Alternative Einstein-Podolsky-Rosen argument based on premises not falsified by Bell's theorem: weak macroscopic realism and weak local realism

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The Einstein-Podolsky-Rosen (EPR) paradox gives an argument for the incompleteness of quantum mechanics based on a plausible criterion for reality, as well as the assumption of locality. A general view is that the argument is compromised, because EPR's premises are falsified by Greenberger-Horne-Zeilinger (GHZ) and Bell experiments. In this paper, we present an EPR argument based on premises not falsified by these experiments. First, we consider macroscopic EPR-Bohm, Bell, and GHZ experiments using spins $\hat{S}_\theta$ defined by two macroscopically distinct states. The analyzers that realize the unitary operations U_θ determining the measurement settings θ are nonlinear devices creating macroscopic superposition states. We note two definitions of macroscopic realism (MR). For a system with two macroscopically distinct states available to it, MR posits a predetermined outcome for the measurement $\hat{S}_\theta$ distinguishing between the states. Deterministic macroscopic realism assumes MR for the system defined *prior* to the interaction U_θ being carried out but is falsifiable by the macroscopic Bell and GHZ proposals. Motivated by arguments to uphold MR (without resorting to decoherence), as well as no-signaling and the properties of a meter, we define a set of premises referred to as weak macroscopic realism (wMR). *Weak macroscopic realism* (wMR) posits MR for the system *after* U_θ, at the time t_f—when the system is prepared with respect to the measurement basis, ready for a "pointer" measurement and readout. For this system, wMR posits that the outcome of $\hat{S}_\theta$ is determined and not changed by interactions that might subsequently occur at a remote system B. The premise wMR also posits a weaker version of EPR's criterion for reality. Importantly, we show that the GHZ and Bell predictions are consistent with wMR. Yet, an EPR paradox arises for a macroscopic EPR-Bohm state, based on wMR. As considered by Schrödinger, it is possible to measure two complementary spins of system A simultaneously, "one by direct, the other by indirect measurement": If we assume wMR, then once both settings are fixed, the outcomes of the two spins are both determined. We revisit the original EPR paradox and find a similar result: An EPR argument can be based on a set of premises, *weak local realism*, which we show are not falsifiable by GHZ or Bell experiments.

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I. INTRODUCTION

In their argument of 1935, Einstein, Podolsky, and Rosen (EPR) introduced premises which if valid suggested quantum mechanics to be an incomplete description of physical reality [1]. In what is often called the EPR paradox, EPR combined two assertions: the assumption of locality and a criterion for reality. The paradox considered two separated particles with correlated positions x and anticorrelated momenta p. From their assertions, EPR argued that the position and momentum of each particle are simultaneously precisely determined prior to measurement, thereby creating an inconsistency with any quantum-state description for the particles.

EPR's premises indicated the existence of local hidden variables to describe the predetermined values [2–4], but in a spin version of the paradox proposed by Bohm [5,6], Bell proved that all local hidden variables theories could be falsified by quantum predictions [2,3,7–10]. Later, Greenberger, Horne, and Zeilinger (GHZ) gave a direct falsification of EPR's premises for three spin-1/2 systems in an "all or nothing" situation [4,11–13]. Experiments verify the quantum mechanics [3,9,14–16]. Consequently, the EPR paradox is most often regarded as an illustration of the incompatibility between the EPR premises and quantum mechanics, rather than as a valid argument for the incompleteness of quantum mechanics [4,17].

In this paper, we present a different perspective on the EPR paradox. We first show that the EPR-Bohm, Bell, and GHZ setups can be mapped onto macroscopic versions, where the relevant states of the system are macroscopically distinct. This allows us to combine the EPR premises with the premise of macroscopic realism (MR) and to show that a very strict form of local realism (which we refer to as *deterministic macroscopic realism*) is falsifiable. Guided by considerations to uphold MR for systems in a macroscopic superposition state (prior to any decoherence) [18], as well as no-signaling and the properties of a meter, we then propose weaker less-restrictive premises which we explain are *not* falsified by the Bell or GHZ setups, but which nonetheless allow an EPR paradox. Two definitions are considered that we classify as *weak macroscopic (local) realism* (wMR) and *weak local realism* (wLR), applying respectively to the macroscopic and standard versions of the EPR, Bell, and GHZ tests. The premises include less restrictive assertions about realism (or MR), along with weaker versions of the EPR assertions. Hence, we show that the situation considered by

Schrödinger, where one measures two noncommuting observables simultaneously, "*one by a direct, the other by indirect measurement*" [19,20], becomes relevant. The wMR and wLR premises allow the conclusion for Bell states that both of two noncommuting operators are simultaneously specified at a time t_f—and hence the conclusions of an EPR paradox. In summary, a modified EPR argument that quantum mechanics is incomplete can be given, based on alternative and (arguably) nonfalsifiable premises.

The alternative premises (wMR and wLR) are motivated by the nature of the measurement process in the EPR and Bell experiments, which occurs in two stages: a unitary operation U_θ achieved by a physical device (e.g., a Stern-Gerlach analyzer) that fixes the choice of measurement setting (whether spin x or y is measured); and a second final stage that includes amplification, a meter, and an irreversible coupling to an environment to give a readout. We refer to the final stage as the "pointer measurement." Related analyses are given in Refs. [21–25].

The premises (wMR and wLR) each involve three assertions. Assertion (1) imposes a weak form of realism, specifying that the outcome of a measurement is determined for the system at a time t_f *after* the experimentalist has performed the unitary operation U_θ to fix the measurement setting [21]. Hence, the pointer measurement can be viewed as passive: the value of the physical quantity being measured is determined at the time t_f and the pointer measurement acts only to reveal its value. Assertion (1) is most strongly justified in the case of wMR, where the relevant states of the system are restricted to be macroscopically distinct. Leggett and Garg gave a definition of macroscopic realism (MR) for a system "with two or more macroscopically distinct states available to it": MR asserts that the system "will at all times be *in* one or other of those states" [26]. In the case of wMR, the system at time t_f has available to it macroscopically distinct states each giving a definite outcome for the pointer measurement, and MR applies, leading to Assertion (1). In the case of wLR, the states immediately after the operation U_θ are not necessarily macroscopically distinct. On the other hand, assuming that the measurement process involves a reversible coupling of the microscopic system to a macroscopic meter at a later time, MR can be applied if the definition specifies the time t_f accordingly. We use the terminology "weak" realism because there is the contrast with Bell's local realistic theories, which consider hidden variables for the system as it exists *prior* to the implementation of operations U_θ that determine the measurement settings, and from which Bell inequalities can be derived.

The premises of wMR and wLR also embody the two original EPR assertions, but in a weakened form. Hence, wMR and wLR posit not only a weak form of realism, but a weak form of locality. Systems A and B are spacelike separated. Assertion (2) states that there is no disturbance to the value λ given [according to Assertion (1)] to the system A (at time t_f) for the outcome of the pointer measurement on system A by any interaction or event at the system B that comes *after* t_f. Since λ is the outcome at A (with the setting at A fixed), Assertion (2) is justified by no-signaling. We also note that the time-order of the interactions at the different sites becomes important, so that we restrict to where systems and observers are at rest.

Assertion (3) is a weaker version of EPR's criterion for reality, motivated by the passive nature of the pointer measurement in the wMR and wLR models, and by properties of a meter in measurement theory. The assertion posits the following: If it is possible to predict with certainty the outcome of a spin measurement S_θ^A on system A, by a spin measurement S_ϕ^B on the system B, then there is a predetermined value (an "element of reality") for the outcome S_θ^A at the time t_{f_B} once the unitary operation U_ϕ^B that fixes the setting ϕ at B has occurred. This is regardless of whether the unitary interaction that fixes the setting θ at A has taken place. A consequence of the assertions of wMR and wLR is that the nonlocal effects contributing to the GHZ and Bell contradictions with local realism emerge when there are changes of measurement setting at *both* systems, A and B. In such models, the violation of Bell inequalities occurs because of a partial failure of *both* realism and locality.

The premise of deterministic local macroscopic realism (dMR) is similar to Bell's form of local realism, because the premise applies to the system as it exists *prior* to the unitary operations U_θ. The macroscopic GHZ setup hence enables an all or nothing falsification of dMR, which supports previous work revealing dMR to be falsifiable by macroscopic Bell tests [21–24,27–30].

It is interesting that the premises of wMR and wLR are also motivated by Bohr's criticism of EPR's 1935 paper [3,31]. Clauser and Shimony state that "*[Bohr's] argument is that when the phrase 'without in any way disturbing the system' is properly understood, it is incorrect to say that system 2 is not disturbed by the experimentalist' s option to measure a rather than a′ on system 1*." This suggests that Bell nonlocality stems from the unitary operations that determine the measurement settings.

The layout of the paper is as follows: In Sec. II, we present the definitions of dMR, wMR, and wLR as well as their motivation. In Sec. III, we review the original EPR-Bohm and GHZ setups and give the macroscopic versions which allow falsification of dMR. In Sec. IV, we present the modified EPR-Bohm argument based on the alternative premises of wMR and wLR. Specific proposals for the macroscopic tests are given in Sec. V and Appendix A. The spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ are realized as macroscopically distinct coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, or else as multimode spin states. The unitary operations U_θ determining the measurements settings are realized by nonlinear interactions and CNOT gates. In Secs. VI and VII, we demonstrate the consistency of wMR and wLR with Bell violations and GHZ contradictions. Further predictions of wMR and wLR that show agreement with quantum predictions are outlined in Sec. VIII.

II. DEFINITIONS

We formalize the definitions of local realism and macroscopic realism relevant to this paper. Several definitions are introduced. The difference between the definitions is clarified once we recognize that there are two stages to a spin measurement S_θ: First, there is the reversible stage involving a unitary operation U_θ which determines the measurement setting θ.

Second, there is the stage that comes after, which includes a final irreversible readout of a meter. We refer to the later stage as the *pointer [stage of] measurement* (since the position of a pointer is ultimately observed).

Consider two separated spin-1/2 systems A and B prepared at time t_0 in the state $|\psi\rangle$. Local unitary operations U_θ^A and U_ϕ^B prepare the systems for spin measurements S_θ^A and S_ϕ^B. The U_θ are realized as reversible interactions of the system with a real device, such as a Stern-Gerlach analyzer or polarizing beam splitter, and are represented by a Hamiltonian H_θ, where $U_\theta = e^{-iH_\theta t/\hbar}$. The U_θ takes place over a time interval t, and the states prior and after the operation U_θ may therefore be regarded as different, in that they define the system at a different time. The state after the interaction at time t_f is

$$|\psi(t_f)\rangle = e^{-iH_\theta t_f/\hbar}|\psi\rangle. \quad (1)$$

Specific examples are given in Sec. IV B and in Appendices A and B, where we note that the "system" may include another (local) set of modes, or a meter that is originally decoupled to the spin system, in which case $|\psi\rangle$ is suitably defined. After the interaction U, there is a final irreversible stage of the measurement, which indicates the measurement outcome. This stage may involve a direct detection of a particle at a given location, as well as amplification and a coupling to a meter, in principle leading to observation of the position of a "pointer" on a measurement apparatus. The local system prepared after the interaction U that fixes the measurement setting, but before the irreversible stage of the measurement, is considered to be *prepared for the pointer measurement*. The system is said to be prepared in the *measurement basis*. We use terminology motivated by earlier studies [32,33], but the analysis of this paper is different, being focused on the unitary interactions and not concerned with decoherence.

A common realization of the spin-1/2 system is given as $|\uparrow\rangle \equiv |1,0\rangle$ and $|\downarrow\rangle \equiv |0,1\rangle$, defined for two orthogonally polarized modes, which we denote by $a_\pm$. Here, $|n_1, n_2\rangle \equiv |n_1\rangle_+|n_2\rangle_-$, where $|n\rangle_\pm$ is a number state for the mode $a_\pm$. A transformation U_θ can be achieved with a polarizing beam splitter, with mode transformations

$$\hat{c}_+ = \hat{a}_+ \cos\theta - \hat{a}_- \sin\theta,$$
$$\hat{c}_- = \hat{a}_+ \sin\theta + \hat{a}_- \cos\theta. \quad (2)$$

The $\hat{c}_\pm$ are boson operators for the outgoing modes emerging from the beam splitter; $\hat{a}_\pm$ are boson operators for the ingoing modes. The interaction is described by the Hamiltonian $H_\theta = i\hbar k(\hat{a}_+\hat{a}_-^\dagger - \hat{a}_+^\dagger\hat{a}_-)$ where $\theta = kt$. The value of S_θ is the outcome of the number difference $\hat{c}_+^\dagger\hat{c}_+^\dagger - \hat{c}_-^\dagger\hat{c}_-^\dagger$. Hence, S_θ is the Pauli spin σ_θ, with outcomes 1 or −1, also indicated by + or − as in Fig. 1. The choice $\theta = 0$ ($\theta = \pi/4$) corresponds to a spin measurement S_z (S_x). A single photon impinges on the beam splitter and is finally detected at one or other locations associated with the outgoing modes [16]. The final detection and readout of the location of the photon constitutes the pointer measurement. Such measurements are used to confirm violation of Bell inequalities and to illustrate Bohm's EPR paradox (Fig. 1), for two spatially separated spin-1/2 systems prepared in the Bell state:

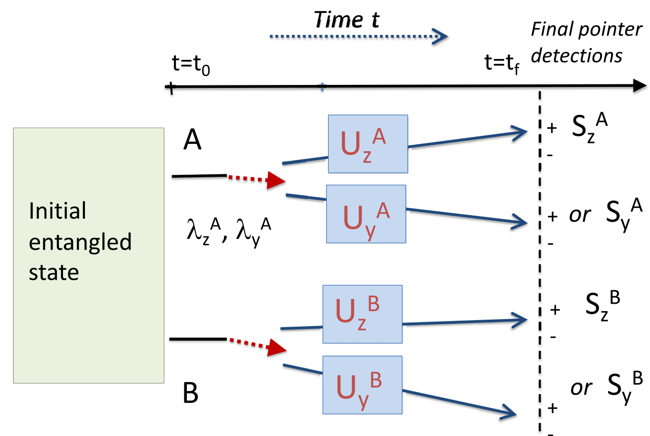

FIG. 1. The setup for an EPR-Bohm paradox involves unitary operations U that fix the measurement settings. Two separated systems A and B are prepared in a Bell state $|\psi_B\rangle$ (Eq. (3)]. A switch (red dashed arrow) gives the choice to measure either S_z or S_y for each of A and B, by interacting with an analyzer symbolized by U. If the same spin component S_θ is measured at each site, the outcomes are always anticorrelated. EPR's premises imply that, because one can predict the outcome for either S_z or S_y by measuring at B, the outcomes for S_z and S_y at A are *both* predetermined at the time t_0 (prior to the choices of measurement setting). The premises of deterministic macroscopic (local) realism apply when the outcomes + and − for both spins S_z and S_y are associated with macroscopically distinct states for the system at the time t_0.

$$|\psi_B\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z|\downarrow\rangle_z - |\downarrow\rangle_z|\uparrow\rangle_z). \quad (3)$$

The two systems and their respective sites are denoted A and B. Here $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ are the eigenstates of the Pauli spin σ_z (i.e. S_z) with eigenvalues +1 and −1, respectively. We use the standard notation, where the first and second states of the product $|\uparrow\rangle|\downarrow\rangle$ refer to the states of system A and B, respectively. The spin operators for the two particles are distinguished by superscripts, e.g., S_z^A and S_z^B. We omit operator "hats" where the meaning is clear. Unitary operations U_θ^A and U_ϕ^B performed at each site determine the measurement settings, preparing the systems in the measurement basis, ready for the pointer measurements of the spin components S_θ^A and S_ϕ^B respectively.

A. Einstein-Podolsky-Rosen premises

The premises presented in the 1935 argument given by EPR are based on the philosophy of local realism [3]. The premises are summarized as two assertions for spacelike separated systems, A and B.

EPR Assertion I: No disturbance (locality). There is no disturbance to system A from a spacelike-separated interaction or event (e.g., a measurement on system B).

EPR Assertion II: EPR's criterion for reality. "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" [1].

This is interpreted as follows: "The 'element of physical reality' is that predictable value, and it ought to exist whether or not we actually carry out the procedure necessary for its prediction, since that procedure in no way disturbs it" [4]. Hence, the second EPR assertion reads: If one can predict with certainty the outcome of a measurement S on system A without disturbing that system, then the outcome of that measurement is predetermined: The system A as it exists at the time prior to the measurement device at A actually being prepared can be ascribed a variable λ^A, the value of which gives the outcome for S [4].

Consider the system of Fig. 1. For any θ, the outcome of a measurement S_θ^A at A can be predicted with certainty by a measurement S_θ^B at B. The EPR assertions hence imply the system A can be ascribed a set of hidden variables $\{\lambda_\theta^A\}$, which give the outcomes of measurements S_θ^A if performed. The assignment of the variables λ_θ^A can be made to the system A as it exists at the time t_0, prior to the unitary interactions U_θ^A and U_ϕ^B that determine any measurement settings [4]. A similar set of variables $\{\lambda_\phi^B\}$ can be assigned to system B. The EPR assertions lead to the EPR-Bohm paradox [5] (Sec. III).

It is well known that Bell derived the inequality [2,3]

$$\left|\langle S_\theta^A S_\phi^B\rangle - \langle S_\theta^A S_{\phi'}^B\rangle\right| \leqslant 1 + \langle S_\phi^A S_{\phi'}^3\rangle \tag{4}$$

based on the assumption of local hidden variable states for which the spin outcome at each location A and B is either $+1$ or -1, as consistent with the local hidden variables $\{\lambda_\theta^A, \lambda_\phi^B\}$. The inequalities were shown by Bell to be violated by quantum mechanics. Hence, the EPR assertions are falsified by quantum mechanics.

B. Deterministic macroscopic realism

The assertions defining *deterministic macroscopic realism* (dMR) include those of EPR, but with the constraint that the assertions are restricted to the subset of systems where the outcomes for all relevant measurements, S_θ^A and S_ϕ^B, are associated with macroscopically distinct states of the system. This means that the systems upon which those measurements are made can be viewed as having two (or more) macroscopically distinct states available to them, so that the Leggett-Garg definition of *macroscopic realism* (MR) [26] can be applied. As a result, we are able to *separately* posit *macroscopic realism*, as below. The importance of examining such macroscopic systems is that the EPR premises are more robustly justified for macroscopically distinct states [20]. We define *deterministic macroscopic realism (dMR)* according to the following three Assertions.

Assertion dMR(1): Leggett-Garg's macroscopic realism. A macroscopic system A which has "two or more macroscopically distinct states available to it will at all times be in one or other of these states" [26]. The system A can be ascribed a variable λ^A, its value indicating which of the macroscopic states the system is in. Hence, λ^A predetermines the outcome of a measurement S that can be made on the system to reveal which of the states the system is in at the given time.

The assertion is sufficient to define macroscopic realism, without specifying details about the nature of the macroscopically distinct states available to the system. It is only necessary to define a macroscopic quantity that is different for the states, so that a suitable measurement S can then *distinguish* between the states.

Assertion dMR(2): No disturbance (macroscopic locality). There is no macroscopic disturbance to a system A from a spacelike-separated interaction or event occurring at another site B, so as to cause a change from one macroscopic state to another macroscopically distinct from it. Hence, the value λ^A is not affected by spacelike-separated measurements (or unitary operations U_ϕ) that may occur at site B.

Assertion dMR(3): EPR's criterion for reality. This reads as for EPR's criterion, Assertion II. However, from Assertion dMR(2), the system can only be assumed to be not disturbed in a macroscopic way.

Referring to Fig. 1, assuming macroscopically distinct spin states can be identified with the outcomes of S_θ^A and S_ϕ^B, deterministic macroscopic realism (dMR) assigns to the systems at the time t_0 (prior to the interactions U_θ^A and U_ϕ^B) the sets of variables $\{\lambda_\theta^A\}$ and $\{\lambda_\phi^B\}$ that predetermine the outcomes of those measurements, should they be performed. Where the outcomes for S_θ^A and S_θ^B are maximally anticorrelated, Assertions dMR(2) and dMR(3) alone are sufficient to imply that the system is in a state with definite outcomes of S_θ^A and S_ϕ^B, for all θ and ϕ (as in the original EPR argument).

The terminology "deterministic MR" is used because the values for the outcomes are assumed to be determined prior to all stages of the measurement. This applies to all measurable observables if macroscopically distinct states are assumed, so that at any given time, a simultaneous predetermination of observables, say, x and p, can exist, as in classical mechanics. The locality Assertion dMR(2) is naturally part of that definition [34]. In this paper, the terms *deterministic macroscopic realism* and *deterministic macroscopic (local) realism* are hence used interchangeably with equivalent meanings.

C. Weak macroscopic realism

The premise defined as deterministic macroscopic realism (dMR) is falsifiable. In Sec. V B, we summarize predictions presented in Ref. [21] for the violation of a Bell inequality using cat states, which falsify dMR. In Sec. III D, we demonstrate a GHZ contradiction with dMR. Motivated by the concept that macroscopic realism should be valid in some form (for a macroscopic superposition state prior to any coupling to the environment) [18], we hence propose a *weaker* form of dMR that we refer to as *weak macroscopic realism (wMR)*. The premise of wMR has been shown consistent with the macroscopic Bell violations [21], which we explain further in Sec. V. Similarly, that wMR is not falsified by the GHZ experiments is explained in Sec. VI. As with dMR, a (weak) locality assumption is naturally part of the definition. We use the term *weak macroscopic realism* (wMR) for brevity, but *weak macroscopic (local) realism* has an equivalent meaning.

