Locally inaccessible hidden quantum correlations

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We prove, modulo a conjecture on quantum steering ellipsoids being true, the existence of the phenomenon of locally inaccessible hidden quantum correlations, that is, the existence of two-particle states whose hidden quantum correlations cannot be revealed by local filters implemented exclusively on one side of the experiment but that can still be revealed when both parties cooperate in applying judiciously chosen local filters. The quantum correlations considered here are the violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality for Bell nonlocality and the violation of the F_3 inequality for Einstein-Podolsky-Rosen steering. Specifically, we provide a necessary criterion for guaranteeing the presence of such phenomena for arbitrary two-qubit states. This criterion in turn relies on the conjecture that the maximal violations of CHSH inequality and F_3 inequality are both upper bounded by functions that depend on the magnitude of the quantum steering ellipsoid center. This latter conjecture, although currently lacking an analytical proof, is supported by numerical results. We use this necessary criterion to explicitly show examples of two-qubit states with locally inaccessible hidden quantum correlations and furthermore two-qubit states with locally inaccessible maximal hidden quantum correlations.

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I. INTRODUCTION

The identification and characterization of quantum correlations [1] such as Bell nonlocality [2,3] and Einstein-Podolsky-Rosen (EPR) steering [4,5] are crucial endeavors for the development of quantum technologies such as device-independent and semi-device-independent information protocols [6]. Quantum states that cannot directly produce these types of correlations can still however exhibit hidden versions of them, which can be revealed by implementing local filtering operations [7–9]. This phenomenon of revealing or extracting quantum correlations by means of local filtering operations, first shown to be possible in the works of Popescu [7] and Gisin [8], has been addressed for revealing various quantum properties such as hidden Bell nonlocality [7,8], hidden EPR steering [10,11], hidden usefulness for teleportation [9,12], and maximally extracting entanglement [13] and have been the subject of extensive study [9,12–20]. Local filtering operations have been successfully implemented and verified in various experimental setups [11,21–26].

Revealing hidden quantum correlations, for instance, hidden Bell nonlocality, refers to the situation in which experimentalists Alice and Bob share a bipartite quantum state, which itself cannot be used to violate any Bell inequality but that can still be transformed (by means of local filtering operations) into a state that can now violate a Bell inequality. However, it happens sometimes that local filters on either Alice's or Bob's side alone are enough to reveal these hidden correlations. This observation then leads one to wonder about the existence of states for which local filters on either Alice's or Bob's side alone are never enough for the procedure to work, but that can still be made to work when both parties cooperate in applying local filters. We refer to the latter phenomenon as locally inaccessible hidden quantum correlations (see Fig. 1 for an schematic illustration). The main questions explored in this paper are whether this type of correlations can actually exist and how to detect them if that is the case.

Before delving into the ways one can propose to tackle this problem, let us first address some physically motivated scenarios in which these one-sided variants of the standard locally filtered Bell test can naturally emerge. First, these one-sided variants can be found useful when the parties do not trust each other; then they are each required to be completely certain that the other party did their job in regard to actually applying a local filter. In other words, if Alice and Bob share a state with locally inaccessible hidden correlations, a locally filtered Bell test displaying a violation of a Bell inequality will guarantee that local filters were indeed implemented on both sides of the experiment (since the state does not allow a violation if only one of the parties acted) and so Alice can be reassured that Bob did his job in applying the required filter and vice versa. Second, from a conceptual point of view, we note that while (hidden) Bell nonlocality is usually regarded

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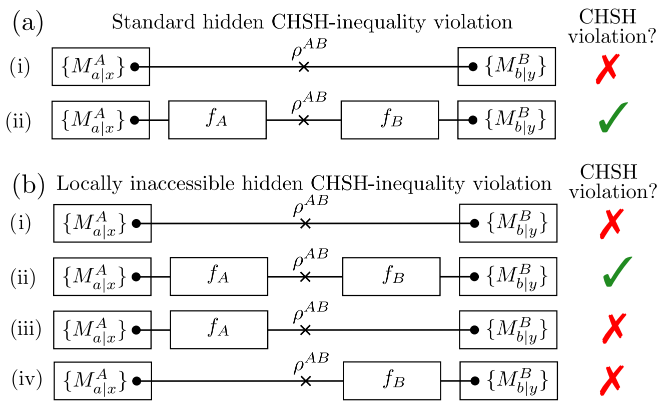

FIG. 1. (a) Standard hidden CHSH-inequality violation scenario. Alice and Bob share an entangled bipartite quantum state ρ^{AB} which (i) does not allow for the violation of the CHSH inequality (for arbitrary sets of measurements $\{M^A_{a|x}\}$ and $\{M^B_{b|y}\}$ in the CHSH scenario) and yet (ii) there exist local filters f_A and f_B so that such a violation is possible. (b) Locally inaccessible hidden CHSH-inequality violation scenario. Two additional properties are satisfied in this scenario: (iii) Alice alone cannot generate any violation by implementing arbitrary local filters and similarly (iv) Bob alone cannot generate any violation via local filters.

as a "symmetric" property, in the sense that a bipartite state, as a whole, is either (hidden) nonlocal or not, the existence of quantum states displaying locally inaccessible hidden quantum correlations suggests that quantum states can still possess an intrinsic asymmetry with respect to their nonlocal behavior, since these states would display a hidden nonlocal behavior when manipulated from Alice's point of view but not from Bob's. Third and finally, we can similarly explore hidden quantum correlations being locally accessible. We can imagine that local filters might actually be infeasible to implement by one of the involved parties, say, Bob, due to experimental limitations, for instance, and therefore it becomes desirable to consider a scenario addressing the hidden quantum correlations that are locally accessible from Alice's point of view.

One key difficulty in approaching the above question is to show that a state cannot display a certain type of quantum correlation after any local filter has been applied on one side [27]. For example, considering Bell nonlocality, this would require methods for finding a local-hidden-variable (LHV) model for a state, after any filtering has been performed on one side. Finding LHV models is a notoriously difficult task, even in the absence of filters [28–30], and the only general techniques known do not seem to generalize readily to the case of arbitrary local filters [27].

