Intense polarized electron beams from chemi-ionization reactions with optically pumped He(2 ${}^{3}S$)[†]

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An optically pumped flowing helium afterglow is evaluated as a potential source of polarized electrons. Spin angular momentum conservation in chemi-ionization reactions involving optically oriented He($2^{3}S$) atoms has been exploited to yield microampere electron beams with 30% polarization and energy spread ≤ 0.5 eV. The beam emittance is on the order of 10 mrad cm at 500 V extraction potential. This source appears well-suited for most spin-dependent scattering experiments that have been proposed.

I. INTRODUCTION

There has been increasing interest during the past decade in the development of polarized electron sources suitable for studies of spin-dependent effects in electron scattering from nuclei,^{1,2} atoms,^{3,4} and molecules.⁴ More recently, there have also been predictions that polarization effects in low-energy electron diffraction (LEED) can provide new information on the properties of solid surfaces,⁵ and Vriens⁶ has further suggested that the momentum and spin-density distributions of bound electrons in magnetic materials can be obtained from scattering experiments with polarized electrons.

Many processes have been proposed as bases for polarized electron sources.⁷ Several that have shown promise are (i) photoionization of polarized atoms,² (ii) photoionization of alkali-metal atoms by circularly polarized light (Fano effect),⁸ (iii) field emission⁹ and photoemission¹⁰ from magnetic materials, and (iv) Mott and low-energy scattering from unpolarized atomic beams.^{4,11}

In the present paper we describe the use of an optically pumped, flowing helium afterglow as a source of polarized electrons. This work is an outgrowth of earlier efforts in this laboratory by McCusker *et al.*, who extracted a polarized electron beam from an optically pumped helium discharge.¹²

II. SOURCE CONFIGURATION AND OPERATING PARAMETERS

The apparatus is basically the same as described in the preceding¹³ paper, henceforth referred to as I. However, in the present work the various component placements and apparatus parameters were adjusted in order to optimize the performance of the apparatus as a source of polarized electrons.

Best results were obtained with the apparatus as configured in Fig. 1; details of the extraction region are shown in Fig. 2. This is similar to the

arrangement used in I except that a second pumping lamp has been added and the position of the reactant injector altered. As in I, helium 2 ^{3}S and 2 ^{1}S metastables are produced by either a microwave discharge or an electron gun. The 2 3S atoms are then optically pumped by circularly polarized resonance radiation to preferentially populate either the $m_s = +1$ or $m_s = -1$ state. Electrons are produced by chemi-ionization of a reactant gas injected near the extraction aperture. These electrons are then extracted and focused by two Einsel lenses before acceleration to 120 keV for spin analysis by Mott scattering. The second Einsel lens can also be operated as a filter lens to measure the energy spread of the beam. In this section the optimization of the various experimental parameters is discussed.

A. Metastable excitation

The microwave cavity was used for most of the data presented here because it produces a low singlet to triplet metastable ratio of 0.06 at the optimum flow-tube pressure of 0.10 Torr compared to a ratio of 0.5 for an electron-gun source. (The electrons produced by chemi-ionization reactions involving singlet metastable atoms are unpolarized and therefore can seriously degrade the polarization of the output beam.)

The 2 ${}^{3}S$ production rate obtained using the microwave cavity source is nearly independent of microwave power for powers above 30 W. The 2 ${}^{3}S$ density at the extractor is then limited by diffusion losses to the flow-tube walls and hence by the distance from the source to extractor for a given flow-tube pressure. For a source-to-extractor separation of less than ~30 cm, the polarization is degraded by source-produced electrons that are not removed by diffusion prior to extraction, and by relaxation of the He(2 ${}^{3}S$) polarization due to electron excitation and exchange. Therefore while operating the source close to the extractor results in an increased He(2 ${}^{3}S$) density

11

1279



FIG. 1. Schematic diagram of the apparatus as configured to perform optimally as a source of polarized electrons (not drawn to scale).

available for production of polarized electrons in chemi-ionization reactions, it also results in the extraction of an increased number of unpolarized electrons from the source. There is therefore a trade-off between polarization and current.

