Quantum theory of loss-induced transparency in coupled waveguides

I. Beder and P. A. Brandão [*]
Instituto de Física, Universidade Federal de Alagoas, Maceió 57072-900, Brazil

(Received 21 May 2024; accepted 21 August 2024; published 3 September 2024)

Several years ago, Guo *et al.* [Phys. Rev. Lett. **103**, 093902 (2009)] demonstrated a counterintuitive phenomenon wherein the transmission of classical light through a coupled pair of waveguides is enhanced as the level of loss in one waveguide surpasses a critical threshold. In this paper, we employ the Heisenberg-Langevin formalism to explore a quantum perspective of this phenomenon. In the case where light is incident in both waveguides, a generally nonzero interference term in the presence of loss appears. Our analysis reveals that loss-induced transparency can manifest at the single-photon level for both separable and entangled photonic states.

DOI: 10.1103/PhysRevA.110.033503

I. INTRODUCTION

The seminal work of Bender and Boettcher suggesting that non-Hermitian Hamiltonians having parity-time (PT) symmetry could have real-valued spectra has initiated an intense debate regarding possible descriptions of open quantum systems [1–4]. The main characteristics of PT-symmetric physical models is that they possess an equilibrium between loss and gain and they highlight in a very clear way the role of exceptional points [5]. In general, for a Hamiltonian that depends on some parameter, say ε, there is a critical value ε_c such that for $\varepsilon > \varepsilon_c$ all eigenvalues are real valued and for $\varepsilon < \varepsilon_c$ they become complex conjugate of each other [6,7]. By introducing a new definition for the inner product between PT-symmetric states, it has been demonstrated that it is possible to retain all the essential characteristics of quantum theory in a consistent framework [8,9].

The concept of open systems achieving equilibrium between gain and loss has found applications in several contexts such as in acoustics [10], optomechanics [11], phase transitions [12], and sensors [13], to cite a few. Notably, in the realm of classical photonic systems, intriguing effects have been reported [14–16]. Optics serves as a convenient platform for exploring PT symmetry largely due to the relative ease of creating dielectric materials with desired dissipative optical properties [17,18]. The pioneering experimental demonstration of a non-Hermitian optical system involved a coupled pair of single-mode waveguides, with a dissipative element (chromium atoms) introduced into one of the waveguides to account for losses [19]. By measuring the transmitted intensities in both waveguides, the authors noted an increase in transmission as the width of the chromium layer passed through a critical value. This counterintuitive phenomenon is commonly referred to as loss-induced transparency.

In the realm of integrated optics, the utilization of arrays of coupled waveguides represents the most promising platform for both classical and quantum photonics applications [20,21]. These arrays are relatively easy to fabricate and enable a description of light propagation (both classical and quantum) analogous to the dynamics of electrons in discrete lattices [22,23]. This analogy has prompted a plethora of classical analog effects that manifest in quantum systems [17]. In fact, the very first experimental demonstration of a truly PT-symmetric optical system was achieved using a pair of coupled waveguides and a two-wave mixing process to attain gain [24].

Recent studies have delved into quantum effects within non-Hermitian optical systems. Utilizing single-photon states, researchers have measured coincidence counts in a pair of coupled waveguides, thereby observing the non-Hermitian version of the Hong-Ou-Mandel dip [25]. The role of loss has been explored in a pair of quantum qubits [26], in arrays of coupled waveguides [23,27–29], in systems displaying high-order exceptional points [30], in anti-parity-time symmetric coupled waveguides [31], and disordered lattices [32].

Drawing inspiration from the counterintuitive phenomenon of loss-induced transparency and the quantum model explaining light propagation in interconnected waveguides, we address the potential for enhancing transmission through the utilization of quantum light states. Specifically, we compute the transmission coefficient for a pair of coupled waveguides incorporating a lossy element. Our analysis reveals that the overall transmission is influenced not solely by the initial number of photons entering each waveguide but also by the nonseparability characteristics of the incoming quantum state. We would like to point out that the formalism developed here is based entirely on conventional quantum mechanics with Hermitian Hamiltonians. The introduction of a dissipative system, consisting of one large waveguide (or several waveguides) generates the effective loss that simulates the PT-symmetric system under consideration [33].

Section II is devoted to a discussion of the Heisenberg-Langevin formalism applied to the coupled waveguide system and Sec. III defines the transmission coefficient and highlights the presence of a loss-induced interference term. Several examples involving entangled and separable states are given in Sec. IV and in Sec. V we present our conclusions.

