

$\gamma_2 = 2\gamma$ is the intermediate-state decay rate, θ and ϕ are the polar angles of the emitted radiation, $\mathbf{R}_{jj'} = \mathbf{R}_j - \mathbf{R}_{j'}$, $\sigma_{11}^{(j)}(t_r)$ is the $m = 1$ sublevel population operator for atom j evaluated at the retarded time $t_r = t - D/c$ (D is the distance to the detector), $\sigma_{1g}^{(j)}(t_r)$ is a raising operator for atom j, and $\sigma_{g1}^{(j')}(t_r)$ is a lowering operator for atom j'.

To simplify the calculations, we make a number of assumptions that are not critical to the overall qualitative nature of the results. The fields are taken to be sufficiently weak or the detunings sufficiently large to justify a perturbation theory approach. Specifically, it is assumed that

$$\gamma T \gg 1, \tag{7a}$$

$$|\Gamma|/\gamma \ll 1, \tag{7b}$$

$$\delta_1 T \gg 1, \tag{7c}$$

$$\frac{\chi_1\chi_2}{\delta_1}T \ll 1, \tag{7d}$$

$$\chi_2 T \ll 1, \tag{7e}$$

where Γ is a complex, interaction-induced transfer rate. Moreover, we assume that the readout pulse is a σ_--polarized π pulse having frequency ω_{e1}, propagation vector $\mathbf{k}_2 = k_2\hat{\mathbf{z}}$, and a duration much less than $1/\gamma_2$. In that limit,

$$\left\langle \sigma_{11}^{(j)}(t)\right\rangle \approx \left\langle \sigma_{ee}^{(j)}(t)\right\rangle, \tag{8a}$$

$$\left\langle \sigma_{1g}^{(j)}(t_r)\sigma_{g1}^{(j')}(t_r)\right\rangle \approx \left\langle \sigma_{eg}^{(j)}(t)\sigma_{ge}^{(j')}(t)\right\rangle e^{-ik_2Z_{jj'}}, \tag{8b}$$

where t is the time that the readout pulse is applied. As a consequence, the time-integrated field intensity is given by

$$I(\theta,\phi) = \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}\sum_{j=1}^{N}\left[\left\langle \sigma_{ee}^{(j)}\right\rangle + \sum_{j'\neq j=1}^{N}\left\langle \sigma_{eg}^{(j)}\sigma_{ge}^{(j')}\right\rangle e^{-ik_2Z_{jj'}}e^{-i\mathbf{k}\cdot\mathbf{R}_{jj'}}\right], \tag{9}$$

where the atomic operators are now taken to be time-independent Schrödinger operators and the expectation values are taken with respect to the state vector of the system, evaluated at the time of the readout pulse. We can define the interference term as

$$I_{\rm int}(\theta,\phi) = \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}\sum_{j=1}^{N} \times \sum_{j'\neq j=1}^{N}\left\langle \sigma_{eg}^{(j)}\sigma_{ge}^{(j')}\right\rangle e^{-i\mathbf{k}\cdot\mathbf{R}_{jj'}}e^{-ik_2Z_{jj'}} \tag{10}$$

and the population term as

$$I_{\rm pop}(\theta) = \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}\sum_{j=1}^{N}\left\langle \sigma_{ee}^{(j)}\right\rangle, \tag{11}$$

with

$$I(\theta,\phi) = I_{\rm pop}(\theta) + I_{\rm int}(\theta,\phi). \tag{12}$$

The interference term represents the contribution to the signal intensity resulting from the interference of the fields radiated by different atoms, whereas the population term is the sum of the field intensity radiated by each atom. The total Rydberg population N_e produced by the excitation scheme is given by

$$N_e = \sum_{j=1}^{N}\left\langle \sigma_{ee}^{(j)}\right\rangle. \tag{13}$$

The requirement that $|\Gamma|/\gamma \ll 1$ implies that $k_1R_{jj'} \gg 1$ (for $j \neq j'$). It is assumed that $R_{jj'}$ is sufficiently large to allow us to neglect Rydberg-Rydberg interactions, which could result in dephasing of the ground-Rydberg state coherence. For such separations, the dipole blockade mechanism does not play a role. In practice, the Rydberg level should be chosen with the lowest possible value of principal quantum number for which decay from the Rydberg level on the timescale of the experiment is negligible. However, we should note that it is also possible to use a highly excited Rydberg level for which the dipole blockade is operational; in this limit, there is only a single Rydberg excitation in the array and ground-Rydberg dephasing does not occur.