Weak macroscopic realism (wMR) involves weaker (i.e., less restrictive) assumptions than dMR. The assertions of dMR imply wMR, but the converse is not true. The assertions of wMR apply to the systems *after* the selection of the measurement settings, at time t_f as in Fig. 2. Here, we specify that the measurements and observations made on systems A and B are defined in a frame where there is no relative motion between the systems, nor between the systems and observers, so

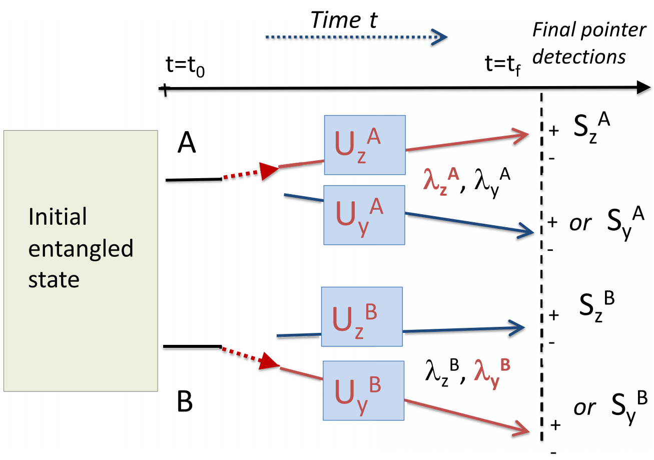

FIG. 2. The assumptions of weak macroscopic realism (wMR) and weak local realism (wLR) can give rise to an EPR-type paradox. The setup is for an EPR-Bohm paradox as in Fig. 1 based on the Bell state $|\psi_B\rangle$. At time t_f, the measurement settings are set to S_z^A and S_y^B, as indicated by the positions of the red dashed arrows. Weak local realism asserts that for the system A at time t_f, after the unitary rotation U_z^A, the outcome for the final pointer measurement S_z at A is determined, given by the variable λ_z^A. Weak local realism also asserts that because one can predict the result for S_y at A by making a final pointer measurement S_y at B, the outcome for S_y at A is also determined at time t_f (after the rotation U_y^B at B). This leads to the paradox. The premise of wMR applies similarly, assuming the outcomes + and − for the spins can be viewed as corresponding to macroscopically distinct states for the system at the time t_f (refer to Sec. IV).

that the time order of any unitary operations is clearly established. The systems A and B are assumed sufficiently spatially separated so as to exclude subluminal influences between them.

Assertion wMR(1): Macroscopic realism for the pointer stage of measurement. Suppose the system A that is prepared at a time t_f for the pointer measurement of S_θ^A can be considered to have two or more macroscopically distinct states available to it, where each of those states has a definite outcome for the pointer measurement. Assertion wMR(1) posits that such a system can be ascribed a predetermined value λ_θ^A for the pointer measurement S_θ^A that will distinguish between these states.

Referring to Fig. 2, the premise wMR implies that the result of the pointer measurement for S_θ^A is predetermined *once* the interaction U_θ^A that fixes the measurement setting at A has taken place. This is a *weaker* assumption than dMR, which posits the predetermined value λ_θ^A of the outcome for S_θ^A at the time t_0, *prior* to the interaction U_θ^A.

Assertion wMR(2): Partial locality. There is no disturbance *to* the predetermined value λ_θ^A for the pointer measurement at A [as described in Assertion wMR(1)] from spacelike-separated interactions or events (e.g., further unitary operations U_ϕ^B) that may *subsequently* occur at B, *after* the time t_f.

Assertion wMR(2) is a partial locality assumption only since it does not imply that the value λ_θ^A is independent of interactions that occurred at B *prior* to t_f. The assumption posits only that there is no disturbance to the *value* λ_θ^A of the *outcome*. It does *not* posit that there is no disturbance to the *state* of the system at A due to interactions or events at B.

This assertion may be extended to posit that there is no disturbance *from* a pointer measurement: i.e., there is no disturbance to the outcomes at A due to a pointer stage of measurement at B. Then we arrive at the assertion below.

Assertion wMR(3): Weak EPR criterion for reality. EPR's criterion for realism (EPR Assertion II) is modified to take into account the weakening of the locality assumption. Assertion wMR(3) posits that, if it is possible to predict with certainty the outcome of a measurement S_φ^A of one system A by a measurement S_ϕ^B on another spacelike-separated system B, then there is a predetermined value (an element of reality) for the outcome S_φ^A at the time t_{f_B}, *once the unitary operation U_ϕ^B that fixes the setting ϕ at B has occurred*. This is regardless of whether the unitary interaction that fixes the setting φ at A has actually taken place.

Assertion wMR(3) originates from the notion that the pointer stage of the measurement can be regarded as passive. By the partial locality assumption, Assertion wMR(2), there is no change *to* a pointer-measurement value, meaning the value λ_θ^A at A is not changed once the setting at A is fixed at θ. By symmetry, we have also supposed there is no change *from* a pointer measurement, meaning that the pointer stage of the measurement of $\hat{S}^B$ can have no impact on the outcomes λ_θ^A at A. Then, there would be no disturbance to the outcomes of system A due to whether or not the pointer stage of measurement at B *actually* takes place. Following EPR's original criterion for realism, this implies the outcome for S_φ^A at A is determined by the element of reality at the time t_{f_B}, *once* the result can be predicted at B by a pointer measurement only.

Considering Fig. 2, Assertion wMR(3) implies that if the outcome of a measurement S_φ^A at A can be predicted with certainty by a pointer measurement on the system B at time $t_{f_B} = t_f$, then the system A at time $t_{f_B} = t_f$ can be ascribed a hidden variable λ_φ^A that predetermines the outcome for S_φ^A. This is true regardless of whether the pointer measurement at B is actually carried out (consistent with the notion that would not disturb the system A) and regardless of whether the interaction U^A at A that fixes the measurement setting φ has actually been carried out at A (and regardless of future unitary interactions at A). However, the predetermination is based on the system B being prepared for a pointer measurement and therefore *only* applies at the times $t > t_{f_B}$ when no further unitary interactions that would cause a change of measurement setting at B have taken place.

D. Motivation leading to the weak macroscopic realism premises

Below, we give the motivation for the premises of weak macroscopic realism. Justifications are also given in Refs. [21,22,24] which examine the consistency of wMR in the context of Bell violations, macrorealism tests, and Wigner friend paradoxes.

1. Motivation for wMR(1): Macroscopic realism and comparison with mixed states

The premise wMR(1) is primarily motivated by the concept that quantum mechanics should be consistent with some form of *macroscopic realism*, even in the absence of decoherence. In fact, motivation for the wMR premises is provided by the formalism and predictions of quantum mechanics (without the need to restrict to macroscopic superposition states) when comparing the predictions of superpositions with those of mixed states.

Consider the bipartite superposition state

$$|\psi\rangle = \sum_{ij} e_{ij}|i\rangle_A|j\rangle_B, \tag{5}$$

where $e_{ij} \neq 0$ are complex amplitudes and $\sum_{i,j}|e_{ij}|^2 = 1$. Here $|i\rangle_A$ ($|j\rangle_B$) are eigenstates of S_θ^A (S_ϕ^B) with eigenvalues denoted λ_i^A (λ_j^B) for measurements S_θ^A and S_ϕ^B made at sites A and B, respectively. First, we consider the corresponding fully mixed state

$$\rho_{\text{mix},f} = \sum_{ij}|e_{ij}|^2|i\rangle_A|j\rangle_B\langle j|_B\langle i|_A \tag{6}$$

and note that the joint probability for outcomes λ_i^A and λ_j^B is $|e_{ij}|^2$ for *both* $\rho_{\text{mix},f}$ and $|\psi\rangle$. This motivates wMR(1), which specifies that the system prepared with respect to this basis at time t_f can be viewed as having a definite outcome, as for the mixed state $\rho_{\text{mix},f}$. The motivation is based on the fact that there is no *inconsistency* with wMR(1), but of course this is not equivalent to the assumption that the systems A (B) is *in* the eigenstate $|i\rangle_A$ ($|j\rangle_B$) at the time t_f.

The definition specifies a value λ_θ^A for the outcome of a measurement S_θ^A at the given time t_k, after the operation fixing the measurement setting has been completed. This definition is consistent with the concept that there is *no retrocausality* at a macroscopic level, since the value is then fixed for the system at that time, independent of future events. The consistency of wMR(1) for delayed choice experiments has been explained in Ref. [23].

2. Motivation for wMR(2): No signaling and comparison with mixed states

If we posit the first premise, wMR(1), then we assign to the system A, at the time t_f when prepared for the pointer stage of measurement of S_θ^A, a definite value λ_θ^A for the *outcome* of S_θ^A. The second premise wMR(2) follows to ensure *no signaling*, defined as the requirement that the outcome at one site is independent of the setting at the other site, conditioned on the setting at the first site being fixed [35]. Hence, at the time after the unitary operation U_θ^A, which fixes the setting at A, there should be no change to λ_θ^A due to unitary operations at B.

The premise wMR(2) is also motivated by consideration of mixed states in quantum mechanics. One can rewrite the expression (5) for the state $|\psi\rangle$ in the form

$$|\psi\rangle = \sum_j d_j|\psi_j\rangle_A|j\rangle_B, \tag{7}$$

where $|\psi_j\rangle = \sum_i\{e_{ij}/d_j\}|i\rangle_A$ ($d_j \neq 0$). Here, the magnitude $|d_j|^2 = \sum_i|e_{ij}|^2$ of d_j is defined uniquely and $\sum_{ij}|e_{ij}|^2 = \sum_j|d_j|^2 = 1$. We consider that the system is prepared with respect to the measurement basis $|i\rangle_A|j\rangle_B$ of S_θ^A and S_ϕ^B. We compare the system in Eq. (7) with the system prepared in the partial mixture $\rho_{\text{mix}B,j}$

$$\rho_{\text{mix}B,j} = \sum_j |d_j|^2|\psi_j\rangle_A|j\rangle_B\langle j|_B\langle\psi_j|_A. \tag{8}$$

Here, for the mixed state description $\rho_{\text{mix}B,j}$, the system can be regarded as *being in* one of the states $|j\rangle_B$ *at* B with probability $|d_j|^2$. Hence, there is justification to assign a variable λ_j^B to the system, where the value λ_j^B gives the outcome of a measurement S_ϕ^B. Suppose a basis rotation (i.e. a unitary operation) U^A takes place at the other site A, in preparation for a different measurement $S_{\theta'}^A$ at A. Then we write the state $|\psi_j\rangle_A$ for system A in terms of the new eigenstates $|k\rangle_A$ of $S_{\theta'}^A$, where $S_{\theta'}^A|k\rangle_A = \lambda_k|k\rangle_A$. Hence $|\psi_j\rangle_A = \sum_k f_k|k\rangle_A$, where $\sum_k|f_k|^2 = 1$. The original superposition state is written

$$|\psi\rangle = \sum_j d_j\sum_k f_k|k\rangle_A|j\rangle_B, \tag{9}$$

and we see that the joint probability for outcomes λ_k and λ_j is $|d_jf_k|^2$. The partial mixture $\rho_{\text{mix}B,j}$ under the same basis rotation is

$$\rho_{\text{mix}B,j} = \sum_j|d_j|^2\sum_k\sum_{k'}f_kf_{k'}^*|k\rangle_A|j\rangle_B\langle j|_B\langle k'|_A, \tag{10}$$

and the probability for the joint outcome is in this case $|d_j|^2|f_k|^2$. We see that, although the *states* $|\psi\rangle$ and $\rho_{\text{mix}B,j}$ are *different* after the single rotation U^A, the *probabilities* for the outcomes of the measurements $S_{\theta'}^A$ and S_ϕ^B are identical. This motivates the *partial locality* assumption, as in the interpretation given by wMR(2), that for the system prepared with respect to the appropriate basis for a measurement of S_ϕ^B at B, there is a predetermined value $\lambda_\phi^B \equiv \lambda_j^B$ for the (pointer) outcome of S_ϕ^B, and that this value is not changed by a unitary interaction $U_{\theta'}^A$ at the other site A.

3. Motivation for wMR(3): Weak locality and meters

The modified EPR criterion for reality is premise wMR(3). This arises on the basis that the original EPR criterion (EPR Assertion II) is correct, but that the locality assumption has been weakened, as explained in Sec. II C.

The motivation for wMR(3) is further strengthened by consideration of a system that acts as a *meter*. Where the measurement outcome of S_ϕ^B implies the result for, say, the outcome of S_φ^A of system A, then B is considered to be a meter for the measurement S_φ^A of system A. For the meter, the outcome for S_φ^A can be obtained from the final readout at B. According to wMR(1), the outcome of S_ϕ^B is predetermined at the time t_f when the measurement setting ϕ is fixed at B. It can then be argued reasonable to posit that the outcome of S_φ^A is also predetermined *at that time*, since it can be obtained correctly (as the measurement S_φ^A directly on A would verify), by accessing a readout from B [22]. We explain in Secs. V and VI of this paper that this assumption does not conflict with the known violation of Bell and GHZ inequalities.

4. A simple analogy: A ball in a box

The motivation for the definition of weak macroscopic realism (wMR) is further conveyed by consideration of a simple analogy, where a macroscopic object (a ball) can be found

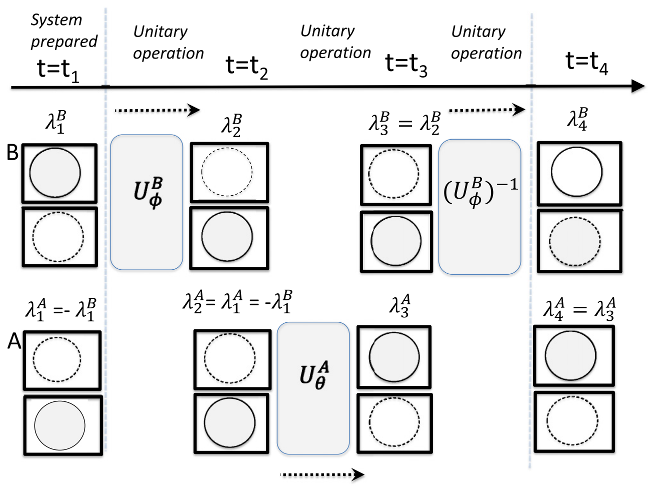

FIG. 3. Motivating weak macroscopic realism, premises wMR(1) and (2): Two macroscopic objects (balls), one at A and one at B, are each shuffled between two boxes. The shuffling at each site is denoted by unitary operations U_θ^A and U_ϕ^B. Assertion wMR(1) is that at each time t_k ($k = 1, 2, 3, 4$), after shuffling, *the ball will be in one of the boxes* prior to an observer opening the boxes. The predetermined outcome (for which box the ball is found in) is denoted λ_k^A (and λ_k^B) for A (and B). According to wMR(2), these values denote the position of the ball at the time t_k, and the position is not changed by any further shuffling that occurs at the other site *at a later time*. For example, the value λ_1^A does not change at time t_2 due to U_ϕ^B and $\lambda_2^A = \lambda_1^A$. Similarly, $\lambda_3^B = \lambda_2^B$ and $\lambda_4^A = \lambda_3^A$.

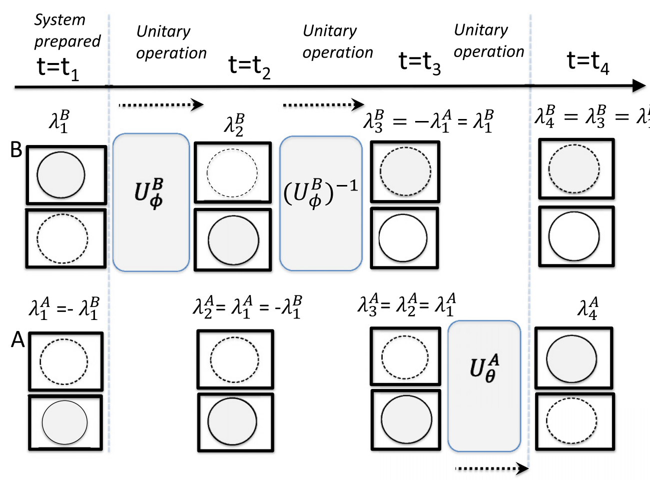

FIG. 4. Motivating weak macroscopic realism, premise wMR(3): Elements of reality: Suppose that at the time t_k the outcome S_k^A of the observer opening the box at A will predict with certainty the outcome of a measurement $S_K^B \equiv S_\Phi^B$ at B, as defined by a certain shuffling operation U_Φ^B. Assertion wMR(3) posits that the value λ_k^A predetermines the outcome of S_Φ^B, even if the shuffling U_Φ^B at B may not yet have occurred. The figure depicts a system prepared in a Bell state (3). Hence, $\lambda_1^A = -\lambda_1^B$. In the figure, where $(U_\phi^B)^{-1}$ is the inverse of U_ϕ^B, wMR(3) implies $\lambda_3^B = -\lambda_1^A = \lambda_1^B$. Also, from wMR(2), $\lambda_4^B = \lambda_3^B$. We note that the predetermination of S_Φ^B need no longer hold if there is further shuffling at A. Hence, in Fig. 3 [also prepared in (3)], wMR does *not* imply $\lambda_4^B = \lambda_3^B = -\lambda_1^A = \lambda_1^B$, because the unitary operation U_θ^A has taken place before $(U_\phi^B)^{-1}$. In the model, what happens at site A has the possibility to affect the outcome at B. Nonlocal effects are not excluded by wMR, but we note that these require unitary rotations at both sites A and B.

in one of two boxes (Figs. 3 and 4) [22,24]. The analogy is motivated by the three-box paradox [25,36–39] and the example of Schrödinger's cat, where the cat can be found in one of two states (alive or dead) when a box is opened [20].

In the analogy, there are two balls, each of which can be placed in one of two boxes at separated sites labeled A and B. Each ball can be independently shuffled between the boxes by an experimentalist. The final pointer measurement S_k^A or S_k^B at A or B respectively is the act of an observer at each site opening the boxes, *after* the shuffling, at a time t_k ($k = 1, 2, 3, 4$), in order to determine which box the ball is in. The outcome of S_k is binary at each site, with outcomes ± 1 in analogy to a spin measurement. The unitary stage U_θ of measurement corresponding to fixing a measurement setting as θ is modeled simplistically by the shuffling, which can take place according to some definite set of operations. In the model, there is a clear distinction between the times t_k after any shuffling, when the ball is going to be observed in one or the other box, and the times in between or prior (refer Figs. 3 and 4). A physical realization relevant to this paper involving cat states is given in Fig. 5.

The Assertion of wMR(1) applies macroscopic realism at the times t_k (only) to posit that there is predetermined outcome for the *pointer* measurement, if the observer were to open the boxes at time t_k, restricted to $k \in \{1, 2, 3, 4\}$. The assumption that there is a predetermined outcome *prior* to the shuffling is a stronger, more restrictive assumption that is not adopted in the wMR model: Assertion wMR(1) posits that, at each time t_k after the shuffling and prior to the observer opening a box (A, say) the ball is in one box or the other, meaning that the outcome of S_k^A is predetermined at this time, given by a value λ_k^A. This contrasts with dMR, which postulates that this value exists at the earlier time, *prior* to the shuffling.

The macroscopic aspect of the model also motivates the partial locality premise wMR(2). In the analogy, Assertion wMR(2) posits that, once the ball is in a box, it is not moved out of the box by any shuffling or observation at the other site (Fig. 3). In other words, at the time t_k, after any preliminary shuffling U^A and prior to any further shuffling at A, the value

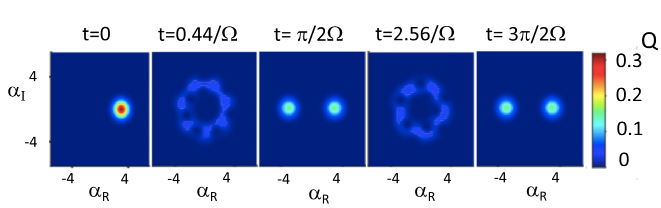

FIG. 5. A depiction of the dynamics associated with a unitary operation U^A that determines a measurement setting, for which the "ball-in-a-box model" is justified. Full details are given in Sec. IV B. The contour graphs show the Q function for a single-mode field. At times $t = 0$, $\pi/2\Omega$, and $t = 3\pi/2\Omega$, a suitable measurement gives outcomes indicating either $\alpha_R \approx 4$ or $\alpha_R \approx -4$ (and $\alpha_I = 0$). The two states associated with these values are considered macroscopically distinct, so that weak macroscopic realism can be applied to the system at these times, as in the "ball-in-a-box model". In between these times, the dynamics is analogous to "shuffling."

of λ_k^A, which determines which box the ball is in, is not changed by any *further* shuffling occurring at B, after the time t_k. The premise posits only that there is no disturbance to the *value* λ_k^A of the pointer outcome, the value referring to the macroscopic position of the ball. It does not posit that there is no disturbance to the *state* of the system (ball) at A due to the interactions at B. The value λ_k^A which gives the position of the ball at time t_k is "stable," and would not be changed by the further coupling to an environment (including decoherence) that models the observer opening the box, although the overall "state" of the ball may well change, in analogy to a final collapse to an eigenstate.

The third premise wMR(3) can also be motivated by the ball-in-a-box analogy (Fig. 4). According to wMR(1), the position of the ball at A at a time t_k (after shuffling has taken place at A) is predetermined, given by λ_k^A. If this value is correlated with an outcome of $S_K^B \equiv S_\Phi^B$ at B, as defined after a certain shuffling operation U_Φ^B, then the observer can open the box at A and correctly predict that outcome for B, regardless of whether the experimentalist at B has actually performed the necessary shuffling. This is consistent with the motivation given above, that system A acts as a meter for the measurement S_ϕ^B. It is implicit in the description of a meter that the setting (as in the shuffling) for the meter A is fixed. If there are further operations performed on A, then the predetermination of the outcome for S_Φ^B need no longer hold.

Figures 3 and 4 show how the wMR model does not exclude that nonlocal effects can be observed. For example, on comparing the figures, the outcome for B at time t_4 may depend on the relative order of unitary operations at A and B. However, in the wMR model, for a system prepared for certain pointer measurements at an initial time t_1, such effects will only emerge after operations (shuffling) are performed at *both* sites, A and B. For completeness, we formalize the result:

Result II D: Nonlocality. The premise of wMR implies that after preparation for pointer measurements, a change of measurement setting at *both* sites is required to realize a violation of a Bell inequality [e.g., Eq. (4)], a prediction consistent with quantum mechanics. The result was proved in Ref. [21] and is proved for GHZ setups in Sec. VII. We summarize for bipartite systems below. Graphical illustrations of this proof are given in Sec. V.

Proof. The result follows from the reasoning given below Eq. (7) that the pointer measurements on the system after a rotation $U_{\theta'}^A$ at a single site A cannot distinguish the system in the superposition from that in a mixture [Eq. (8)]. Here, at the initial time t_1, the system is prepared for pointer measurements S_θ^A and S_ϕ^B. The mixed state (8) is equivalent to a probabilistic mixture of the system B being in one of the states $|j\rangle_B$ with a definite outcome for the pointer measurement S_ϕ^B at B. The mixed state can be assigned a hidden variable λ_j^B throughout the entire unitary operation $U_{\theta'}^A$. After the operation, the wMR model gives a change in variable (to $\lambda_{\theta'}^A$) at A but that can be explained in a local realistic model by the operation at A. Hence, there is no inconsistency between wMR and Bell's local hidden variables. However, if we consider a second rotation $U_{\phi'}^B$ performed at B, to prepare for a pointer measurement of $S_{\phi'}^B$ at B, then wMR assigns a new variable $\lambda_{\phi'}^B$ to system B after the rotation. While wMR predicts that the variable $\lambda_{\theta'}^A$ is now fixed, wMR does *not* predict that the variable $\lambda_{\phi'}^B$ is independent of the earlier operation $U_{\theta'}^A$, as would Bell's local hidden variable theories. ■

Comment. While the assertions of wMR are well motivated and consistent with quantum predictions (refer to Secs. V–VII), the asymmetry introduced by wMR(2) with respect to the time order of operations at different sites introduces potential paradoxes if we consider different frames of reference. For spacelike-separated events the time order can change. It may then seem that the value of λ_4^B in Figs. 3 and 4 can be frame dependent (see also Sec. V C). In this paper, we restrict the definition to the laboratory frame where all observers and systems are stationary. The difficulty of reconciling the requirement of no-signaling in Bell experiments with classical casual models has been pointed out in Ref. [35].