[*]Contact author: paulo.brandao@fis.ufal.br

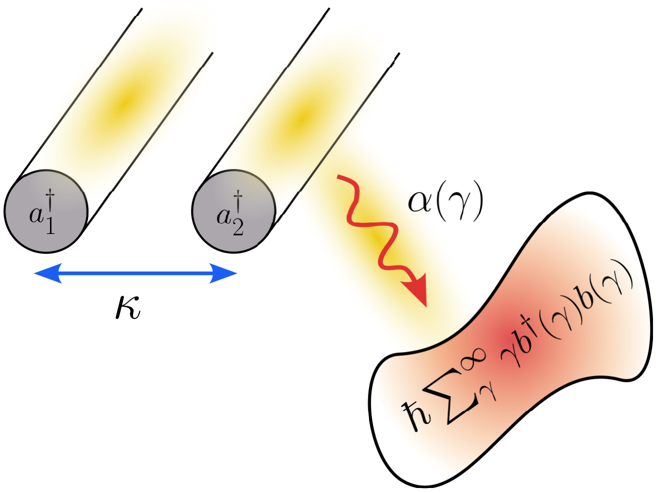

FIG. 1. Two coupled waveguides are represented by creation operators $a_1^\dagger$ and $a_2^\dagger$, featuring a coupling constant κ. The waveguide $a_2^\dagger$ interacts with a reservoir $b^\dagger(\gamma)$, characterized by frequencies γ, through the coupling constant $\alpha(\gamma)$ at each frequency.

II. HEISENBERG-LANGEVIN EQUATIONS

The coupled waveguide system is described by the total Hamiltonian $H = H_0 + H'$, where

$$H_0 = \hbar\beta \sum_{j=1}^{2} a_j^\dagger a_j + \hbar \sum_{\gamma}^{\infty} \gamma b^\dagger(\gamma) b(\gamma) \tag{1}$$

and

$$H' = \hbar\kappa(a_1^\dagger a_2 + a_2^\dagger a_1) + \hbar \sum_{\gamma}^{\infty} \alpha(\gamma)[a_2^\dagger b(\gamma) + a_2 b^\dagger(\gamma)]. \tag{2}$$

The Hamiltonian H describes two coupled waveguides, represented by the operators a_1 and a_2, where the second waveguide is coupled to a reservoir [it could be another large waveguide, whose annihilation operator is $b(\gamma)$, having several frequencies γ]. The parameter $\alpha(\gamma)$ characterizes the interaction between waveguide 2 and the mode γ of the reservoir, and the parameter κ describes the interaction between the two waveguides (see Fig. 1) with frequency β.

In the Heisenberg picture, the operators $a_j^\dagger(z)$ and $b^\dagger(\gamma, z)$ satisfy the following set of coupled differential equations:

$$\frac{da_1^\dagger}{dz} = i\beta a_1^\dagger + i\kappa a_2^\dagger, \tag{3}$$

$$\frac{da_2^\dagger}{dz} = i\beta a_2^\dagger + i\kappa a_1^\dagger + i\sum_\gamma \alpha(\gamma) b^\dagger(\gamma), \tag{4}$$

$$\frac{db^\dagger(\gamma)}{dz} = i\gamma b^\dagger(\gamma) + i\alpha(\gamma) a_2^\dagger. \tag{5}$$

After transforming Eq. (5) to an integral relation and substituting the result into Eq. (4), we obtain

$$\frac{da_2^\dagger}{dz} = i\beta a_2^\dagger + i\kappa a_1^\dagger + i\sum_\gamma \alpha(\gamma) b^\dagger(\gamma, 0) e^{i\gamma z} - \sum_\gamma [\alpha(\gamma)]^2 \int_0^z e^{i\gamma(z-s)} a_2^\dagger(s)\, ds. \tag{6}$$

The last term in the above expression can be written as

$$\begin{aligned}
&\sum_\gamma [\alpha(\gamma)]^2 \int_0^z e^{i\gamma(z-s)} a_2^\dagger(s) ds \\
&= \int_{-\infty}^{\infty} \rho(\gamma)[\alpha(\gamma)]^2 d\gamma \int_0^z e^{i\gamma(z-s)} a_2^\dagger(s) ds \\
&= \int_{-\infty}^{\infty} \int_0^z \rho(\beta+\gamma')[\alpha(\beta+\gamma')]^2 \\
&\quad \times e^{i(\beta+\gamma')\tau} a_2^\dagger(z-\tau) d\tau\, d\gamma' \\
&= \int_0^z \left[\int_{-\infty}^{\infty} S(\beta+\gamma') e^{i\gamma'\tau}\, d\gamma'\right] e^{i\beta_k\tau} a_2^\dagger(z-\tau) d\tau \\
&= 2\pi \int_0^z \Gamma(\tau) e^{i\beta_k\tau} a_2^\dagger(z-\tau) d\tau,
\end{aligned} \tag{7}$$