III. CALCULATION OF THE SIGNAL

Since we are using a perturbation theory approach, the calculation is fairly straightforward. First we assume the ground-state amplitude of atom j' to be $c_g^{(j')} \approx 1$. In an interaction representation, the intermediate-state amplitude $c_{-1}^{(j')}$ created by the first pulse is then governed by the evolution equation,

$$\dot{c}_{-1}^{(j')} \approx -\gamma c_{-1}^{(j')} - i\chi_1(t)e^{ik_1Z_{j'}}e^{-i\delta_1 t}. \tag{14}$$

When the inequalities given in Eqs. (7) are satisfied, the solution of this equation is

$$c_{-1}^{(j')} \approx -i\frac{\chi_1(t)e^{ik_1Z_{j'}}e^{-i\delta_1 t}}{(\gamma - i\delta_1)}. \tag{15}$$

For our atom-field geometry, all the atoms are excited by the pulse, with the state amplitudes differing only by a spatial phase factor.

To lowest order in the dipole-dipole interaction and neglecting retardation in the atom-atom interactions, the state amplitude created by the first field in atom j' can be transferred the $m = 1$ sublevel in atom j according to

$$\dot{c}_1^{(j)} = -\gamma c_1^{(j)} - \gamma e^{2i\omega_B t}\sum_{j'\neq j=1}^{N} G_{1,-1}(\mathbf{R}_{jj'})c_{-1}^{(j')}, \tag{16}$$

where [21]

$$G_{1,-1}(\mathbf{R}_{jj'}) = -\frac{3}{2}\sqrt{\frac{8\pi}{15}}h_2(k_1R_{jj'})Y_{22}(\theta_{jj'},\phi_{jj'}) = -\frac{3}{4}h_2(k_1R_{jj'})\sin^2\theta_{jj'}e^{2i\phi_{jj'}}, \tag{17}$$

h_2 is a spherical Hankel function, Y_{22} is a spherical harmonic, and $(\theta_{jj'},\phi_{jj'})$ are the spherical angles of $\mathbf{R}_{jj'}$. Again, assuming that inequalities (7) are satisfied and taking $\delta_1 = 2\omega_B$, we find

$$c_1^{(j)} \approx i\frac{\chi_1(t)}{(\gamma - i\delta_1)}\sum_{j'\neq j=1}^{N} G_{1,-1}(\mathbf{R}_{jj'})e^{ik_1Z_{j'}}. \tag{18}$$

Now the second field acts to result in a Rydberg state amplitude whose time derivative is determined by

$$\dot{c}_e^{(j)} = -i\chi_2(t)e^{ik_2Z_j}e^{-i\delta_2 t}c_1^{(j)} \approx \frac{\chi_1(t)\chi_2(t)e^{ik_2Z_j}}{(\gamma - i\delta_1)}\sum_{j'\neq j=1}^{N} G_{1,-1}(\mathbf{R}_{jj'})e^{ik_1Z_{j'}}. \tag{19}$$

When inequalities (7) hold, it then follows that, following the pulse,

$$c_e^{(j)} \approx BF_je^{i(k_1+k_2)Z_j}, \tag{20}$$

where

$$F_j = \sum_{j'\neq j=1}^{N} G_{1,-1}(\mathbf{R}_{jj'})e^{-ik_1Z_{jj'}} \tag{21}$$

is an interaction-induced structure factor and

$$B = \int_{-\infty}^{\infty} dt\,\frac{\chi_1(t)\chi_2(t)e^{-i\delta_2 t}}{(\gamma - i\delta_1)} \tag{22}$$

is an amplitude for two-photon excitation.

