Quantum battery with interactive atomic collective charging

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The collective charge of N two-level atoms interacting in an open system is investigated. In contrast to a no atomic dissipation quantum battery, the battery remains at a steady state after reaching the moment of full charge. Considering the interactions between atoms, we find that the energy storage of quantum batteries will be significantly enhanced with the increase of atomic repulsion, and the attraction between atoms will exacerbate the dissipation of batteries in the environment. We extend this conclusion to two-level batteries in the drive field and three-level atom quantum batteries. In addition, we also investigate the dissipative charging process of multiatom quantum batteries and find that the number of atoms during the charging process can to some extent affect the energy conversion efficiency during the charging process. Under extreme conditions, the inverse temperature β can disrupt the energy storage of the battery.

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I. INTRODUCTION

With the development of quantum science, quantum informatics and quantum thermodynamics have gradually become the focus of attention. As an important part of quantum informatics, optical quantum information has been widely studied, for example, the preparation of the squeezed state light field [1–3] and quantum teleportation [4–9], which are closely related to quantum entanglement [10,11] and steering [12–14]. Quantum thermodynamics and quantum informatics are inseparable. Traditional thermodynamics cannot describe quantum-scale devices, and a new understanding of concepts such as work, heat, and entropy is required. This led to the field of quantum thermodynamics, which explores new understandings of those quantities and also involves the study of quantum machines such as heat engines and refrigerators [15–19]. In the process of studying quantum information and quantum thermodynamics, the potential advantages of quantum effects in some applications, such as in quantum sensing [20], cryptography [21], and computation [22], were discovered. One scenario which features both of these aspects of quantum science is that of a possible quantum enhancement in thermodynamic tasks, such as the charging of batteries [23–32].

Quantum batteries (QBs), first introduced by Alicki and Fannes [23], seek to use nonclassical effects such as quantum coherence or quantum entanglement to impart an advantage compared with classical batteries. It is different from the traditional sense of the battery. Generally speaking, it is composed of a two-level system and an external field. The interaction between the external field and the two-level system is used to achieve the charging effect. Recently, solid-state collective charging has been proven to greatly increase the energy storage of the battery [26], and quantum entanglement or quantum coherence is an indispensable resource in charging [33]. In particular, Alicki and Fannes [23] found that global entangling operations could extract more work from a QB than local operations. Hovhannisyan *et al.* [34] found that a series of N global entangling operations can extract the maximum work without creating any entanglement in QB. This scenario corresponds to taking a time-consuming indirect path such that the QB only traverses the space of separable states. By contrast, the direct path taken under the action of a global entangling operation does generate entanglement during operation. This led to the conjecture that the rate of work extraction, that is, the power, is linked to quantum entanglement [34]. This was supported by Binder *et al.* [35], who showed that N interacting QBs traversing through entangled subspaces can charge N times faster than the same number of noninteracting batteries confined to uncorrelated subspaces.

On the other hand, Barra *et al.* [27,36] proposed the concept of dissipative quantum batteries in open systems and showed that quantum collective charging offered unique advantages. Pirmoradian and Mølmer [28] proposed the concept of quantum aging. Rolandi *et al.* [37] showed that the collective agreement of a multibody system in thermodynamics is effectively suppressed in the slow time process. Dou *et al.* [38] showed that both atomic interactions and an external driving field can lead to faster charging and greater energy storage of quantum batteries. Liu and Segal [39] found that the coherence between spins can effectively inhibit the aging of quantum batteries in open systems. Song *et al.* [40] proposed a QB scheme to realize a remote charging via coupling the QB and the charger to a rectangular hollow metal waveguide, which provides an effective way to realize the antiaging of QB. Therefore, quantum batteries in open systems will be popular in the future.

In this paper, as shown in Fig. 1(a), we propose a charging model in which N ring interacting two-level atoms are charged

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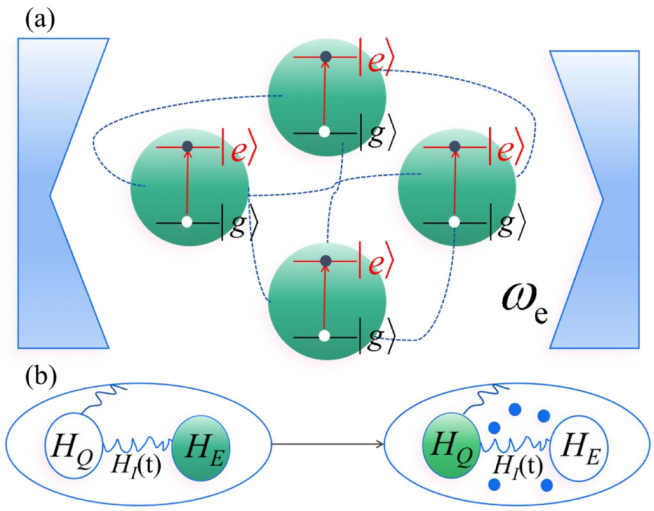

FIG. 1. (a) A Dicke QB considering interatomic interactions, where the same array of two-level systems is now embedded into a single cavity and interacting with a common photonic mode. (b) A quantum dissipative charge in an open system, with the blue origin representing the lost energy.

in the environment by coupling with a single optical cavity. In addition to this, we also discuss two models: a drive field battery and a three-level battery. We fully relax the constraint on the unitary evolution of the QB. Such a unitary evolution regime occurs only when the dynamics of the energy source is very slow compared to the QB dynamics (i.e., in the Born-Oppenheimer limit). Although certainly interesting, this is motivated more by mathematical convenience than adherence to reality. We consider the more realistic situation in which no timescale separation exists between the QB and energy source subsystem. Therefore, we treat the system "QB + energy source" in a fully quantum mechanical fashion, which generally results in a nonunitary reduced dynamics of the QB alone.