where $\rho(\gamma)$ is the density of states of the reservoir, $S(\gamma) = \rho(\gamma)[\alpha(\gamma)]^2$ is the spectral density, $\Gamma(\tau) = (1/2\pi)\int_{-\infty}^{\infty} S(\beta+\gamma') e^{i\gamma'\tau}\, d\gamma'$ is obtained from $S(\gamma)$ by using the Wiener-Khinchin theorem, and the substitutions $\gamma = \beta + \gamma'$ and $\tau = z - s$ were used. Assuming that the spectral density $S(\gamma)$ of the reservoir has a large distribution consisting of many oscillator modes, it is natural to approximate $S(\beta_k + \gamma') \approx S(\beta_k)$ such that $\Gamma(\tau) = S(\beta_k)\delta(\tau)$ (Markov approximation) and write for $z > 0$

$$\sum_\gamma [\alpha(\gamma)]^2 \int_0^z e^{i\gamma(z-s)} a_2^\dagger(s) ds \approx \sigma a_2^\dagger(z), \tag{8}$$

where $\sigma = 2\pi S(\beta_k)$ is a positive constant describing the loss rate.

We finally obtain the system of coupled equations for the dynamics of propagating photons in two coupled waveguides with dissipation,

$$\frac{da_1^\dagger}{dz} = i\beta a_1^\dagger + i\kappa a_2^\dagger, \tag{9}$$

$$\frac{da_2^\dagger}{dz} = i\beta a_2^\dagger - \sigma a_2^\dagger + i\kappa a_1^\dagger + f^\dagger(z), \tag{10}$$

where $f^\dagger(z) = i\sum_\gamma \alpha(\gamma) b^\dagger(\gamma, 0) e^{i\gamma z}$ is the noise operator describing the reservoir fluctuations. The fact that it appears together with the dissipation parameter σ is a manifestation of the fluctuation-dissipation theorem.

Assuming that both waveguides are identical and have the same propagation constant β, it is possible to simplify the equations by using the transformation $a_{1,2}^\dagger = A_{1,2}^\dagger e^{i\beta z}$, where $A_{1,2}^\dagger$ are slowly varying operators compared to $a_{1,2}^\dagger$. They satisfy

$$\frac{dA_1^\dagger}{dz} = i\kappa A_2^\dagger, \tag{11}$$

$$\frac{dA_2^\dagger}{dz} = -\sigma A_2^\dagger + i\kappa A_1^\dagger + F^\dagger(z), \tag{12}$$

where $F^\dagger(z) = e^{-i\beta z} f^\dagger(z) = i\sum_\gamma \alpha(\gamma) b^\dagger(\gamma, 0) e^{i(\gamma-\beta)z}$. This system can be written as a matrix equation of the form

$d\mathcal{A}^\dagger/dz = \mathcal{M}\cdot\mathcal{A}^\dagger + \mathcal{F}^\dagger$, where $\mathcal{A}^\dagger = (A_1^\dagger \quad A_2^\dagger)^T$ and $\mathcal{F}^\dagger = (0 \quad F^\dagger)^T$ are one column matrices and

$$\mathcal{M} = \begin{pmatrix} 0 & i\kappa \\ i\kappa & -\sigma \end{pmatrix}. \tag{13}$$

The solution $\mathcal{A}^\dagger(z)$ to this nonhomogeneous system of equations is given by

$$\mathcal{A}^\dagger(z) = e^{z\mathcal{M}}\cdot\mathcal{A}^\dagger(0) + \int_0^z e^{(z-s)\mathcal{M}}\cdot\mathcal{F}^\dagger(s)\,ds. \tag{14}$$

The exponential of the matrix $\mathcal{M}$ is easily evaluated,

$$e^{z\mathcal{M}} = e^{-\sigma z/2}\begin{bmatrix} \cos(z\Delta) + \dfrac{\sigma}{2\Delta}\sin(z\Delta) & \dfrac{i\kappa}{\Delta}\sin(z\Delta) \\ \dfrac{i\kappa}{\Delta}\sin(z\Delta) & \cos(z\Delta) - \dfrac{\sigma}{2\Delta}\sin(z\Delta) \end{bmatrix}, \tag{15}$$

where $\Delta = \sqrt{\kappa^2 - \sigma^2/4}$. Finally, the solutions for $A_1^\dagger(z)$ and $A_2^\dagger(z)$ are given by