In the case of the electron-gun source the 2 ^{3}S density increases linearly with emission current,



FIG. 2. Detail of the gas injector and electron extractor. The injection ring has 12 holes on the inside diameter and 28 on the outside, each 0.25 mm in diameter.

so that higher densities can be obtained by use of a larger filament or multiple filaments. (The filament used was 0.005-in. diameter by 1.5-in. length operated at an emission current of 0.12 A and accelerating voltage of 75 V.) However, the lateral dimension of the excitation region, which normally extends some 10 cm from the filament, also tends to increase with emission current, thus increasing the number of unpolarized source electrons extracted.

A trade-off must also be made between polarization and current as a function of pressure for a given 2 ${}^{3}S$ source position. For the source-extractor separation shown in Fig. 1, the unpolarized source-produced electrons account for less than 5% of the extracted current for flow-tube pressures below 0.15 Torr, but their contribution to the extracted current increases rapidly for higher pressures because of their slower diffusion to the walls. These electrons can be removed, at least partially, by the use of rf heating near the source to enhance the ambipolar diffusion rate. However the rf must be shielded from the chemi-ionization region to avoid a similarly enhanced diffusion loss of the polarized electrons produced there.

B. Optical pumping

The 2³S state is optically pumped in the usual manner¹⁴ by the absorption of circularly polarized 2³S-2³P resonance radiation with subsequent return to the 2 ³S state by spontaneous emission. The use of two pumping lamps instead of one to extend the length of the pumping region (but not the pumping radiation intensity) increases the electron polarization by a factor of about 1.2, suggesting that steady-state $He(2^{3}S)$ polarization is nearly attained in the $\sim 10^{-3}$ second traversal time of the pumping region. Each lamp contains helium at 10 Torr and is excited in the tank circuit of a 100-MHz multivibrator which dissipates 400 W.^{12,15} Typical photon flux in the $2 {}^{3}S - 2 {}^{3}P$ resonance line is about 10^{17} photons/sec. The lamps are shielded to prevent rf heating of the electrons in the afterglow.

A weak magnetic field of 5 G in the direction of the pumping light is used to overcome Earth's magnetic field and thereby provide a unique quantization axis. The electrons are extracted along the field direction to avoid magnetic deflection and are therefore longitudinally polarized. However, transverse polarization should easily be obtainable, simply by rotating the pumping lamps 90° about the flow-tube axis or by extracting the electrons along the flow-tube axis. In either case, large magnetic deflections of the extracted beam can be avoided by first nulling Earth's field and then superimposing a field along the opticalpumping axis that is much smaller than the 5 G used in the experiments reported here.

The attainable $He(2^{3}S)$ polarization, and hence the polarization of electrons they produce in chemi-ionization reactions, is limited to approximately 30% by $He(2^{3}S)$ spin-randomizing processes that compete with the optical-pumping process.

C. Gas injection

The extracted current and polarization that can be realized depend somewhat on the particular gas used. Results for N_2 and CO_2 are presented in Sec. III. (The polarizations obtained for several other common gases are described in I.) However, some general comments apply to all gases.

First by placing the injector downstream of the extractor as shown in Fig. 1, the helium metastables can be completely reacted very near the extraction orifice by back-streaming gas. The flow rate of gas injected in this configuration is typically 4 Torr liters/sec, about 10% of the He flow. This injector location also avoids the loss of metastables and electrons associated with dif-fusion to the gas injector surface when it is placed upstream of the extractor.

Second the gas should be chosen to have a large cross section for both chemi-ionization and for rotational and vibrational excitation by low-energy electrons. In this way electrons produced during chemi-ionization are rapidly thermalized thus reducing the electron loss by ambipolar diffusion while also reducing the energy spread of the extracted beam. For example, from the measured fractional energy loss per collision¹⁶ a simple calculation shows that the energy of an electron will be reduced from 1.5 to 0.05 eV in an average of 60 collisions with CO₂ compared to 1.2×10^4 collisions for He.