II. DICKE BATTERIES WITH INTERACTING ATOMS IN AN OPEN ENVIRONMENT

QB takes the form of a Dicke model [41], which considers the atom interaction in an open system. However, in an open system, both the cavity and the two-level atoms have attenuation, so the battery dissipation and the charger dissipation cannot be ignored. In order to investigate the influences of the interatomic interaction on the model, we may activate or deactivate the interatomic interaction. As a result, the Hamiltonian of the model comprises a battery, a charger, and a dissipative term, expressed as

$$H_Z = H_{IN} + \lambda(t)P, \tag{1}$$

where $\lambda(t)P$ is the dissipative term, and then one can use the master equation to characterize it. H_{IN} is the total Hamiltonian without considering dissipation, which can be written as

$$H_{IN} = H_Q + H_E + H_I. \tag{2}$$

For N identical two-level atoms, long-range forces between all atoms can be mediated by the electric field. Such long-range interactions can be engineered and controlled using atoms trapped in a photonic crystal waveguide [42] and Bose-Einstein condensed atoms [43], which highlights the practical relevance for the interacting Hamiltonian considered here. Involving the infinite-range dipole-dipole interactions, the Hamiltonian H_Q of N interacting atoms can be described by [44,45]

$$H_Q = \frac{\omega_q}{2}\sum_{i=1}^{N}\sigma_i^z + \frac{g}{2N}\sum_{i\neq j}^{N}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right), \tag{3}$$

where i refers to the ith spin in the chain and σ_i^k denotes the Pauli spin operator with $k = x, y, z$. In general, $|g\rangle$ is the ground state and $|e\rangle$ is the excited state. The quantity ω_q is the energy splitting between $|g\rangle$ and $|e\rangle$ of each two-level system. Here, the magnetic moment, lattice spacing, and $\hbar$ are all set to 1.

The second term H_E of the Hamiltonian in Eq. (2) is constituted by the bosonic annihilation (creation) operator which represents the annihilation (creation) of a cavity photon with frequency ω_e and is written as

$$H_E = \omega_e\hat{a}^\dagger\hat{a}. \tag{4}$$

The third term H_I of the Hamiltonian in Eq. (2) represents the interaction between the atoms and the optical cavity, which can be expressed as

$$H_I = g_c\omega_e\sum_{i=1}^{N}\sigma_i^x(a^\dagger + \hat{a}), \tag{5}$$

where g_c represents the interaction constant between the optical cavity and the atoms. The energy storage of QBs is best in the resonance region and worse in the nonresonance region. Therefore, we will focus on discussing the characteristics of QBs under the resonance of $\omega_e = \omega_q$.

For an isolated QB, the dynamics would simply be determined by the Hamiltonian H_{IN}. However, to account for dissipation and decoherence on the QB due to its coupling with the environment, a master equation is needed where a dissipator superoperator $L[\cdot]$ is added to the Liouville–von Neumann equation, giving [36]

$$\frac{\partial\rho_\tau}{\partial t} = -i[H_{IN}, \rho_\tau] + L[\rho_\tau]. \tag{6}$$

The specific form of $L[\rho_\tau]$ represents the dissipation of QB, which is given by

$$L[\rho_\tau] = \kappa(a\rho_\tau a^\dagger - \{a^\dagger a, \rho_\tau\}/2), \tag{7}$$

where κ is the decay rate of the cavity mode. We do not consider the collective dissipation and local dissipation of atoms. Here, we employ the Markovian assumption for considering the weak coupling between the environment and the system [28].

For the Tavis-Cummings QB [46], the charger dissipator is $L_E[\cdot] = \kappa(n_{th}+1)(2a\cdot a^\dagger - a^\dagger a\cdot - \cdot a^\dagger a) + \kappa n_{th}(2a^\dagger\cdot a - aa^\dagger\cdot - \cdot aa^\dagger)$. Here, we use $n_{\rm th} = 0$ to solely study the dissipation from the charger avoiding the repopulation effects of a nonzero temperature.