In order to gain insight into this problem, we first consider more specialized scenarios. In particular, the most important Bell inequality is arguably the Clauser-Horne-Shimony-Holt (CHSH) inequality [31,32]. This is the most basic of all Bell inequalities, which has been the subject of extensive study over the years and now also plays a key role in numerous applications in the domain of device-independent (DI) quantum information [6]. Our central point here is thus primarily to explore locally inaccessible hidden violations of the CHSH inequality. Explicitly, we focus on quantum states that do not violate the CHSH inequality and explore whether they can violate it after appropriate one-sided local filters. Such a phenomenon is relevant, for example, to any DI protocol built upon violations of the CHSH inequality [6].

We also apply the same reasoning to the case of EPR steering. Showing that a state is unsteerable, i.e., that it has a local-hidden-state (LHS) model, is again a difficult problem. We thus focus on the simplest fixed steering inequality, the F_3 inequality, where Bob measures the three Pauli operators on his qubit system [33,34]. This inequality is again important from the perspective of one-sided device-independent quantum information and it is therefore relevant to ask whether there are states that can have locally inaccessible hidden violations of this inequality, meaning that we concentrate on states that do not violate the F_3 inequality but that can do so after local filtering operations.

We obtain preliminary positive results in this direction by deriving a necessary criterion for witnessing states with locally inaccessible hidden quantum correlations, specifically for the violation of the CHSH inequality and the F_3 inequality. This criterion relies on the validity of a conjecture on quantum steering ellipsoids, first described in the work of Milne *et al.* [35], which is supported by numerical results [35]. We use this criterion to provide examples of two-qubit states displaying the phenomena of B-inaccessible hidden correlations, AB-inaccessible hidden correlations, and AB-inaccessible maximal hidden correlations.

This paper is organized as follows. We start in Sec. II with some preliminaries, notation, and the measures of quantum correlations that we are interested in: the violation of the CHSH inequality for Bell nonlocality and the violation of the F_3 inequality for EPR steering. This is followed by a succinct description of QSEs. In Sec. III we address standard hidden quantum correlations and formalize what we mean by locally accessible and inaccessible hidden quantum correlations. We then derive our main result in the form of a necessary criterion for detecting locally inaccessible hidden quantum correlations for arbitrary two-qubit states. In Sec. IV we use this criterion to explicitly show examples of states with various such properties. In Sec. V we summarize as well as discuss conclusions, open problems, and perspectives for future work.

II. PRELIMINARIES

We focus on the scenario where experimentalists Alice and Bob share an arbitrary two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$, which can be written as $\rho = \frac{1}{4}\sum_{i,j=0}^{3} R_{ij}\sigma_i \otimes \sigma_j$, where $R_{ij} = \mathrm{Tr}[(\sigma_i \otimes \sigma_j)\rho]$, with $\sigma_0 = \mathbb{1}$ and $\{\sigma_i\}$ $(i = 1, 2, 3)$ the Pauli matrices. It is convenient to write this real matrix as

$$R = \begin{pmatrix} 1 & \boldsymbol{b}^T \\ \boldsymbol{a} & T \end{pmatrix}, \tag{1}$$

where $\boldsymbol{a} = [a_i]$, with $a_i = \mathrm{Tr}[(\sigma_i \otimes \mathbb{1})\rho]$, and $\boldsymbol{b} = [b_i]$, with $b_i = \mathrm{Tr}[(\mathbb{1} \otimes \sigma_i)\rho]$, are the Bloch vectors of the reduced states and $T_{ij} = \mathrm{Tr}[\rho(\sigma_i \otimes \sigma_j)]$, with $i, j = 1, 2, 3$, is the correlation matrix. Bob's reduced state is given by $\rho_B = \mathrm{Tr}_A[\rho] = \frac{1}{2}(\mathbb{1} + \boldsymbol{b}\cdot\boldsymbol{\sigma})$, with $\boldsymbol{\sigma} = [\sigma_i]$, and similarly for Alice's. We are interested in the CHSH inequality, which accounts for Bell nonlocality, and the F_3 inequality, which accounts for EPR steering; these two inequalities read

$$\mathcal{B}(\rho, \{A^i, B^j\}) = \frac{1}{2}(\langle A^1B^1\rangle + \langle A^1B^2\rangle + \langle A^2B^1\rangle - \langle A^2B^2\rangle) \leqslant 1,$$

$$F_3(\rho, \{A^i\}) = \frac{1}{\sqrt{3}}(\langle A^1\sigma_1\rangle + \langle A^2\sigma_2\rangle + \langle A^3\sigma_3\rangle) \leqslant 1,$$

with the expectation values $\langle A^iB^j\rangle = \mathrm{Tr}[(A^i \otimes B^j)\rho]$ and observables $A^i = \boldsymbol{\sigma} \cdot \boldsymbol{\alpha}^i$ and $B^j = \boldsymbol{\sigma} \cdot \boldsymbol{\beta}^j$, with $\boldsymbol{\alpha}^i$ and $\boldsymbol{\beta}^j$ real unit vectors. We want to maximize these functions over all possible measurements so as to maximally violate these inequalities. The Horodecki criterion [32] and the Costa-Angelo criterion [34] solve these optimization problems as

$$\mathcal{B}(\rho) = \max_{\{A^i,B^j\}} \mathcal{B}(\rho, \{A^i, B^j\}) = \sqrt{s_1^2 + s_2^2}, \tag{2}$$

$$F_3(\rho) = \max_{\{A^i\}} F_3(\rho, \{A^i\}) = \sqrt{s_1^2 + s_2^2 + s_3^2}, \tag{3}$$

where $\{s_i\}$ are the singular values in decreasing order of the correlation matrix T (1). Let us now consider that Alice performs a general positive operator-valued measure (POVM) measurement with O_A outcomes as $E = \{E_e\}$, $e = 1, \ldots, O_A$. The POVM effects can be written as $E_e = \frac{1}{2}(\mathbb{1} + \boldsymbol{\gamma}_e \cdot \boldsymbol{\sigma})$, with $|\boldsymbol{\gamma}_e| \leqslant 1$ and $|\boldsymbol{\gamma}_e| = 1$ for projective measurements. After the implementation of a particular POVM effect, Bob's reduced state is "steered" to a state of the form