$$A_1^\dagger(z) = \Theta_+(z)A_1^\dagger(0) + \Phi(z)A_2^\dagger(0) + \sum_\gamma \Omega_1(\gamma,z)b^\dagger(\gamma,0), \tag{16}$$

$$A_2^\dagger(z) = \Phi(z)A_1^\dagger(0) + \Theta_-(z)A_2^\dagger(0) + \sum_\gamma \Omega_2(\gamma,z)b^\dagger(\gamma,0), \tag{17}$$

where

$$\Theta_\pm(z) = e^{-\sigma z/2}\Big[\cos(z\Delta) \pm \frac{\sigma}{2\Delta}\sin(z\Delta)\Big], \tag{18}$$

$$\Phi(z) = \frac{i\kappa e^{-\sigma z/2}}{\Delta}\sin(z\Delta), \tag{19}$$

$$\Omega_1(\gamma,z) = \frac{\kappa\alpha(\gamma)}{\Delta}\int_0^z e^{-\sigma(z-s)/2}e^{i(\gamma-\beta)s}\sin[(s-z)\Delta]ds, \tag{20}$$

and

$$\Omega_2(\gamma,z) = i\alpha(\gamma)\int_0^z e^{-\sigma(z-s)/2}e^{i(\gamma-\beta)s} \times\left\{\cos[(z-s)\Delta] - \frac{\sigma}{2\Delta}\sin[(z-s)\Delta]\right\}ds. \tag{21}$$

Given our primary interest lies in the averaged number of photons departing from both waveguides, namely the transmission coefficient, the noise operator $F^\dagger$ will not be relevant for the analysis, as noted in Ref. [29]. Hence, the integrals represented by Eqs. (20) and (21) need not be explicitly computed. All the pertinent information regarding the total transmission through the waveguides is encapsulated in Eqs. (18) and (19) along with the initial state ket.

III. TRANSMISSION COEFFICIENT FOR QUANTUM AND CLASSICAL LIGHT

To connect the formalism developed in the previous section to the loss-induced transparency effect verified in classical settings, we define the transmission coefficient as

$$T = \frac{n_1(z) + n_2(z)}{n_1(0) + n_2(0)}, \tag{22}$$

where $n_j(z) = \langle\Psi_0|A_j^\dagger(z)A_j(z)|\Psi_0\rangle$ is the average number of photons in waveguide j at propagation distance z. They are given explicitly by

$$n_1(z) = |\Theta_+(z)|^2 n_1(0) + |\Phi(z)|^2 n_2(0) + 2\,\mathrm{Re}[\Theta_+(z)\Phi^*(z)\langle A_1^\dagger(0)A_2(0)\rangle], \tag{23}$$

$$n_2(z) = |\Phi(z)|^2 n_1(0) + |\Theta_-(z)|^2 n_2(0) + 2\,\mathrm{Re}[\Theta_-^*(z)\Phi(z)\langle A_1^\dagger(0)A_2(0)\rangle]. \tag{24}$$

Substitution of Eqs. (23) and (24) into Eq. (22) gives

$$T = \frac{P(z)n_1(0) + Q(z)n_2(0) + I(z)}{n_1(0) + n_2(0)}, \tag{25}$$

where $P(z) = |\Theta_+(z)|^2 + |\Phi(z)|^2$, $Q(z) = |\Theta_-(z)|^2 + |\Phi(z)|^2$, and

$$I(z) = 2\,\mathrm{Re}[(\Theta_+\Phi^* + \Theta_-^*\Phi)\langle A_1^\dagger(0)A_2(0)\rangle] = \frac{\sigma\kappa e^{-\sigma z}}{\Delta^2}[1-\cos(2z\Delta)]\mathrm{Re}\left[\frac{\langle A_1^\dagger(0)A_2(0)\rangle}{i}\right]. \tag{26}$$

The transmission T evidently depends on the type of input state $|\Psi_0\rangle$ provided to the waveguide system. The presence of an "interference term" $I(z)$ suggests that loss can introduce a more intricate dynamic, where the transmission is influenced not only by the averaged number of incident photons $n_j(0)$ but also by the nonseparability properties of the initial state $|\Psi_0\rangle$, as depicted in Eq. (26). It is noteworthy that in the absence of loss ($\sigma = 0$), $I(z) = 0$ as well, and consequently, the overall transmission would solely rely on the initial averaged number of photons. For an input state expanded in terms of the Fock basis,

$$|\psi(0)\rangle = \sum_{N_1,N_2} c_{N_1,N_2}|N_1\rangle_1|N_2\rangle_2|0\rangle_R,$$

where $A_j^\dagger A_j|N_j\rangle_j = N_j|N_j\rangle_j$ and $\sum_{N_1,N_2}|c_{N_1,N_2}|^2 = 1$, it is straightforward to demonstrate that

$$\langle A_1^\dagger(0)A_2(0)\rangle = \sum_{N_1,N_2} c^*_{N_1+1,N_2-1}c_{N_1,N_2}\sqrt{N_2(N_1+1)}. \tag{27}$$

Thus, the interference term also vanishes if the expansion coefficients c_{N_1,N_2} are real valued.