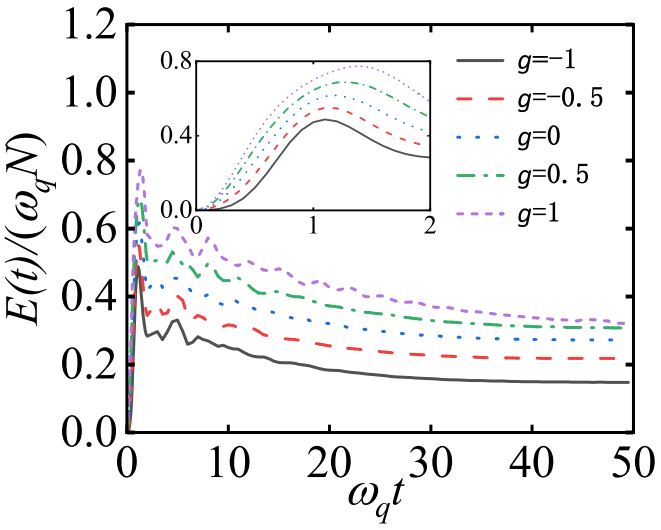

FIG. 2. $E(t)$ (in units of $\omega_q N$) vs the dimensionless charging time $\omega_q t$ with $g_c = 0.5$ for $g = 1, 0.5, 0, -0.5$, and -1, respectively. The small illustration is an enlarged view of $\omega_q t$ from 0 to 2.

The single-mode cavity is in the Fock state at the beginning of battery charging $|N\rangle$ [47]. The initial state of the system is

$$|\psi^{(N)}(0)\rangle = |N\rangle \otimes \underbrace{|g, g, \ldots, g\rangle}_{N}. \tag{8}$$

The density matrix at the corresponding initial moment is

$$\rho_\tau(0) = |\psi^{(N)}(0)\rangle\langle\psi^{(N)}(0)|. \tag{9}$$

We calculate the dissipated charge energy stored by the battery using the following equation [26,32],

$$E(t) = \mathrm{Tr}(\rho_\tau H_Q) - \mathrm{Tr}[\rho_\tau(0) H_Q]. \tag{10}$$

Here, we use the large spin operator to depict the Hamiltonian, and the corresponding Hamiltonian will be expressed as

$$H_Q = \frac{\omega_q}{2}\sum_{i=1}^{N}\sigma_i^z + \frac{g}{2N}\sum_{i\neq j}^{N}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right)$$

$$= \omega_q S_z + \frac{g}{N}\left(S^2 - S_z^2 - \frac{N}{2}\right), \tag{11}$$

$$H_I = g_c\omega_e S_x(a^\dagger + a), \tag{12}$$

where $S_k = \sum_{i=1}^{N}\sigma_i^k$ and $S^2 = \sum_k S_k^2$. A convenient basis set for representing the Hamiltonian is $|n, j, m\rangle$. Here, n indicates the number of photons, $j(j+1)$ is the eigenvalue of $\hat{J}^2$, and m denotes the eigenvalue of $\hat{J}_z$. With this notation, the initial state reads $|\psi^{(N)}(0)\rangle|N, \frac{N}{2}, -\frac{N}{2}\rangle$. The corresponding calculation relationship for Hamiltonian is [48]

$$a^\dagger|n, j, m\rangle = \sqrt{n+1}|n+1, j, m\rangle,$$

$$a|n, j, m\rangle = \sqrt{n}|n-1, j, m\rangle,$$

$$J_\pm|n, j, m\rangle = \sqrt{j(j+1) - m(m\pm 1)}|n, j, m\pm 1\rangle. \tag{13}$$

In this paper, we select the maximum number of photons as $N_{\mathrm{ph}} = 20$. The calculation process uses QUTIP encoding [49].

Figure 2 shows that interatomic repulsion ($g > 0$) has a quantum advantage in the energy storage of QB. The energy stored in an open environment significantly increases with the increase of the repulsion force. The interatomic force of attraction ($g < 0$) exacerbates the dissipation of QBs in an open system. One can find from H_Q that the low energy of the first noninteracting term is responsible for charging between the states $|g\rangle$ and $|e\rangle$ for each atom. As the attractive coupling strength increases, the evolution is dominated by the high-energy part of the second g term. The high-energy eigenstates can influence the charging states of the many-body battery, which play a negative role in the charging. This result is similar to that of harmonic charging in a closed system [29].

In addition, regardless of whether there is an interaction between atoms in the system, the system will ultimately be in a state of constant energy storage. The maximum capacity of QB is in an open system, with no energy exchange with the outside world. As the dissipation coefficient increases, the final quantum system will be in a stable state.

III. DRIVE FIELD BATTERY IN AN OPEN ENVIRONMENT

In order to make the results more convincing, we considered a system that uses a drive field H_C to charge the battery H_Q in an open environment. H_Q is shown in Eq. (11). The specific Hamiltonian of the drive field H_C is written as [25]

$$H_C = \omega S_x. \tag{14}$$

We consider the collective dissipation of atoms and express it by the main equation as [50]

$$\frac{\partial\rho_\tau}{\partial t} = -i[H_Q + H_C, \rho] + \gamma(S_-\rho_\tau S_+ - \{S_+S_-, \rho\}/2), \tag{15}$$

where γ is the collective decay rate. ρ_τ is the density-matrix operator of the whole system. The N spins are prepared in the ground state $|g\rangle$. A convenient basis set for representing the Hamiltonian is $|j, m\rangle$. With this notation, the QB's initial state reads

$$|\psi^{(N)}(0)\rangle = |N/2, -N/2\rangle. \tag{16}$$

The matrix elements of the $H_Q + H_C$ can be evaluated over the basis set $|j, m\rangle$ using the following relations for the ladder operator,

$$J_\pm|j, m\rangle = \sqrt{j(j+1) - m(m\pm 1)}|j, m\pm 1\rangle. \tag{17}$$

We plotted the energy storage image of the system by Eq. (10). As shown in Fig. 3, the repulsive force between the atoms increases the energy storage of the battery when the external field is used to charge the battery in an open environment. The attraction force between atoms reduces battery energy storage.