$$\rho_B^e = \frac{1}{2}[\mathbb{1} + \boldsymbol{b}(\boldsymbol{\gamma}_e)\cdot\boldsymbol{\sigma}], \quad \boldsymbol{b}(\boldsymbol{\gamma}_e) = \frac{1}{2p_e}(\boldsymbol{b} + T^T\boldsymbol{\gamma}_e),$$

with probabilities $p_e = \mathrm{Tr}[(E_e \otimes \mathbb{1})\rho] = \frac{1}{2}(1 + \boldsymbol{a}\cdot\boldsymbol{\gamma}_e)$. The Bloch vectors $\boldsymbol{b}(\boldsymbol{\gamma}_e)$ turn out to lie on the surface of an ellipsoid $\mathcal{E}_B$, Bob's quantum steering ellipsoid (QSE) [35–38], which is characterized by an ellipsoid matrix Q_B and a center $\boldsymbol{c}_B$, which depend on the two-qubit state and are given by [36]

$$\boldsymbol{c}_B = \gamma_a^2(\boldsymbol{b} - T^T\boldsymbol{a}), \quad \gamma_a = \frac{1}{\sqrt{1-a^2}}, \quad a = |\boldsymbol{a}|, \tag{4}$$

$$Q_B = \gamma_a^2(T^T - \boldsymbol{b}\boldsymbol{a}^T)(\mathbb{1} + \gamma_a^2\boldsymbol{a}\boldsymbol{a}^T)(T - \boldsymbol{a}\boldsymbol{b}^T), \tag{5}$$

with $\boldsymbol{a}$, $\boldsymbol{b}$, and T as in (1). The square root of the eigenvalues of Q_B corresponds to the lengths of the semiaxes of the ellipsoid, while the eigenvectors correspond to the ellipsoid's orientation [36]. Therefore, for a given two-qubit state ρ, Bob's QSE is specified by the pair $\mathcal{E}_B = \{Q_B, \boldsymbol{c}_B\}$. Similarly, we can calculate Alice's QSE $\mathcal{E}_A = \{Q_A, \boldsymbol{c}_A\}$ by switching $\boldsymbol{a} \leftrightarrow \boldsymbol{b}$ and replacing $T \to T^T$ in (4) and (5) [36]. We now address, in the spirit of Ref. [35], a conjecture that relates the QSE's centers and the violation of both the CHSH inequality and the F_3 inequality. More specifically, this conjecture says that the maximal violation of the CHSH and F_3 inequalities can be upper bounded by a function of the QSE center magnitude alone, without reference to the QSE matrix. The following is the QSE conjecture.

Conjecture. For any two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$, its maximal violations of the CHSH inequality (2) and the F_3 inequality (3) are upper bounded by functions that depend on the magnitude of the QSE's center as $\mathcal{B}(\rho) \leqslant f_{\mathrm{CHSH}}(c_B)$ and $F_3(\rho) \leqslant f_{F_3}(c_B)$, respectively, with $c_B = |\boldsymbol{c}_B|$.

The part of the conjecture concerning the CHSH inequality was first introduced in [35], where Milne *et al.* arrived at this phenomenon via numerical simulations which can be appreciated in Fig. 2 in [35], reproduced in Fig. 2(a) herein. Milne *et al.* [35] additionally conjectured a potential function $f_{\mathrm{CHSH}}(c_B) = \max\{\sqrt{2(1-c_B)}, 1\}$. This is an interesting upper bound which tells us that quantum states with QSE centers $c_B > 0.5$ [$f_{\mathrm{CHSH}}(c_B) = 1$] cannot violate the CHSH inequality. Since QSEs are contained within the Bloch ball, this leads to the appealing geometric observation that states with QSE centers close ($c_B > 0.5$) to the surface cannot violate the CHSH inequality. In this work we report, also via numerical simulations, that the violation of the F_3 inequality displays behavior similar to that of the CHSH inequality, as it can be appreciated in Fig. 2(b). In Fig. 2 we show that the conjecture holds true for 10^8 randomly generated arbitrary two-qubit states. In particular, we highlight that there seem to exist values $c_{\mathrm{CHSH}} = 0.5$ and $c_{F_3} = 0.66$ such that states with QSE center magnitudes greater than these values can no longer lead to a violation of the CHSH inequality and the F_3 inequality, respectively. We then address the regions $[c_{\mathrm{CHSH}}, 1]$ and $[c_{F_3}, 1]$ as impossibility regions, which are extracted from the numerical results in Fig. 2. Exploring a potential upper bound $f_{F_3}(c_B)$ is an interesting task which we leave however for future research, as here we only require the

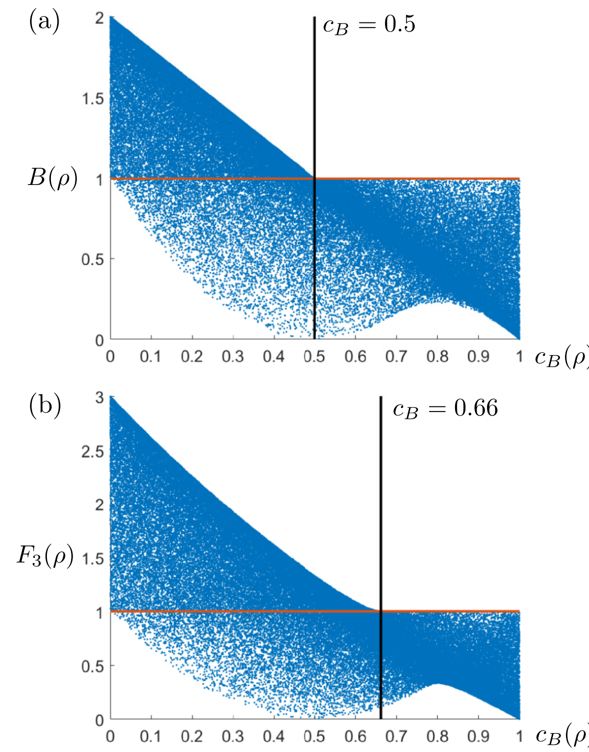

FIG. 2. Conjecture on the relationship between two measures of quantum correlations versus the QSE's center: (a) CHSH-inequality violation (2) and (b) F_3-inequality violation (3) for 10^8 randomly generated arbitrary two-qubit states against the magnitude of their respective QSE's center c_B (4). Similar plots can be obtained when analyzing Alice's QSE center c_A (not shown). The horizontal orange lines at 1 depict the classical bound for these inequalities. Vertical black lines depict the values $c_{\mathrm{CHSH}} = 0.5$ and $c_{F_3} = 0.66$, beyond which states appear to no longer violate the respective inequalities.