D. Electron extraction

The electrons extracted from the flow tube pass through the small extraction aperture and are accelerated by a voltage V_a typically of 500 V. This beam energy is achieved by floating the entire flow tube and extraction aperture negative with respect to the grounded extraction anode. The beam currents that are reported here were measured to the inner cup (12-mm diameter) of a movable collector shown in detail in Fig. 2. The current to the inner cup was typically five times the current to the outer one (25-mm diameter). An upper limit on the beam emittance¹⁷ for the current to the inner cup can be estimated by the product of the cup radius and the maximum angular divergence, which is determined in this case by the diameter of the cylindrical focus electrode. This upper limit is 20 mrad cm. A lower limit is provided by the product of the radius of the ex-traction anode (0.16 cm) and the minimum angular divergence given by the ratio of axial and per-pendicular velocities v_a and v_{\perp} , such that $\theta \approx v_{\perp}/v_a = (E_{\perp}/E_a)^{1/2}$. If E_{\perp} is equal to the measured energy spread of the beam ($\leq 0.5 \text{ eV}$) and $E_a = 500 \text{ eV}$ then $(r\theta)_{\min} \cong 5 \text{ mrad cm at 500 eV}$. No attempt was made to improve the electron optical quality of the beam or increase the extracted current by changes in the geometry of the extraction anode or the first lens element.

According to the extraction scheme just described, the electrons in the flow tube simply diffuse through the extraction aperture before being accelerated by the extraction field. However, it was observed that a large increase in current can be obtained if the afterglow tube is biased with a potential V_b of 10-20 V negative with respect to the extraction aperture. For the largest aperture used (2.0-mm orifice) an observed maximum in the extracted current as a function of bias voltage indicates some type of focusing effect is present. Since there is very little degradation of polarization for bias potentials up to 20 V, it is obvious that the bias increases extraction efficiency rather than initiating a discharge. The manner in which the extraction and bias potentials are established is indicated in Fig.2.

Since most of the potential drop in a gas diode occurs at the cathode and not the anode, it was not anticipated that electrons could be accelerated out of the afterglow in this manner. However, the energy distribution of the extracted beam shows that the electrons are in fact extracted from a plasma at the potential of the flow tube, independent of the extraction aperture potential. It should be noted that the entire extraction cone that protrudes into the flow tube, except for the 0.5-in.diam aperture disc, was coated with PVC insulation. It is likely that a larger exposed metallic surface area would have an adverse effect on the extraction characteristics, but this was not explored experimentally.

Electron polarizations are observed to decrease rapidly at pressures above 0.125 or 0.15 Torr depending on the extraction aperture size. This effect is attributed to ionization or excitation of the background gas by the electron beam in the immediate post-extraction region. Elimination of this problem by increasing the effective post-extraction pumping speed or by lowering the accelerating voltage should improve source performance.

E. Filter lens

The filter lens used in this experiment to measure the energy distribution of the electron beam was designed to provide an energy resolution of about 0.3-0.5 eV. It is very similar to the lens described by Kessler and Lindner,¹⁸ and Simpson and Marton.¹⁹ The principle of operation for this type of lens is discussed in detail by Simpson.²⁰ It is best viewed as two back-to-back short-focus lenses symmetrically placed about a retarding plane. The first lens focuses the incoming beam on a small aperture in the retarding plane; those electrons with sufficient energy to pass through the retarding plane are then imaged by the second lens onto an exit aperture. The energy distribution of the beam is obtained from the dependence of the transmitted current on the retarding plane potential.

For proper operation of the lens, the 5-G field used for optical pumping is turned off. The presence of Earth's magnetic field results in a slight degradation of the energy resolution of the lens.

III. RESULTS

A. Efficiency of electron extraction

The current extracted from the afterglow, while reproducible, is dependent upon several parameters such as flow-tube pressure and bias potential, extraction-aperture geometry and choice of reactant gas. These dependences are evident in a summary of typical data presented in Table I, which is discussed below.

As expected, the extracted current for a given set of experimental parameters was found to be essentially independent of the post-extraction accelerating potential V_a . The value $V_a = 500$ V was chosen only to minimize deflections of the extracted beam by Earth's magnetic field.

However, the extraction efficiency is increased by a large factor—as much as 200 for some combinations of reactant gas and extractor geometry by biasing the extraction aperture a few volts positive with respect to the afterglow tube.