IV. THREE-LEVEL ATOM BATTERY IN AN OPEN ENVIRONMENT

In addition, we extended this conclusion to three-level atom QBs. We consider a battery system consists of a single-mode cavity and two identical cascaded three-level atoms directly driven by a pump field Ω_a (the angular frequency is

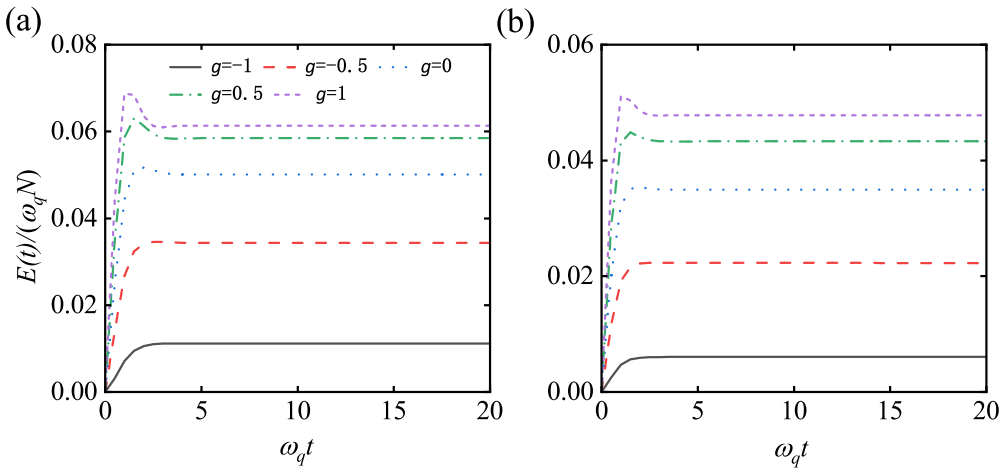

FIG. 3. $E(t)$ (in units of $\omega_q N$) vs the dimensionless charging time $\omega_q t$ with $g_c = 0.5$, (a) $\gamma = 0.4$ and (b) $\gamma = 0.5$, for $g = -1$, -0.5, 0, 0.5, and 1, respectively.

ω_p) and a control field Ω_b (the angular frequency is ω_c) simultaneously. We assume that the pump (control) field drives the $|g\rangle \leftrightarrow |m\rangle$ ($|m\rangle \leftrightarrow |e\rangle$) transition, while the cavity mode is only coupled with the $|g\rangle \leftrightarrow |m\rangle$ transition.

We use the master equation to describe the dynamic behavior of the system as

$$\frac{d\rho_\tau}{dt} = -i[H, \rho_\tau] + \mathcal{L}_\kappa \rho_\tau + \mathcal{L}_\gamma \rho_\tau, \tag{18}$$

where ρ_τ is the density-matrix operator of the whole system. Under the rotating-wave and electric-dipole approximations, the system Hamiltonian can be written as $H = H_Q + H_I + H_L$ with [51]

$$H_Q = \sum_{i=1,2} \left(\Delta_e \sigma_{ee}^i + \Delta_m \sigma_{mm}^i\right) + g\left(\sigma_{mg}^{(1)}\sigma_{gm}^{(2)} + \text{H.c.}\right) + g\left(\sigma_{me}^{(1)}\sigma_{em}^{(2)} + \text{H.c.}\right), \tag{19}$$

$$H_I = \sum_{i=1,2} g_i\left(a\sigma_{mg}^i + a^\dagger \sigma_{gm}^i\right) + \Delta_{\text{cav}} a^\dagger a, \tag{20}$$

$$H_L = \Omega_a \sum_{i=1,2} \left(\sigma_{mg}^i + \sigma_{gm}^i\right) + \Omega_b \sum_{i=1,2} \left(\sigma_{me}^i + \sigma_{em}^i\right), \tag{21}$$

where H_Q is the energy of atoms and the interaction between the atoms. H_I represents the energy of the cavity field itself and the interaction between the atoms and the cavity field. H_L is the coherent driving term involving the pump field and control field. g_i represents the interaction constant between the optical cavity and the atoms. g is the interaction coupling constant between the atoms. $\sigma_{jk}^i = |j\rangle^i\langle k|$ ($j, k = \{g, m, e\}$) denotes the atomic operator of the ith atom. The detunings are defined as $\Delta_{\text{cav}} = \omega_{\text{cav}} - \omega_p$, $\Delta_m = \omega_m - \omega_g - \omega_p$, $\Delta_e = \omega_e - \omega_g - (\omega_p + \omega_c) = \Delta_m + \Delta_c$ with $\Delta_c = \omega_e - \omega_m - \omega_c$ and ω_j ($j = \{g, m, e\}$) being the energy of state $|j\rangle$.