existence of the impossibility regions. It is also worth mentioning that Milne *et al.* [35] proposed an additional conjecture for the measure of fully entangled fraction which, although interesting in its own right, is omitted in this work since it does not display an impossibility region like CHSH or F_3. In the next section we exploit the existence of these impossibility regions for detecting locally inaccessible hidden quantum correlations. We now move on to introduce this phenomenon more formally.

III. HIDDEN QUANTUM CORRELATIONS

In this section we start by addressing the standard procedure for revealing hidden quantum correlations. We move on to more restrictive versions, where only one-sided local filters are allowed, to then formalize what we mean by locally accessible and inaccessible hidden quantum correlations. We use the previous conjecture on QSEs to derive a sufficient criterion for detecting locally inaccessible hidden quantum correlations, specifically, locally inaccessible hidden CHSH-inequality violation and F_3-inequality violation.

A. Standard hidden quantum correlations

As described in the preceding section, consider experimentalists Alice and Bob sharing an arbitrary two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$ but now, before implementing a standard Bell test, they implement a local filtering procedure as follows. Alice and Bob can each perform a local binary POVM measurement given by $E_W = \{E_W^0, E_W^1\}$, $W \in \{A, B\}$, where $E_W^1 = \mathbb{1} - E_W^0$ and $E_W^0 = f_W^\dagger f_W$, with f_W satisfying the property $f_W^\dagger f_W \leqslant \mathbb{1}$. This last property guarantees that the procedure can be performed as a valid measurement $E_W^1 \geqslant 0$. After the implementation of this measurement, the postmeasured state is the unnormalized state $(f_A \otimes f_B)\rho(f_A \otimes f_B)^\dagger$ with probability $\mathrm{Tr}[(f_A^\dagger f_A \otimes f_B^\dagger f_B)\rho]$. They then keep the postmeasured state only when they obtain this desired target state, discarding the system otherwise, and repeating until success. This procedure is also known as stochastic local operations with classical communication (SLOCC) and the operators f_A and f_B are referred to as local filters or SLOCC. This local filtering procedure then can effectively be seen as transforming the initial state to a filtered state of the form

$$\rho' = \frac{(f_A \otimes f_B)\rho(f_A \otimes f_B)^\dagger}{\mathrm{Tr}[(f_A^\dagger f_A \otimes f_B^\dagger f_B)\rho]}, \tag{6}$$

where $f_W \in \mathrm{GL}(2, \mathbb{C})$, the group of invertible 2×2 complex matrices. This latter condition guarantees that the transformation does not destroy quantum correlations [9], CHSH nonlocality and F_3 steering in particular [16]. Among all possible local filtering operations, there exists a particular one, which we address here as the Kent-Linden-Massar (KLM) SLOCC transformation [13], the KLM SLOCC from now on, which has the important property of transforming the state ρ into its Bell-diagonal unique normal form [13], which we denote by $\rho_{\mathrm{UNF}}^{\mathrm{BD}}$ [9,16]. It has been proven that the KLM SLOCC is the optimal local filtering transformation that simultaneously maximizes the quantum correlations of concurrence [9], usefulness for teleportation [16], and violation of the CHSH inequality for Bell nonlocality [16]. The KLM SLOCC then effectively acts as $\rho \to \rho_{\mathrm{UNF}}^{\mathrm{BD}}$, which in terms of the R matrix (1) reads $R \to R_{\mathrm{UNF}}^{\mathrm{BD}} = \mathrm{diag}(1, -\sqrt{\nu_1/\nu_0}, -\sqrt{\nu_2/\nu_0}, -\sqrt{\nu_3/\nu_0})$, with $\{\nu_{i=0,1,2,3}\}$ the eigenvalues of the operator $\eta R \eta R^T$ in decreasing order and $\eta = \mathrm{diag}(1, -1, -1, -1)$ [39]. Hence, the quantum correlations of the Bell-diagonal unique normal form, which defines the hidden quantum correlations of the initial state ρ, are given by

$$H\mathcal{B}(\rho) := \mathcal{B}(\rho_{\mathrm{UNF}}^{\mathrm{BD}}) = \sqrt{\frac{\nu_1 + \nu_2}{\nu_0}}, \tag{7}$$

$$HF_3(\rho) := F_3(\rho_{\mathrm{UNF}}^{\mathrm{BD}}) = \sqrt{\frac{\nu_1 + \nu_2 + \nu_3}{\nu_0}}. \tag{8}$$

As mentioned before, the KLM SLOCC is the optimal SLOCC for maximizing the CHSH inequality and therefore $H\mathcal{B}^*(\rho) := \max_{\{f_A, f_B\}} \mathcal{B}(\rho') = H\mathcal{B}(\rho)$, with ρ' defined as in (6). It is not known whether this is also the case for the F_3 inequality, but we nevertheless have the inequality $HF_3^*(\rho) := \max_{\{f_A, f_B\}} F_3(\rho') \geqslant HF_3(\rho)$.

