Comment on "Theory of the thermomagnetic force"

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It is pointed out that the thermomagnetic force observed to date is a magnetic-field–induced change of the thermal force which is proportional to the first spatial derivative of the temperature. This is in contradistinction to the point of view expressed by Fleming.

Some years ago, Larchez and Adair[1] reported measurements of the "thermomagnetic force" (TMF), i.e., of a magnetic-field-induced change of the thermal force exerted by a heat-conducting polyatomic gas on a disk. Recently, Fleming presented a theory[2] of the TMF based on the assumption that the observed force is due to thermal stresses and consequently proportional to $\vec{\nabla}\vec{\nabla} T$ (T is the temperature of the gas). He comments on an article[3] by the present author which also dealt with the kinetic theory of the TMF: "His [Hess's] theory utilized a direct coupling between the force and $\vec{\nabla}T$. This type of coupling is forbidden because ordinary transport coefficients always couple quantities which have different time-reversal properties.[4] Since $\vec{F}$ and T are both even under time reversal, there can be no such coupling (in the bulk)." However, it is a well-established experimental fact that a thermal force proportional to $\vec{\nabla}T$ exists.[5] (For reviews of the experimental and theoretical work on this subject see, e.g., Refs. 6 and 7.)

Half a century ago Einstein[8] presented a mean-free-path theory for the thermal force exerted on a sphere which is small compared with the mean free path l of a molecule in the gas. The kinetic theory was developed by Waldmann.[9] The case of a sphere large compared with l was treated theoretically by Epstein.[10] These theories have been extended to the thermal force exerted on obstacles of nonspherical shapes.[10–12]

Of course, now the question arises as to why Fleming's argument quoted above does not apply to the situation described in Refs. 3, 5–12. To simplify the discussion, the limiting cases of obstacles which are small and of those which are large compared with the mean free path l of a molecule are considered. In the first case, Fleming's argument does not apply since the force acting on a small obstacle is certainly not a "bulk" effect. The situation is different for obstacles large compared with l. Here the coefficient which couples $\vec{F}$ with $\vec{\nabla}T$ is proportional to the product of two "ordinary" transport coefficients, viz., the viscosity and the heat conductivity (cf. Refs. 6, 10 and 13). Thus there is no conflict with time-reversal arguments. After these remarks on the thermal force, some comments on the thermomagnetic force are in order.

In Ref. 3 I reported theoretical results for the thermomagnetic force exerted by a polyatomic gas on a sphere which are based on rather straightforward extensions of the theories of Einstein[8] and Epstein[10] as refined by Waldmann.[9,13] The TMF, more precisely, the quantity $\Delta F/F$, where ΔF is the magnetic-field-induced change of the thermal force and F is its field-free value, was related to the magnetic-field-induced change of the heat conductivity and the viscosity (Senftleben-Beenakker effect[14–16]). Thus it is possible to calculate the magnitude of $\Delta F/F$ (its maximum value is typically of the order of 10^{-3}–10^{-2}) and its dependence on H/P, the ratio of the magnetic field strength H and the pressure P. In the meantime, Vestner and Adair[17] have extended this theory to nonspherical obstacles (including the disk as used in the measurements[1,18]) which are small compared with the mean free path.

It must be emphasized that the H/P dependence of the TMF as calculated according to the Fleming[2] and the Hess-Vestner-Adair theories[3,17] is very similar. Thus a comparison of experimental and theoretical curves of the TMF normalized to its maximum value, as was done by Fleming,[2] cannot be used to discriminate between the theories. No order-of-magnitude estimate of the $\vec{\nabla}\vec{\nabla} T$ contribution to the TMF was given in Ref. 2. On the other hand, the magnitude and the H/P dependence of $\Delta F/F$ as calculated according to the Hess-Vestner-Adair theory is in good agreement with the measurements.[18]

Some further remarks are in order. The constitutive law which links a force with the second-rank tensor must contain a polar vector (or a tensor of rank 2 or 3 with negative parity) which characterizes the shape of the obstacle. Fleming[2] correctly states that there is no force proportional to $\vec{\nabla}\vec{\nabla} T$ acting on a flat disk (or a sphere) in a gas confined by flat walls. An experimental situation where the thermal force proportional to $\vec{\nabla}T$

alone matters can be realized rather easily.[5]

In summary, there is ample evidence that a thermal force proportional to $\vec{\nabla}T$ does exist and that the TMF observed so far is a magnetic-field-induced change of this force. Some further theoretical and experimental[19] studies on the field-free force proportional to $\vec{\nabla}\vec{\nabla}T$, as well as an investigation of the TMF under experimental conditions where the force, as calculated by Fleming, gives the dominant contribution, would certainly be worthwhile.

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[19]It is very likely that the motion of dust particles in a laser cavity, as observed by Rawson and May, was caused by a force proportional to $\vec{\nabla}\vec{\nabla}T$; cf. E. G. Rawson and A. D. May, Appl. Phys. Lett. 8, 93 (1966); A. D. May, E. G. Rawson, and E. H. Hara, J. Appl. Phys. 38, 5290 (1967).