At the time of the readout pulse, the only component of the state vector that contributes to the expectation values in Eqs. (10) and (11) is $\sum_{j=1}^{N} c_e^{(j)}|e\rangle_j$, where $c_e^{(j)}$ is given by Eq. (20) and $|e\rangle_j$ is a ket corresponding to atom j in state $|e\rangle$ and all the other atoms in state $|g\rangle$. It then follows that the contribution from the interference term to the radiated field intensity, given by Eq. (10), can be evaluated as

$$\begin{aligned} I_{\rm int}(\theta,\phi) &= \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}\sum_{j=1}^{N} \\ &\times \sum_{j'\neq j=1}^{N} c_e^{(j)}\big[c_e^{(j')}\big]^* e^{-i\mathbf{k}\cdot\mathbf{R}_{jj'}} e^{-ik_2 Z_{jj'}} \\ &= \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}|B|^2\sum_{j=1}^{N} \\ &\times \sum_{j'\neq j=1}^{N} F_j F_{j'}^* e^{ik_1 Z_{jj'}} e^{-i\mathbf{k}\cdot\mathbf{R}_{jj'}} \end{aligned} \tag{23}$$

and contribution from the population term can be evaluated as

$$\begin{aligned} I_{\rm pop}(\theta) &= \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}\sum_{j=1}^{N}\left|c_e^{(j)}\right|^2 \\ &= \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}|B|^2 S, \end{aligned} \tag{24}$$

where

$$S = \sum_{j=1}^{N}|F_j|^2. \tag{25}$$

The total intensity is given by

$$I(\theta,\phi) = \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}|B|^2\left|\sum_{j=1}^{N} F_j e^{ik_1 Z_j} e^{-i\mathbf{k}\cdot\mathbf{R}_j}\right|^2, \tag{26}$$

and the total Rydberg population is given by

$$N_e = \sum_{j=1}^{N}\left|c_e^{(j)}\right|^2 = |B|^2 S. \tag{27}$$

It is important to recall that we have assumed that $k_1 R_{jj'} \gg 1$ (for $j \neq j'$) or, equivalently, that $|G_{1,-1}(\mathbf{R}_{jj'})| \ll 1$. The actual dependence of the signal on N depends on the array geometry.

For example, if the atoms form a linear chain in the z direction, $G_{1,-1}(Z_{jj'}) = 0$, $F_j = 0$, and both the interference and population terms vanish. Moreover, for arrays that are invariant under a rotation $\delta\phi < \pi$ about the z axis, even though $F_j \neq 0$, it is possible to show that the *total* intensity in the phase-matched direction, which is proportional to $|\sum_{j=1}^{N} F_j|^2$, vanishes. To prove that $\sum_{j=1}^{N} F_j = 0$, we expand the factor $h_2(k_1 R_{jj'})Y_{22}(\hat{\mathbf{R}}_{jj'})e^{-ik_1 Z_{jj'}}$ (for $j' \neq j$) appearing in Eq. (17) as [21]

$$\begin{aligned} &h_2(k_1 R_{jj'})Y_{22}(\hat{\mathbf{R}}_{jj'})e^{-ik_1 Z_{jj'}} \\ &= i^{l_1+l_2}(-1)^{l_2}e^{-ik_1 Z_{jj'}}\sqrt{20\pi(2l_1+1)(2l_2+1)} \\ &\times \begin{pmatrix} l_1 & 2 & l_2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_1 & 2 & l_2 \\ m_1 & -2 & m_2 \end{pmatrix} \\ &\times h_{l_1}(k_0 R_>)j_{l_2}(k_0 R_<)Y_{\ell_1 m_1}(\hat{\mathbf{R}}_>)Y_{\ell_2 m_2}(\hat{\mathbf{R}}_<), \end{aligned} \tag{28}$$

where $\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$ is a 3-j symbol, $j_l(x)$ is a spherical Bessel function, $R_>$ ($R_<$) is the larger (smaller) of R_j and $R_{j'}$, and a summation convention is implicit in the right-hand side of the equation. The three-j symbol vanishes unless $m_2 + m_1 = 2$. Consequently, under a rotation $\delta\phi$ about the z axis, the right-hand side of Eq. (28) is multiplied by a factor of $\exp(2i\delta\phi)$. However, if the distribution is invariant under the rotation $\delta\phi$, Eq. (28) must be also be invariant under this rotation when summed over j and j' (with $j' \neq j$). This implies that $\sum_{j=1}^{N} F_j = \exp(2i\delta\phi)\sum_{j=1}^{N} F_j$. For $\delta\phi < \pi$, the only way this equality can be satisfied is if $\sum_{j=1}^{N} F_j = 0$. In other words, if there exists an azimuthal rotation $\delta\phi < \pi$ for which the array is unchanged, then $\sum_{j=1}^{N} F_j = 0$.