B. One-sided hidden quantum correlations

We are now interested in restricting the standard locally filtered Bell-test scenario to the case when only one of the parties is allowed or capable of implementing local filters. We define one-sided filtered states ρ'_{FW}, $W \in \{A, B\}$, as

$$\rho'_{FA} = \frac{(f_A \otimes \mathbb{1})\rho(f_A \otimes \mathbb{1})^\dagger}{\mathrm{Tr}[(f_A^\dagger f_A \otimes \mathbb{1})\rho]}, \quad \rho'_{FB} = \frac{(\mathbb{1} \otimes f_B)\rho(\mathbb{1} \otimes f_B)^\dagger}{\mathrm{Tr}[(\mathbb{1} \otimes f_B^\dagger f_B)\rho]}.$$

These two locally filtered states can alternatively be seen as imposing the condition $f_B = \mathbb{1}$ or $f_A = \mathbb{1}$ in the standard definition (6), respectively. We can now define two measures for locally accessible hidden CHSH-inequality violation and two measures for locally accessible F_3-inequality violation as follows:

$$H\mathcal{B}_W(\rho) := \max_{\{f_W\}} \mathcal{B}(\rho'_{FW}), \quad W \in \{A, B\}, \tag{9}$$

$$HF_{3W}(\rho) := \max_{\{f_W\}} F_3(\rho'_{FW}), \quad W \in \{A, B\}. \tag{10}$$

These measures define the amount of hidden correlations that each party (either Alice or Bob) can extract by working unilaterally while the other party does nothing. It follows from these definitions that we have the inequalities $\mathcal{B}(\rho) \leqslant H\mathcal{B}_W(\rho) \leqslant H\mathcal{B}(\rho)$ and similarly for the F_3 inequality. It would be desirable to have analytical expressions for these asymmetric measures, as it is the case for their symmetric counterparts. We are now interested in the scenarios where it is possible to reveal quantum correlations when they were not initially present. In Table I we define five fine-grained hidden quantum correlation phenomena. We now describe the cases in Table I. The first case addresses standard hidden CHSH-inequality violation, which was first introduced by Popescu in [7] and specialized to two-qubit states by Gisin in [8]. The second case deals with maximal hidden CHSH-inequality violation. An example of this phenomenon was shown to be present in a qutrit-qubit state, referred to as an erasure state [40,41]. A case for two-qubit states was considered in [42], as states coming from the dynamics of open quantum

TABLE I. Definitions and results on fine-grained hidden quantum correlations regarding the violation of the CHSH inequality for Bell nonlocality. We can similarly define these cases for the F_3-inequality violation for EPR steering.

Case	Definition: ρ displays ...	$\mathcal{B}(\rho)$ (2)	$H\mathcal{B}_A(\rho)$ (9)	$H\mathcal{B}_B(\rho)$ (9)	$H\mathcal{B}(\rho)$ (7)	Reference
1	hidden CHSH	$\leqslant 1$	–	–	>1	[7,8]
2	maximal hidden CHSH	$\leqslant 1$	–	–	2	[40–42]
3	B-inaccessible hidden CHSH	$\leqslant 1$	>1	$\leqslant 1$	>1	this work (Sec. IV A)
4	AB-inaccessible hidden CHSH	$\leqslant 1$	$\leqslant 1$	$\leqslant 1$	>1	this work (Sec. IV B)
5	AB-inaccessible maximal hidden CHSH	$\leqslant 1$	$\leqslant 1$	$\leqslant 1$	2	this work (Sec. IV C)

systems. The third case specifies locally B-inaccessible hidden CHSH-inequality violation, which encapsulates the idea of the quantum state not allowing Bob to enhance its correlations, no matter what local filter he is using, so that the cooperation of Alice is indispensable. Here we also naturally define locally A-accessible hidden CHSH-inequality violation, meaning that Alice alone can extract some amount of correlations. The fourth case considers the stronger notion of a state whose hidden quantum correlations are locally inaccessible from both sides. This encapsulates the idea of the quantum state not allowing Alice and Bob to act individually, but forcing them to cooperate, and hence the wording locally AB-inaccessible hidden CHSH-inequality violation. We emphasize here that the difference between standard hidden quantum correlations [for which we have $H\mathcal{B}(\rho) > 1$] and AB-inaccessible hidden quantum correlations [for which we also have $H\mathcal{B}(\rho) > 1$] is that in the latter we can additionally guarantee that $H\mathcal{B}_A(\rho) \leqslant 1$ and $H\mathcal{B}_B(\rho) \leqslant 1$. Finally, the fifth case considers an extreme case in which the correlations are locally inaccessible and yet they can still be revealed to be the maximum amount allowed by quantum theory.

These one-sided versions here introduced can be found useful when considering semi-device-independent protocols, as it has been the case for EPR steering [4,5]. Unlike the standard hidden correlations, for which we have the closed formulas in (7) and (8), there are currently no closed formulas for the one-sided versions defined in (9) and (10). In this work we start exploring these alternative one-sided measures. The first step we take is to address whether there actually exist states with the properties depicted in these three definitions. This is because, *a priori*, it might well be the case that the hidden correlations of all states are actually already always locally accessible (by both parties) and therefore the previous definitions are unnecessary. The main challenge in tackling these questions is that currently there are no efficient tools for guaranteeing that the hidden correlations of a state are locally inaccessible or explicitly that $H\mathcal{B}_A(\rho) \leqslant 1$ or that $HF_{3A}(\rho) \leqslant 1$. In this work we take a step in this direction by providing a sufficient criterion for guaranteeing that a state has locally W-inaccessible hidden quantum correlations with $W \in \{A, B\}$ so that when considered together it also allows us to explore locally AB-inaccessible hidden quantum correlations.

C. Sufficient criterion for locally W-inaccessible hidden quantum correlations $W \in \{A, B\}$

We now provide a sufficient criterion for guaranteeing that a state possesses locally W-inaccessible hidden quantum correlations. We first need the following lemma about quantum steering ellipsoids [36].

Lemma. Consider a two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$ with associated QSEs given by $\mathcal{E}_W = \{\boldsymbol{c}_W, Q_W\}$, $W \in \{A, B\}$. Consider also that the state is locally filtered to a state ρ' with QSEs given by $\mathcal{E}'_W = \{\boldsymbol{c}'_W, Q'_W\}$, $W \in \{A, B\}$. If we consider local filters of the form $\mathbb{1} \otimes f_B$, then $\mathcal{E}'_A = \mathcal{E}_A$. Conversely, if we consider local filters of the form $f_A \otimes \mathbb{1}$, then $\mathcal{E}'_B = \mathcal{E}_B$.

