# Higher-order relativistic contributions to the Zeeman effect in helium and heliumlike ions* 

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#### Abstract

The higher-order relativistic contributions to the Zeeman effect in triplet states in helium and heliumlike ions have been calculated to order $\alpha^{3} \mu_{B} H$.


We have recently given a calculation of higherorder relativistic contributions to the Zeeman effect, to order $\alpha^{2} \mu_{B} H$, in the $2^{3} P, 2^{3} S, 3^{3} P$, $4^{3} P$, and $5^{3} P$ states of helium and the $2{ }^{3} P_{1}, 3^{3} P_{1}$, $4^{3} P_{1}$, and $5^{3} P_{1}$ states of the helium isoelectronic series from LiII through Ne IX. ${ }^{1}$ In this addendum we calculate the contributions to the Zeeman effect of order $\alpha^{3} \mu_{B} H$.

The higher-order corrections to the $g$ factor are contained in $g_{S}^{\prime}$. To order $\alpha^{2}$, we find ${ }^{1,2}$

$$
\begin{equation*}
g_{S}^{\prime}=g_{S}-2 \alpha^{2}\left(\frac{1}{3}\langle T\rangle+\frac{1}{6}\left\langle 1 / r_{12}\right\rangle\right), \tag{1.1}
\end{equation*}
$$

where $g_{s}$ is the gyromagnetic ratio of the free
electron, $T$ is the total kinetic energy of both electrons, and $r_{12}$ is the interelectron distance. Equation (1.1) agrees to order $\alpha^{2}$ with a new result, ${ }^{3}$ including terms to order $\alpha^{3}$ :

$$
\begin{equation*}
g_{s}^{\prime}=g_{S}\left[1-\alpha^{2}\left(\frac{1}{3}\langle T\rangle+\frac{1}{6}\left\langle 1 / r_{12}\right\rangle\right)-\left(\alpha^{3} / 4 \pi\right) E\right], \tag{1.2}
\end{equation*}
$$

where $E$ is the nonrelativistic energy eigenvalue. Table I presents the relativistic corrections to the $g$ factors in helium and heliumlike ions calculated from Eq. (1.2) with the matrix elements of $T$ and $1 / r_{12}$ computed by Accad, Pekeris, and Schiff. ${ }^{4}$ We have neglected terms of order $\alpha^{2}(m / M)$ because they contribute less than $10^{-9} g_{S}$ to $g_{S}^{\prime}$.

TABLE I. Relativistic contributions, to order $\alpha^{3}$, to the $g$ factors of helium and heliumlike ions. $\left[\left(g_{s}^{\prime} / g_{s}\right)-1\right] \times 10^{6}$ is tabulated.

|  |  | $\alpha^{2}$ | $\alpha^{3}$ | Total |  |  | $\alpha^{2}$ | $\alpha^{3}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| He | $2^{3} S$ | -40.99161 | 0.06727 | -40.924 34 | $\mathrm{C}^{12} \mathrm{~V}$ | $2^{3} P_{1}$ | -387.334 4 | 0.6563 | -386.6781 |
|  | $2^{3} \boldsymbol{P}$ | -40.23112 | 0.06596 | -40.16516 |  | $3^{3} P_{1}$ | -349.448 6 | 0.6004 | -348.8482 |
|  | $3^{3} P$ | -37.560 75 | 0.06364 | -37.49711 |  | $4^{3} P_{1}$ | -336.2971 | 0.5811 | -335.716 0 |
|  | $4^{3} P$ | -36.647 17 | 0.06285 | -36.584 32 |  | $5^{3} P_{1}$ | -330.223 6 | 0.5723 | -329.6513 |
|  | $5^{3} \boldsymbol{P}$ | -36.229 79 | 0.06248 | -36.167 31 | $\mathrm{N}^{14} \mathrm{VI}$ | $2^{3} P_{1}$ | -529.558 5 | 0.9005 | -528.658 0 |
|  |  |  |  |  |  | $3^{3} P_{1}$ | -476.7218 | 0.8205 | -475.9013 |
| $\mathrm{Li}^{7}{ }^{7}$ II | $2^{3} P_{1}$ | -93.760 64 | 0.15547 | -93.605 17 |  | $4^{3} P_{1}$ | -458.3569 | 0.7929 | -457.564 0 |
|  | $3^{3} P_{1}$ | -85.958 23 | 0.14628 | -85.81195 |  | $5^{3} P_{1}$ | -449.8845 | 0.7801 | -449.104 4 |
|  | $4^{3} P_{1}$ | -83.27375 | 0.14313 | -83.13062 | $0^{16} \mathrm{VII}$ | $2^{3} P_{1}$ | -693.9692 | 1.1834 | -692.785 8 |
|  | $5^{3} P_{1}$ | -82.04195 | 0.14168 | -81.900 27 |  | $3^{3} P_{1}$ | -623.7174 | 1.0700 | -622.647 4 |
|  |  |  |  |  |  | $4^{3} P_{1}$ | -599.2766 | 1.0374 | -598.2392 |
| $\mathrm{Be}^{9} \mathrm{III}$ | $2^{3} P_{1}$ | -169.440 1 | 0.2837 | - -169.156 4 |  | $5^{3} P_{1}$ | -587.995 9 | 1.0201 | -586.975 8 |
|  | $3^{3} P_{1}$ | -154.068 0 | 0.2633 | -153.804 7 | $\mathrm{F}^{19} \mathrm{VIII}$ | $2^{3} P_{1}$ | -880.567 0 | 1.5050 | -879.062 0 |
|  | $4^{3} P_{1}$ | -148.7560 | 0.2563 | -148.499 7 |  | $3^{3} P_{1}$ | -790.4354 | 1.3638 | -789.0716 |
|  | $5^{3} P_{1}$ | -146.312 8 | 0.2531 | -146.059 7 |  | $4^{3} P_{1}$ | -759.055 9 | 1.3149 | -757.7410 |
|  |  |  |  |  |  | $5^{3} P_{1}$ | -744.567 7 | 1.2923 | -743.275 4 |
| $B^{11}$ IV | $2^{3} P_{1}$ | -267.295 8 | 0.4507 | -266.8451 | Ne ${ }^{20}{ }_{\text {IX }}$ | $2^{3} P_{1}$ | -1089.352 | 1.865 | -1087.487 |
|  | $3^{3} P_{1}$ | -241.8976 | 0.4147 | -241.4829 |  | $3^{3} P_{1}$ | -976.8761 | 1.6870 | -975.1891 |
|  | $4^{3} P_{1}$ | -233.096 8 | 0.4023 | -232.6945 |  | $4^{3} P_{1}$ | -937.6951 | 1.6252 | -936.069 9 |
|  | $5^{3} P_{1}$ | -229.043 0 | 0.3966 | -228.646 4 |  | $5^{3} P_{1}$ | -919.600 0 | 1.5966 | -918.003 4 |

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${ }^{1}$ M. L. Lewis and V. W. Hughes, Phys. Rev. A 8, 2845 (1973).
${ }^{2}$ W. Perl and V. W. Hughes, Phys. Rev. 91, 842 (1953). ${ }^{3}$ H. Grotch and R. A. Hegstrom, Phys. Rev. A 8, 1166 (1973).
${ }^{4}$ Y. Accad, C. L. Pekeris, and B. Schiff, Phys. Rev. A $\underline{4}$, 516 (1971).