It is possible to calculate a value for the maximum current $I_{\rm max}$ which may be extracted from the afterglow. This is accomplished by determining the number of metastables per second which cross a fixed plane perpendicular to the afterglow using an optical absorption technique. The maximum current which may be extracted can then be estimated by assuming that each metastable produces one electron. Extraction efficiencies in Table I are expressed as percentages of $I_{\rm max}$. All data were obtained with the gas injector placed 1 cm downstream of the extractor.

It is evident that the extraction efficiency is extremely sensitive to the flow-tube bias V_b , and also highly dependent on the choice of reactant gas. For the 2-mm orifice and CO₂ as a reactant gas, a small bias voltage—in this case 5 V—increases the current by a factor of 200 when some 40% of all electrons created in the tube are ex-

TABLE I. Electron extraction efficiencies. He $(2^{3}S)$ decay lengths are determined by diffusion to the flow-tube walls, and quoted densities \overline{N} are averages across the diameter of the flow tube in the optical-pumping region. I_{max} and V_{b} are defined in the text. Efficiencies quoted where no reactant gas is present refer to the extraction of electrons ejected from the brass extractor surface by He $(2^{3}S)$ metastable atoms.

			0.90	Pressur 0.100	e (Torr) 0.125	0.150
He(2 3 S) Decay length (cm) He(2 3 S) Density: \overline{N} (cm ${}^{-3}$) I_{max} (A)			$5.0 \\ 1.1 \times 10^{7} \\ 5.8 \times 10^{-7}$	5.5 2.7×10 ⁷ 1.3×10 ⁻⁶	6.5 1.3×10^{8} 7.0×10^{-6}	$8.2 \\ 5.1 \times 10^8 \\ 2.7 \times 10^{-5}$
Extraction aperture	Optimum bias ^a voltage V_b	Reactant gas	Extraction Efficiency % of I _{max}			
2.0 mm×0.2 mm	Biased	CO ₂ N ₂ None	17	$37 \\ 2.6 \\ 0.64$	$43 \\ 8.1 \\ 0.45$	37
	No bias	CO ₂ N ₂ None		1.0 0.6 0.19	$0.2 \\ 0.74 \\ 0.12$	3.7 0.11
1.5 mm×1.5 mm	Biased Biased No bias	$\begin{array}{c} \mathrm{CO}_2 \\ \mathrm{N}_2 \\ \mathrm{CO}_2 \\ \mathrm{N}_2 \end{array}$		1.4	2.9 0.71 0.11 0.08	2.7 0.45 0.27
		None			0.05	0.03

^aOptimum bias voltage varies with pressure but is between 5 and 25 V.

tracted into a usable beam. It may thus be concluded that no major increases in extracted current can be obtained by either larger extraction apertures or redesigned extraction optics, although the latter may improve the electron optical quality of the beam. The order-of-magnitude difference in the currents obtained with CO_2 and N_2 is probably due to the slower thermalization of electrons in N_2 resulting in a larger diffusion loss.

B. Energy spread of the extracted beam

The filter lens described in Sec. II was used to measure the energy distributions of extracted electrons. The transmitted current through the filter lens as a function of retarding voltage is shown in Fig. 3 both for electrons produced during ionization of CO_2 injected 1 cm downstream from the extractor and for electrons ejected from the extractor orifice surface when no reactant gas is present. The measured energy spread ΔE for surface ejected electrons is typically 3-5 eV. On the other hand, $\Delta E = 0.35 - 0.5$ eV for electrons produced by chemi-ionization of the molecular gases $CO_2,\ N_2,\ and\ C_2H_2.$ The initial energies of electrons from these gases are primarily 1-3 eV for CO_2 , 1-5 eV for N₂, and 9 eV for C_2H_2 . The electrons are rapidly thermalized by rotational and vibrational excitation of their parent gases. Since the measured energy spreads approximate the calculated resolution of the filter lens, the true energy spread of the extracted beam may be substantially less.

When the afterglow is biased negatively relative to the extraction aperture, the energy of the extracted beam increases correspondingly but the energy spread remains unchanged. Therefore, for the same conditions of electron extraction at



FIG. 3. Transmitted electron current through the filter lens vs retarding voltage: (a) electrons from chemi-ion-ization of CO_2 and (b) electrons ejected from the surface of the extraction aperture by $He(2^{3}S)$ atoms when no reactant gas is introduced.