The last two terms in Eq. (18) denote the decay of the atoms and cavity, which are given by

$$\mathcal{L}_\kappa \rho_\tau = \kappa(2a\rho_\tau a^\dagger - a^\dagger a\rho_\tau - \rho_\tau a^\dagger a), \tag{22}$$

$$\mathcal{L}_\gamma \rho_\tau = \sum_{i=1,2} \left[\gamma_e\left(2\sigma_{me}^i \rho_\tau \sigma_{em}^i - \sigma_{em}^i \sigma_{me}^i \rho_\tau - \rho_\tau \sigma_{em}^i \sigma_{me}^i\right) + \gamma_m\left(2\sigma_{gm}^i \rho_\tau \sigma_{mg}^i - \sigma_{mg}^i \sigma_{gm}^i \rho_\tau - \rho_\tau \sigma_{mg}^i \sigma_{gm}^i\right)\right], \tag{23}$$

where κ is the cavity decay rate and γ_α ($\alpha = m, e$) is the spontaneous emission rate of the state $|\alpha\rangle$.

We set the initial state of QB to the lowest-energy state,

$$|\psi^{(N)}(0)\rangle = |N\rangle \otimes |g, g\rangle, \tag{24}$$

where N is the number of photons. Assuming $\Delta_c = 0$ and $\omega_{\text{cav}} = \omega_m - \omega_g$ for simplicity, we have $\Delta_m = \Delta_e = \Delta_{\text{cav}} \equiv \Delta_p$.

Now we focus on the case where two atoms are coupled with the same cavity field, i.e., $g_1 = g_2$. We can also use Eq. (10) to find the energy storage of the battery, as shown in Fig. 4. It can be seen that the interaction forces between atoms can affect the energy storage of QBs in an open environment. The maximum interception number of photons in the calculation process $n_{\text{ph}} = 10$, and the number of photons used for charging $N = 2$.

V. RECHARGING PROCESS

If the battery has reached a fully charged state, it will not do any work to the outside world afterwards. Otherwise, the battery will dissipate the energy it wants to store. From a thermodynamic perspective, the steady state of dissipative dynamics is either a nonequilibrium state of dissipative energy or an equilibrium state without dissipation. Next, we take battery H_Q as an example to discuss the process of extracting work from the battery reaching an equilibrium state under the influence of the environment. It can be given by active equilibrium conditions and equilibrium schemes [27].

A. Condition for active equilibrium

We set the environment to a hot bath, and the battery is in equilibrium $\omega_\beta(H_0)$ under the action of the hot bath, with H_0 as an operator on the Hilbert space of the system. First, note

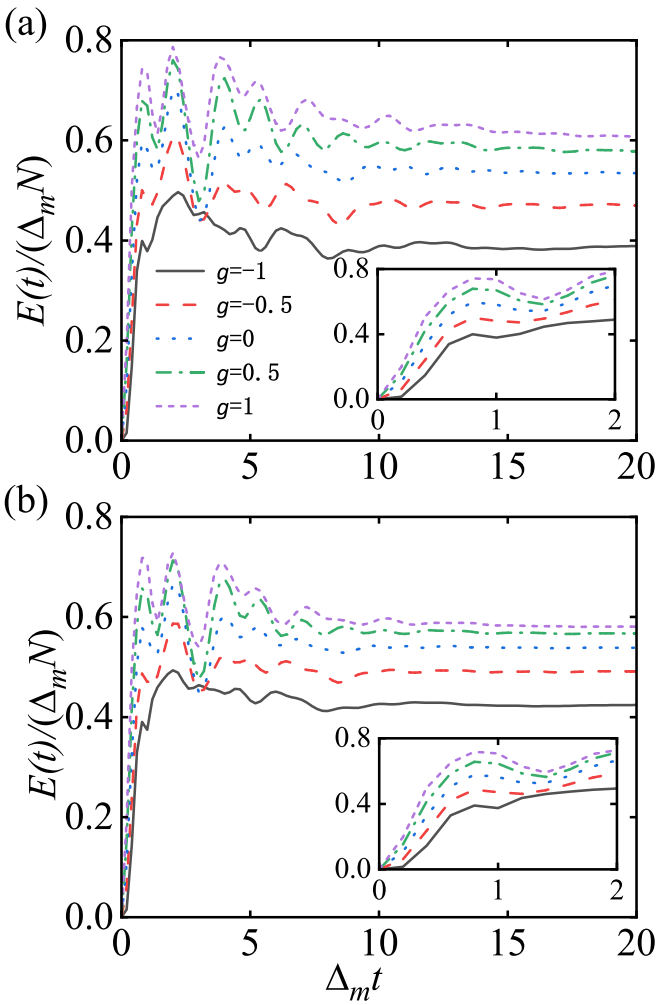

FIG. 4. $E(t)$ (in units of $\Delta_m N$) vs the dimensionless charging time $\Delta_m t$ with g_1, $\Omega_a = \Omega_b = 1$, $\kappa = 0.5$, (a) $\gamma_e = 0.1$, $\gamma_m = 0.01$ and (b) $\gamma_e = 0.2$, $\gamma_m = 0.1$, for $g = -1$, -0.5, 0, 0.5, and 1, respectively. The small illustration is an enlarged view of $\Delta_m t$ from 0 to 2.