It is valuable to conduct a comparative analysis between quantum and classical descriptions. Let $E_1(z)$ and $E_2(z)$ represent the single-mode *classical* electric fields in waveguides

1 and 2, respectively. They satisfy the following system of equations:

$$i\frac{dE_1}{dz} + \kappa E_2 = 0, \quad (28)$$

$$i\frac{dE_2}{dz} + i\sigma E_2 + \kappa E_1 = 0. \quad (29)$$

The general solution is expressed as $E_1(z) = \Theta_+(z)E_1(0) + \Phi(z)E_2(0)$ and $E_2(z) = \Phi(z)E_1(0) + \Theta_-(z)E_2(0)$, where $\Theta_\pm$ and Φ are defined by Eqs. (18) and (19). The transmission for this classical system can be defined analogously to Eq. (25), except that $n_j(0) \to I_j(0)$, with $I_j(0) = |E_j(0)|^2$ representing the initial intensities, and the interference term is now given by

$$I_{\rm cl}(z) = \frac{\sigma\kappa e^{-\sigma z}}{\Delta^2}[1 - \cos(2z\Delta)]{\rm Re}\left[\frac{E_1^*(0)E_2(0)}{i}\right]$$
$$= \frac{\sqrt{I_1(0)I_2(0)}\sigma\kappa e^{-\sigma z}}{\Delta^2}[1 - \cos(2z\Delta)]\sin\phi, \quad (30)$$

where the last line is written assuming the general representations $E_1(0) = \mathcal{E}_1$ and $E_2(0) = \mathcal{E}_2 e^{i\phi}$ with $\mathcal{E}_1$ and $\mathcal{E}_2$ real and positive numbers. The interference term is thus proportional to the sine of the phase difference between the incident field components.

In classical experiments, it is typically assumed that light initially couples only to one of the waveguides (usually the lossless one). Consequently, the absence of the interference term $I_{\rm cl}(z)$ in these analyses becomes evident.

IV. EXAMPLES

This section is dedicated to applying the formalism developed in the preceding sections to clarify the enhancement of transmission of quantum light through lossy structures. We examine the loss-induced transparency effect for coherent states, separable Fock states, and entangled Fock states.

A. Coherent states

It should be expected that for an initial input state given in terms of coherent states, the overall behavior of the system must be quasiclassical. To see that this is indeed the case, consider the initial state $|\Psi_C\rangle = |\eta\rangle_1 |\zeta\rangle_2 |0\rangle_R$, where $A_1(0)|\eta\rangle_1 = \eta|\eta\rangle_1, A_2(0)|\zeta\rangle_2 = \zeta|\zeta\rangle_2$, and $|0\rangle_R$ is the initial state of the reservoir. In this case, the interference term has exactly the same form as Eq. (30) by making the identifications $I_1(0) \to |\eta|^2 = n_1(0)$, $I_2(0) \to |\zeta|^2 = n_2(0)$, and $\phi \to \arg(\zeta)$. The quantum and classical versions thus coincide regarding the behavior of the transmission coefficient.

Notice that the interference term vanishes in the classical and quasiclassical cases if the phase difference between $\mathcal{E}_1(0)$ and $\mathcal{E}_2(0)$ (or η and ζ) is a multiple of π. In this case, the transmission only depends on the initial intensities (or the initial average number of photons) present in both waveguides.

B. Separable Fock states

Before delving into the implications of a genuinely quantum input state, it is worth noting that if no photons are initially coupled in the lossy waveguide, resulting in $I_2(0) = 0$ or $|\Psi_0\rangle = |\cdots\rangle_{1,R}|0\rangle_2$, both the classical and quantum interference terms disappear. In either scenario, the transmission coefficient remains determined solely by $T = P(z)$, regardless of the nature of the field input in the lossless waveguide.