The proof of this lemma can be found in [36]. We now use this lemma to establish our main result.

Result 1. Consider a two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$. If Alice's (Bob's) QSE center magnitude satisfies $c_A > c_{\text{CHSH}}$ ($c_B > c_{\text{CHSH}}$) with $c_{\text{CHSH}} = 0.5$, then, modulo the QSE conjecture being true, it follows that $H\mathcal{B}_B(\rho) \leqslant 1$ [$H\mathcal{B}_A(\rho) \leqslant 1$]. This means that Bob (Alice) cannot locally achieve any hidden CHSH-inequality violation.

Proof. Consider a two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$ with Alice's QSE center satisfying $c_A > c_{\text{CHSH}} = 0.5$. Then, because of the QSE conjecture, the state cannot violate the CHSH inequality. Moreover, if we consider local filters of the form $\mathbb{1} \otimes f_B$ and the associated filtered state ρ'_{FB}, the Lemma guarantees that Alice's QSE $\mathcal{E}_A$ remains unchanged as $\mathcal{E}'_A = \mathcal{E}_A$. In particular, the magnitude of the QSE's center remains unchanged and therefore, again by the QSE conjecture, the filtered state ρ'_{FB} cannot violate the CHSH inequality, meaning that $\mathcal{B}(\rho'_{\text{FB}}) \leqslant 1$. Since this holds for any f_B, it therefore follows that $H\mathcal{B}_B(\rho) = \max_{f_B} \mathcal{B}(\rho'_{\text{FB}}) \leqslant 1$, thus completing the proof. The same argument holds with Bob's QSE center satisfying $c_B > c_{\text{CHSH}} = 0.5$, but now for $\mathcal{B}_A(\rho)$. ■

Taking these two criteria together allows us to look for locally AB-inaccessible hidden CHSH-inequality violation by calculating the QSE's center magnitudes c_A and c_B. The F_3 inequality also displays an impossibility region, and so we have an analogous result.

Result 2. Consider a two-qubit state $\rho \in D(\mathbb{C}^2 \otimes \mathbb{C}^2)$. If the QSE center's magnitude satisfies $c_A > c_{\text{F}_3}$ ($c_B > c_{\text{F}_3}$) with $c_{\text{F}_3} = 0.66$, then, modulo the QSE conjecture being true, we have that $HF_{3B}(\rho) \leqslant 1$ [$HF_{3A}(\rho) \leqslant 1$]. This means that Bob (Alice) cannot locally access any hidden F_3-inequality violation.

With these sufficient criteria in place, it becomes a straightforward exercise to look for both locally inaccessible hidden CHSH-inequality and F_3-inequality violation, since it is essentially calculating the QSE's centers, which are explicitly given as per (4), and comparing these values to $c_{\text{CHSH}} = 0.5$ and $c_{F_3} = 0.66$, respectively. We now proceed to use these sufficient criteria to explore locally W-inaccessible ($W \in \{A, B\}$) and locally AB-inaccessible hidden quantum correlations of two-qubit states, showing that these sufficient criteria are enough detect the existence of states displaying such properties.

IV. EXAMPLES: LOCALLY INACCESSIBLE HIDDEN CORRELATIONS

We now consider specific two-qubit states and calculate the properties of entanglement by means of the positive partial transpose (PPT) criterion [43], CHSH-inequality violation (2), F_3-inequality violation (3), hidden CHSH-inequality violation (7), hidden F_3-inequality violation (8), unsteerability [28,29], and regions for which the QSEs' centers (4) satisfy the conditions $c_A, c_B > c_{\rm CHSH} = 0.5$ and $c_A, c_B > c_{\rm F_3} = 0.66$ so as to guarantee the local W inaccessibility $W \in \{A, B\}$ of either CHSH nonlocality or F_3 steering, respectively, as per Results 1 and 2. We address three examples: first, an asymmetric case which displays A-accessible, B-inaccessible hidden quantum correlations; second, a symmetric case which displays locally AB-inaccessible hidden quantum correlations; and third, an extreme case of states that display locally AB-inaccessible maximal hidden quantum correlations.

A. Locally *A*-accessible, *B*-inaccessible hidden quantum correlations

We show evidence of the existence of this phenomenon with a particular family of states, which is usually called partially entangled states with (asymmetric) colored noise,

$$\rho_{\rm M}(\theta, p) = p\phi^+(\theta) + (1-p)\rho_A(\theta) \otimes \mathbb{1}, \tag{11}$$

where $\phi^+(\theta) = |\phi^+(\theta)\rangle\langle\phi^+(\theta)|$, $\rho_A(\theta) = {\rm Tr}_B[\phi^+(\theta)]$ $(0 \leqslant \theta \leqslant \pi/4$ and $0 \leqslant p \leqslant 1)$, and $|\phi^+(\theta)\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$. In Fig. 3(a) we address CHSH-inequality-related correlations and in particular distinguish regions for $c_A > c_{\rm CHSH} = 0.5$, that is, A-accessible, B-inaccessible hidden CHSH-inequality violation. In Fig. 3(b) we address F_3-inequality-related correlations with regions for locally A-accessible, B-inaccessible hidden F_3-inequality violation ($c_A > c_{F_3} = 0.66$). In both Figs. 3(a) and 3(b) the shaded region refers to states displaying A-accessible, B-inaccessible hidden CHSH-inequality and F_3-inequality violation, respectively. The KLM SLOCC for these states can explicitly be written as

$$f_A(\theta, p) = \sin\theta \begin{pmatrix} \frac{1}{\cos\theta} & 0 \\ 0 & \frac{1}{\sin\theta} \end{pmatrix}, \quad f_B(\theta, p) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We have that these states are locally A accessible as the optimal local filters do not require that Bob acts on his part, in agreement with Fig. 3. In this example, it is worth pointing out that the states below the black curve (in both plots) admit a LHS model and so, beyond hidden CHSH-inequality violation, we more generally have instances of hidden Bell nonlocality.