that the equilibrium condition $[U, H_0 + H_E] = 0$ with $U = e^{-itH_{IN}}$ is satisfied if $[H_0, H_Q] = 0$ and $[H_0 + H_E, H_I] = 0$. On the basis of common eigenvectors of the nondegenerate H_Q and H_0, the equilibrium state is

$$\omega_\beta(H_0) = \sum_{i=1}^{N} \frac{e^{-\beta E_i^0}}{Z_0} |E_i\rangle \langle E_i| , \tag{25}$$

where $Z_0 = \mathrm{Tr} e^{-\beta H_0}$, and β is the inverse temperature of the environment when in thermal equilibrium. E_i is the eigenvalue of H_Q. E_i^0 is the eigenvalue of H_0. If a pair (j, k) exists such that $(E_j - E_k)(E_j^0 - E_k^0) \leqslant 0$, the state is active. Then, its ergotropy is extracted by a process described by a permutation unitary matrix u associated to the permutation p of $\{1, \ldots, N\}$ such that $E_{p_1}^0 \leqslant \cdots \leqslant E_{p_N}^0$, leaving the battery in the passive state [52,53],

$$\sigma_{\omega_\beta(H_0)} = \sum_{i=1}^{N} \frac{e^{-\beta E_{p_i}^0}}{Z_0} |E_i\rangle \langle E_i| , \tag{26}$$

with

$$u = \mathcal{T}_+ e^{-i \int dt [H_Q + V_Q(t)]}, \tag{27}$$

where $\mathcal{T}_+$ denotes the time-ordering operator, and $V_Q(t)$ is a time-dependent potential vanishing at the beginning and end of the process, accounting for a cyclic external work source.

Heat Q_R and work W_R obtained by characterizing a recharging process $\sigma_{\omega_\beta(H_0)} \to \omega_\beta(H_0)$ are [27]

$$Q_R = \mathrm{Tr}_Q\{H_0[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}]\}, \tag{28}$$

$$W_R = \mathrm{Tr}_Q\big\{(H_Q - H_0)\big[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}\big]\big\}. \tag{29}$$

The ergotropy of the state $\omega_\beta(H_0)$ can be obtained from Eqs. (28) and (29) as

$$\mathcal{W}[\omega_\beta(H_0)] = Q_R + W_R. \tag{30}$$

Note that $Q_R \leqslant 0$ [see Eq. (28)] since $\omega_\beta(H_0)$ is the state with a minimum average of H_0 among states with the same entropy. It follows that $W_R \geqslant \mathcal{W}[\omega_\beta(H_0)] \geqslant 0$, and thus no perpetuum mobile of the second kind can be built. We quantify the efficiency of the charging process by the ratio

$$\eta \equiv \frac{\mathcal{W}[\omega_\beta(H_0)]}{W_R} = 1 - \frac{|Q_R|}{W_R}, \quad 0 \leqslant \eta \leqslant 1. \tag{31}$$

B. A protocol for active equilibrium [27]

A particularly interesting equilibrating processes with an active equilibrium state is obtained with interaction $V = \sum_\nu F_\nu \otimes O_\nu$ where the system operators F_ν and auxiliary bath operators O_ν satisfy $[H_Q, F_\nu] = \lambda_\nu F_\nu$ and $[H_E, O_\nu] = \lambda_\nu O_\nu$. In this case, $[H_E - H_Q, V] = 0$.

Generally, if $H_0 = nH_Q$ ($n = 1, 2, 3, \ldots$), the thermal equilibrium process will be incapable of generating work. If $H_0 = H_Q$, the efficiency of the recharging process will be $\eta = 1/2$.

Here, we omit the fast oscillation term as a result of the conservation of the number of particles in space. We employ the equilibrium protocol in conjunction with Eq. (11) to acquire a practical $H_0 = \omega_q S_z$. A more advantageous approach is the use of the eigenstates that correspond with the large spin operator [54]. This produces consistent results with the spin states in an actual sense and can significantly decrease the computational burden.

In fact, the efficiency of charging is not negative. This relationship is related to the selected atomic coupling constant g and the energy level difference ω_q of two-level atoms. To effectively resist such situations, the coupling constant between the atoms is typically set to $g > \omega_q$.

Figure 5 shows that the corresponding recharge efficiency will decrease with the increase of the number of particles N. When $g/\omega_q = 60$ or more, the recharge efficiency will not fluctuate very much. When $g/\omega_q < 60$, the interaction between atoms is weak, the binding ability is weak, and multiple particles lose more energy, resulting in a corresponding decrease in effective energy. When the number of particles is small, the energy loss between particles is not significant, so the relative recharge efficiency increases. It can be seen that when we need a QB with high conversion efficiency but less energy storing [55], we can effectively reduce the number of particles to achieve this effect. When we need a battery with high-energy storage capacity, we can effectively increase