E. Weak local realism

The assertions of weak local realism (wLR) are as for weak macroscopic realism (wMR), except there is no longer the restriction that outcomes correspond to macroscopically distinct states of the system being measured. The motivation for the wLR premises is based on the reasoning given in Sec. II D 1-3 for wMR which (apart from the argument in favor of macroscopic realism) is formulated to apply to superpositions of states that need not be macroscopically distinct. As with wMR, the pointer measurement is interpreted to constitute a passive stage of the measurement. The wLR assertions distinguish between the systems prepared before and after the measurement settings are fixed (Fig. 2). Weak forms of realism and locality are defined that apply at the time t_f, after local unitary interactions are performed. This allows an argument to be put forward that wLR is justified by wMR (refer to Sec. II F).

The premise of wLR contrasts with that of *local realism* [2,3], which introduces hidden variables that apply to the system at time t_0, *prior* to the entire measurement process (Fig. 2). Local realistic theories (alternatively called local hidden variable theories) are falsified by the violation of Bell inequalities [3]. By contrast, weak local realism, as with wMR, does *not* exclude Bell violations. We show in Secs. V, VI, and VII how wLR (and wMR) can be viewed consistently with the quantum predictions for Bell and GHZ experiments. Regardless, the assertions when applied to the setup of Fig. 2 lead to an EPR-type paradox (Sec. IV).

Assertion wLR(1): Realism for the pointer stage of measurement. The outcome of the pointer stage of the measurement S_θ for system A is predetermined (as given by a variable λ_θ^A, say) once the local operation U_θ^A that determines the measurement setting θ at A has taken place.

Consider the system A at time t_f, after the unitary rotation U_θ^A, as in Fig. 2. The premise wLR asserts that the system A at time t_f can be assigned a variable by λ_θ^A, the value of which determines the outcome of the pointer measurement for S_θ^A at A, if the pointer stage of measurement were to be carried out.

Assertion wLR(2): Partial locality. The assertion reads as for Assertion wMR(2).

Assertion wLR(3): Weak EPR criterion for reality. The assertion reads as for Assertion wMR(3).

Comment. We use the terminology "weak local realism" because the form of local realism defined by the wLR assertions is not sufficient to imply a Bell inequality. However, we emphasize that in general the assertions of wLR are not strictly a subset of the assertions of local realism, which do not always require that the value for the outcome be predetermined prior to the pointer stage of the measurement. On the other hand, for the case of ideal EPR-Bohm anticorrelation, local realistic theories are consistent with the EPR premises and imply a predetermined value for the spin outcome at the time t_0 (Fig. 1) [2].

F. Link between weak local realism and weak macroscopic realism

At first glance, weak local realism (wLR) is seen to be a stronger assumption than weak macroscopic realism (wMR), meaning it is a more restrictive (less convincing) assumption. However, if the time t_f is carefully specified, we show that there is link between wLR and wMR, which gives reason to justify wLR.

Consider the system at time t_f after the unitary rotations U_θ^A and U_ϕ^B that determine the measurement settings, θ and ϕ, respectively at A and B. At this stage, or later, in the measurement process, there is a coupling of each local system to a macroscopic meter, via an interaction H_M. The final state after coupling is of the form

$$|\psi_M\rangle = c_1|p_+\rangle_A|\uparrow\rangle_\theta|p_-\rangle_B|\downarrow\rangle_\phi + c_2|p_-\rangle_A|\downarrow\rangle_\theta|p_+\rangle_B|\uparrow\rangle_\phi + c_3|p_+\rangle_A|\uparrow\rangle_\theta|p_+\rangle_B|\uparrow\rangle_\phi + c_4|p_-\rangle_A|\downarrow\rangle_\theta|p_-\rangle_B|\downarrow\rangle_\phi, \tag{11}$$

where c_i are probability amplitudes and $|p_+\rangle_{A/B}$ and $|p_-\rangle_{A/B}$ are macroscopic states for the pointer of the meter, indicating Pauli spin outcomes of $+1$ and -1, respectively, at A and B. Here, $|\uparrow\rangle_\theta$ and $|\downarrow\rangle_\theta$ are the eigenstates of S_θ^A, and similarly $|\uparrow\rangle_\phi$ and $|\downarrow\rangle_\phi$ are the eigenstates of S_ϕ^B.

We see that $|\psi_M\rangle$ is a macroscopic superposition state. Weak macroscopic realism implies predetermined values λ_M^A and λ_M^B for the outcomes of measurements on the meter systems—the pointers are in some kind of definite state that will indicate the result of the measurement to be either "spin up" or "spin down." In view of the correlation, it can be argued that the systems A and B (which may be microscopic) are similarly specified to have a definite outcomes for the pointer stage of the measurements, hence justifying wLR. Hence the definition of wLR can be rephrased to apply to the system at the time t_f where it is assumed that the stage of the measurement that couples each system to a meter has already occurred, just after or in association with the interactions U_θ^A and U_ϕ^B. Due to the reversibility of H_M, this would not change the results of this paper.

III. EPR-BOHM PARADOXES, GHZ NONLOCALITY, AND MACROSCOPIC VERSIONS

A. Bohm's version of the EPR paradox

Bohm generalized the EPR paradox to spin measurements by considering two spatially separated spin-1/2 particles (labeled A and B) prepared in the Bell state [2,5]:

$$|\psi_B\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z|\downarrow\rangle_z - |\downarrow\rangle_z|\uparrow\rangle_z),$$

as in Eq. (3). We consider the Pauli spins $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\vec{S} = (S_x, S_y, S_z) = \hbar\vec{\sigma}/2$, noting that we use the notation S and σ interchangeably.

1. Two-spin version

From $|\psi_B\rangle$ of (3), it is clear that the outcomes of spin-z measurements on each particle are anticorrelated. Similarly, we may measure the component σ_y of each particle. To predict the outcomes, we transform the state into the y basis, noting the transformation

$$|\uparrow\rangle_y = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z + i|\downarrow\rangle_z),$$
$$|\downarrow\rangle_y = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z - i|\downarrow\rangle_z), \tag{12}$$

where $|\uparrow\rangle_y$ and $|\downarrow\rangle_y$ are the eigenstates of σ_y, with respective eigenvalues $+1$ and -1. Hence $|\uparrow\rangle_z = (|\uparrow\rangle_y + |\downarrow\rangle_y)/\sqrt{2}$ and $|\downarrow\rangle_z = -i(|\uparrow\rangle_y - |\downarrow\rangle_y)/\sqrt{2}$. The state in the new basis becomes

$$|\psi_B\rangle = \frac{i}{\sqrt{2}}(|\uparrow\rangle_y|\downarrow\rangle_y - |\downarrow\rangle_y|\uparrow\rangle_y). \tag{13}$$

The spin-y outcomes at A and B are also anticorrelated.

An EPR-Bohm paradox follows (Fig. 1) [5]. By making a measurement of σ_z^B on particle B, the outcome for the measurement σ_z^A on particle A is known with certainty. EPR's assertions [1] are summarized in Sec. II A. Invoking EPR's Assertion I that there is no disturbance to system A due to the measurement at B, EPR's Assertion II implies the existence of a hidden variable λ_z^A, which predetermines the outcome for the measurement σ_z^A should that measurement be performed [4]. However, the outcome of σ_y^A can also be predicted with certainty by measurement of σ_y^B at B. EPR's premises therefore assert the existence of two hidden variables λ_z^A and λ_y^A, which simultaneously predetermine the outcomes of σ_z and σ_y for particle A. This description is not compatible with any quantum state $|\psi\rangle$ for the spin-1/2 system A. Hence, we arrive at an EPR paradox, where the EPR assertions imply an inconsistency with the completeness of quantum mechanics.

The above conclusions based on just two spin directions draw on the assumption that the system A is described quantum mechanically as a spin-1/2 system. For such a system, the Pauli spin variances defined by $(\Delta\sigma_i)^2 = \langle\sigma_i^2\rangle - \langle\sigma_i\rangle^2$ satisfy the uncertainty relation $(\Delta\sigma_x)^2 + (\Delta\sigma_y)^2 + (\Delta\sigma_z)^2 \geqslant 2$ [40]. Since $(\Delta\sigma_z)^2 \leqslant 1$, this implies [40]

$$(\Delta\sigma_y)^2 + (\Delta\sigma_z)^2 \geqslant 1. \tag{14}$$

Hence, for a quantum-state description of the system A, the values of σ_y and σ_z cannot be simultaneously precisely defined. A realization has been given for two spin-1/2 particles (photons) which showed near-perfect correlation for both of two orthogonal spins (orthogonal linear polarizations) [16].

2. Three-spin version

A stricter argument not dependent on the assumption of a spin-1/2 system is possible, if the experimentalist can measure the correlation of all three spin components [5,17,41]. Consider the spin-x measurements σ_x^A and σ_x^B. The eigenstates of σ_x are

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z + |\downarrow\rangle_z),$$
$$|\downarrow\rangle_x = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z - |\downarrow\rangle_z). \quad (15)$$

The state $|\psi_B\rangle$ [Eq. (3)] becomes in the spin-x basis:

$$|\psi_B\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle_x|\uparrow\rangle_x - |\uparrow\rangle_x|\downarrow\rangle_x). \quad (16)$$

The spin-x outcomes at A and B are also anticorrelated. According to the EPR premises, it is therefore possible to assign a hidden variable λ_x^A to the system A that predetermines the outcome of the measurement σ_x^A. This implies that the system A would at any time be described by three precise values, λ_x^A, λ_y^A, and λ_z^A, which predetermine the outcomes of measurements σ_x, σ_y, and σ_z, respectively. Each of λ_x, λ_y, and λ_z has the value $+1$ or -1. Since always $|\lambda_z^A| = 1$, such a hidden variable description cannot be given by a local quantum state $|\psi\rangle$ of A, since that would contradict the quantum uncertainty relation

$$\Delta\sigma_x\Delta\sigma_y \geqslant |\langle\sigma_z\rangle|, \quad (17)$$

which applies to *all* quantum states. Hence, we arrive at an EPR paradox.

B. Macroscopic EPR-Bohm paradox based on deterministic macroscopic realism

Consider the set-up depicted in Fig. 6, based on a macroscopic Bell state $|\psi_B\rangle$ [Eq. (3)] where the states $|\uparrow\rangle$ and $|\downarrow\rangle$ are macroscopically distinct, so that outcomes $+1$ and -1 imply macroscopically distinct states for the system defined at time t_1. Pseudospin observables are defined based on the two orthogonal states

$$S_z^A = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|,$$
$$S_x^A = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|,$$
$$S_y^A = \frac{1}{i}(|\uparrow\rangle\langle\downarrow| - |\downarrow\rangle\langle\uparrow|). \quad (18)$$

Examples are given in Sec. IV B and Appendix A, where the states $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ are $|\alpha\rangle$ and $|-\alpha\rangle$, or else $|\uparrow\rangle^{\otimes N}$ and $|\downarrow\rangle^{\otimes N}$, which can be regarded as macroscopically distinct for large α and N. The system is prepared in the macroscopic Bell state $|\psi_B\rangle$, called an entangled "cat" state, in such a way that at an initial time t_1, it is prepared for the pointer stage of the measurement S_z.

Deterministic macroscopic realism (dMR) incorporates the original EPR Assertions I and II as dMR(2) and dMR(3), applying to the macroscopic system (Sec. II B). The locality Assertion dMR(2) is more convincing than EPR's original locality assertion, since with dMR(2) it is only assumed there can be no *macroscopic* change to the outcome at A, due to

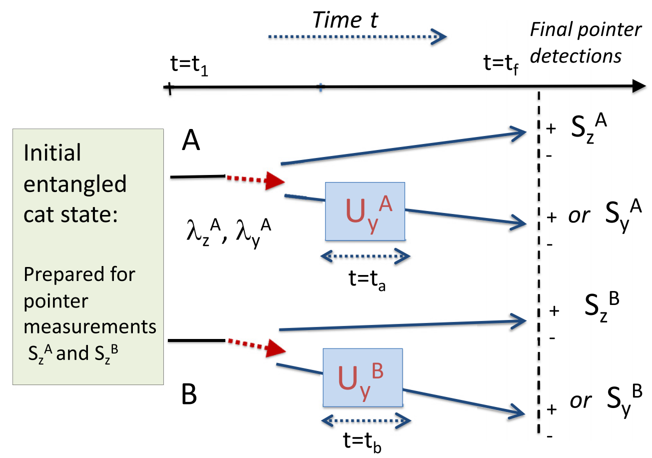

FIG. 6. Macroscopic version of the EPR-Bohm paradox: The system is prepared in a macroscopic entangled superposition state (a cat state) where the spin states associated with each basis are considered macroscopically distinct. At each site A and B, a switch (dashed arrow) allows the independent and random choice to evolve the systems by U_y or not. The initial preparation is such that with no evolution, S_z is measured. If the rotation U_y takes place, S_y is measured. The outcomes for S_z^A and S_z^B (and S_y^A and S_y^B) are anticorrelated.

the measurement at B. An EPR-Bohm argument follows as in Sec. III A, to describe a macroscopic EPR-Bohm paradox. The application of dMR(2) and dMR(3) to the system where both sets of spin states ($|\uparrow\rangle_y$ and $|\downarrow\rangle_y$ as well as $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$) are macroscopically distinct leads to the conclusion that system A is described simultaneously by two hidden variables λ_z^A and λ_y^A at the time t_1 (and similarly a variable λ_x^A exists to determine the outcomes of S_x^A). The macroscopic EPR-Bohm paradox therefore indicates inconsistency between dMR and the completeness of quantum mechanics.

The realization of the macroscopic EPR-Bohm paradox requires, by analogy with the microscopic example, applying the transformation given by (12) for U_y,

$$U_y^{-1}|\uparrow\rangle_z \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle),$$
$$U_y^{-1}|\downarrow\rangle_z \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle), \quad (19)$$

which creates macroscopic eigenstates $|\uparrow\rangle_y$ and $|\downarrow\rangle_y$ according to (12). The transformation U_x given by (15),

$$U_x^{-1}|\uparrow\rangle_z \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle),$$
$$U_x^{-1}|\downarrow\rangle_z \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle). \quad (20)$$

is necessary for the three-spin paradox. The transformations are more difficult to achieve, because the spin states are macroscopically distinct. Realizations are given in Sec. IV and Appendices A and B.

A subtlety is that the application of the dMR premises to the setup requires that the states $|\uparrow\rangle_y$ and $|\downarrow\rangle_y$ distinguished by the measurement S_y be regarded as macroscopically distinct at the time t_1 (Fig. 6). The eigenstates $|\uparrow\rangle_y$ and $|\downarrow\rangle_y$ can be represented as superpositions, $|\uparrow\rangle_z + i|\downarrow\rangle_z$ and $|\uparrow\rangle_z - i|\downarrow\rangle_z$ of the macroscopically distinct states $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$. It is hence argued that the $|\uparrow\rangle_y$ and $|\downarrow\rangle_y$ represented by the different probability amplitudes (i or $-i$) can also be regarded as macroscopically distinct.

In summary, the macroscopic EPR-Bohm paradox illustrates inconsistency between dMR and the notion that quantum mechanics is a complete theory. The question becomes whether dMR can be falsified directly, thus undermining the logic of the macroscopic EPR-Bohm argument. Leggett and Garg motivated tests of macroscopic realism [26]. However, in order to establish a test, the additional assumption of noninvasive measurability was introduced. Therefore, reports of violations of Leggett-Garg inequalities (see, e.g., Refs. [42–47]) do not imply falsification of macroscopic realism. Rather the violations falsify the combined premises of "macrorealism." We show below how dMR can be falsified in a macroscopic GHZ setup using multimode spin states.

C. Greenberger-Horne-Zeilinger nonlocality

The Greenberger-Horne-Zeilinger (GHZ) argument shows that EPR's assertions can be falsified if quantum mechanics is correct [4,11–13]. The argument is well known. The GHZ state

$$|\psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z|\uparrow\rangle_z|\uparrow\rangle_z - |\downarrow\rangle_z|\downarrow\rangle_z|\downarrow\rangle_z) \quad (21)$$

involves three spatially separated spin-1/2 systems A, B, and C. We denote the Pauli spin measurement σ_θ on system J by σ_θ^J, where $J \in \{A, B, C\}$, and we consider $\theta \in \{x, y, z\}$. The eigenstates of σ_θ^J are denoted $|\uparrow\rangle_{\theta,J}$ and $|\downarrow\rangle_{\theta,J}$, but we write $|m\rangle_{\theta,A}|m'\rangle_{\theta',B}|m''\rangle_{\theta'',C} \equiv |m\rangle_\theta|m'\rangle_{\theta'}|m''\rangle_{\theta''}$ dropping the subscripts J for convenience. Consider measurements of σ_x at each site. To obtain the predicted outcomes according to quantum mechanics, we rewrite in the x basis,

$$|\psi_{\text{GHZ}}\rangle = \tfrac{1}{2}(|\downarrow\rangle_x|\uparrow\rangle_x|\uparrow\rangle_x + |\uparrow\rangle_x|\downarrow\rangle_x|\uparrow\rangle_x + |\uparrow\rangle_x|\uparrow\rangle_x|\downarrow\rangle_x + |\downarrow\rangle_x|\downarrow\rangle_x|\downarrow\rangle_x). \quad (22)$$

From this we see that $\langle\sigma_x^A\sigma_x^B\sigma_x^C\rangle = -1$. Now we also consider the measurement $\sigma_x^A\sigma_y^B\sigma_y^C$ on the system in the GHZ state. The GHZ state in the spin-y basis is

$$|\psi_{\text{GHZ}}\rangle = \tfrac{1}{2}(|\uparrow\rangle_x|\uparrow\rangle_y|\uparrow\rangle_y + |\downarrow\rangle_x|\downarrow\rangle_y|\uparrow\rangle_y + |\downarrow\rangle_x|\uparrow\rangle_y|\downarrow\rangle_y + |\uparrow\rangle_x|\downarrow\rangle_y|\downarrow\rangle_y). \quad (23)$$

This shows $\langle\sigma_x^A\sigma_y^B\sigma_y^C\rangle = 1$. Similarly, $\langle\sigma_y^A\sigma_x^B\sigma_y^C\rangle = 1$ and $\langle\sigma_y^A\sigma_y^B\sigma_x^C\rangle = 1$.

The outcome for σ_x at A can be predicted with certainty by performing measurements σ_x^B and σ_x^C. The measurements do not disturb the system A because the system at A and those at B and C are spacelike-separated. Similarly, the outcome for σ_y^A can be predicted, without disturbing the system A, by measurements of σ_y^B and σ_x^C. EPR's assertions imply hidden variables λ_x^A and λ_y^A that can be simultaneously ascribed to system A, these variables predetermining the outcome for measurements σ_x^A and σ_y^A at A. The variables assume values of $+1$ or -1. A similar argument can be made for particles B and C. The contradiction with EPR's assertions arises because the product $\lambda_x^A\lambda_x^B\lambda_x^C$ must equal -1 in order that the prediction $\langle\sigma_x^A\sigma_x^B\sigma_x^C\rangle = -1$ holds. Similarly, $\lambda_x^A\lambda_y^B\lambda_y^C = \lambda_y^A\lambda_x^B\lambda_y^C = \lambda_y^A\lambda_y^B\lambda_x^C = 1$ in order that the prediction $\langle\sigma_x^A\sigma_y^B\sigma_y^C\rangle = \langle\sigma_y^A\sigma_x^B\sigma_y^C\rangle = \langle\sigma_y^A\sigma_y^B\sigma_x^C\rangle = 1$ holds. Yet, we see algebraically that $\lambda_x^A\lambda_x^B\lambda_x^C = \lambda_x^A\lambda_x^B\lambda_x^C(\lambda_y^B)^2(\lambda_y^A)^2(\lambda_y^C)^2$, and hence

$$\lambda_x^A\lambda_x^B\lambda_x^C = (\lambda_x^A\lambda_y^B\lambda_y^C)(\lambda_x^B\lambda_y^A\lambda_y^C)(\lambda_x^C\lambda_y^B\lambda_y^A) = 1, \quad (24)$$

which leads to the prediction $\langle\sigma_x^A\sigma_x^B\sigma_x^C\rangle = 1$, the opposite sign to that of quantum mechanics. This gives an "all or nothing" contradiction between quantum mechanics and predictions based on EPR's Assertions. The conclusion (based on experiments) is that EPR's assertions (or at least one of them) do not hold.

D. Macroscopic Greenberger-Horne-Zeilinger nonlocality

The GHZ argument becomes macroscopic when the spins $|\uparrow\rangle$ and $|\downarrow\rangle$ correspond to macroscopically distinct states. The macroscopic setup begins with the preparation at time t_1 of the GHZ state:

$$|\psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{z,A}|\uparrow\rangle_{z,B}|\uparrow\rangle_{z,C} - |\downarrow\rangle_{z,A}|\downarrow\rangle_{z,B}|\downarrow\rangle_{z,C}), \quad (25)$$

where $|\uparrow\rangle_{z,J} \equiv |\uparrow\rangle_{z,J}^{\otimes N}$ and $|\downarrow\rangle_{z,J} \equiv |\downarrow\rangle_{z,J}^{\otimes N}$ are multimode spin states defined as eigenstates of $S_z^J = \prod_{k=1}^N \sigma_z^k$ with eigenvalues $+1$ and -1, respectively. Here, $J \equiv A, B, C$ denotes the site. The system is prepared at t_1 for a pointer measurement of $S_z^A S_z^B S_z^C$.