FIG. 4. Measured electron polarizations vs extracted currents. The electrons were produced by chemi-ionization of CO_2 or N_2 with the apparatus configuration as shown in Figs. 1 and 2. Numbers in parentheses are flow-tube pressure in mTorr (upper) and bias voltage V_b (lower): (a) Extraction aperture 2-mm diameter $\times 0.2$ mm and (b) Extraction canal 1.5-mm diameter $\times 1.5$ mm. (Multiple data points taken under seemingly identical conditions are typical of the day-to-day variation in source performance.)

which the polarization and current measurements were made, we conclude that the energy spread of the beam is less than 0.35-0.5 eV and may approach thermal energies (0.03 eV).

C. Polarization and current

Polarizations and extracted currents were measured for a variety of afterglow conditions and for several reactant gases. Best results were obtained with a microwave 2^{3} S excitation source, a 2.0mm-diam by 0.2-mm extraction orifice, and using CO₂ as the reactant gas. Typical data obtained under optimum operating conditions are presented in Fig. 4(a). The reactant gas was admitted 1 cm downstream of the extractor. For each data point the flow-tube pressure (in mTorr) and bias potential are given in parentheses. Two data points taken with N₂ as the reactant gas are shown for comparison. The fall-off in polarization at higher pressures results from ionization of the background gas in the immediate post-extraction region. Better differential vacuum pumping of this region should allow extraction of currents on the order of 10^{-5} A at higher flow-tube pressures without degradation of the polarization.

For purposes of comparison, Fig. 4(b) presents typical data for a 1.5-mm $\times 1.5$ -mm extraction aperture. The results are similar to those presented in Fig. 4(a) except that the extracted currents for comparable polarizations are about an order of magnitude lower.

The extracted currents and polarizations were found to be insensitive to the geometry of the nozzles used in the microwave excitation source. As expected, substantially lower polarizations were obtained with the electron-gun source because it produces a much higher proportion of 2^{1} S metastables.

Finally, when *no* reactant gas is introduced, polarized electrons are ejected from the extractor surfaces by the optically pumped helium metastables, as discussed in I. Under these conditions, polarizations and currents comparable to those shown in Fig. 4(b) can be extracted through a $1.5\text{-mm} \times 1.5\text{-mm}$ aperture 30 cm from the $\text{He}(2\,^{3}\text{S}_{1})$ source. It should be kept in mind, however, that the energy spread of the extracted beam when the apparatus is operated in this mode is several electron volts, while the chemi-ionization-produced beam has an energy spread $\leq 0.5 \text{ eV}$.

IV. CONCLUSIONS

In this section the basic performance characteristics of the optically pumped, flowing helium afterglow as a source of polarized electrons are summarized, and several areas of possible improvement are discussed.

Electron polarizations of about 30% can be obtained routinely so long as the flow-tube pressure is kept low enough to avoid ionization of the background helium gas immediately behind the extractor. In the present apparatus the attainable current at 30% polarization is limited to about 5×10^{-7} A. Substantially higher currents can be extracted at some sacrifice in polarization; operation at 0.125 Torr with CO₂ reactant gas yielded the best values for the P^2I quality factor (P = 21%, $I = 3 \times 10^{-6}$ A).²¹ Better differential pumping of the post-extraction region should allow the flow tube to be operated at somewhat higher pressures and allow extracted currents substantially in excess of 10^{-6} A at 30% polarization.

The degree of polarization is limited by $He(2^{3}S)$ spin thermalization processes that compete with the optical pumping of the 2³S helium metastables. The helium resonance lamps used in these experiments are optically thick and cannot be substantially improved upon. However, advances in tunable infrared laser technology may yield superior optical-pumping sources in the future.

In the present work the beam was extracted at a potential of 500 eV with longitudinal polarization, a measured energy spread of ≤ 0.5 eV, and a beam emittance of 5–20 mrad cm. As previously discussed it should be possible to extract a comparable beam with transverse polarization. Efficient electron extraction is possible with a potential of only ~30 V; 500 V is used only to reduce magnetic deflections of the beam.

An added and distinct advantage of this source is that the electron-spin direction can be reversed (or modulated at frequencies up to about 50-100 Hz) without affecting the beam trajectory, simply by rotating a quarter-wave plate to reverse the sense of circular polarization of the optical-pumping radiation.