As an example, consider a cubic array of $N = n^3$ atoms located at positions

$$\mathbf{R}_j(j_x, j_y, j_z) = d(j_x\hat{\mathbf{x}} + j_y\hat{\mathbf{y}} + j_z\hat{\mathbf{z}}), \tag{29}$$

with j_x, j_y, and j_z each taking on values from $-(n-1)/2$ to $(n-1)/2$. This array is invariant under a rotation of $\pi/2$ about the z axis, implying that $\sum_{j=1}^{N} F_j = 0$. As a consequence, in the phase-matched direction ($\theta = \phi = 0$), $\mathbf{k} = k_1\hat{\mathbf{z}}$ and $I(0,0) = 0$. That is, in the phase-matched direction, $I_{\rm pop}(\theta)$ and $I_{\rm int}(\theta,\phi)$ have equal magnitudes but opposite signs, exactly canceling one another. For this array, the most direct way to establish that the OCT-M has occurred is to measure the Rydberg population $N_e = |B|^2 S \neq 0$. In Fig. 2 we plot S as a function of $k_1 d$ for $n = 4$ (64 atoms). The positions of the local maxima and minima in this figure are determined by a complicated interplay of all the phase factors appearing in the summation in Eq. (21). That is, if we write

$$F_j = \sum_{j'\neq j=1}^{N} A_{jj'}e^{i\Phi_{jj'}}, \tag{30}$$

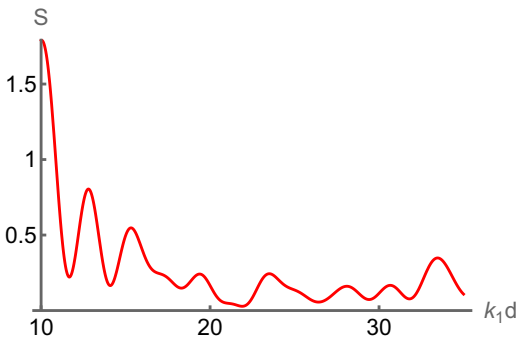

FIG. 2. A plot of $S = \sum_{j=1}^{N}|F_j|^2$ versus $k_1 d$ for a cubic array of 64 atoms. In this case, $\sum_{j=1}^{N} F_j = 0$, but there is still a contribution to the Rydberg population that serves as a measure of the OCT-M.

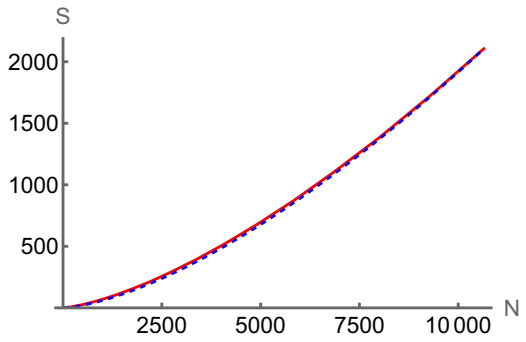

FIG. 3. The solid red curve is a plot of S versus N for a cubic array of N atoms with $k_1 d = 2\pi$. The dashed blue curve is a plot of $0.001\,91 N^{1.5}$. The dependence on N is greater than linear owing to the fact that the OCT-M rate depends on N.

where

$$A_{jj'} = |G_{1,-1}(\mathbf{R}_{jj'})| e^{-ik_1 Z_{jj'}}, \tag{31a}$$

$$\Phi_{jj'} = \mathrm{Arg}[G_{1,-1}(\mathbf{R}_{jj'}) e^{-ik_1 Z_{jj'}}], \tag{31b}$$

there are certain values of $k_1 d$ that lead to maxima and minima.