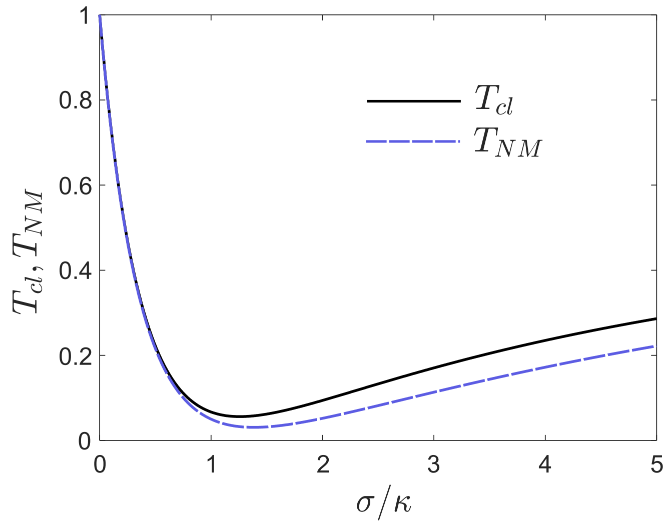

FIG. 2. Loss-induced transparency comparison between classical and separable Fock states. The initial classical state is characterized by $I_1(0) = 10$, $I_2(0) = 5$, and $\theta = \pi/2$. The input quantum state is given by $|\psi_{NM}\rangle = |10\rangle_1 |5\rangle_2 |0\rangle_R$, where $N = 10$ ($M = 5$) photons are coupled to the lossless (lossy) waveguide.

Consider now an input quantum state where N photons are coupled to the lossless waveguide and M photons are coupled to the lossy one, the reservoir being in the vacuum state $|0\rangle_R$. The state ket is given by the separable product of number states $|\Psi_0\rangle = |\psi_{NM}\rangle = |N\rangle_1 |M\rangle_2 |0\rangle_R$, where $A_1^\dagger(0)A_1(0)|N\rangle_1 = N|N\rangle_1$ and $A_2^\dagger(0)A_2(0)|M\rangle_2 = M|M\rangle_2$. In this case, the interference term vanishes $I(z) = 0$, regardless of the initial presence of photons in the waveguides, and the transmission coefficient is given by

$$T_{NM} = \frac{NP(z) + MQ(z)}{N + M}. \quad (31)$$

Hence, the transmission coefficient for separable number states solely relies on the input number of photons at each waveguide and is independent of the interference term $I(z)$. This stands in stark contrast to the classical scenario, where the interference term $I_{\rm cl}(z)$ can be made nonzero. Naturally, both transmission profiles coincide in scenarios where ϕ (the phase of the classical field components) is a multiple of π. However, in cases where an experiment is designed such that $\phi \neq \pi$, the quantum and classical versions produce different results. Figure 2 displays an example of this situation.

C. Entangled states

The preceding example addressed an input state in which the location and the number of photons is precisely known at $z = 0$. If this condition is relaxed, implying a state of uncertainty regarding the initial photons' localization, we can then consider the entangled initial state

$$|\psi_{NM,e}\rangle = \frac{1}{\sqrt{2}}(|N\rangle_1 |M\rangle_2 + e^{i\theta}|M\rangle_1 |N\rangle_2)|0\rangle_R, \quad (32)$$

where θ is the phase angle. The initial averaged number of photons in the waveguides is given by $n_1(0) = n_2(0) = (N + M)/2$ and it can be demonstrated that the interference term is nonzero if $M = N - 1$, where it is given by

$$I(z) = \frac{N\sigma\kappa e^{-\sigma z}}{2\Delta^2}[1 - \cos(2z\Delta)]\sin\theta. \quad (33)$$

Hence, states of the form $(|N\rangle_1 |N-1\rangle_2 + e^{i\theta} |N-1\rangle_1 |N\rangle_2)/\sqrt{2}$ interfere (in the sense of the transmission coefficient) if $\theta \neq n\pi$ for integer n.

To show that this produces different experimental outcomes even when a single photon is present in the waveguides, consider three input states of the form $|\Psi_a\rangle = |1\rangle_1 |0\rangle_2 |0\rangle_R$, $|\Psi_b\rangle = |0\rangle_1 |1\rangle_2 |0\rangle_R$, and $|\Psi_c\rangle = (1/\sqrt{2})(|1\rangle_1 |0\rangle_2 + e^{i\theta} |0\rangle_1 |1\rangle_2) |0\rangle_R$, the difference being that the initial photon localization is known with unit probability in $|\Psi_a\rangle$ and $|\Psi_b\rangle$ but not in the state ket $|\Psi_c\rangle$. Even though in all cases there is one photon propagating in the system, Fig. 3 reveals that (1) the transmittance is enhanced for the case where the photon is initially coupled to the lossless waveguide and (2) there is a critical value of $\theta = \pi/2$ for which the transmission is enhanced in comparison with $|\Psi_b\rangle = |0\rangle_1 |1\rangle_2 |0\rangle_R$ and other values of θ.