One considers measurements of $S_x^A S_x^B S_x^C$ and $S_x^A S_y^B S_y^C$. By analogy with the microscopic example, this involves applying the transformations U_x or U_y given by (20) and (19) at each site. [The transformations define macroscopic eigenstates $|\uparrow\rangle_{y,J}$, $|\downarrow\rangle_{y,J}$, $|\uparrow\rangle_{x,J}$ and $|\downarrow\rangle_{x,J}$ in terms of $|\uparrow\rangle_{z,J}$ and $|\downarrow\rangle_{z,J}$ according to (12) and (15).] After the interactions U_x^A, U_x^B, and U_x^C, the system is prepared for the pointer measurement of $S_x^A S_x^B S_x^C$. The state in the new basis is expressed as Eq. (22), where the new eigenstates $|\uparrow\rangle_{x,J}$ and $|\downarrow\rangle_{x,J}$ (abbreviated as $|\uparrow\rangle_x$ and $|\downarrow\rangle_x$) are now regarded as macroscopically distinct. The product of the spins is $S_x^A S_x^B S_x^C = -1$. Details of a physical example of the necessary transformations using CNOT gates are given in the Appendix A.

If we evolve the state (25) with U_x^A, U_y^B, and U_y^C given by (20) and (19), the system is prepared for a pointer measurement of $S_x^A S_y^B S_y^C$. In the new basis, the state is given as Eq. (23), where the states $|\uparrow\rangle_{y,J}$ and $|\downarrow\rangle_{y,J}$ (abbreviated as $|\uparrow\rangle_y$ and $|\downarrow\rangle_y$) are now regarded as macroscopically distinct. Always, $S_x^A S_y^B S_y^C = 1$. Similarly, we consider $S_y^A S_y^B S_x^C$ and $S_y^A S_x^B S_y^C$ and arrive at the GHZ contradiction, as for the microscopic case.

The macroscopic GHZ setup enables a falsification of dMR and hence is a stronger version of the GHZ experiment. This is because the states $|\uparrow\rangle_{z,J} \equiv |\uparrow\rangle_{z,J}^{\otimes N}$ and $|\downarrow\rangle_{z,J} \equiv |\downarrow\rangle_{z,J}^{\otimes N}$ are macroscopically distinct for large N. Applying the justification given in Sec. III B that the eigenstates $|\uparrow\rangle_{y,J}$ and $|\downarrow\rangle_{y,J}$ (and $|\uparrow\rangle_{x,J}$ and $|\downarrow\rangle_{x,J}$) are also macroscopically distinct, the hidden variables λ_x^A, λ_y^A, λ_x^B, λ_y^B, λ_x^C, λ_y^C defined for the GHZ system in Sec. III C are hence deduced based on dMR. The setup is as in Fig. 7, where switches control whether S_x^A or S_y^A will be inferred at A by measuring either $S_x^B S_x^C$ or $S_y^B S_y^C$. This

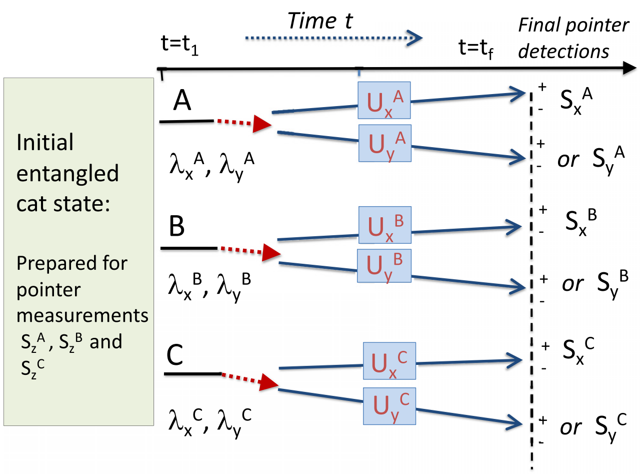

FIG. 7. Setup for the GHZ paradox with cat states based on multimode spin states. The outcome of S_x (and S_y) at each of the sites A, B, and C can be predicted with certainty by choosing certain measurements at the other two sites. Deterministic macroscopic realism (dMR) implies the outcomes are predetermined by variables λ_x and λ_y at the time t_1, as indicated on the diagram, which leads to the GHZ contradiction. The GHZ contradiction is hence a falsification of dMR.

is a strong result, giving an "all or nothing" contradiction with the assertions of dMR.

A macroscopic GHZ experiment which validates the predictions of quantum mechanics thus falsifies dMR. Macroscopic GHZ tests have been previously proposed [48–51] but, referring to multidimensional systems, these tests have not directly addressed the macroscopic distinction between the spin states. The falsification of dMR undermines the macroscopic EPR-Bohm argument given in Sec. III B for the incompleteness of quantum mechanics, which is based on the assumption that these premises are valid.

IV. EPR PARADOX BASED ON WEAK MACROSCOPIC REALISM AND WEAK LOCAL REALISM

An EPR-Bohm argument for the incompleteness of quantum mechanics can be formulated based on the weak macroscopic realism (wMR) premises (Secs. II C and II D). An identical argument follows based on the weak local realism (wLR) premises (Sec. II E). Both arguments are possible because of the additional first premise wMR(1) [or wLR(1)], which posits a "realism" for the system *after* it has been prepared with respect to the measurement basis, for the pointer stage of measurement. The importance of the arguments presented here is that the premises wMR and wLR are not falsifiable by the Bell or GHZ experiments.

A. The modified EPR argument

The modified argument is depicted in Fig. 2. The system at time t_0 is prepared in the Bell state

$$|\psi_B\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z|\downarrow\rangle_z - |\downarrow\rangle_z|\uparrow\rangle_z)$$

of Eq. (3). The preparation is such that the systems A and B are both prepared for the pointer measurement of spins S_z^A and S_z^B. For the application of wLR, the standard Bell state created from polarized photons can be used. For the application of wMR, $|\psi_B\rangle$ requires macroscopically distinct spin states. The argument based on wMR is stronger because the premise wMR(1) is less restrictive than wLR(1), considering only the application of realism to a macroscopic superposition state, as opposed to arbitrary superpositions. The wMR argument, however, requires realization of the unitary operations (20) and (19). Below, we explain the modified EPR argument that is based on wLR. The argument based on wMR is identical provided the unitary operations are selected appropriately.

The modified EPR argument is as follows. After the initial preparation, the measurement setting of system B is adjusted by applying the unitary operation U_y^B at that site. After the interaction U_y^B, the system B is prepared for the pointer measurement of σ_y^B (denoted S_y^B in Fig. 2). From the anticorrelation of state $|\psi_B\rangle$ given by Eq. (13) as written in the spin-y basis, the outcome for σ_y^A (i.e., S_y^A) *can be predicted with certainty* by measurement on system B. This constitutes Schrödinger's "indirect measurement" of σ_y^A (i.e., S_y^A) [19,20]. Therefore, we can apply EPR's criterion for reality but weakened according to the definitions of wLR: By Assertion wLR(3), the system A at time $t_{f_B} = t_f$ *after* U_y^B has been performed can be ascribed a definite value λ_y^A for the outcome of S_y^A. We note that a further unitary interaction U^A is required at A after time t_f if the measurement σ_y^A is to be carried out. Regardless, the final outcome for σ_y^A is already determined at time t_{f_B} by the value λ_y^A. This inferred variable is depicted as λ_y^A (in black) in Fig. 2.

However, at the time t_f, the system A is *itself* prepared for a pointer measurement of σ_z^A (i.e., S_z^A). Hence, by Assertion wLR(1), there is a hidden variable λ_z^A that predetermines the value for the pointer measurement σ_z^A should it be performed. This constitutes Schrödinger's "direct measurement," of σ_z^A (i.e., S_z^A) [19,20]. This variable is depicted as λ_z^A (in red) in Fig. 2. According to the premises, the system A at the time t_f therefore can be ascribed *two* definite spin values, λ_z^A and λ_y^A. This assignment cannot be given by any localized quantum state for a spin-1/2 system, and hence the argument can be put forward, as for the original EPR-Bohm argument (Sec. III A), that quantum mechanics is incomplete.

The Assertion wMR(1) when applied to the spin system might be regarded as dubious. This is the motivation for a macroscopic version of the modified paradox, where the assertion is replaced by that of *macroscopic realism,* wMR(1). However, Assertion wLR(1) is *not* falsified by the violation of a Bell inequality. The quantum correlations of the Bell and GHZ paradoxes can be consistent with wLR (and similarly, wMR) because the systems are prepared for a pointer measurement of S_z at *one* time t_1 and then prepared for a pointer measurement of S_θ at a *later* time t_f after the relevant unitary operation U_θ. The hidden variables for the modified EPR-Bohm paradox are tracked in Figs. 2 and 8. The premise wMR(1) does not assert that at the time t_f the value of an arbitrary third measurement S_φ is determined, because the unitary operation U that fixes the measurement setting φ has not yet been performed at site A or B.

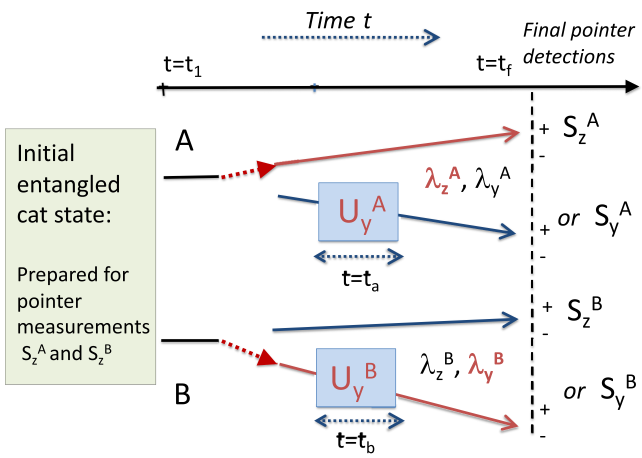

FIG. 8. A macroscopic EPR-Bohm paradox based on weak macroscopic realism (wMR): The system is prepared in the state (26). At time t_f, the settings are fixed, depicting the choice to measure S_z^A and S_y^B. The unitary operations U_Y involve interacting the system with a nonlinear medium for a given time (Fig. 9). The outputs at t_f are macroscopic superposition states with respect to the chosen measurement basis at each site. Hence, wMR(1) ascribes to system A at time t_f a predetermined value λ_z^A for the outcome for S_z^A, since the system has been prepared for a pointer measurement of S_z^A. The outcomes for S_y^A and S_y^B are anticorrelated. Hence, wMR(3) ascribes a hidden variable λ_y^A for system A that predetermines the outcome S_y^A. Hence, there is a two-spin EPR-Bohm paradox based on the premises of wMR.

For a bipartite system, it is the introduction of a third measurement setting that leads to the falsification of the original EPR premises (refer Sec. II A), and of local hidden variable theories in general. This is evident by the Bell tests, which require three or more different measurement settings [2,21].

In an ideal experiment, it would be demonstrated that the result of S_y^A can be inferred from the measurement at B with certainty. Requirements for a realistic experiment are given in Appendices C and D. In Appendix D, conditions are derived for the demonstration of the paradox where the inference of S_y^A cannot be made with certainty. It would also be necessary to establish that system A *is* given quantum mechanically as a spin-1/2 system. In the macroscopic version based on wMR, a third criterion is to justify that the spin states for systems A and B are macroscopically distinct. Note there is no actual violation of the uncertainty principle because the quantum state defined at the time t_f differs from the quantum state defined *after* the further interaction necessary to change the measurement setting from z to y at A.

Comment. The modified EPR-Bohm argument is based on the two-spin version of the EPR-Bohm paradox. The three-spin version cannot be formulated using wLR, because this would require preparation for three "pointer" measurements, which is not possible for the bipartite system. The two-spin version paradox presented above is based on wLR (or wMR) which has not been falsified. On the other hand, the two-spin version of the EPR-Bohm paradox allows a counterargument against the incompleteness of quantum mechanics: It could be proposed that a local quantum state description is possible for A, but that the description is more complex than for a spin-1/2 system.

B. Cat state proposal for the paradox

We now present a proposal for a realization of the two-spin macroscopic EPR-Bohm paradox (Fig. 8). Here, the spin states are realized using as a basis two macroscopically distinct coherent states. The unitary operations that fix the measurement settings preserve the macroscopic two-state nature of the system and are realized by Kerr interactions. Two other realizations which enable mesoscopic realizations for both the two- and three-spin EPR-Bohm paradoxes and the GHZ nonlocality are presented in Appendices A and B.

We consider the system to be prepared at time t_1 in the entangled cat state [52,53]

$$|\psi_{\rm Bell}\rangle = \mathcal{N}(|\alpha\rangle|-\beta\rangle - |-\alpha\rangle|\beta\rangle). \tag{26}$$

Here $|\alpha\rangle$ and $|\beta\rangle$ are coherent states for single-mode fields A and B, and we take α and β to be real, positive, and large. $\mathcal{N} = \frac{1}{\sqrt{2}}\{1-\exp(-2|\alpha|^2-2|\beta|^2)\}^{-1/2}$ is the normalization constant. The phases of the coherent amplitudes α and β are defined as real relative to a fixed axis, which is usually defined by a phase specified in the preparation process. For example, this is may be fixed by the phase of a pump field, as in the coherent-state superpositions generated by nonlinear dispersion [54].

For each system A and B, one may measure the field quadrature phase amplitudes $\hat{X}_A = \frac{1}{2}(\hat{a}+\hat{a}^\dagger)$, $\hat{P}_A = \frac{1}{2i}(\hat{a}-\hat{a}^\dagger)$, $\hat{X}_B = \frac{1}{2}(\hat{b}+\hat{b}^\dagger)$, and $\hat{P}_A = \frac{1}{2i}(\hat{a}-\hat{a}^\dagger)$, which are defined in a rotating frame [54]. The boson destruction mode operators for modes A and B are denoted by $\hat{a}$ and $\hat{b}$. As $\alpha\to\infty$, the probability distribution $P(X_A)$ for the outcome X_A of the measurement $\hat{X}_A$ consists of two distinct Gaussians, associated with the distributions for the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$. (A central component due to interference vanishes for large α,β). Hence, the outcome X_A distinguishes between the states $|\alpha\rangle$ and $|-\alpha\rangle$. Similarly, $\hat{X}_B$ distinguishes between the states $|\beta\rangle$ and $|-\beta\rangle$. We define the outcome of the "spin" measurement $\hat{S}^A$ to be $S^A=+1$ if $X_A\geqslant 0$, and -1 otherwise. Similarly, the outcome of the measurement $\hat{S}^B$ is $S^B=+1$ if $X_B\geqslant 0$, and -1 otherwise. The result is identified as the spin of the system i.e. the qubit value. For each system, the coherent states become orthogonal in the limit of large α and β, in which case the superposition (26) maps to the two-qubit Bell state

$$|\psi_{\rm Bell}\rangle = \frac{1}{\sqrt{2}}(|+\rangle_z|-\rangle_z - |-\rangle_z|+\rangle_z)$$

given by (3), where we write $|\uparrow\rangle$ ($|\downarrow\rangle$) as $|+\rangle$ ($|-\rangle$). At time t_1, the outcomes S^A and S^B are anticorrelated.

To realize the EPR-Bohm paradox, it is necessary to identify the noncommuting spin observables and the appropriate unitary rotations U at each site required to measure these. For this purpose, we examine the systems A and B as they evolve independently according to local transformations $U_A(t_a)$ and $U_B(t_b)$, defined as

$$U_A(t_a) = e^{-iH_{NL}^A t_a/\hbar}, \quad U_B(t_b) = e^{-iH_{NL}^B t_b/\hbar}, \tag{27}$$

where

$$H_{NL}^A = \Omega\hat{n}_a^k, \quad H_{NL}^B = \Omega\hat{n}_b^k. \tag{28}$$

Here, t_a and t_b are the times of evolution at each site, $\hat{n}_a = \hat{a}^\dagger\hat{a}$, $\hat{n}_b = \hat{b}^\dagger\hat{b}$, and Ω is a constant. We consider $k=2$. The

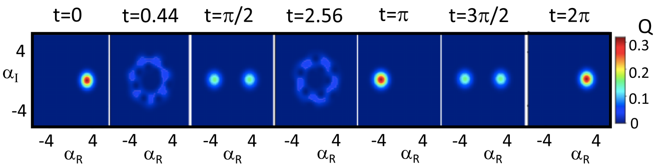

FIG. 9. The dynamics associated with the unitary operation determining the measurement setting. For the subset of times when the system is in a macroscopic superposition state, weak macroscopic realism can be applied, as in the ball-in-a-box model given by Fig. 3. The system A prepared in a coherent state $|\alpha\rangle$ evolves according to H_{NL}^{A} of Eq. (28). Shown are the contour plots for the Q function $Q(\alpha_0)$ of the single-mode field ($\alpha_0=\alpha_R+i\alpha_I$). Here, $\alpha=4$ and time is in units of Ω^{-1}. After time $t=\pi/2$, the system is in the cat state $U_A|\alpha\rangle\equiv U_{\pi/4}^{A}\equiv U_A(\pi/2\Omega)|\alpha\rangle$ [Eq. (29)]. The operation $(U_{\pi/4}^{A})^{-1}$ gives the rotation of basis necessary for measurement of spin $S_y^A\equiv\sigma_y^A$. We find $U_A^{-1}\equiv(U_{\pi/4}^{A})^{-1}=U_A(3\pi/2\Omega)$.

dynamics of this evolution is well known [54–57] and is depicted in Fig. 9 in terms of the Husimi Q function, defined for the M-mode quantum state ρ as $Q(\alpha_0)=\langle\alpha_0|\rho|\alpha_0\rangle/\pi^M$ where $|\alpha_0\rangle$ is a coherent state [58]. If the system A is prepared in a coherent state $|\alpha\rangle$, then after a time $t_a=\pi/2\Omega$ the state of the system A becomes [54]

$$U_{\pi/4}^{A}|\alpha\rangle=e^{-i\pi/4}(|\alpha\rangle+i|-\alpha\rangle)/\sqrt{2}. \tag{29}$$

Here we define $U_{\pi/4}^{A}=U_A(\pi/2\Omega)$. The state $U_{\pi/4}^{A}|\alpha\rangle$ is a superposition of two macroscopically distinct states and is referred to as a "cat state" after Schrödinger's paradox [20,59]. After a time $t_a=3\pi/2\Omega$, the evolved state is

$$U_{3\pi/4}^{A}|\alpha\rangle=e^{i\pi/4}(|\alpha\rangle-i|-\alpha\rangle)/\sqrt{2}. \tag{30}$$

Similar transformations $U_{\pi/4}^{B}$ are defined at B for $t_b=\pi/2\Omega$ and $t_b=3\pi/2\Omega$. Further interaction for the whole period $t_a=2\pi/\Omega$ returns the system to the coherent state $|\alpha\rangle$ (Fig. 9) [54].

The macroscopic version of the EPR-Bohm paradox is depicted in Fig. 8. We consider the spin-1/2 observables S_z, S_x, and S_y defined by Eq. (18) for orthogonal states $|\pm\rangle$ of a two-level system, which we also denote by $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$. Here, we identify the eigenstates $|\pm\rangle$ of S_z^A (S_z^B) as the coherent states $|\pm\alpha\rangle$ ($|\pm\beta\rangle$), respectively, with α and β real, and in the limit of large α and β where orthogonality is justified. In this limit, we define

$$\begin{aligned}S_z^A&=|\alpha\rangle\langle\alpha|-|-\alpha\rangle\langle-\alpha|,\\ S_x^A&=|\alpha\rangle\langle-\alpha|+|-\alpha\rangle\langle\alpha|,\\ S_y^A&=\frac{1}{i}(|\alpha\rangle\langle-\alpha|-|-\alpha\rangle\langle\alpha|)\end{aligned} \tag{31}$$

for system A. The scaling corresponds to Pauli spins $\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$. The spins S_z^B, S_x^B, and S_y^B for system B are defined in identical fashion on replacing α with β. We have omitted operator "hats" where the meaning is clear.

The EPR-Bohm paradox requires measurement of S_z^A and S_z^B. The system in the state (26) is *prepared for the pointer stage of the measurements* of S_z. This is because, for this system, a measurement of (the sign of) $\hat{X}_A$ and $\hat{X}_B$ is all that is required to complete the S_z^A and S_z^B measurement. The local pointer measurement constitutes an optical homodyne, in which the fields are combined with a strong field across a beam splitter with a relative phase shift ϑ, followed by direct detection in the arms of the beam splitter [54]. Here, ϑ is chosen to measure $\hat{X}_A$ ($\hat{X}_B$), the axis so that α (β) is real. The ϑ is defined by the preparation process.

The EPR-Bohm argument also requires measurements of S_y^A and S_y^B on the Bell state (26) prepared at time t_1 (Fig. 6). Here, it is required to adjust the measurement setting by applying a local unitary transformation U_y at each site. The eigenstates of S_y are $|\uparrow\rangle_y=(|\uparrow\rangle_z+i|\downarrow\rangle_z)/\sqrt{2}$ and $|\downarrow\rangle_y=(|\uparrow\rangle_z-i|\downarrow\rangle_z)/\sqrt{2}$, but the normalization can vary by a phase factor. We can abbreviate as $|\pm\rangle_y=\frac{1}{\sqrt{2}}(|\pm\rangle+i|\mp\rangle)$, denoting $|\uparrow\rangle$ as $|+\rangle$, and $|\downarrow\rangle$ as $|-\rangle$, interchangeably. We choose

$$\begin{aligned}|\uparrow\rangle_y&=\frac{e^{-i\pi/4}}{\sqrt{2}}(|\uparrow\rangle_z+i|\downarrow\rangle_z),\\ |\downarrow\rangle_y&=\frac{e^{-i\pi/4}}{\sqrt{2}}(|\downarrow\rangle_z+i|\uparrow\rangle_z)\\ &=\frac{e^{i\pi/4}}{\sqrt{2}}(|\uparrow\rangle_z-i|\downarrow\rangle_z).\end{aligned} \tag{32}$$

We temporarily drop for convenience the superscripts and subscripts indicating the A and B since the transformations are local and apply independently to both sites. It is readily verified that $S_y|\uparrow\rangle_y=|\uparrow\rangle_y$ and $S_y|\downarrow\rangle_y=|\downarrow\rangle_y$, i.e., $S_y^A|\pm\rangle_{y,A}=\pm|\pm\rangle_{y,A}$ and $S_y^B|\pm\rangle_{y,B}=\pm|\pm\rangle_{y,B}$.