This source of polarized electrons is inexpensive and easy to operate. It appears well-suited for most, if not all, spin-dependent scattering experiments that have been proposed to date.

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- *Present address: Texas A &M University, College Station, Texas.
- ¹G. Domokos, S. Kovesi-Domokos, and E. Schonberg, Phys. Rev. D <u>3</u>, 1191 (1971).
- ²V. W. Hughes, R. L. Long, Jr., M. S. Lubell,
- M. Posner, and W. Raith, Phys. Rev. A 5, 195 (1972).
- ³H. Kleinpoppen, Phys. Rev. A <u>3</u>, 2015 (1971).
- ⁴J. Kessler, Rev. Mod. Phys. <u>41</u>, 3 (1969).

- ⁵P. J. Jennings, Surf. Sci. <u>26</u>, 509 (1971); P. J. Jennings and B. K. Sim, Surf. Sci. <u>33</u>, 1 (1972); R. Feder, Phys. Status. Solidi. B 49, 699 (1972).
- ⁶L. Vreins, Phys. Rev. B <u>4</u>, 3088 (1971).
- ⁷J. Kessler, *Atomic Physics 3*, edited by S. J. Smith and G. K. Walters (Plenum, New York, 1973), p. 523.
- ⁸J. Kessler and J. Lorenz, Phys. Rev. Lett. <u>24</u>, 87 (1970); U. Heinzmann, J. Kessler, and J. Lorenz, Z. Phys. <u>240</u>, 42 (1970).
- ⁹N. Muller, W. Eckstein, W. Heiland, and W. Zinn, Phys. Rev. Lett. 29, 1651 (1972).

- ¹⁰G. Busch, M. Campagna, and H. C. Siegmann, J. Appl. Phys. <u>41</u>, 1044 (1970) and <u>42</u>, 1781 (1971); G. Busch, M. Campagna, P. Cotti, and H. C. Siegmann, Phys. Rev. Lett. <u>22</u>, 597 (1969); U. Banninger, G. Busch, M. Campagna, and H. C. Siegmann, Phys. Rev. Lett. <u>25</u>, 585 (1970).
- ¹¹M. Wilmers, R. Haug, and H. Deishsel, Z. Angew. Phys. <u>27</u>, 204 (1969); H. D. Zeman, K. Jost, and S. Gilad, in Proceedings of the Seventh International Conference on the Physics of Electronic and Atomic Collisions, Abstract of Papers, Amsterdam, 1971, edited by L. M. Branscomb et al. (North-Holland, Amsterdam, 1972), p. 1005.
- ¹²M. V. McCusker, L. L. Hatfield, and G. K. Walters, Phys. Rev. Lett. <u>22</u>, 817 (1969); Phys. Rev. A <u>5</u>, 177 (1972); and M. V. McCusker, Ph.D. thesis (Rice University, 1969) (unpublished).
- ¹³P. J. Keliher, F. B. Dunning, M. R. O'Neill, R. D. Rundel, and G. K. Walters, preceding paper, Phys. Rev. A <u>11</u>, 1271 (1975).
- ¹⁴L. D. Schearer, Advances in Quantum Electronics,

edited by J. R. Singer (Columbia U. P., New York, 1961), pp. 239-251.

- ¹⁵R. L. Gamblin and T. R. Carver, Phys. Rev. <u>138</u>, A946 (1965).
- ¹⁶H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena*, Vol. I (Clarendon, Oxford, 1969).
- ¹⁷Beam emittance is defined as the product of the minimum beam radius (i.e., at the extraction aperture focal point) and the angular spread of the beam at that position.
- ¹⁸J. Kessler and H. Lindner, Z. Angew. Phys. <u>18</u>, 7 (1964).
- ¹⁹J. A. Simpson and L. Marton, Rev. Sci. Instrum. <u>32</u>, 802 (1961).
- ²⁰J. A. Simpson, Rev. Sci. Instrum. <u>32</u>, 1283 (1961).
- ²¹For scattering experiment applications, P^2I is a measure of the effective intensity of a polarized electron beam, since the signal-to-noise ratio for spin-dependent scattering varies linearly with the polarization and as the square root of the counting rate.