We conclude this section by noting that the propagation of photons through dissipative waveguides has already been demonstrated under laboratory conditions [25]. The authors observed the non-Hermitian HOM dip effect using coincident measurements of arriving photons. Therefore, in principle, the theoretical predictions presented here can be verified in the near future with existing technologies.

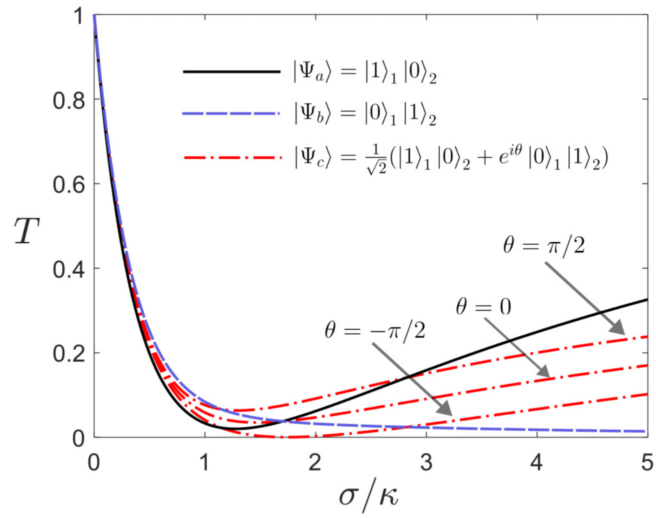

FIG. 3. Loss-induced transparency with single-photon states is demonstrated here. The continuous black curve illustrates the enhancement of transmission when a single photon is coupled to the lossless waveguide, represented by the ket $|\Psi_a\rangle$. The dashed blue curve depicts the initial coupling in the lossy waveguide, represented by $|\Psi_b\rangle$, while the dotted-dashed red curve represents the transmission for the entangled state $|\Psi_c\rangle$ for three values of the phase θ.

V. CONCLUSIONS

Our findings show that using the quantum formalism, it is also possible to observe the phenomenon of loss-induced transparency. We have demonstrated that incorporating a dissipative element into a pair of coupled waveguides yields an interference term, denoted as $I(z)$, within the transmission coefficient. This coefficient's behavior hinges upon the inseparable attributes of the initial quantum state, presenting an avenue to probe the phase characteristics of entangled states. Through our investigation, we have illustrated that loss-induced transparency phenomena manifest particularly under the circumstances where dissipation adheres to the Markov approximation. Furthermore, in scenarios where strong dissipative propagation dominates ($\sigma/\kappa \gg 1$), distinct single-photon states emerge with heightened transmittance, indicating a preferential state transmission. Given the indispensable role of loss in large-scale quantum computational tasks, our findings hold promise for tailoring waveguide applications in quantum information and computation. In real life situations, dissipation is always present. Thus, the present theory could be used to devise optimal incident photonic states that give more transmitted energy and, therefore, information.

ACKNOWLEDGMENT

The authors acknowledge the financial support of the Brazilian agencies CNPq and FAPEAL.

[1] C. M. Bender and S. Boettcher, Real spectra in non-Hermitian Hamiltonians having $\mathcal{PT}$ symmetry, Phys. Rev. Lett. **80**, 5243 (1998).
[2] C. M. Bender, Making sense of non-Hermitian Hamiltonians, Rep. Prog. Phys. **70**, 947 (2007).
[3] A. Mostafazadeh, Pseudo-Hermiticity versus PT symmetry: The necessary condition for the reality of the spectrum of a non-Hermitian Hamiltonian, J. Math. Phys. **43**, 205 (2002).
[4] A. Mostafazadeh, Pseudo-Hermitian representation of quantum mechanics, Int. J. Geom. Methods Mod. Phys. **07**, 1191 (2010).
[5] W. Heiss, The physics of exceptional points, J. Phys. A: Math. Theor. **45**, 444016 (2012).
[6] C. M. Bender, *PT Symmetry: In Quantum and Classical Physics* (World Scientific, Singapore, 2018).
[7] C. M. Bender, Introduction to $\mathcal{PT}$-symmetric quantum theory, Contemp. Phys. **46**, 277 (2005).
[8] C. M. Bender, D. C. Brody, and H. F. Jones, Complex extension of quantum mechanics, Phys. Rev. Lett. **89**, 270401 (2002).
[9] C.-Y. Ju, A. Miranowicz, G.-Y. Chen, and F. Nori, Non-Hermitian Hamiltonians and no-go theorems in quantum information, Phys. Rev. A **100**, 062118 (2019).
[10] X. Zhu, H. Ramezani, C. Shi, J. Zhu, and X. Zhang, $\mathcal{PT}$-symmetric acoustics, Phys. Rev. X **4**, 031042 (2014).
[11] X.-W. Xu, Y.-X. Liu, C.-P. Sun, and Y. Li, Mechanical $\mathcal{PT}$ symmetry in coupled optomechanical systems, Phys. Rev. A **92**, 013852 (2015).
[12] M. Fruchart, R. Hanai, P. B. Littlewood, and V. Vitelli, Non-reciprocal phase transitions, Nature (London) **592**, 363 (2021).