Now we consider how to perform the measurement of S_y. As explained in Sec. II, the first stage of measurement involves a unitary operation U_y, giving a transformation to the measurement basis, so that the system is then prepared for the second stage of measurement, which is the pointer [stage of] measurement of S_y. The pointer stage constitutes a measurement of the sign S of $\hat{X}$, which for large α and β will (after U_y has been applied) directly yield the outcome ±1 for the system prepared in $|\pm\rangle_y$. To establish U_y, following the procedure leading to Eqs. (13) and (23), any state

$$|\psi\rangle=c_+|\uparrow\rangle_z+c_-|\downarrow\rangle \tag{33}$$

written in the z basis can be transformed into the y basis by the following substitutions:

$$\begin{aligned}|\uparrow\rangle_z&\rightarrow(e^{i\pi/4}|\uparrow\rangle_y+e^{-i\pi/4}|\downarrow\rangle_y)/\sqrt{2},\\ |\downarrow\rangle_z&\rightarrow-i(e^{i\pi/4}|\uparrow\rangle_y-e^{-i\pi/4}|\downarrow\rangle_y)/\sqrt{2}.\end{aligned} \tag{34}$$

This gives

$$|\psi\rangle=d_+|+\rangle_y+d_-|-\rangle_y, \tag{35}$$

where $d_\pm=(c_\pm\mp ic_-)e^{\pm i\pi/4}$. To obtain the transformed state (35), ready for the pointer stage of measurement of S_y, the system is thus evolved by applying the operation U_y at each site, where

$$U_y\equiv U_{\pi/4}^{-1}=U^{-1}(\pi/2\Omega)=U_{3\pi/4}, \tag{36}$$

as given by Eq. (30). We explain this result further in the Appendix E for clarity. The U_y is the inverse of the transformation $e^{-iH_{NL}t/\hbar}$ where $t=\pi/2\Omega$, given by (29). U_y is achieved by evolving the local system for a time $t=-\pi/2\Omega\equiv3\pi/2\Omega$ [Eq. (30)].

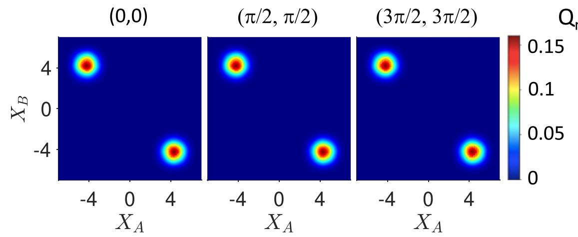

FIG. 10. The contour plots depict the state for systems A and B after evolution according to H_{NL}^A and H_{NL}^B at each site, for a time t_a and t_b, respectively. The systems are prepared in the entangled Bell cat state $|\psi_{\text{Bell}}\rangle$ [Eq. (26)] at time $t = 0$, where $\alpha = \beta$. The times are given as (t_a, t_b) above each snapshot. The contour plots are for the marginal $Q_m(X_A, X_B)$ of the Q function $Q(\alpha_0, \beta_0)$ of the quantum state, where $X_A = \text{Re}(\alpha_0)$ and $X_B = \text{Re}(\beta_0)$. Here, $\alpha = 4$, $\Omega = 1$. The spin S_z^A (S_z^B) of system A (B) is measured as the sign of $\hat{X}_A$ ($\hat{X}_B$) at (0, 0), so that a ± sign indicates a spin of ±1. The relative weightings of the peaks of Q_m indicate the joint probability for obtaining the spin outcomes. The spin S_y^A (S_y^B) is given by the sign of $\hat{X}_A$ ($\hat{X}_B$) after the evolution $U_Y^A = U_{3\pi/4}^A$ ($U_Y^B = U_{3\pi/4}^B$), corresponding to $t_a = 3\pi/2$ ($t_b = 3\pi/2$), which gives the rotation of basis necessary to measure S_y^A (S_y^B). We see that the outcomes of S_z^A and S_z^B (and of S_y^A and S_y^B) are anticorrelated. Hence, a two-spin EPR-Bohm paradox as in Figs. 6 and 8 is possible. The paradox is a signature of entanglement and is not observed for the mixed state ρ_{mix} given by Eq. (48).

Comment. The states $|+\rangle_y$ and $|-\rangle_y$ refer to the macroscopically distinct coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ defined at the time t *after* the local unitary rotation U_y has taken place. This is important in identifying the macroscopic nature of the paradox and justifies that the premises of wMR defined in Sec. II C will apply.

The macroscopic modified EPR-Bohm argument based on the entangled cat state (26) can now be formulated (Fig. 8). The measurements of S_z^A and S_z^B are made as pointer measurements at the time t_1 by measuring X_A and X_B, respectively. The anticorrelation is evident from the Bell cat state $|\psi_{\text{Bell}}\rangle$ (26) for large α and β. The measurements of S_y^A (S_y^B) are made by first applying the local unitary rotation U_y^A (U_y^B) to the Bell state prepared at t_1 followed by a measurement of the sign of X_A (X_B). The state of the system prepared after the unitary rotations U_y^A and U_y^B is also of the form of a Bell cat state. After a time $t_a = 3\pi/2\Omega$, the evolved state is [use Eq. (30)]

$$\begin{aligned} |\psi_{\text{Bell}}\rangle_{y,y} &= \frac{U_y^A U_y^B}{\sqrt{2}}(|\alpha\rangle|-\beta\rangle - |-\alpha\rangle|\beta\rangle) \\ &= \frac{i}{\sqrt{2}}(|-\alpha\rangle|\beta\rangle - |\alpha\rangle|-\beta\rangle) \\ &\rightarrow \frac{i}{\sqrt{2}}(|-\rangle_y|+\rangle_y - |+\rangle_y|-\rangle_y), \end{aligned} \tag{37}$$

where we have taken α, β large. Hence, as with the original paradox given in Sec. III A, there is an anticorrelation between S_y^A and S_y^B. The dynamics that generates the anticorrelation evident by (37) is depicted in the Fig. 10. An anticorrelation is present between the spins S_z^A and S_z^B and also between the spins S_y^A and S_y^B. These correlations form the basis for the two-spin EPR-Bohm paradox explained in Secs. III A 1 and III B based on dMR (Fig. 6), and also the modified two-spin version based on wMR, explained in Sec. IV A (Fig. 8).

The proposed two-spin paradox assumes that the system A is described in quantum mechanics as a spin-1/2 system, which is valid as $\alpha \rightarrow \infty$, where the two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ become orthogonal. The spin outcomes for S_z^A and S_z^B, and also for S_y^A and S_y^B, remain perfectly anticorrelated as $\alpha \rightarrow \infty$, so that this realization of the EPR-Bohm paradox holds in the macroscopic limit. An account of the effect of finite α is presented in Appendix C.

V. CONSISTENCY OF WEAK LOCAL REALISM AND WEAK MACROSCOPIC REALISM WITH BELL VIOLATIONS

It has been shown possible to falsify deterministic macroscopic (local) realism (dMR) for the cat-state system described in Sec. IV B [21]. This was demonstrated by a violation of Bell inequalities constructed for the cat states involving $|\alpha\rangle$ and $|-\alpha\rangle$. It has also been pointed out that weak macroscopic realism (wMR) is not falsified by the macroscopic Bell violations nor by violations of Leggett-Garg inequalities [21,22] and can be found consistent with Wigner friend paradoxes [24]. The Assertions of wLR are weaker (less restrictive) than those of Bell's local realistic theories [3,8] and do not imply Bell-CHSH inequalities. For clarity, we demonstrate below the consistency of wLR and wMR with Bell violations.

A. Consistency with Bell violations: Tracking the hidden variables

Pauli spin components given as

$$\begin{aligned} S_\theta^A &= S_x^A \sin\theta + S_z^A \cos\theta, \\ S_\phi^B &= S_x^B \sin\phi + S_z^B \cos\phi, \end{aligned} \tag{38}$$

can be measured by adjusting the analyzer (e.g., a Stern-Gerlach apparatus or polarizing beam splitter) for each of the spacelike-separated systems A and B. According to the EPR-Bohm argument based on the original EPR Assertions, if the composite system is in the Bell state (3), then each spin component S_θ^A and S_ϕ^B is represented by a hidden variable (λ_θ^A and λ_ϕ^B). This is because its value can be predicted with certainty by a measurement at the other site [2–4]. Defining the expectation value $E(\theta, \phi) = \langle S_\theta^A S_\phi^B\rangle$, this leads to the constraint $-2 \leqslant S \leqslant 2$, where

$$S = E(\theta, \phi) - E(\theta, \phi') + E(\theta', \phi) + E(\theta', \phi'), \tag{39}$$

known as the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality, which is violated for the Bell state (3) [3,7,8]. The violation therefore falsifies EPR's assertions. More generally, the violation shows failure of all local realistic theories defined as those satisfying Bell's hidden variable assumptions [7,8].

In Figs. 11 and 12, we track the hidden variables that predetermine the values of the spin measurements at each time, based on weak local realism (wLR). We illustrate without loss of generality with one possible time sequence, based on the preparation at the initial time for pointer measurements in the directions θ and ϕ. Assuming wLR(1), the values of S_θ^A and

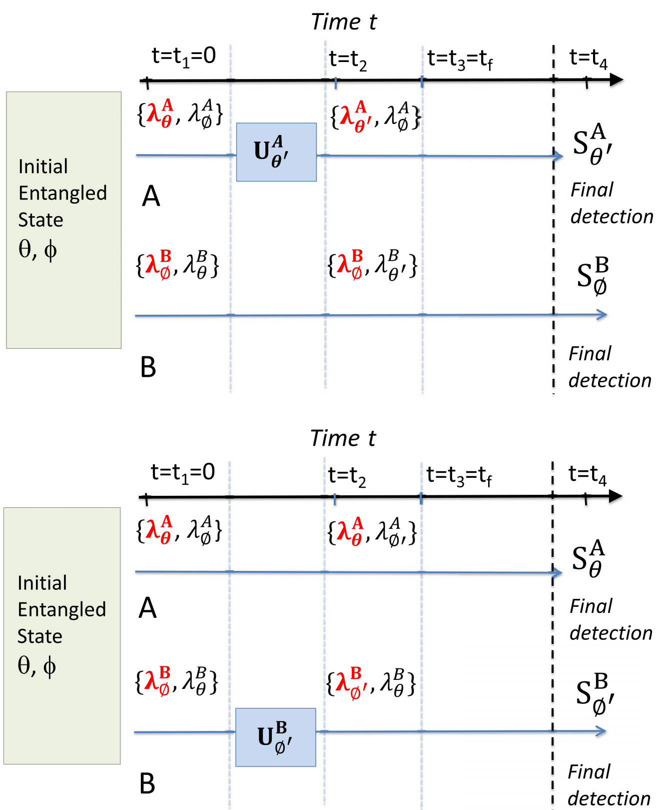

FIG. 11. Tracking the hidden variables according to weak local realism (wLR) through the dynamics of the Bell test, which shows violation of the Bell inequality (39) for the system prepared in the Bell state $|\psi_B\rangle$, Eq. (3). First, the moment $\langle S_\theta^A S_\phi^B\rangle$ is measured. We take the initial time t_1 to be after the passage through the analyzers set at θ (at A) and ϕ (at B), so that measurement settings θ and ϕ have been determined and the system prepared for the pointer stage of measurements S_θ^A and S_ϕ^B at this time. At each time t_i, wLR implies that certain hidden variables λ are valid, depending on the preparation of the system at that time. The hidden variables are depicted in the brackets. Those in red (bold) are implied by Assertion wLR(1). Those in black (not bold) are implied by Assertion wLR(3). To measure $\langle S_{\theta'} S_\phi\rangle$, for example, there is a further rotation $U_{\theta'}^A$ at A (top figure). According to wLR, the system A at time t_2 is described by a new hidden variable $\lambda_{\theta'}^A$, but the value for the outcome of S_ϕ remains specified by the variable λ_ϕ^B.

S_ϕ^B that are realized by the pointer stage of measurement (if made at that time) are predetermined and given by λ_θ^A and λ_ϕ^B at the time t_1. Hence,

$$E(\theta,\phi) = \langle \lambda_\theta^A \lambda_\phi^B\rangle. \tag{40}$$

The values λ_θ^A and λ_ϕ^B are depicted in red (bold) in the Figs. 11 and 12. To measure $E(\theta',\phi)$, there is a further rotation $U_{\theta'}^A$ at A (Fig. 11, top). At time t_2, the state is prepared for the pointer measurements of $S_{\theta'}^A$ and S_ϕ^B. The hidden variables $\lambda_{\theta'}^A$ and λ_ϕ^B specify the outcomes for those pointer measurements, should they be performed. Based on Assertion wMR(1), these variables are assigned to describe the state of the system at the time t_2. The prediction for wLR is

$$E(\theta',\phi) = \langle \lambda_{\theta'}^A \lambda_\phi^B\rangle. \tag{41}$$

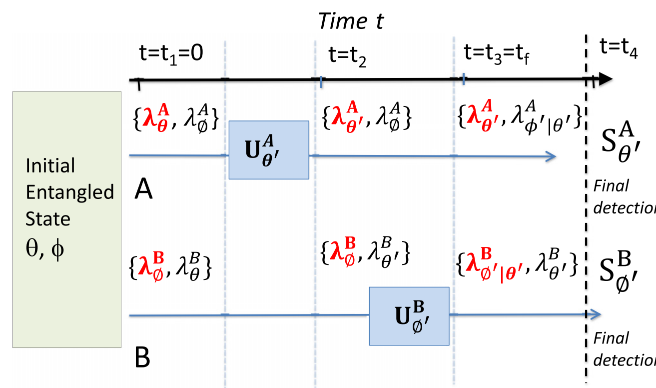

FIG. 12. Tracking the hidden variables through the dynamics of the Bell test, which shows violation of the Bell inequality. The description is as for Fig. 11, but here the moment $\langle S_{\theta'}^A S_{\phi'}^B\rangle$ is measured. We note that the conditioning for $\lambda_{\phi'}$ necessitates that a rotation $U_{\phi'}^B$ has also occurred at B, as well as $U_{\theta'}^A$. The nonlocality emerges after rotations at both sites.

We note that [because of the anticorrelation evident for spins prepared in the Bell state (3)], Assertion wLR(3) implies that the hidden variable λ_ϕ^B *also* specifies the outcome of a measurement S_ϕ^A, if performed on the system defined at time t_2.

Similarly, the measurements of S_θ^A and $S_{\phi'}^B$ require a further rotation $U_{\phi'}^B$ at B after the initial preparation at t_1, with no rotation at A (Fig. 11, lower). A variable $\lambda_{\phi'}^B$ is defined to give the outcome for $S_{\phi'}^B$ if that measurement were to be performed at t_2 after the rotation $U_{\phi'}^B$. Hence, wLR implies

$$E(\theta,\phi') = \langle \lambda_\theta^A \lambda_{\phi'}^B\rangle. \tag{42}$$

The difference between Bell's local hidden variable theories and the assertions of wLR are evident when considering measurement of $S_{\theta'}^A$ and $S_{\phi'}^B$. This measurement requires *two further rotations* after the preparation at time t_1, consistent with the nonlocality Result II D presented earlier. A possible sequence is given in Fig. 12. Suppose the rotation $U_{\theta'}^A$ is performed first, and at time t_2, the hidden variables defining the state are $\lambda_{\theta'}^A$ and λ_ϕ^B. The pointer measurements are not actually performed, and a rotation $U_{\phi'}^B$ is then made at B. The final state at time t_3 is given by hidden variables $\lambda_{\theta'}^A$ and $\lambda_{\phi'|\theta'}^B$. Here, we use subscripts $|\theta'$ to specify that $\lambda_{\phi'|\theta'}^B$ is the variable defined for the state specified at the time t_3, conditioned on the first rotation $U_{\theta'}^A$ at A. This is necessary in the context of a wLR model, because the premise of wLR specifies the necessity of a *partial* locality, defined for the hidden values of the *pointer* measurements only. The value $\lambda_{\theta'}^A$ is defined for the pointer measurement of $S_{\theta'}^A$, and is independent of the choice for ϕ'. The value $\lambda_{\theta'}^A$ is [according to wLR(2)] not affected by the unitary rotation $U_{\phi'}^B$ at the other site B, which comes *after* t_2. However, we cannot conclude from the wLR Assertions that the value of $\lambda_{\phi'}^B$ defined for the measurement $S_{\phi'}^B$ on the state after the rotation $U_{\theta'}^A$ is the same as that defined for pointer measurement $S_{\phi'}^B$ in Fig. 11, lower, where there was no rotation at A. Hence we write

$$E(\theta',\phi') = \langle \lambda_{\theta'}^A \lambda_{\phi'|\theta'}^B\rangle. \tag{43}$$

The conditional symbol $|\theta'$ indicates that the value λ_ϕ^B need not be independent of the operation $U_{\theta'}^A$, e.g., it may depend on θ' or the value of $\lambda_{\theta'}^A$. Hence, wLR does not imply the CHSH-Bell inequality, which is derived based on Bell's full locality assumption that $\lambda_{\phi'}^B$ is independent of the setting θ', i.e., is independent of whether the rotation $U_{\theta'}^A$ has been performed at A.

It is well known that where the values for λ_θ, λ_ϕ, $\lambda_{\theta'}$, and $\lambda_{\phi'}$ are either $+1$ or -1, and if Bell's locality is assumed so that $\lambda_{\phi'|\theta}^B = \lambda_{\phi'}^B$, then the value of S is bounded by -2 and $+2$, leading to the CHSH-Bell inequality [3]. However, where we consider $\lambda_{\phi'|\theta}^B$ to be an independent variable, $+1$ or -1, the bound for S becomes the algebraic bound of four. Hence, wLR does not constrain S to be bounded by the CHSH-Bell inequality.

B. Consistency of macroscopic Bell violations with weak macroscopic realism: The Bell-cat example

We now give an example of the dynamics associated with the choice of measurement setting in a proposed macroscopic Bell experiment, in order to illustrate the consistency with wMR. We summarize the violations of the Bell inequality predicted for the Bell cat state (26) in Ref. [21], showing how these violations negate dMR but are consistent with weak macroscopic realism (wMR). This illustrates Result II D. A similar summary is given in Ref. [24], which examines wMR in the context of Wigner's friend paradoxes.

Consider the system is prepared at time t_1 in the Bell-cat state [52]

$$|\psi_{\rm Bell}\rangle = \mathcal{N}(|\alpha\rangle|-\alpha\rangle - |-\alpha\rangle|\alpha\rangle) \tag{44}$$

of Eq. (26), where $\alpha = \beta$ is real and very large. As with the system studied in Sec. IV B, the local unitary operations equivalent to the rotations of analyzers in a Bell test are achieved by evolving the systems independently according to a nonlinear interaction. Here, the interactions are $H_{nl}^A = \hbar\Omega\hat{n}_A^4$ at A and $H_{nl}^B = \hbar\Omega\hat{n}_B^4$ at B. If system A is initially in the coherent state $|\alpha\rangle$, then after a time $t_a = t_{m+1} = mT$, where $m = 0, 1, 2, 3$ and $T = \pi/4\Omega$, the system A evolves under H_{nl}^A to [21]

$$U_{m\pi/8}^A|\alpha\rangle = e^{-im\pi/8}[\cos(m\pi/8)\,|\alpha\rangle + i\sin(m\pi/8)\,|-\alpha\rangle]. \tag{45}$$

A similar interaction at B leads to the transformation $U_{m'\pi/8}^B$, with an evolution time t_b. The interaction times t_a and t_b are independent and determine the local measurement setting for a Bell test. The unitary operations equivalent to the rotations of analyzers are $U_{m\pi/8}^A$ and $U_{m'\pi/8}^B$.

The spin measurement S_k^A ($S_{k'}^B$) is determined by the sign of the measured quadrature phase amplitude $\hat{X}_A$ ($\hat{X}_B$) *after* the interaction time $t_a = t_k$ ($t_b = t_{k'}$). The outcome is $+1$ or -1 if the sign is non-negative or negative, respectively. Here t_a, $t_b \in \{t_{m+1}, m = 0, 1, 2, 3\}$, so that S_1^A is the outcome measured after an interaction time $t_a = t_1 = 0$, S_2^A is measured after an interaction time $t_a = t_2 = \pi/4\Omega$, and S_3^A is measured after an interaction time of $t_a = t_3 = \pi/2\Omega$. Similarly, spins S_1^B, S_2^B, and S_3^B are measured after interactions times of $t_b = 0$, $t_b = \pi/4\Omega$, and $t_b = \pi/2\Omega$, respectively.

The Bell inequality (4) becomes [2]

$$-\langle S_1^A S_2^B\rangle + \langle S_1^A S_3^B\rangle - \langle S_2^A S_3^B\rangle \leqslant 1. \tag{46}$$

The inequality applies to the macroscopic case, for the Bell-cat state (26) provided there is the restriction to the measurements S_k^A and $S_{k'}^B$, as specified above. The assertions of dMR imply the systems to have predetermined spins S_k^A and $S_{k'}^B$, with values of either $+1$ or -1. Hence, dMR implies that the Bell inequality (46) holds.

The dynamics associated with the spin measurements on the Bell-cat state is depicted and explained in Fig. 13. The figure depicts the evolution due to the unitary interactions that determine the measurement settings, which are equivalent to the rotations of the analyzers in a standard Bell experiment. The predictions are $\langle S_1^A S_2^B\rangle = \langle S_2^A S_3^B\rangle = -1/\sqrt{2}$, $\langle S_1^A S_3^B\rangle = 0$, leading to a violation of (46). Details are given in Ref. [21]. We note that the CHSH-Bell inequality (39) can be written as

$$|\langle S_1^A S_2^B\rangle + \langle S_3^A S_2^B\rangle + \langle S_3^A S_4^B\rangle - \langle S_1^A S_4^B\rangle| \leqslant 2. \tag{47}$$

This inequality is also derived from the dMR assertions and is also violated for the macroscopic Bell-cat system, where S_4 is measured after an interaction time $t_4 = 3\pi/4\Omega$ [21]. Details are summarized in Ref. [24]. Hence, dMR is negated by the quantum predictions. The Bell violations are unchanged for arbitrarily large α, which demonstrates the macroscopic nature of the Bell test.

As with weak local realism (wLR), the assertions of wMR are weaker (less restrictive) than those of dMR and do not imply the Bell inequalities (46) and (47). In Fig. 13, we track the variables λ_k^A and λ_k^B ($k = 1, 2, 3, 4$) that in a wMR model correspond to the outcomes of the spin measurements S_k^A and S_k^B respectively. The center diagram depicts these values λ_k using the ball-in-a-box model explained in Sec. II D 4. There is consistency of the Bell violations with wMR, as explained in Sec. V A for wLR for the standard Bell state. This is because Assertion wMR(1) assigns the hidden variables λ_k only at the fixed times t_k *once* the system is prepared with respect to the measurement basis for S_k. We observe that the prediction of a Bell violation occurs over the time span associated with *two* rotations, one for each system, as predicted by Result II D and illustrated by the ball-in-a-box model.