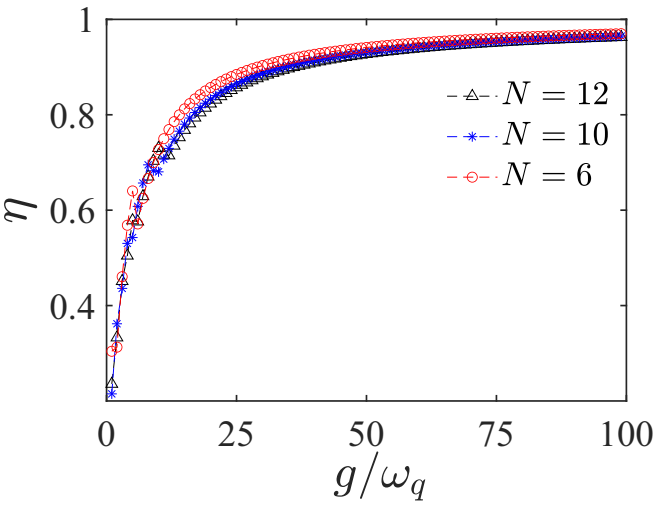

FIG. 5. The relationship between the energy conversion rate η and g/ω_q during collective charging. The inverse temperature $\beta = 1$.

the number of particles and excite them through an external field, thereby increasing the interaction between atoms and ultimately achieving high efficiency and large energy storage.

Figure 6 indicates that the work and ergotropy of the battery will be related to the inverse temperature β, and there will be a great value (the maximum capacity of the battery). When the inverse temperature β is high, the work and ergotropy will approach 0. This is related to the cells that make up the battery. The more cells there are, the more energy is stored. When the inverse temperature is very high, it will destroy the energy storage of the battery and damage its lifespan. Therefore, choosing an appropriate thermal steady-state temperature is reasonable.

Figure 7 shows that the ergotropy and work have a linear relationship with g, which is a very important factor affecting the amount of thermal equilibrium work. Their scale law is

$$\mathcal{W}[\omega_\beta(H_0)], \quad W_R \propto g. \tag{32}$$

VI. CONCLUSIONS

In this paper, we investigate atomic interactions using an optical cavity as a charger for dissipative charging in the environment. Compared with the open system where atomic interactions were not considered before, we find that atomic

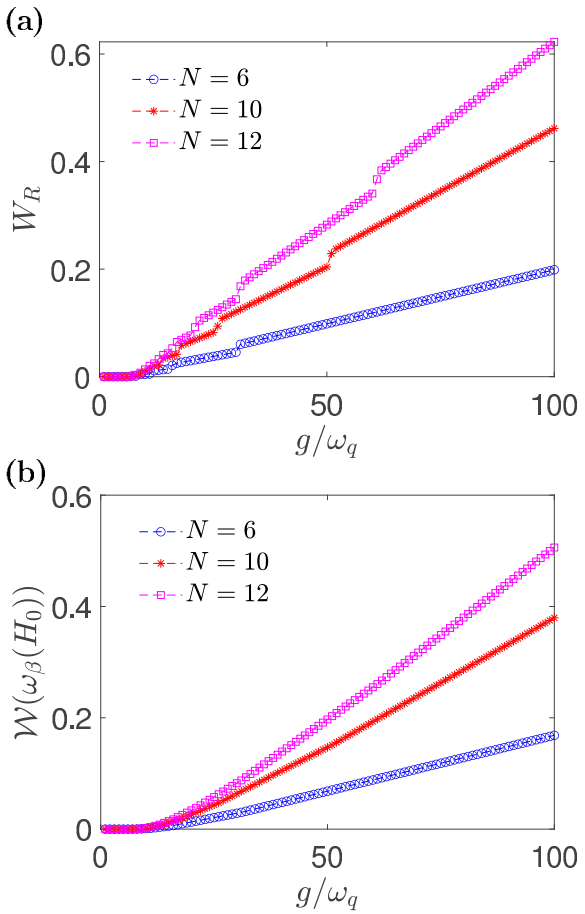

FIG. 7. (a) The work and (b) the ergotropy vs g/ω_q with $\beta = 1$ for different N, respectively.

repulsion suppressed the energy dissipation stored by the battery in the environment to a greater extent, while attraction increased the energy dissipation of the QB. This conclusion is also applicable to drive field QBs and three-level atom QBs. We also find that in the thermal steady state, as the interatomic interaction g increases, the recharging efficiency maintains an increasing trend and tends to 1 in the limit $g \to \infty$, but it cannot be equal to 1. This is consistent with the second law of quantum thermodynamics, which states that there is always energy dissipation during the charging process of a battery, making it impossible to generate a second type of perpetual

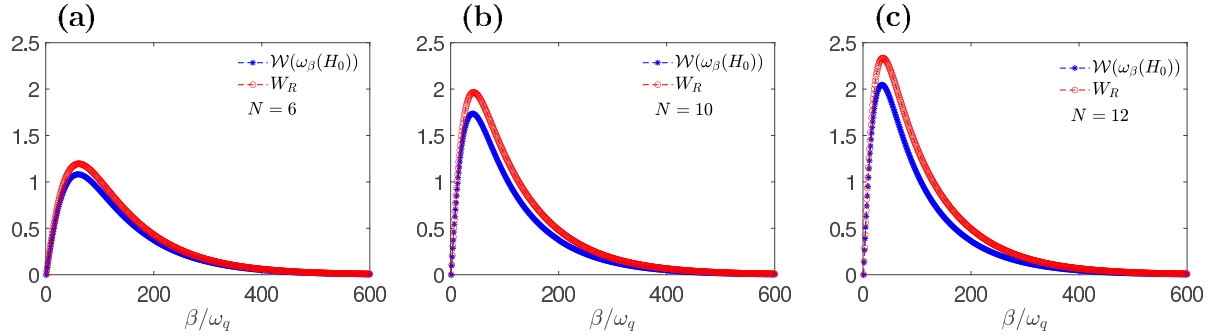

FIG. 6. Values of ergotropy and work as a function of β/ω_q with $g = 4$ for (a) $N = 6$, (b) $N = 10$, and (c) $N = 12$, respectively.