[13] J. Wiersig, Review of exceptional point-based sensors, Photonics Res. **8**, 1457 (2020).

[14] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Beam dynamics in $\mathcal{PT}$ symmetric optical lattices, Phys. Rev. Lett. **100**, 103904 (2008).

[15] M.-A. Miri and A. Alu, Exceptional points in optics and photonics, Science **363**, eaar7709 (2019).

[16] R. El-Ganainy, M. Khajavikhan, D. N. Christodoulides, and S. K. Ozdemir, The dawn of non-Hermitian optics, Commun. Phys. **2**, 37 (2019).

[17] S. Longhi, Parity-time symmetry meets photonics: A new twist in non-Hermitian optics, Europhys. Lett. **120**, 64001 (2017).

[18] L. Feng, R. El-Ganainy, and L. Ge, Non-Hermitian photonics based on parity-time symmetry, Nat. Photonics **11**, 752 (2017).

[19] A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, Observation of $\mathcal{PT}$-symmetry breaking in complex optical potentials, Phys. Rev. Lett. **103**, 093902 (2009).

[20] H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, Diffraction management, Phys. Rev. Lett. **85**, 1863 (2000).

[21] S. Somekh, E. Garmire, A. Yariv, H. Garvin, and R. Hunsperger, Channel optical waveguide directional couplers, Appl. Phys. Lett. **22**, 46 (1973).

[22] Y. Chen, X. Chen, X. Ren, M. Gong, and G.-c. Guo, Tight-binding model in optical waveguides: Design principle and transferability for simulation of complex photonics networks, Phys. Rev. A **104**, 023501 (2021).

[23] Y. N. Joglekar, C. Thompson, D. D. Scott, and G. Vemuri, Optical waveguide arrays: Quantum effects and PT symmetry breaking, Eur. Phys. J. Appl. Phys. **63**, 30001 (2013).

[24] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Observation of parity–time symmetry in optics, Nat. Phys. **6**, 192 (2010).

[25] F. Klauck, L. Teuber, M. Ornigotti, M. Heinrich, S. Scheel, and A. Szameit, Observation of PT-symmetric quantum interference, Nat. Photonics **13**, 883 (2019).

[26] C. A. Downing and O. I. R. Fox, Unbalanced gain and loss in a quantum photonic system, J. Opt. **25**, 095201 (2023).

[27] J. Zhou, Characterization of PT-symmetric quantum interference based on the coupled mode theory, Opt. Express **30**, 23600 (2022).

[28] J. Zhou, Analytical formulation of quantum interference inside coupled waveguides with unequal losses, Opt. Express **30**, 38357 (2022).

[29] M. Gräfe, R. Heilmann, R. Keil, T. Eichelkraut, M. Heinrich, S. Nolte, and A. Szameit, Correlations of indistinguishable particles in non-Hermitian lattices, New J. Phys. **15**, 033008 (2013).

[30] K. Tschernig, K. Busch, D. N. Christodoulides, and A. Perez-Leija, Branching high-order exceptional points in non-Hermitian optical systems, Laser Photonics Rev. **16**, 2100707 (2022).

[31] Y. Qin, H. Chen, D. Luo, C. Pan, H. Hu, Y. Zhang, and D. Wei, Quantum interference in anti-parity-time symmetric coupled waveguide system, Opt. Express **29**, 29175 (2021).

[32] L. Xu, Y. Dou, F. Bo, J. Xu, and G. Zhang, Two-photon correlation and photon transport in disordered passive parity-time-symmetric lattices, Phys. Rev. A **91**, 023817 (2015).

[33] M. Ornigotti and A. Szameit, Quasi-symmetry in passive photonic lattices, J. Opt. **16**, 065501 (2014).