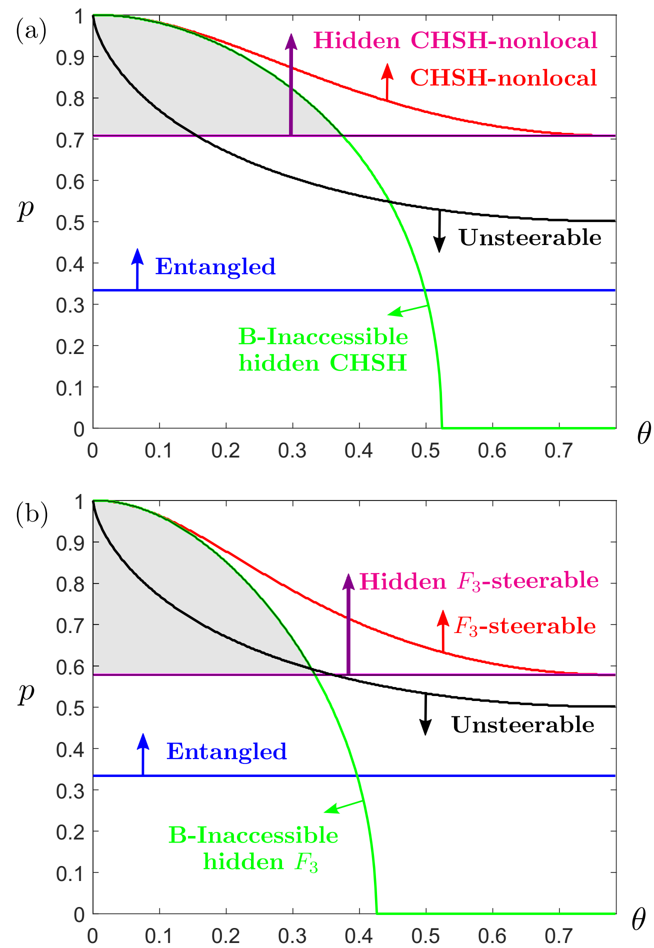

FIG. 3. Quantum correlation measures of partially entangled states with colored noise (11). In both plots the entanglement in blue is calculated by means of the PPT criterion and the unsteerability in black by means of the sufficient criterion from [44]. (a) CHSH-inequality-related correlation measures: in red, the CHSH-inequality violation (2); in magenta, the hidden CHSH-inequality violation (7); and in green, the QSE center condition $c_A > c_{\rm CHSH} = 0.5$. The shaded region shows states with locally B-inaccessible (yet locally A-accessible) hidden CHSH-inequality violation as per Result 1 (see case 3 in Table I). (b) F_3-inequality-related correlation measures: in red, the F_3-inequality violation (3); in magenta, the hidden F_3-inequality violation (8); and in green, the QSE center condition $c_A > c_{F_3} = 0.66$. The shaded region depicts states with locally B-inaccessible F_3-inequality violation as per Result 2.

B. Locally *AB*-inaccessible hidden quantum correlations

We now modify the previous states to be partially entangled states with symmetric colored noise as

$$\rho_{\rm MM}(\theta, p) = p\phi^+(\theta) + (1-p)\rho_A(\theta) \otimes \rho_B(\theta), \tag{12}$$

with $\rho_A(\theta) = {\rm Tr}_B[\phi^+(\theta)]$, $\rho_B(\theta) = {\rm Tr}_A[\phi^+(\theta)]$, and $0 \leqslant p \leqslant 1$. In Fig. 4(a) we address CHSH-inequality-related correlations, while Fig. 4(b) deals with the F_3-inequality-related correlations. Unlike the previous case, we now have regions for locally AB-inaccessible CHSH-inequality and F_3-inequality violation, as opposed to only B-inaccessible hidden quantum correlations. Furthermore, in the shaded regions we have

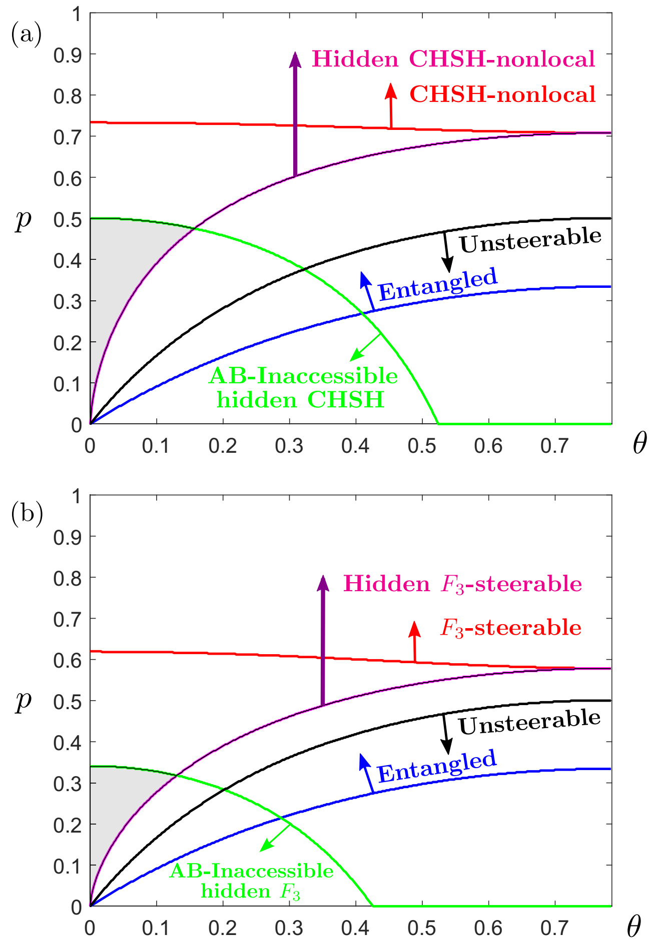

FIG. 4. Quantum correlation measures of partially entangled states with symmetric colored noise (12). In both plots the entanglement, unsteerability, CHSH inequality, F_3 inequality, and hidden inequalities are labeled as in Fig. 3. (a) The green curve indicates the QSE center condition $c_A, c_B > c_{\mathrm{CHSH}} = 0.5$. In this case, the shaded region corresponds to states with locally AB-inaccessible hidden CHSH-inequality violation as per Result 1. This means that neither Alice nor Bob can extract unilaterally hidden correlations, but they are required to cooperate (case 4 in Table I). (b) The green curve indicates the QSE center condition $c_A, c_B > c_{\mathrm{CHSH}} = 0.66$ such that the shaded region corresponds to states with locally AB-inaccessible hidden F_3-inequality violation as per Result 2.