Following Refs. [21,22,24], we illustrate the latter point by comparing the dynamics of $|\psi_{\rm Bell}\rangle$ with that for the system prepared in the mixed state

$$\rho = \tfrac{1}{2}(|\alpha\rangle_A|-\alpha\rangle_B\langle-\alpha|_B\langle\alpha|_A + |-\alpha\rangle_A|\alpha\rangle_B\langle\alpha|_B\langle-\alpha|_A). \tag{48}$$

On comparing the top and lower sequences of Fig. 13, we see that the plots for $|\psi_{\rm Bell}\rangle$ and $\rho_{\rm mix}$ are visually *indistinguishable* up to the time t_3, corresponding to the single rotation at B. After t_3, there is a second rotation at A, and the distributions shown by the plots diverge. The predictions for $\rho_{\rm mix}$ are consistent with a Bell local hidden variable theory and do not violate the Bell inequality [21]. The state $\rho_{\rm mix}$ is an example of the type $\rho_{{\rm mix}A,j}$ defined by (10) in Sec. II D 1 which, as predicted from that analysis, gives for the pointer measurements predictions *indistinguishable* (at a macroscopic level) from those of the Bell-cat state $|\psi_{\rm Bell}\rangle$ when there is a rotation U_ϕ^B at the site B only (in agreement with Fig. 13).

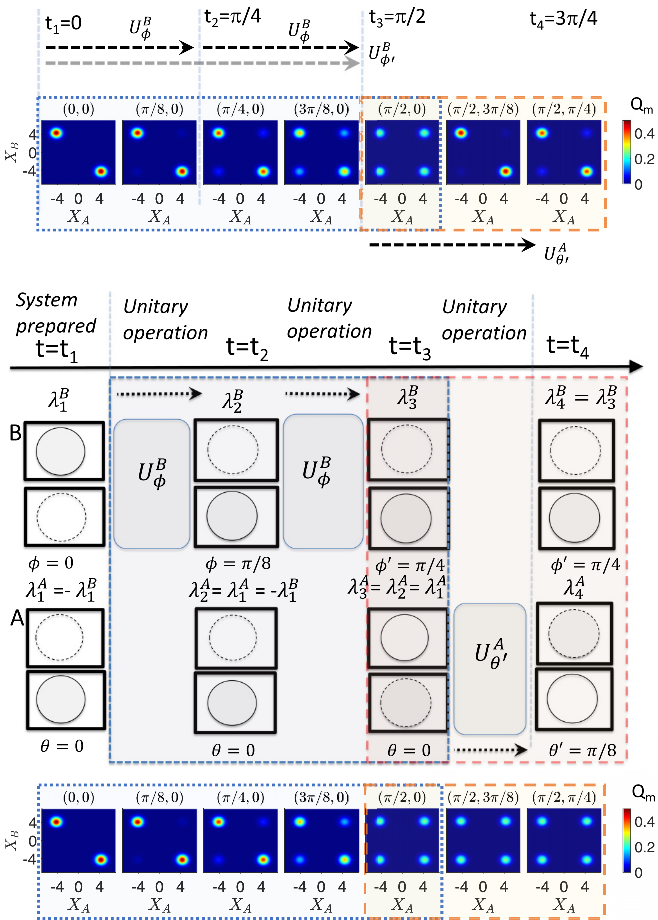

FIG. 13. Illustration showing how weak macroscopic realism (wMR) is consistent with the violation of the Bell inequality (46). The top plot shows the quantum predictions for the Bell-cat system (44) as it evolves under H_{nl}^A and H_{nl}^B for times t_a and t_b, respectively, given as the pair (t_b, t_a) above each snapshot, where $\Omega = 1$. The measurement settings $\theta = 0$, $\phi = \pi/8$, $\phi' = \pi/4$, and $\theta' = \pi/8$ are realized by unitary operations U^A and U^B. The signs of $\hat{X}_A$ and $\hat{X}_B$ at t_k give the spin outcomes. At time t_2, spins S_1^A and S_2^B are measured; at time t_3, spins S_1^A and S_3^B are measured; and at time t_4, spins S_3^B and S_2^A are measured. The quantum predictions are depicted by the contour plots showing the marginal $Q_m(X_A, X_B)$ of the Q function $Q(\alpha_0, \beta_0)$ where $X_A = \mathrm{Re}(\alpha_0)$ and $X_B = \mathrm{Re}(\beta_0)$. The joint probabilities for the spin outcomes at time t_k ($k = 1, 2, 3, 4$) are given by the relative weighting of the peaks visible at -4 and $+4$. Here, $\alpha = 4$ but results are unchanged as $\alpha \to \infty$. The predictions violate the Bell inequality but there is no inconsistency with wMR, which assigns predetermined values for the spins according to the center panel. According to wMR, the spin values S_k^A and S_k^B at the time t_k are given by λ_k^A and λ_k^B. The lower sequence shows the dynamics for the mixed state ρ_{mix}, Eq. (48). Consistent with wMR(2), there is no distinction between the predictions for $|\psi_{\mathrm{Bell}}\rangle$ and ρ_{mix} at times t_1, t_2, and t_3 (blue dotted rectangle) where there is a unitary operation only at one site, B. The predictions are macroscopically different at time t_4, after rotations at both sites (orange dashed rectangle).

This prediction is proved in Result II D. To be explicit, the mixed state ρ_{mix} gives a model in which system A *does* have a predetermined value λ_k^A for the spin outcome of S_z^A, since there is the probabilistic interpretation of the system being in the state $|\alpha\rangle$ *or* $|-\alpha\rangle$. This is true over the entire evolution due to U_ϕ^B, since the evolving state over this time period is

$$\rho_{\mathrm{mix}}(t) = e^{-iH_{nl}^B t}\rho_{\mathrm{mix}}e^{iH_{nl}^B t}, \tag{49}$$

which does not change the interpretation that the system A is in one or other state, $|\alpha\rangle$ or $|-\alpha\rangle$. Hence, for the system prepared in ρ_{mix}, there is a model assigning a single value λ_k^A to system A throughout the evolution associated with the operation of U_ϕ^B at B, implying the outcome for S_k^A does not change with U_ϕ^B in this model. Hence, there is consistency of the predictions for the Bell-cat state with a model satisfying Assertion wMR(2).

A detailed analysis in fact reveals terms distinguishing the dynamics for the two *states* (ρ_{mix} and $|\psi_{\mathrm{Bell}}\rangle$) over the single rotation U^B but which decay with increasing α, thereby vanishing in the macroscopic limit [21]. On the other hand, the Bell violations are maintained for arbitrarily large α. Figure 13 shows that the predictions of the Bell-cat state and ρ_{mix} differ *macroscopically* over the time span where there are *two* rotations, $U_{\theta'}^A$ at A and $U_{\phi'}^B$ at B. This difference leads to the Bell violation for the Bell-cat state and yet does not contradict wMR, as illustrated by the ball-in-a-box diagram of Fig. 13.

Similarly, the violation of the Bell inequality for the Bell-cat state $|\psi_{\mathrm{Bell}}\rangle$ is consistent with Assertion wMR(3). Consider where the interaction H_{nl} is applied to each site, to realize the operations U_θ^A and U_ϕ^B, where $\theta, \phi \in \{0, \pi/8, \pi/4, 3\pi/8\}$ as above, and depicted in Fig. 13. If we select $t_a = t_b$, there is an anticorrelation between the spin outcomes. The *original* EPR criterion for reality, EPR Assertion II, allows one to deduce that each spin value λ_θ^A, λ_ϕ^B, $\lambda_{\theta'}^A$, and $\lambda_{\phi'}^B$ is predetermined at the time t_1, prior to the unitary operations being applied. The original EPR premises are falsified by the violation of (46), as shown in Bell's original paper [2]. However, the weaker premise wMR(3) does not imply that each of the spin values is predetermined at time t_1. The Assertion wMR(3) implies instead that a spin value of B (say) is predetermined *once* it can be predicted by a *pointer* measurement at A. Referring to Fig. 13, we see that at time t_1, the system is prepared for pointer measurements of S_1^A and S_1^B, for which the spin outcomes will be anticorrelated, given in the wMR model by variables λ_1^A and $\lambda_1^B = -\lambda_1^A$ respectively (Fig. 13, center). The Assertion wMR(3) implies (for example) that the outcome for the spin defined as $S_1^B = S_{\phi=0}^B$ is determined at time t_3 (even though there has been evolution for system B) because the outcome of S_1 (if that evolution is reversed) can be predicted by making a pointer measurement on system A at time t_3 to obtain the value of S_1^A as $-\lambda_1^A$. However, the value for S_1^B can no longer be assumed determined at the time t_4, after the *further* operation $U_{\theta'}^A$ at A (Fig. 13, center). We note that wMR(3) justifies the meaning that the value of S_1^B defined for the system at time $t_1 = 0$ is measurable by measuring S_1^A, despite that there have been further unitary interactions at B. The system A at time t_3 is a meter for S_1^B.

C. Nonlocality and deeper models

It is clear that the wLR and wMR premises allow for nonlocal effects, in the sense of a violation of a Bell inequality, as evident in Fig. 13. Nonlocal effects are also evident in the

Figs. 3 and 4. The premises do not encompass the full local realistic assumptions of Bell.

The nonlocal effect arises in the analysis given in Sec. V A (Fig. 12) because it cannot be assumed that the value $\lambda^B_{\phi'}$ is independent of the value θ', which determines the unitary rotation at A. In Fig. 12, $\lambda^A_{\theta'}$ is independent of ϕ' because the setting θ' is fixed (the unitary rotation $U^A_{\theta'}$ has occurred before $U^B_{\phi'}$ at A), but it cannot be excluded that $\lambda^B_{\phi'}$ can depend on θ' because $U^A_{\theta'}$ occurred earlier. The order of the unitary rotations becomes relevant despite the two rotations occurring at spatially separated locations. At first, this seems to be a strong nonlocal effect that might induce signaling. However, there is no inconsistency with Assertions wMR(2) and wLR(2), which do not allow signaling. According to the quantum predictions, the joint moments do *not* depend on the order of rotation. Consistency of wMR and wLR with quantum mechanics (and nosignaling) is possible because of the symmetry induced by the assumptions: Any conditioning of $\lambda^B_{\phi'}$ on θ' necessitates that a rotation $U^B_{\phi'}$ has also occurred at B, since the preparation time t_1. The joint distribution for values $\lambda^A_{\theta'}, \lambda^B_{\phi'}$ hence depends on both θ' and ϕ', the final settings. Basis rotations at *both* sites are required for the nonlocal effect to emerge (Result II D).

We note that paradoxes arise for wMR and wLR models, if we were to consider different frames of reference, since the time order of spacelike-separated events can change from the perspective of the different observers. It might hence appear that in one frame, the real property $\lambda^A_{\theta'}$ is independent of ϕ', while in another frame it is not, creating apparent conflict with the principle of Lorentz invariance. As before, symmetry plays a role in a possible resolution of the paradox. Suppose in one frame the setting is first changed to ϕ' at B (Fig. 11, lower panel), so that in the wLR model the spin outcome $\lambda^B_{\phi'}$ for B emerges, where $\lambda^B_{\phi'} = +1$ or -1. Next, the unitary rotation at A occurs to change the setting to θ', after which the spin outcome $\lambda^A_{\theta'}$ at A is specified. The outcome $\lambda^A_{\theta'}$ is dependent on the operation θ', but so as to satisfy the observed (quantum) conditional distribution $P_{\theta',\phi'}(\lambda^A_{\theta'}|\lambda^B_{\phi'})$ for the outcome $\lambda^A_{\theta'}$, given the outcome $\lambda^B_{\phi'}$ at B (and the two settings θ' and ϕ'). However, for the Bell state, the marginal probability distributions $P(\lambda^B_{\phi'})$ and $P(\lambda^A_{\theta'})$ for the outcome of the spin are independent of the angles θ' and ϕ'. Hence, the conditional probability for $\lambda^A_{\theta'}$ given $\lambda^B_{\phi'}$ is the same as that for $\lambda^B_{\phi'}$ given $\lambda^A_{\theta'}$, being determined by the joint distribution $P_{\theta',\phi'}(\lambda^A_{\theta'}, \lambda^B_{\phi'})$. Hence, it can be argued that there is no detectable difference between the statistics (associated with the different time order) for the observers in different frames.

In summary, the wLR and wMR premises thus rule out the possibility of any *instantaneous* (or strong) nonlocal effect. In models where wMR (or wLR) hold, the choice to measure ϕ' instead of ϕ at B *does not change* the value of λ^A_θ at A once the unitary rotation has occurred at A to fix the measurement setting as θ at A. A *further* unitary operation is required at A for nonlocality to manifest (which requires a time duration). As explained in Sec. II D, this ensures *no-signaling*, defined as the requirement that the outcome at one site is independent of the setting at the other site conditioned on the setting at the first site being fixed [35]. Similarly, consider the system prepared in the Bell state ready for pointer measurements corresponding to $\theta = \phi = 0$. If the operation U^B_ϕ then occurs at B, then the value for S^A_ϕ at A is fixed, because [according to Assertion wMR(3)] the value for S^A_ϕ can be predicted with certainty by completing the measurement S^B_ϕ at B. While this gives a nonlocal effect, a *further* local interaction U^A_ϕ is required at A for the effect to actually be observed.

There is motivation to examine models and interpretations of quantum mechanics (see, e.g., Refs. [31–33,60–85]) for consistency with wLR and wMR. An early study that gives a decomposition of Bell's locality condition was presented by Jarrett [84], who distinguishes between a relativistic locality constraint (no signaling) and a stronger assumption linked to a completeness of state descriptions. However, the feature of the different stages of measurement process was not explicitly included. Maroney and Timpson proposed models for macroscopic realism (MR) that are consistent with the violation of Leggett-Garg inequalities [36,62]. In Ref. [36], these authors examine the ball-in-a-box model explained in Sec. II, and define a "supra eigenstate support MR model" that allows (weak) MR to hold (by our definitions). The authors gave the nonlocal de Broglie-Bohm theory as an example of such a model [61].

Another model of interest is the "objective field model," or "Q-based model" for quantum mechanics, motivated by the Q function in quantum optics [60,68–70,85]. Solutions have been given where the pointer stage of measurement is modeled as amplification [60,68,85]. These solutions reveal retrocausality and hidden causal loops based on future boundary conditions. In one example [85], the measurement settings are phase shifts, the pointer stage of measurement being the amplification of a field quadrature amplitude. The amplification transforms microscopic superposition states into macroscopic superpositions states so that wMR can be applied at a suitable time t_f, which is prior to further amplification and a final readout (as described in Sec. II F). We note that consistency with wMR requires there is *no retrocausality at a macroscopic level* because the hidden variables λ associated with wMR(1) are fixed at the given time, being independent of any future event. Recent solutions for the objective field model suggest how the retrocausality that is present microscopically can be consistent with wMR [85].

VI. CONSISTENCY OF GHZ QUANTUM PREDICTIONS WITH WEAK MACROSCOPIC REALISM AND WEAK LOCAL REALISM

The premise wLR can be applied to the GHZ setup to show there is no inconsistency of wLR with the quantum predictions. This is in agreement with the results summarized in Sec. V [21,23], where consistency with wLR is shown for Bell violations. The same analysis applies to a macroscopic GHZ setup for wMR.

To demonstrate the consistency with wLR, we consider the state (25) at time t_1 and then suppose the systems B and C are further prepared so that pointer measurements of S^B_x and S^C_x at the time t_f will yield the outcomes of S^B_x and S^C_x (as in Fig. 14). Weak local realism asserts that the systems are each

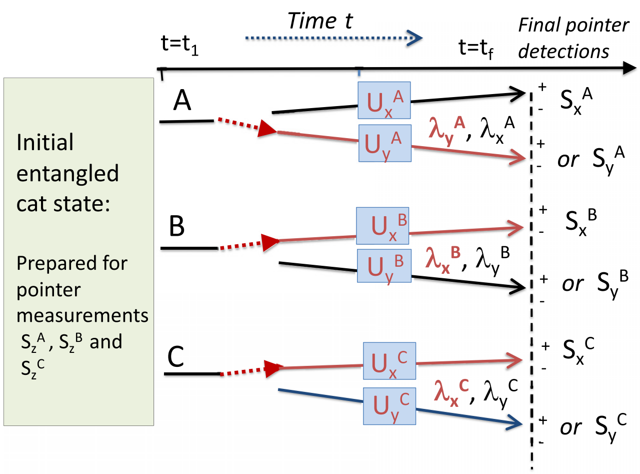

FIG. 14. Weak macroscopic realism (wMR) applied to the macroscopic GHZ experiment. The premise wMR asserts validity of hidden variables for systems at time t_f prepared for the *pointer* measurements. Sketched is the setup where S_y is measured at A, and S_x at B and S_x at C. The wMR premise asserts hidden variables for the system A at the time t_f that predetermine the final outcome of S_y at A, and also predetermine the outcome of S_x at A. This is because the prediction for S_x at A can be given with certainty by the pointer measurements at B and C. Similar logic implies hidden variables that predetermine the outcomes for both S_x and S_y for sites B and C. However, there is no GHZ contradiction with wMR. This is because the hidden variables λ_x^A, λ_y^B, and λ_y^C give the outcomes of pointer measurements to be made after a further local unitary interaction U, assuming there are no further unitary interactions at the other sites. The same analysis applies to the standard GHZ setup for weak local realism (wLR), with the initial state being $|\psi_{\text{GHZ}}\rangle$ of Eq. (21).

assigned a predetermined value λ_x^B and λ_x^C, respectively, for the outcomes of those pointer measurements, at the time t_f. The premise of wLR also assigns an inferred value

$$\lambda_x^A \equiv \lambda_{x,inf}^A = \lambda_x^B \lambda_x^C \tag{50}$$

to the system A since the values λ_x^B and λ_x^C enable a prediction with certainty for the outcome of the measurement S_x^A, if performed at A.

It is also the case, however, that wLR applies directly to A. If the system A undergoes rotation U_y, as depicted in Fig. 14, then it is prepared with respect to the measurement basis of S_y^A. Hence the system A is ascribed a hidden variable λ_y^A to predetermine the outcome for a pointer measurement S_y^A at A, if performed.

At first glance, this seems to suggest a GHZ contradiction for wLR. Suppose one prepares the systems B and C for the pointer measurements of S_x^B and S_x^C, at time t_f (Fig. 14). Hence, for the systems B and C at time t_f, the outcomes for S_x^A, S_x^B, and S_x^C are all predetermined, and given by variables $\lambda_x^A \equiv \lambda_{x,inf}^A$, λ_x^B, and λ_x^C, respectively. Additionally, one can prepare system A in pointer measurement for y, and the outcome for S_y^A is also determined (Fig. 14). Then, one can infer the values for the outcomes of measurements S_y^B and S_y^C, should they be performed by carrying out the appropriate unitary interaction at B and C. We have for the inferred values

$$\begin{aligned} \lambda_{x,inf}^A &= -\lambda_x^B \lambda_x^C, \\ \lambda_{y,inf}^C &= \lambda_x^B \lambda_y^A, \\ \lambda_{y,inf}^B &= \lambda_x^C \lambda_y^A. \end{aligned} \tag{51}$$

For each system, the value of either S_x or S_y is determined (by the pointer preparation), and the value of the other measurement is determined by inference of the other (pointer) values. Hence, it appears that there is the GHZ contradiction, because it is as though the outcomes of both S_x and S_y are determined at each site (at the same time), and these outcomes are either $+1$ or -1, hence creating the contradiction of Eq. (24).

However, there is no GHZ contradiction with wMR. The value for either S_x or S_y (the one that is inferred at each site) will require a local unitary rotation U (a change of measurement setting) before the final readout given by a pointer measurement. The unitary interaction U occurs over a time interval. The unitary rotation means that the value λ that predetermines the outcome of the pointer measurement at time t_f no longer (necessarily) applies at the later time, t_m, after the interaction U. The system at t_m is prepared with respect to a *different* pointer measurement. Hence, at time t_m, the earlier predictions of the inferred values λ_{inf} for the other sites no longer apply. The paradox as arising from Eq. (24) assumes all values of λ apply simultaneously, to the state at time t_f.

In Fig. 15, we give more details of the way in which the hidden variables implied by wLR can be tracked and found consistent with the predictions of quantum mechanics. Suppose the system is prepared ready for pointer measurements S_y^A, S_x^B, and S_x^C at the time t_k, and the hidden variables λ_y^A, λ_x^B, and λ_x^C (in bold red) determine those pointer outcomes. The decision is then made to measure instead S_y^A, S_y^B, and S_x^C. This requires a further unitary rotation $\mathbf{U}_y^B = U_y U_x^{-1}$ at site B. At time t_m, after $\mathbf{U}_y^B$ has taken place, the system is described by a different set of hidden variables, λ_y^A, λ_y^B, and λ_x^C. The outcome of the measurement of S_y is however determined with certainty by the pointer measurements for A and C, as $\lambda_y^B = \lambda_{y,inf}^B = \lambda_y^A \lambda_x^C$. At time t_m, we then see that system B is no longer prepared in a pointer state for S_x. Hence, at time t_m, the earlier value of the inferred result $\lambda_{x,inf}^A$ at A (which depended on λ_x^B) is not relevant. A further unitary rotation $\mathbf{U}_x^A = U_x U_y^{-1}$ at A that prepares the system A for a final pointer measurement S_x will not (necessarily) give the results that applied at time t_k (which was prior to the U_y at B taking place). Consider the hidden variables that are defined (based on the premise of wLR) at the time, t_4, after the evolution $\mathbf{U}_x^A$. At the time t_4, the system is ascribed the variables λ_x^A, λ_y^B, and λ_x^C, with λ_y^A also determined, for a future single unitary transformation at A. The outcome for S_y at A is predetermined (by the pointer outcomes at B and C) according to wLR, given by

$$\begin{aligned} \lambda_{y,inf,4}^A &= \lambda_y^B \lambda_x^C = \lambda_{y,inf}^B \lambda_x^C \\ &= \lambda_x^C \lambda_y^A \lambda_x^C \\ &= \lambda_y^A, \end{aligned} \tag{52}$$

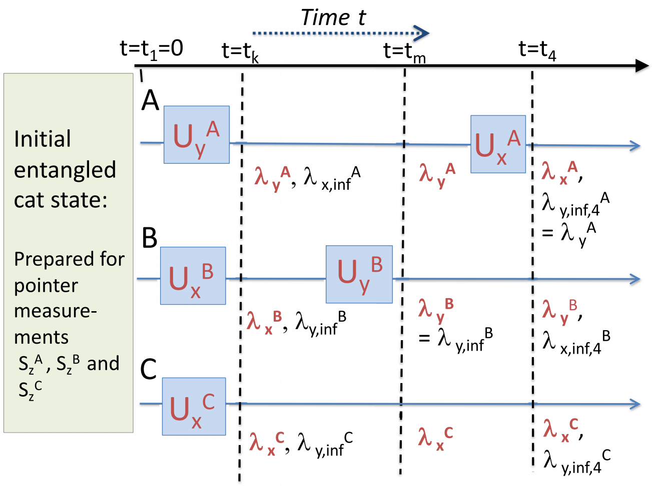

FIG. 15. Tracking the hidden variables as given by the premise of weak macroscopic realism (wMR). The dynamics is entirely consistent with the predictions of wMR, despite there being a GHZ paradox. These variables λ give the outcomes for the appropriate measurement if performed. The variables in bold red are implied by Assertion wMR(1), since the system is prepared at that time t_i at that site, with respect to the measurement basis for a specific pointer measurement. The variables indicated by the subscript inf are deduced by Assertion wMR(3), that the outcome for the measurement can be predicted by the pointer measurements at the other sites, at that time t_i. The hidden variables that are implied by wMR at the times t_i are indicated beside the dashed vertical line labeled t_i. The same analysis for wLR applies to the standard GHZ setup, with the initial state being $|\psi_{\mathrm{GHZ}}\rangle$ of Eq. (21).