In Fig. 3 we plot S as a function N for $k_1 d = 2\pi$. As can be seen the dependence of S on N is greater than linear, varying approximately as $0.001\,91 N^{1.5}$ (dashed blue curve) over this range of N, reflecting the fact that interactions are needed to produce a nonvanishing S. For a cubic array the departure from linear dependence is significant since the falloff of $h_2^2(k_1 R_{jj'})$ as $R_{jj'}^{-2}$ is partially canceled by the $R_{jj'}^2 d\Omega$ volume element measured from a particular $\mathbf{R}_j$.

The conclusive evidence for the transfer of coherence is provided by the factor $|B|^2$ factor in Eq. (24), which varies as $e^{-\delta_2^2 T^2/2}$. If we had used a density matrix approach to calculate the intensity, there would be both "two-photon" and "stepwise" contributions to the signal [19], both of which are maximum when $\delta_2 = 0$. The two-photon contribution corresponds to OCT and the stepwise contribution corresponds to a population transfer. For our atom-field parameters, the population transfer term is $(\gamma_2/\omega_B)^2$ times smaller than the coherence transfer term and can be neglected.

A. Linear chain in the *x* direction

One way to obtain a nonvanishing phase-matched signal is to take an equally spaced linear chain in the x direction in which the spacing d of adjacent atoms is an integral multiple q of $\lambda_1 = 2\pi/k_1$. In that limit, $e^{ik_1 Z_{jj'}} = 1$, $\mathbf{k}\cdot\mathbf{R}_{jj'} = 2\pi(j - j')q\sin\theta\cos\phi$, and the expression for the radiated intensity reduces to

$$I(\theta,\phi) = \frac{3}{8\pi}\frac{1+\cos^2\theta}{2}|B|^2\left|\sum_{j=1}^{N} F_j e^{-i2\pi jq\sin\theta\cos\phi}\right|^2,$$

with

$$F_j = \sum_{j'\neq j=1}^{N} G_{1,-1}(\mathbf{R}_{jj'}) = -\frac{3}{4}\sum_{j'\neq j=1}^{N} h_2[2\pi|(j-j')|q]. \tag{32}$$

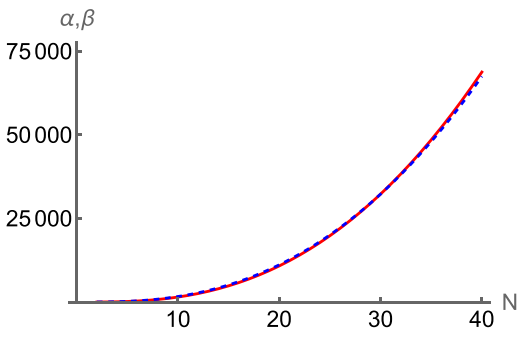

FIG. 4. A plot of α (solid red curve) and β (dashed blue curve) as a function N for N atoms on the x axis with $k_1 d = 20\pi$.

In addition, if $k_1 d = 2\pi q \gg 1$, we can approximate

$$F_j \sim -i\frac{3}{8\pi q}\sum_{j'\neq j=0}^{N-1}\frac{1}{|(j-j')|} = -i\frac{3}{4k_1 d}[2\mathrm{Eu} + \mathrm{PG}(1+j) + \mathrm{PG}(N-j)], \tag{33}$$

where Eu is Euler's constant and PG is the polygamma function. For our perturbative approach to remain valid in this limit, we must require that $|F_j| \ll 1$. For large values of N, $\mathrm{PG}(N) \sim \ln(N)$. In the forward direction, $(\theta,\phi) = (0,0)$; we might expect therefore that

$$\alpha = \left|\frac{4k_1 d}{3}\sum_{j=1}^{N} F_j\right|^2 \propto [N\ln(N)]^2. \tag{34}$$

In Fig. 4 we plot α and $\beta = 3.1N^2[\ln(N)]^2$ as a function of N for $k_1 d = 20\pi$ (the result is independent of $k_1 d$ for $k_1 d \gg 1$); as can be seen the results agree to a good approximation over this range of N. We can see there is a $[\ln(N)]^2$ enhancement over an N^2 dependence owing to the dependence of the OCT-M transfer on the number of atoms. There is a similar $[\ln(N)]^2$ enhancement of the Rydberg population; that is, S varies as $N[\ln(N)]^2$. For this linear array, the enhancement factor is smaller than that of the three-dimensional cubic array.