B-inaccessible hidden CHSH-inequality and F_3-inequality violation, respectively.

C. Locally *AB*-inaccessible maximal hidden quantum correlations

We now address the so-called quasidistillable states [16], which can be parametrized by $0 \leqslant p \leqslant 1$ as

$$\rho_{\mathrm{QD}}(p) = p|\Psi^-\rangle\langle\Psi^-| + (1-p)|00\rangle\langle 00|, \qquad (13)$$

with $|\Psi^-\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$. These states belong to the SLOCC orbit of the singlet, so in this sense they can be distilled [16]. This procedure however has a success probability that decreases with $p \to 0$, hence the name quasidistillable. In Fig. 5(a) we address CHSH-inequality-related correlations; in particular, in the shaded region ($0 < p < 0.66$) they evidence locally AB-inaccessible maximal hidden CHSH-inequality violation. In Fig. 5(b) we address F_3-inequality-related correlations with the shaded region ($0 < p < 0.50$) evidencing locally AB-inaccessible maximal hidden F_3-inequality violation. We emphasize that the states in the shaded region go from having weak entanglement, CHSH inequality, and F_3-inequality violation to being the singlet state and therefore having the maximum amount of these correlations that is allowed by quantum theory. Overall, these simple families of states show that this phenomenon is not hard to find, and so we believe it is in fact a ubiquitous aspect of the correlations allowed by quantum theory.

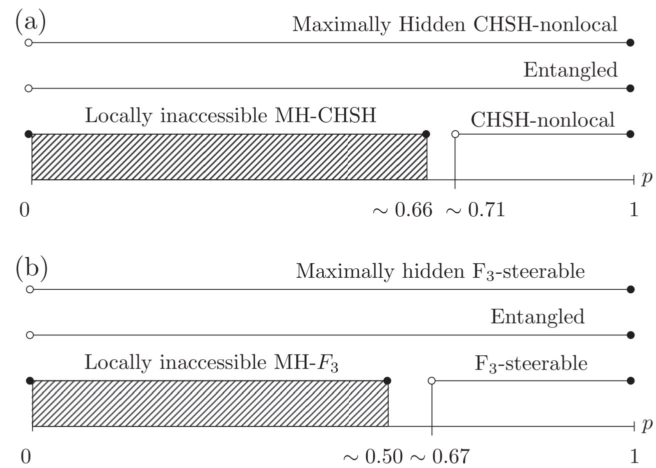

FIG. 5. Quantum correlations of quasidistillable states (13). In both plots the entanglement is calculated by means of the PPT criterion. (a) CHSH-inequality-related measures: CHSH-inequality violation (2), hidden CHSH-inequality violation (7), and locally AB-inaccessible maximal hidden CHSH-inequality violation (shaded region) as per Result 1. (b) F_3-inequality-related measures: F_3-inequality violation (3), hidden F_3-inequality violation (8), and locally AB-inaccessible maximal hidden F_3-inequality violation (shaded region) as per Result 2.

V. CONCLUSION

In this work we proved, modulo a conjecture on quantum steering ellipsoids being true, the existence of the phenomenon of locally inaccessible hidden quantum correlations, in particular, locally inaccessible hidden CHSH-inequality violation for Bell nonlocality and F_3-inequality violation for EPR steering. The case of simultaneous A-inaccessible and B-inaccessible hidden quantum correlations can alternatively be regarded as a type of "super" hidden nonlocality, in the sense that it is a type of nonlocality that is actually more hidden than its standard counterpart, since it explicitly requires the active intervention of both parties, as opposed to potentially only one of them. Moreover, we reported on a stronger version of this phenomenon in the form of locally inaccessible maximal hidden quantum correlations, meaning that the local filters reveal the maximal amount of correlations allowed by quantum theory. The relatively simple families of states that display these phenomena we provided here were not difficult to find, and so this leads us to believe that this is in actuality a generic feature of the type of correlations allowed by quantum theory.

We believe that the results found in this work introduce several questions for future research. First, although the conjectures regarding the QSEs in question are supported by numerical results, they lack analytical proof. It would therefore be desirable to have analytical proofs for these conjectures in order to completely guarantee the existence of this phenomenon. Second, it would also be interesting to derive closed formulas for these locally accessible hidden quantum correlations measures, as it has been done for their standard hidden counterparts. Third, the idea of quantum correlations being revealed by filters on only one side of the experiment can naturally be extended to other setups such as higher dimensions and multipartite scenarios as well as to other measures of correlations such as entanglement, quantum obesity, and quantum discord. We remark, however, that the cases for entanglement and quantum obesity, for instance, do not display a hidden phenomenon but a rather an extractable counterpart [13], in the sense that local filters cannot take separable states into entangled states, but they can nonetheless take entangled states into states with a larger amount of entanglement. It would nonetheless be desirable to have closed formulas for these cases as well. Fourth, in addition to the hidden CHSH-inequality (F_3-inequality) violation, one can define stronger versions of these phenomena in the form of hidden Bell nonlocality (EPR steering) by guaranteeing that the prefiltered state allows LHV and LHS models. This can be explored, for instance, for two-qubit states, by means of the sufficient unsteerability criterion derived in [44] and, for general states, by means of the numerical codes from [28–30]. Fifth and finally, from a practical point of view, the results found in this work could also find application in semi-device-independent information-processing protocols, as it has already proven to be the case for EPR steering.

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