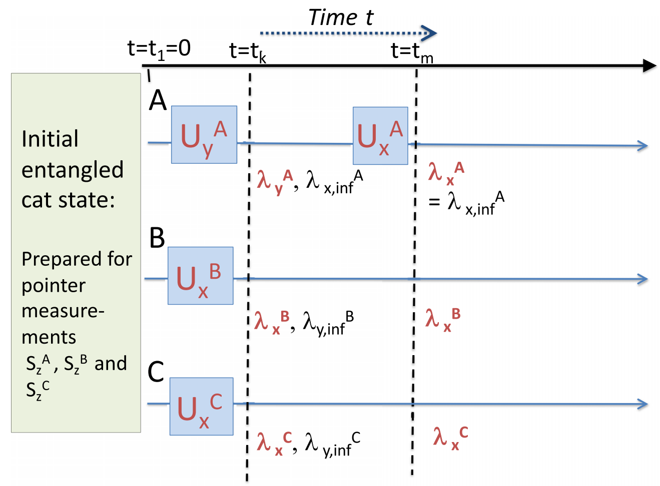

FIG. 16. Predictions of weak macroscopic realism (wMR) are consistent with EPR's assertions and Bell's local realism for the single rotation $\mathbf{U_x^A}$ after preparation at time t_k. The notation is as for Fig. 15. The system at time t_k is prepared such that pointer measurements will yield outcomes for measurements of S_y^A, S_x^B, and S_x^C. The prediction for S_x^A can be inferred from the pointer measurements at B and C and hence is also predetermined at the time t_k, according to wMR. This measurement requires a further unitary interaction U_x^A. The wMR model identifies at time t_k hidden variables for outcomes of S_y^A, S_x^B, S_x^C and also S_x^A (if U_x^A is performed). These variables are unchanged by the unitary operation U_x^A and hence there is consistency with Bell's local hidden variable models. Quantum mechanics also predicts consistency with Bell's local realism for the single rotation (see text).

which gives consistency with the earlier value at t_k. However, the outcome for S_x at B is inferred from the pointer values at time t_4:

$$\lambda_{x,inf,4}^{B} = -\lambda_x^A\lambda_x^C. \tag{53}$$

For consistency with the values defined at t_k, we could propose $\lambda_x^A = \lambda_{x,inf}^A = -\lambda_x^B\lambda_x^C$, in which case we would obtain $\lambda_{x,inf,4}^B = -\lambda_x^A\lambda_x^C = \lambda_x^B(\lambda_x^C)^2 = \lambda_x^B$, giving an apparent consistency with the earlier value. However, the value of S_y at C at time t_4 is inferred to be

$$\begin{aligned}\lambda_{y,inf,4}^{C} &= \lambda_y^B\lambda_x^A = \lambda_{y,inf}^B\lambda_x^A\\ &= \left(\lambda_x^C\lambda_y^A\right)\lambda_x^A.\end{aligned} \tag{54}$$

Now if we propose $\lambda_x^A = \lambda_{x,inf}^A = -\lambda_x^B\lambda_x^C$, we obtain $\lambda_{y,inf,4}^C = (\lambda_x^C\lambda_y^A)(-\lambda_x^B\lambda_x^C) = -\lambda_y^A\lambda_x^B$. We see here that this differs from the earlier value $\lambda_{y,inf}^C = \lambda_y^A\lambda_x^B$. Hence, it is not possible to gain consistency between wLR and the values λ asserted by the premise of the original EPR Assertions. While the EPR Assertions are falsified by the GHZ paradox, we see that the GHZ contradiction does not apply to wLR. Similarly, for the macroscopic realization, there is falsification of dMR but consistency with wMR.

We note that, according to wLR, the value λ_y^A for system A prepared for the pointer measurement S_y^A, for example, is not changed by unitary rotations that may take place at B or C (Fig. 15, at time t_m). However, if there is a further unitary rotation at A, and also at B (i.e., two unitary rotations, at different sites), then the outcome for S_x is no longer (necessarily) given by the inferred value $\lambda_{x,inf}^A$ defined at time t_k (Fig. 15, at time t_4).

VII. PREDICTIONS OF WEAK MACROSCOPIC REALISM AND WEAK LOCAL REALISM

We present further predictions for wLR and wMR. These provide a means to experimentally test wLR and wMR. The predictions are identical to those of quantum mechanics. The analyses below are presented for wMR but apply in identical fashion to wLR for the state $|\psi_{\mathrm{GHZ}}\rangle$ of Eq. (21) where the spin states need not be macroscopic.

A. Moments involving a single unitary rotation are consistent with EPR's assertions and Bell's local realism

Prediction of wMR .We consider the entangled cat-state GHZ system $|\psi\rangle$ [Eq. (25)], which is then prepared at time t_k for pointer measurements S_y^A, S_x^B, and S_x^C at the respective sites (as in Fig. 16). The GHZ contradiction with EPR's assertions is realized by first further changing the measurement settings, to measure S_x^A, S_x^B, and S_x^C, which involves one unitary rotation $\mathbf{U}_x^A$. Also required are measurements S_x^A, S_y^B, and S_y^C, which involve two further rotations, one at each site B and C, as well as $\mathbf{U}_x^A$ (Fig. 17). The prediction is that results violating dMR (i.e., EPR's assertions) do not arise from the correlations involving only one unitary U_x^A after the preparation at t_k. The violations arise from the correlations involving the two further rotations. A similar result was proved for Bell violations [21].

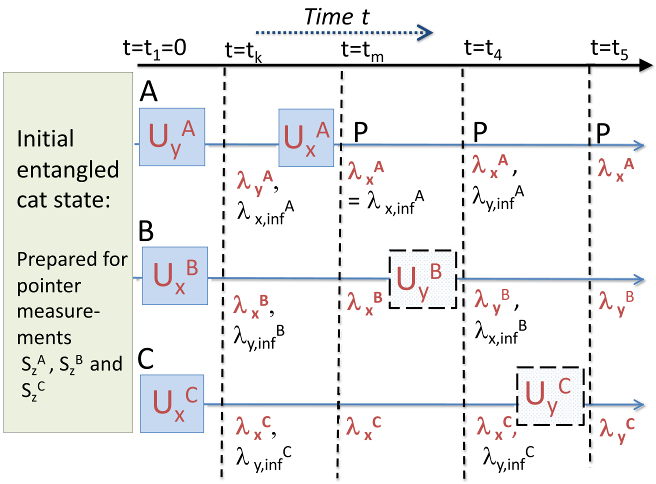

FIG. 17. Predictions of weak macroscopic realism (wMR) are consistent with those of quantum mechanics for multiple rotations. The notation is as for Fig. 15. Here, further unitary interactions $\mathbf{U}_y^B$ and $\mathbf{U}_y^C$ prepare the system for measurement of $S_x^A S_y^B S_y^C$ at the time t_5. However, at the time t_m, there is no longer the pointer preparation for S_y^A, and the values that were inferred for measurements S_y^B and S_y^C no longer apply. The predictions lead to the GHZ contradiction but are consistent with wMR. The premise wMR also predicts that the results for $S_x^A S_y^B S_y^C$ are independent of when the final irreversible stage of the pointer measurement for S_x^A is performed, relative to the unitary transformations at B and C. The different timings are indicated by the P symbol.

Proof. We denote the state prepared at the time t_k by $|\psi\rangle_{y,x,x}$. Pointer measurements if conducted at time t_k at A, B, and C give the outcomes for S_y^A, S_x^B, and S_x^C respectively. According to wMR, at time t_k, the values for S_x^A, S_y^A, S_x^B, and S_x^C are hence each predetermined, being given by the variables $\lambda_{inf,x}^A$, λ_y^A, λ_x^B, and λ_x^C. A single unitary rota tion $\mathbf{U}_x^A = U_y^{-1}U_x$ at A will enable the result $\lambda_{inf,x}^A$ to be revealed consistently with the predictions at B and C, if those pointer measurements are carried out. According to wMR, the prediction for $S_x^A S_x^B S_x^C$ is predetermined at time t_k, for the system prepared in the state $|\psi\rangle_{y,x,x}$. There are hidden variables for each measurement, defined for the system at t_k, the value $\lambda_{inf,x}^A$ not being changed by whether or not the pointer measurements at B and C occur. Therefore, the prediction in the wMR model for $S_x^A S_x^B S_x^C$, conditioned on the initial state $|\psi\rangle_{y,x,x}$, is consistent with EPR's assertions (and Bell's local realism). ■

Proof of agreement with quantum prediction. Here, we prove that the prediction of quantum mechanics also shows consistency with Bell's local realism for the setup of just one rotation after preparation (Fig. 16). To do this, we compare the predictions of quantum mechanics for the prepared state $|\psi\rangle_{y,x,x}$ against those of a mixed state $\rho_{\text{mix}}^{A\text{-}BC}$. The state $|\psi\rangle_{y,x,x}$ prepared at time t_k in the basis for S_y^A, S_x^B, and S_x^B is

$$
\begin{aligned}
|\psi\rangle_{y,x,x} &= \frac{1}{4}\{(|\uparrow\rangle_y + |\downarrow\rangle_y)(|\uparrow\rangle_x + |\downarrow\rangle_x)(|\uparrow\rangle_x + |\downarrow\rangle_x) \\
&\quad + i(|\uparrow\rangle_y - |\downarrow\rangle_y)(|\uparrow\rangle_x - |\downarrow\rangle_x)(|\uparrow\rangle_x - |\downarrow\rangle_x)\} \\
&= \frac{1}{\sqrt{2}}(|\psi_-^A\rangle|\psi_+^{BC}\rangle + |\psi_+^A\rangle|\psi_-^{BC}\rangle),
\end{aligned} \tag{55}
$$

where

$$
\begin{aligned}
|\psi_+^{BC}\rangle &= (|\uparrow\rangle_x|\uparrow\rangle_x + |\downarrow\rangle_x|\downarrow\rangle_x)/\sqrt{2}, \\
|\psi_-^{BC}\rangle &= (|\downarrow\rangle_x|\uparrow\rangle_x + |\uparrow\rangle_x|\downarrow\rangle_x)/\sqrt{2},
\end{aligned}
$$

and

$$
\begin{aligned}
|\psi_+^A\rangle &= \{(1-i)|\uparrow\rangle_y + (1+i)|\downarrow\rangle_y\}/2, \\
|\psi_-^A\rangle &= \{(1+i)|\uparrow\rangle_y + (1-i)|\downarrow\rangle_y\}/2.
\end{aligned}
$$

The state $|\psi\rangle_{y,x,x}$ is a superposition with entanglement between the system A and the composite system BC, which comprises the systems B and C. If the unitary rotation $\mathbf{U}_x^A$ is performed at A, then the prediction for the pointer measurements is $S_x^A S_x^B S_x^C = -1$. Now we compare with the system initially prepared in the mixture

$$\rho_{\text{mix}}^{A\text{-}BC} = |\psi_-^A\rangle\langle\psi_-^A|\rho_+^{BC} + |\psi_+^A\rangle\langle\psi_+^A|\rho_-^{BC}. \tag{56}$$

Here, $\rho_+^{BC} = |\psi_+^{BC}\rangle\langle\psi_+^{BC}|$ and $\rho_-^{BC} = |\psi_-^{BC}\rangle\langle\psi_-^{BC}|$. This mixture has no entanglement between the system A and the combined systems B and C, i.e., it is fully separable with respect to the bipartition that we denote by A-BC. If we transform to the x basis at A, then we write

$$\rho_{\text{mix}}^{A\text{-}BC} = |\downarrow\rangle_x\langle\downarrow|_x\rho_+^{BC} + |\uparrow\rangle_x\langle\uparrow|_x\rho_-^{BC}. \tag{57}$$

The prediction is $S_x^A S_x^B S_x^C = -1$, which is identical to the prediction for the system prepared in $|\psi\rangle_{y,x,x}$. The quantum prediction for the single unitary interaction $\mathbf{U}_x^A$ on $|\psi\rangle_{y,x,x}$ is therefore consistent with Bell's local realism—since the prediction for $\rho_{\text{mix}}^{A\text{-}BC}$ is fully local with respect to A, arising from a local interaction at A. ■

The GHZ test showing violation of the EPR assertions (and Bell's local realism) requires two further rotations, $\mathbf{U}_y^B$ and $\mathbf{U}_y^C$ at the sites B and C. This allows measurement of $S_x^A S_y^B S_y^C$ (Fig. 17). However, wMR does not predict for the system at time t_k a predetermination of the outcomes for both S_x^A and S_y^B. Hence, there is no contradiction between the predictions of quantum mechanics and wMR. The hidden variables that are predicted by wMR are tracked in Fig. 17. An experiment could be performed, by comparing the observed moments for the GHZ state with those generated by the appropriate mixed states, as in the above proof.

B. The timing of the pointer stage of measurement

Prediction of wMR. Consider the system of Fig. 17, prepared at time t_k so that pointer measurements at A, B, and C will give the outcomes for S_y^A, S_x^B, and S_x^C. At A, the system is then prepared for a pointer measurement of S_x. At B, a unitary rotation then prepares system B for a pointer measurement of S_y^B, and then similarly at C. If wMR is valid, then the predictions for the correlations are not dependent on whether the final pointer stages P of the measurement for S_x^A at A occur before or after the unitary rotations at B and C. Here, the final pointer stages of the measurement (denoted P in the figure) involve a coupling to an environment, whereby the measurement becomes irreversible.

Proof. The premises wMR(1) and (2) assert that the value λ for the outcome of the pointer measurement is fixed locally for the appropriately prepared system, provided there is no further unitary U on that system which changes the measurement setting. This prediction agrees with that of quantum mechanics. Quantum calculations do not distinguish the timing of the measurement stage P. ■

VIII. CONCLUSION

The main conclusion of this paper is that the negation of local realism, as evidenced by a Bell or GHZ experiment, does not fully resolve the EPR paradox. We have proposed how EPR-Bohm, Bell and GHZ experiments may be realized in mesoscopic and macroscopic regimes, using cat states and suitable unitary interactions. These are tests in a setting where all relevant measurements can be coarse-grained, distinguishing only between two macroscopically distinct states. The macroscopic EPR-Bohm test illustrates an incompatibility between the assumptions of deterministic macroscopic realism (dMR) and the notion that quantum mechanics is a complete description of physical reality. We explain that it is also possible to consider the weaker assumption, *weak macroscopic realism* (wMR), and to demonstrate a similar inconsistency with the notion that quantum mechanics is a complete theory, using a two-spin version of the EPR-Bohm argument. Yet, while dMR can be falsified by the macroscopic GHZ or Bell experiments, the predictions for these experiments are consistent with those of wMR. In defining wMR, it is necessary to consider that the measurement occurs in *two* stages, a reversible stage establishing the measurement setting, and an irreversible stage referred to as the pointer stage of measurement.

Similar conclusions can be drawn for the original EPR and GHZ paradoxes. This paper motivates consideration of a weaker assumption, *weak local realism* (wLR), in the set-up of the original paradox. The EPR argument can be modified to show inconsistency between wLR and the notion that quantum mechanics is a complete description of physical reality. Yet, we show that the predictions of quantum mechanics for the GHZ and Bell experiments are consistent with those of wLR. The definitions of wMR and wLR apply to systems after the choice of measurement basis and hence are consistent with the contextuality of quantum mechanics [86]. Our work may be seen as a supplement to other arguments presented for the incompleteness of quantum mechanics (see, e.g., Refs. [18,20,87]) and may motivate a study of alternative models.

In addition to EPR-Bohm and GHZ experiments, we propose further tests of wLR and wMR. These tests examine correlations after single unitary rotations and adjust the timing of the unitary interactions that lead to the GHZ contradiction. The predictions of wMR and wLR agree with those of quantum mechanics. The EPR and GHZ paradoxes apply where one can predict *with certainty* the outcome of a measurement, given measurements at spacelike-separated sites. Experimental factors may prevent the realization of predictions that are certain. The tests can nonetheless be carried out using inequalities [17,41,51,88]. Proposals for realistic tests are given in the Appendices.

The proposed experiments could be realized in the microscopic regime where wLR is applicable using standard techniques e.g., polarising beam splitters. Macroscopic realizations are given in Sec. IV B and Appendices A and B. The two-mode cat states involving coherent states have been generated in cavities [52,53], and GHZ states have been generated for $N \approx 20$ [89]. Mesoscopic realizations of the unitary transformations are in principle feasible using dynamical interactions involving a nonlinear medium, or else CNOT gates.

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APPENDIX A: REALIZATION OF MACROSCOPIC TWO- AND THREE-SPIN EPR-BOHM PARADOX WITH SPINS AND CNOT GATES

A useful mesoscopic or macroscopic realization of both the EPR-Bohm and GHZ setups uses multimode spin states and CNOT gates. This allows a realization of both types of EPR-Bohm paradoxes presented in Sec. III A, the two- and three-spin versions, at an increasingly macroscopic level depending on the number of modes.

By analogy with the microscopic example of Sec. III A 2, the three-spin paradox requires a transformation U_x at each site, where (apart from phase factors)

$$\begin{aligned} U_x^{-1}|\uparrow\rangle_z &\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \\ U_x^{-1}|\downarrow\rangle_z &\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle), \end{aligned} \tag{A1}$$

as well as that for U_y, given as

$$\begin{aligned} U_y^{-1}|\uparrow\rangle_z &\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle), \\ U_y^{-1}|\downarrow\rangle_z &\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle). \end{aligned} \tag{A2}$$

The important step is to find Hamiltonians that give realizations of U_x and U_y. In Sec. IV B, for the cat states involving the coherent states, a transformation U_y was specified, but none was given for U_x. We note that cat-state superpositions have been created (see, e.g., Refs. [53,90–92]), but for superpositions $|\alpha\rangle \pm |-\alpha\rangle$ methods have proposed using conditional measurements [90], or open dissipative systems (see, e.g., Refs. [93–96]). We prefer here to use simple unitary (reversible) transformations. A realization based on NOON states is given in Appendix B.

A realization can be achieved using an array of spins. The qubits of (A1) and (A2) become the macroscopically distinct states $|\Uparrow\rangle \equiv |\uparrow\rangle^{\otimes N}$ and $|\Downarrow\rangle \equiv |\downarrow\rangle^{\otimes N}$, for large N, so that the initial Bell state (3) becomes the two-site GHZ state

$$|\psi_{\rm Bell}\rangle_{z,z} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{z,A}^{\otimes N}|\uparrow\rangle_{z,B}^{\otimes N} - |\downarrow\rangle_{z,A}^{\otimes N}|\downarrow\rangle_{z,B}^{\otimes N}\right). \tag{A3}$$

The premises of macroscopic realism can be applied to the macroscopically distinct states. Here, $|\uparrow\rangle_{z,J}^{\otimes N}=\prod_{k=1}^{N}|\uparrow\rangle_{J,k}$, where $|\uparrow\rangle_{J,k}$ is the eigenstate of the Pauli spin σ_z^k for the mode labeled k at site J, the collection of modes $k=1,\dots,N$ forming the system labeled J. The $|\uparrow\rangle_{z,J}^{\otimes N}$ and $|\downarrow\rangle_{z,J}^{\otimes N}$ represent macroscopically distinct states, with collective Pauli spin values of N or $-N$, and are eigenstates of the spin product $S_z^J=\prod_{k=1}^{N}\sigma_z^k$.

To realize the paradox, the transformations U needed at each site J are, for U_x and U_y, of the form (A1) and (A2), but where $|\uparrow\rangle\equiv|\uparrow\rangle^{\otimes N}$ and $|\downarrow\rangle\equiv|\downarrow\rangle^{\otimes N}$. Generally, one can first consider how to achieve

$$|\uparrow\rangle^{\otimes N}\rightarrow\cos\frac{\theta}{2}|\uparrow\rangle^{\otimes N}+e^{i\vartheta}\sin\frac{\theta}{2}|\downarrow\rangle^{\otimes N}. \qquad \text{(A4)}$$

Following the experiment described in Ref. [44], the unitary transformations U_x and U_y are made in two steps.

The first step is a rotation on the single-mode spin

$$|\uparrow\rangle_1\equiv\begin{pmatrix}1\\0\end{pmatrix},\quad |\downarrow\rangle_1\equiv\begin{pmatrix}0\\1\end{pmatrix},$$

given by the unitary matrix

$$U_{\theta,\vartheta}=\begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ e^{i\vartheta}\sin\frac{\theta}{2} & e^{i\vartheta}\cos\frac{\theta}{2}\end{pmatrix},$$

where $\vartheta=0$ or $\pi/2$, which transforms the spin as

$$\begin{aligned}|\uparrow\rangle_1&\rightarrow U_{\theta,\vartheta}|\uparrow\rangle_1=\cos\frac{\theta}{2}|\uparrow\rangle_1+e^{i\vartheta}\sin\frac{\theta}{2}|\downarrow\rangle_1,\\ |\downarrow\rangle_1&\rightarrow U_{\theta,\vartheta}|\downarrow\rangle_1=-\sin\frac{\theta}{2}|\uparrow\rangle_1+e^{i\vartheta}\cos\frac{\theta}{2}|\downarrow\rangle_1.\end{aligned} \qquad \text{(A5)}$$

Here, we drop the subscript J representing the site, for notational simplicity. Choosing $\theta=\pi/2$ gives the starting point for the transformation U_x or U_y, at each site, with $\vartheta=0$ or $\pi/2$, respectively.