motion machine. In addition, the inverse temperature β under a thermal steady state is a key factor affecting battery energy storage. At high inverse temperatures, the battery's energy storage will be destroyed, and the battery will no longer store energy.

The two-level system is physically experimentally feasible. Recently, there has been some experimental work dedicated to QBs, such as a QB using star-topology NMR spin systems [56], a QB based on room temperature [57] and a QB made of a harmonic oscillator [58]. With the development of science and technology, QBs will be gradually explored. QBs considering dissipation are of great significance for the study of many-body systems and the study of thermodynamics.

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APPENDIX: RECHARGING PROCESS AND ITS EFFICIENCY

We set the eigenstates of H_0 to be arranged in ascending order, i.e., $E_1 < E_2 \cdots < E_N$. Because $[H_0, H_Q] = 0$, the equilibrium state of the battery is

$$\omega_\beta(H_0) = \frac{e^{-\beta H_0}}{Z_0} = \sum_{i=1}^{N} \frac{e^{-\beta E_i^0}}{Z_0} |E_i\rangle \langle E_i| , \tag{A1}$$

which can be ordered in the form $\rho = \sum_i r_i |r_i\rangle \langle r_i|$ with $r_1 > r_2 > \cdots > r_N$ by finding a permutation p of $\{1, \ldots, N\}$ such that $E_{p_1}^0 \leqslant E_{p_2}^0 \leqslant \cdots \leqslant E_{p_N}^0$, i.e.,

$$\omega_\beta(H_0) = \sum_i \frac{e^{-\beta E_{p_i}^0}}{Z_0} |E_{p_i}\rangle \langle E_{p_i}| . \tag{A2}$$

The passive state $\sigma_{\omega_\beta(H_0)} = \sum_i r_i |E_i\rangle \langle E_i|$ is given by

$$\sigma_{\omega_\beta(H_0)} = \sum_i \frac{e^{-\beta E_{p_i}^0}}{Z_0} |E_i\rangle \langle E_i| . \tag{A3}$$

A simple physical explanation for it is that the largest occupancy occupies the state with the lowest-energy level, and the smallest occupancy occupies the state with the highest-energy level. The unitary matrix that characterizes the process of extracting the ergotropy $\sigma_{\omega_\beta(H_0)} = u\omega_\beta(H_0)u^\dagger$ is given by the permutation matrix associated to p, i.e., its elements are $u_{ij} = \langle E_i| u |E_j\rangle = \delta_{p_i,j}$.

We have the following equalities for the ergotropy,

$$\begin{aligned} \mathcal{W}(\omega_\beta(H_0)) &= \mathrm{Tr}\{H_S[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}]\} \\ &= \sum_{i=1}^{N} E_i \left(\frac{e^{-\beta E_i^0}}{Z_0} - \frac{e^{-\beta E_{p_i}^0}}{Z_0} \right) \\ &= \sum_{i=1}^{N} (E_{p_i} - E_i) \frac{e^{-\beta E_{p_i}^0}}{Z_0} . \end{aligned} \tag{A4}$$

In the recharging process $\sigma_{\omega_\beta(H_0)} \to \omega_\beta(H_0)$, the work W_R performed by the agent and the heat flow Q_R from the bath are

$$\begin{aligned} W_R &= \mathrm{Tr}\{(H_S - H_0)[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}]\} \\ &= \sum_{i=1}^{N} (E_i - E_i^0) \left(\frac{e^{-\beta E_i^0}}{Z_0} - \frac{e^{-\beta E_{p_i}^0}}{Z_0} \right), \end{aligned} \tag{A5}$$

$$\begin{aligned} Q_R &= \mathrm{Tr}\{H_0[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}]\} \\ &= \sum_{i=1}^{N} E_i^0 \left(\frac{e^{-\beta E_i^0}}{Z_0} - \frac{e^{-\beta E_{p_i}^0}}{Z_0} \right), \end{aligned} \tag{A6}$$

and they satisfy $\mathcal{W}[\omega_\beta(H_0)] = W_R + Q_R$. The efficiency of the recharging process

$$\eta = \frac{\mathcal{W}[\omega_\beta(H_0)]}{W_R} = 1 - \frac{|Q_R|}{W_R}, \tag{A7}$$

which satisfies $0 \leqslant \eta \leqslant 1$ because $\mathcal{W}[\omega_\beta(H_0)] \geqslant 0$, $Q_R \leqslant 0$, and $W_R \geqslant 0$. These relationships exclude the existence of the second type of perpetual motion machine.

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