For $N \gg 1$, the intensity pattern consists of many narrow resonances. In effect the signal is a maximum for any values of θ and ϕ for which $q\sin\theta\cos\phi = n$, where n is a non-negative integer. There are resonances that occur for both continuous and discrete values of the spherical angles. In Fig. 5, we plot $I(\theta,\phi)/|B|^2$ as a function of θ and ϕ for $q = 2$ and $N = 10$. The interference term is negative and cancels most of the population at all points *except* those giving rise to constructive interference. This is similar to what occurs in an N-slit diffraction pattern when the slit widths are much less than a wavelength—the background constant intensity pattern from each of the slits is canceled at all points *except* those giving rise to constructive interference. In other words, most of the radiated intensity is concentrated in the constructive interference peaks. For the parameters of Fig. 5 this corresponds to approximately 98% of the energy.

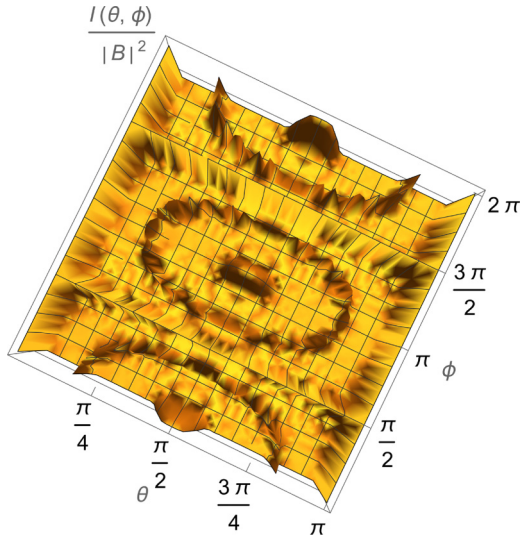

FIG. 5. A plot of the total intensity, $I(\theta,\phi)/|B|^2$, as a function of θ and ϕ for $k_1 d = 2\pi q = 4\pi$ and $N = 10$.

IV. DISCUSSION

We have studied the transfer of optical coherence between atoms in an array. In particular, we have determined under what conditions it is possible to transfer coherence between the $m = -1$ state and the ground state in one atom to a coherence between the $m = +1$ state and the ground state in another atom. For atoms on a line, there is a dramatic effect that could be tested experimentally. If the atoms are located on the z axis, there is no transfer, but if they are on the x axis, many constructive interference peaks should be seen. We have considered only the simplest experimental schemes for detection of the OCT-M. In doing so, we found that, for arrays that are invariant under a rotation $\delta\phi < \pi$ about the z axis, the total intensity in the phase-matched direction vanishes. To observe OCT-M-induced phase-matched emission using such an array, one could modify the readout procedure. If one uses a σ_- readout pulse whose frequency is resonant with the e to $m = -1$ transition frequency rather than with the e to $m = +1$ transition frequency, a second OCT-M can resonantly transfer this excitation from the $m = 1$ level back to the $m = -1$ level. In this *double* OCT-M protocol, there can be σ_- phase-matched radiation which is nonvanishing.

In the limiting case of two atoms, population transfer can occur provided that both the atoms are not on the z axis. This brings up an interesting point. Suppose that one of the atoms is excited to its $m = -1$ sublevel and there is no magnetic field. Some of the initial excitation is readily transferred to the $m = +1$ sublevel in the other atom. The question then arises how angular momentum can be conserved, given the fact that the average dipole moment of each atom vanishes. If the atoms are separated by less than a wavelength and if the atom-atom interaction is modeled as a *static* dipole-dipole interaction with the neglect of spontaneous emission, there is a simple explanation for angular momentum conservation. The change in the internal angular momentum of the atoms is converted to orbital angular momentum of the atoms about their center of mass. On the other hand, if the interaction between the atoms is assumed to arise from interactions of each atom with the transverse vacuum field (as in this paper), the situation becomes more complex since the angular momentum of the field must be taken into account to ensure overall angular momentum conservation [22].

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