A common physical realization of the spin qubit involves two polarization modes: $|\uparrow\rangle\equiv|1,0\rangle$ and $|\downarrow\rangle\equiv|0,1\rangle$ defined for two modes $a_{\pm}$ as in Appendix B. The transformation $U_{\theta,\vartheta}$ can then be achieved with a polarizing beam splitter, with mode transformations ($\hat{a}_{\pm}$ are boson operators defining the modes)

$$\begin{aligned}\hat{c}_+&=\hat{a}_+\cos\theta-\hat{a}_-\sin\theta,\\ e^{i\vartheta}\hat{c}_-&=\hat{a}_+\sin\theta+\hat{a}_-\cos\theta.\end{aligned} \qquad \text{(A6)}$$

The $\hat{c}_{\pm}$ are boson operators for the outgoing modes emerging from the beam splitter. The interaction is described by the Hamiltonian $H=i\hbar k(\hat{a}_+\hat{a}_-^\dagger-\hat{a}_+^\dagger\hat{a}_-)$ where $\theta=kt$, for $\vartheta=0$. The addition of a $\vartheta=\pi/2$ phase shift (or not) relative to the two outputs gives the mode transformations with the dependence on $\vartheta=0$ or $\pi/2$. If the input is $|\uparrow\rangle$, the output state is

$$\begin{aligned}|1,0\rangle_{in}&=\hat{a}_+^\dagger|0\rangle\\ &=\cos\theta|1,0\rangle_{\text{out}}+e^{i\vartheta}\sin\theta|0,1\rangle_{\text{out}}.\end{aligned} \qquad \text{(A7)}$$

If the input is $|\downarrow\rangle$, the output is found according to

$$\begin{aligned}|0,1\rangle_{in}&=\hat{a}_-^\dagger|0\rangle\\ &=-\sin\theta|1,0\rangle_{\text{out}}+e^{i\vartheta}\cos\theta|0,1\rangle_{\text{out}},\end{aligned} \qquad \text{(A8)}$$

which gives a starting point for the transformation U_x (where $\vartheta=0$) and U_y (where $\vartheta=\pi/2$), at each site J.

The second step of the transformations U_x and U_y involves a sequence of CNOT gates. Consider the example of two qubits, with the initial state $|00\rangle\equiv|\uparrow\rangle|\uparrow\rangle$. The transformation $U_{\theta,\vartheta}$ on the first qubit evolves the state into

$$U_{\theta,\vartheta}|\uparrow\rangle|\uparrow\rangle=\cos\frac{\theta}{2}|\uparrow\rangle|\uparrow\rangle+e^{i\vartheta}\sin\frac{\theta}{2}|\downarrow\rangle|\uparrow\rangle. \qquad \text{(A9)}$$

The subsequent CNOT gate then flips the second (target) qubit to $|1\rangle\equiv|\downarrow\rangle$ if the first (control) qubit is $|1\rangle$. For $n>2$, the CNOT gates will be performed between the first qubit and all other qubits. This gives

$$U_{\theta,\vartheta}|\uparrow\rangle^{\otimes N}=\cos\frac{\theta}{2}|\uparrow\rangle^{\otimes N}+e^{i\vartheta}\sin\frac{\theta}{2}|\downarrow\rangle^{\otimes N}. \qquad \text{(A10)}$$

In this way, the transformations (A1) and (A2) for U_x and U_y can be achieved macroscopically (for large N) for each site.

In the two-spin experiment, either U_y or U_z is selected at each site in order to measure S_y^J or S_z^J. We specify that the initial state $|\psi_{\text{Bell}}\rangle_{z,z}$ [Eq. (A3)] has been prepared for the pointer measurement of S_z^J. This means that a direct detection of the qubit value (such as a direct detection of a photon in the mode a_+ or a_-) is all that is required to complete the measurement of S_z^J.

The experiment of Ref. [44] used the IBM quantum computer to perform the operations with $N=2$–6, enabling a test of macrorealism. In a macroscopic realization, similar operations have been performed using Rydberg atoms, for $N\approx 20$ [89].

The analyses given in Secs. III A and IV B follow for this mesoscopic realization, which allows tests involving both two and three spins. The analyses of Secs. V and VI also follow, on replacing the macroscopically distinct states $|\alpha\rangle$ and $|-\alpha\rangle$ with $|\uparrow\rangle^{\otimes N}$ and $|\downarrow\rangle^{\otimes N}$. One can define the macroscopic spins and the eigenstates $|\pm\rangle_y$ and $|\pm\rangle_x$ of S_y and S_x similarly. Hence, most importantly, the premises of weak macroscopic realism (wMR) defined in Sec. II C apply so that an EPR-Bohm paradox can be realized for finite N, based on premises that are not falsified by the corresponding Bell and GHZ tests (as in Fig. 8).

APPENDIX B: EXAMPLE OF MESOSCOPIC QUBITS: NOON STATES

We may also consider where the macroscopic pseudospin states are two-mode number states $|N\rangle|0\rangle$ and $|0\rangle|N\rangle$, for N large. We denote two distinct modes by symbols $+$ and $-$ and simplify the notation so that $|N\rangle|0\rangle\equiv|N,0\rangle$ and $|0\rangle|N\rangle\equiv|0,N\rangle$. The macroscopic qubits become $|\uparrow\rangle\rightarrow|N,0\rangle$ and $|\downarrow\rangle\rightarrow|0,N\rangle$. For the GHZ paradoxes, we consider three sites, labeled A, B, and C. There are two modes (labeled $J+$ and $J-$) identified for each site $J\equiv A,B,C$. The initial state would be of the form (25). For each site, we use the transformation [27]

$$\begin{aligned}\left(U_y^J\right)^{-1}|N,0\rangle_J&=e^{i\varphi}(\cos\theta|N,0\rangle_J+i\sin\theta|0,N\rangle_J),\\ \left(U_y^J\right)^{-1}|0,N\rangle_J&=ie^{i\varphi}(\sin\theta|N,|0\rangle_J-i\cos\theta|0,N\rangle_J),\end{aligned} \qquad \text{(B1)}$$

where $|N,0\rangle_J$ and $|N,0\rangle_J$ are the two-mode number states at site J, and φ is a phase shift. The transformation has been realized to an excellent approximation for $N \lesssim 100$ [27], using the interaction [97,98]

$$H_{nl}^J = \kappa(\hat{a}_{J+}^\dagger \hat{a}_{J-} + \hat{a}_{J+}\hat{a}_{J-}^\dagger) + g\hat{a}_{J+}^{\dagger 2}\hat{a}_{J+}^2 + g\hat{a}_{J-}^{\dagger 2}\hat{a}_{J-}^2, \quad \text{(B2)}$$

so that $U_y^J = e^{-iH_{nl}^J t/\hbar}$. Here, $\hat{a}_{J+}$, $\hat{a}_{J-}$ are the boson destruction operators for the field modes $J+$ and $J-$, and κ and g are the interaction constants. θ is a function of the interaction time t and can be selected so that $\theta = \pi/4$. To realize U_x^J at each site $J \equiv A, B, C$, we suppose the field modes $J+$ and $J-$ are spatially separated at the site J, so that a phase shift θ_p can be applied along one arm, that of mode $J-$, as used in the detection of NOON states [99–101]. For a suitable choice of θ_p, this induces an overall relative phase shift between the modes, allowing realization of the final transformation

$$\left(U_x^J\right)^{-1}|N,0\rangle_J \rightarrow \cos\theta|N,0\rangle_J + \sin\theta|0,N\rangle_J. \quad \text{(B3)}$$

APPENDIX C: CONSIDERATIONS FOR A REALISTIC TEST OF THE EPR-BOHM PARADOX

The EPR-Bohm paradox for the two-spin setup of Sec. III A 1 can be signified when

$$\left(\Delta_{inf}\hat{\sigma}_y^A\right)^2 + \left(\Delta_{inf}\hat{\sigma}_z^A\right)^2 < 1, \quad \text{(C1)}$$

where $(\Delta_{inf}\hat{\sigma}_\theta^A)^2$ is the variance associated with the estimate inferred for the outcome of $\hat{\sigma}_\theta^A$ given a result for a measurement of $\hat{\sigma}_\phi^B$ on system B. The value of ϕ is chosen optimally to minimize the error [17,88]. Hence, for the Bell state (3), $\phi = \theta$. A sufficient condition that the inequality be satisfied is that

$$\left[\Delta\left(\hat{\sigma}_y^A + \hat{\sigma}_y^B\right)\right]^2 + \left[\Delta\left(\hat{\sigma}_z^A + \hat{\sigma}_z^B\right)\right]^2 < 1, \quad \text{(C2)}$$

where $(\Delta\hat{O})^2 = \langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2$ is the variance of $\hat{O}$. Then the estimate of the outcomes for $\hat{\sigma}_\theta^A$ is taken to be $-\sigma_\theta^B$, where σ_θ^B is the outcome of the measurement $\hat{\sigma}_\theta^B$, for $\theta = x, y$. The upper bound of one is half that given by Hofmann and Takeuchi for the entanglement criterion, Eq. (24) of their paper [40], as expected for an EPR-steering inequality [17,102]. Clearly, for the EPR-Bohm test given in Secs. III and Appendix A, the inequality is satisfied since there is a perfect anticorrelation between the outcomes, implying the left-side has a value of zero. Similar inequalities can be derived for the three-spin setup [17].

For the EPR-Bohm test of Sec. V, the inequality is also satisfied in the limit of α large. However, for finite α, the states $|\alpha\rangle$ and $|-\alpha\rangle$ are not truly orthogonal. We consider a realistic experiment as follows: Two orthogonal states $|+\rangle$ and $|-\rangle$ are defined. The spin operators are $\hat{\sigma}_z = |+\rangle\langle+| - |-\rangle\langle-|$ and $\hat{\sigma}_y = (|+\rangle\langle-| - |-\rangle\langle+|)/i$, where $+$ indicates a state with an outcome for $\hat{X}$ that is non-negative ($x \geqslant 0$) and $-$ indicates a state with an outcome for $\hat{X}$ that is negative ($x < 0$). The notation $x \in +$ implies $x \geqslant 0$; the notation $x \in -$ implies $x < 0$. We expand the coherent state as follows:

$$\begin{aligned}|\alpha\rangle &= \sum_{x\geqslant 0}\langle x|\alpha\rangle|x\rangle + \sum_{x<0}\langle x|\alpha\rangle|x\rangle \\ &= c_+(\alpha)|+\rangle_\alpha + c_-(\alpha)|-\rangle_\alpha, \end{aligned} \quad \text{(C3)}$$

where

$$|\pm\rangle_\alpha = \frac{\sum_{x\in\pm}\langle x|\alpha\rangle}{\left[\sum_{x\in\pm}|\langle x|\alpha\rangle|^2\right]^{1/2}}|x\rangle \quad \text{(C4)}$$

so that $c_\pm(\alpha) = [\sum_{x\in\pm}|\langle x|\alpha\rangle|^2]^{1/2}$. We note also

$$|-\alpha\rangle = c_+(-\alpha)|+\rangle_{-\alpha} + c_-(-\alpha)|-\rangle_{-\alpha}, \quad \text{(C5)}$$

where $c_-(-\alpha) = c_+(\alpha)$ and $c_+(-\alpha) = c_-(\alpha)$ from symmetry (we take α real). We rewrite the Bell-cat state as

$$\begin{aligned}|\psi_{\text{Bell}}\rangle &= \mathcal{N}(|\alpha\rangle|-\alpha\rangle - |-\alpha\rangle|\alpha\rangle) \\ &= \mathcal{N}\{(c_+(\alpha)|+\rangle_\alpha + c_-(\alpha)|-\rangle_\alpha) \\ &\quad (c_+(-\alpha)|+\rangle_{-\alpha} + c_-(-\alpha)|-\rangle_{-\alpha}) \\ &\quad - (c_+(-\alpha)|+\rangle_{-\alpha} + c_-(-\alpha)|-\rangle_{-\alpha}) \\ &\quad (c_+(\alpha)|+\rangle_\alpha + c_-(\alpha)|-\rangle_\alpha)\}. \end{aligned} \quad \text{(C6)}$$

This is expanded as

$$\begin{aligned}|\psi_{\text{Bell}}\rangle &= \mathcal{N}\{c_+(\alpha)c_-(-\alpha)(|+\rangle_\alpha|-\rangle_{-\alpha} - |-\rangle_{-\alpha}|+\rangle_\alpha) \\ &\quad + c_-(\alpha)c_+(-\alpha)(|-\rangle_\alpha|+\rangle_{-\alpha} - |+\rangle_{-\alpha}|-\rangle_\alpha) \\ &\quad + c_+(\alpha)c_+(-\alpha)(|+\rangle_\alpha|+\rangle_{-\alpha} - |+\rangle_{-\alpha}|+\rangle_\alpha) \\ &\quad + c_-(\alpha)c_-(-\alpha)(|-\rangle_\alpha|-\rangle_{-\alpha} - |-\rangle_{-\alpha}|-\rangle_\alpha)\} \\ &\equiv L + E, \end{aligned} \quad \text{(C7)}$$

where we write L as the first term, proportional to $c_+(\alpha)c_-(-\alpha) = c_+(\alpha)^2$, and E as the remaining terms. The measurement of σ_z corresponds to determining whether the outcome x of $\hat{X}$ is non-negative or negative: The overlap function for $\hat{X} = (\hat{a} + \hat{a}^\dagger)/2$, where α is real and positive, is $\langle x|\alpha\rangle \sim e^{-(x-\alpha)^2}/\pi^{1/4}$ [54]. We see that

$$|c_+(\alpha)|^2 = \sum_{x\in+}|\langle x|\alpha\rangle|^2 \quad \text{(C8)}$$

corresponds to the integral over positive values of x of the Gaussian $e^{-2(x-\alpha)^2}/\pi^{1/2}$ with mean $\mu = -\alpha$ and standard deviation $\sigma = 1/2$, which approaches one as $\alpha \rightarrow \infty$. Taking a conservative and experimentally realizable value of $\alpha > 2$, we see that the term L dominates. For $\alpha > 2$, we estimate $|c_+(\alpha)|^2 > 0.97$. The remaining terms E involve expressions such as $c_\pm(\mp\alpha)$ which correspond to an integral in the negative-valued tail of the Gaussian, $e^{-2(x-\alpha)^2}/\pi^{1/2}$, giving small contributions to the probabilities: $|c_-(\alpha)|^2 < 0.03$ for $\alpha = 2$. The probability $P(0)$ of obtaining $\sigma_z^A + \sigma_z^B = 0$ is given by the first two terms, with L dominating. Hence,

$$P(0) = 2\mathcal{N}^2|c_+(\alpha)c_+(\alpha)|^2, \quad \text{(C9)}$$

which approaches one as $\alpha \rightarrow \infty$. For $\alpha > 2$, $P(0) > 0.9$. The maximum magnitude possible for the sum of the two spins is two, with a probability of less than 0.1, which gives a bound on $[(\Delta(\hat{\sigma}_z^A + \hat{\sigma}_z^B)]^2$ of below 0.4.

The measurement of σ_y requires a rotation:

$$\begin{aligned}U|+\rangle_{\alpha,z} &= (e^{i\pi/4}|+\rangle_{\alpha,y} + e^{-i\pi/4}|-\rangle_{-\alpha,y})/\sqrt{2}, \\ U|-\rangle_{-\alpha,z} &= (e^{-i\pi/4}|+\rangle_{\alpha,y} + e^{i\pi/4}|-\rangle_{-\alpha,y})/\sqrt{2}, \end{aligned} \quad \text{(C10)}$$

where the basis is denoted by the subscript. The actual rotations are

$$U|\alpha\rangle_z = (e^{i\pi/4}|\alpha\rangle_y + e^{-i\pi/4}|-\alpha\rangle_y)/\sqrt{2}$$
$$= c_+(\alpha)(e^{i\pi/4}|+\rangle_{\alpha,y} + e^{-i\pi/4}|-\rangle_{-\alpha,y})/\sqrt{2}$$
$$+ c_-(\alpha)(e^{i\pi/4}|-\rangle_{\alpha,y} + e^{-i\pi/4}|+\rangle_{-\alpha,y})/\sqrt{2}, \quad \text{(C11)}$$

and similarly

$$U|-\alpha\rangle_z = c_-(-\alpha)(e^{i\pi/4}|-\rangle_{-\alpha,y} + e^{-i\pi/4}|+\rangle_{\alpha,y})/\sqrt{2}$$
$$+ c_+(-\alpha)(e^{i\pi/4}|+\rangle_{-\alpha,y} + e^{-i\pi/4}|-\rangle_{\alpha,y})/\sqrt{2}. \quad \text{(C12)}$$

We consider the transformed Bell state:

$$U|\psi_{\text{Bell}}\rangle = \mathcal{N}\{U_A|\alpha\rangle_z U_B|-\alpha\rangle_z - U_A|-\alpha\rangle_z U_B|\alpha\rangle_z\}$$
$$= \frac{\mathcal{N}}{2}\{ic_+(\alpha)c_-(-\alpha)$$
$$\times [|+\rangle_{\alpha,y}|-\rangle_{-\alpha,y} - |-\rangle_{-\alpha,y}|+\rangle_{\alpha,y}$$
$$- |-\rangle_{-\alpha,y}|+\rangle_{\alpha,y} + |+\rangle_{\alpha,y}|-\rangle_{-\alpha,y}] + \mathcal{E}\}. \quad \text{(C13)}$$

As above, the leading term contributes to the outcome $\sigma_y^A + \sigma_y^B = 0$ and is proportional to $c_+(\alpha)c_-(-\alpha) = c_+(\alpha)^2$, giving a probability $P(0) > 0.9$ for $\alpha > 2$. The terms $\mathcal{E}$, depending on terms proportional to $c_-(\alpha)c_+(\alpha)$ or $c_-(\alpha)^2$, become negligible for large α. As above, the upper bound on $[\Delta(\hat{\sigma}_y^A + \hat{\sigma}_y^B)]^2$ is ≈ 0.4. Since the uncertainty bound for the inequality is one, it is possible to signify the paradox for $\alpha > 2$. The error due to estimating $|+\rangle$ as $|\alpha\rangle$, and $|-\rangle$ as $|-\alpha\rangle$, or vice versa, becomes negligible.

APPENDIX D: CONSIDERATIONS FOR A REALISTIC TEST OF THE EPR-BOHM PARADOX BASED ON WEAK LOCAL REALISM

The EPR-Bohm paradox based on wLR (or wMR) as described in the setups of Figs. 2 and 8 can be signified when

$$\left(\Delta_{inf}\hat{\sigma}_y^A\right)^2 + \left(\Delta_d\hat{\sigma}_z^A\right)^2 < 1 \quad \text{(D1)}$$

for measurements on the system prepared at the time t_f. Here $(\Delta_{inf}\hat{\sigma}_y^A)^2$ is the square of the error in the inferred value for $\hat{\sigma}_y^A$ given the result for the measurement $\hat{\sigma}_y^B$ on system B. The $(\Delta_d\hat{\sigma}_z^A)^2$ is the square of the error in distinguishing the spin states for the state of system A as prepared at the time t_f. The inference variance can be measured using standard techniques, as in Appendix C and Refs. [17,88]. It is also necessary to confirm that system A is given quantum mechanically as a spin-1/2 system, which includes defining the two spin eigenstates and demonstrating both spin measurements $\hat{\sigma}_y^A$ and $\hat{\sigma}_z^A$ (and their noncommutativity) for the entangled system at time t_f, as well as confirming the lower bound of the inequality (14): $(\Delta\hat{\sigma}_y^A)^2 + (\Delta\hat{\sigma}_z^A)^2 \geqslant 1$. The experiment of Ref. [19] reports simultaneous measurement along these lines, but for $\hat{X}$ and $\hat{P}$.

In the macroscopic proposals, the pseudospin states are the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, or else $|\uparrow\rangle_z \equiv |\uparrow\rangle_z^{\otimes N}$ and $|\downarrow\rangle_z \equiv |\downarrow\rangle_z^{\otimes N}$. The latter are clearly distinguishable with $\Delta_d\hat{\sigma}_z^A = 0$. Noise can diminish the effectiveness of the measurement $\hat{\sigma}_z^A$, increasing $(\Delta_d\hat{\sigma}_z^A)^2$. The analysis in Appendix C indicates that the error due to the overlap of the coherent states becomes negligible for $\alpha > 2$, so that $(\Delta_d\hat{\sigma}_z^A)^2 \to 0$.

APPENDIX E: THE UNITARY OPERATION U_y FOR MEASUREMENT OF S_y

Consider the system A originally in the eigenstate for S_y:

$$|\uparrow\rangle_y = \frac{e^{-i\pi/4}}{\sqrt{2}}(|\uparrow\rangle_z + i|\downarrow\rangle_z), \quad \text{(E1)}$$

which is

$$|\uparrow\rangle_y \equiv \frac{e^{-i\pi/4}}{\sqrt{2}}(|\alpha\rangle_z + i|-\alpha\rangle_z) \quad \text{(E2)}$$

in our realization. The state after the operation U_y is $|\alpha\rangle_y$, since we see from (29) that

$$U_y|\uparrow\rangle_y = U_{\pi/4}^{-1}\frac{e^{-i\pi/4}}{\sqrt{2}}(|\uparrow\rangle_z + i|\downarrow\rangle_z)$$
$$= |\alpha\rangle. \quad \text{(E3)}$$

The pointer measurement $\hat{S}$ on this state (for large α) gives $+1$, corresponding to the outcome required for the eigenstate $|\uparrow\rangle_y$. Similarly, consider the system prepared in $|\downarrow\rangle_y$

$$|\downarrow\rangle_y = \frac{e^{-i\pi/4}}{\sqrt{2}}(|\downarrow\rangle_z + i|\uparrow\rangle_z, \quad \text{(E4)}$$

which is

$$|\downarrow\rangle_y \equiv \frac{e^{-i\pi/4}}{\sqrt{2}}(|-\alpha\rangle_z + i|\alpha\rangle_z). \quad \text{(E5)}$$

The state after the operation U_y is $|-\alpha\rangle_y$, since from (29), we see that

$$U_y|\downarrow\rangle_y = |-\alpha\rangle, \quad \text{(E6)}$$

for which the pointer measurement X gives the outcome -1, as required for this eigenstate. Hence, the system that is originally in the linear superposition (35) transforms after U_y to

$$U_y|\psi\rangle = d_+U_y|+\rangle_y + d_-U_y|-\rangle_y$$
$$\to d_+|\alpha\rangle + d_-|-\alpha\rangle. \quad \text{(E7)}$$

As $\alpha \to \infty$, the probability of an outcome $+1$ (-1) for the measurement $\hat{S}$ of the sign of $\hat{X}_A$ is $|d_+|^2$ $(|d_-|^2)$, as required.

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