Controllable generation of vortices with varied charges by dark-soliton seeds through adiabatic manipulation

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We investigate the transition from a dark soliton to vortices in a Bose-Einstein condensate, achieved through the adiabatic manipulation of external potentials. Our primary focus is on the fundamental transition process, where a single dark soliton generates a single vortex with a ± 1 topological charge in a symmetric configuration. We reveal the emergence of this transition near the condition that the transverse characteristic length exceeds twice the healing length of the condensate, where some higher excited states are always involved in the transition process. The two topological charges of a fundamental vortex are generated with equal probability, and the final charge can be effectively manipulated by introducing some weak asymmetric operations. Our results provide an alternative approach for generating vortices with controllable charges and numbers from dark solitons, and open an avenue for studying the interactions of vortices based on dark-soliton-permitted systems.

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I. INTRODUCTION

Bose-Einstein condensates provide a good platform for investigating the dynamics of topological excitations, benefiting from their remarkable capacity to manipulate and control coherent matter waves [1,2]. Dark solitons stand out as well-known one-dimensional topological excitations characterized by a density dip with a phase shift across it [3,4]. Vortices, as typical two-dimensional topological excitations, exhibit a density of zero and possess a quantized phase charge [5–7]. These vortices can appear as vortex lines or rings in three dimensions. While the central cross-sectional profiles of quasi-two-dimensional vortices exhibit similarities with those of stationary dark solitons in their density and phase characteristics, this correspondence breaks down when the dark soliton possesses a nonzero velocity. Additionally, vortices can vanish via annihilation with vortices exhibiting opposite charge, whereas dark solitons do not adhere to such constraints. Meanwhile, the well-established transverse instability mechanism, known as the snaking instability, has been employed to describe and explain the generation of a single vortex or multiple vortex pairs from line (or planar) or ring dark solitons, both theoretically and experimentally [8–17]. However, the transition process and how to control the generated single vortex charge have yet to be clearly addressed.

Generation of vortices holds great significance in the investigation of vortex dynamics, vortex interactions, and even turbulence phenomena. Based on the phase characteristics of vortices, phase-imprinting technique has been employed to create vortices in Bose-Einstein condensates [18–20]. This method provides precise control over the topological charge of each generated vortex, but effectively managing the number of generated vortices presents challenges due to the substantial requirement of large-scale phase imprinting for each individual vortex. With the knowledge that the vortex is the fundamental excitation in quasi-two-dimensional condensates, various techniques have been explored to generate one or more vortices, such as rotating the trapping potential [21–23], moving a laser beam through the condensate [24–28], and employing other methods to excite the condensate [29,30]. Comparing with the phase-imprinting technique, these methods typically generate one or more vortices with the same charge (either 1 or -1) [21–23], or vortex dipoles (a pair of vortices with charges of 1 and -1) [24–28], and even multiple vortices with random charges [25,29,30]. It is obvious that the above methods cannot effectively and easily control the charge of each vortex generated. As such, there is a significant demand for an excitation method capable of generating vortices with both highly controllable charges and numbers.

In this paper, we investigate the transition from a dark soliton to a vortex in a Bose-Einstein condensate. By adiabatically manipulating the external potentials, we observe this transition occurring near the critical condition where the transverse characteristic length exceeds twice the healing length of the condensate, wherein a dark soliton generates a vortex with a topological charge of ± 1. Notably, we observe the involvement of higher excited states in the fundamental transition process. In symmetric cases, the two charges of a single fundamental vortex are randomly generated with equal probabilities. By introducing weak asymmetric operations on the external potentials, we can effectively control the final charge of the generated vortex. These results significantly deepen our

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understanding of the transition process from dark solitons to fundamental vortices. Furthermore, the controllable generation of vortices, including both their numbers and individual charges, provides an excellent platform for investigating the vortex interactions.

The remainder of the present paper is organized as follows. In Sec. II, we formulate the theoretical model for a quasi-two-dimensional Bose-Einstein condensate confined within a time-varying harmonic potential, and perform the numerical simulations to explore the transition from a dark soliton to vortices through the adiabatic manipulation. In Sec. III, we focus primarily on the fundamental transition, wherein a single dark soliton generates a single vortex with an equiprobable topological charge of ± 1 in a symmetrical configuration. We observe that this transition occurs when the transverse characteristic length exceeds twice the healing length of the condensate, where some higher excited states are always involved in the transition process. In Sec. IV, we introduce some weak asymmetric operations on the trapping potential to demonstrate the generation of vortices with controllable charges and numbers from dark solitons. Finally, in Sec. V, the main results of the present paper are summarized and discussed.

II. PHYSICAL MODEL AND ADIABATIC MANIPULATION

We consider a zero-temperature weakly interacting Bose-Einstein condensate as a platform to investigate the transition from a dark soliton to a vortex. The condensate is composed of atoms of mass m and confined by a harmonic potential $m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)/2$, where $\omega_{x,y,z}$ denote the confining frequencies in the respective directions. The atomic interactions are modeled by the contact pseudopotential $g\delta(r - r')$, where $g = 4\pi\hbar^2 a_s/m$ and a_s is the atomic s-wave scattering length. In this paper, we assume that the condensate is strongly confined along the z direction, which renders the condensate dynamics as quasi-two-dimensional. The condensate dynamics can be effectively described by a Gross-Pitaevskii equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2\right) + \frac{g}{\sqrt{2\pi}\,l_z}|\psi|^2\right]\psi, \tag{1}$$

where $\psi(x, y, t)$ represents the wave function, which is normalized to the total particle number N. The parameter $l_z = \sqrt{\hbar/m\omega_z}$ is the harmonic oscillator length along the z direction. The energy scale of the condensate is characterized by the chemical potential μ, which is the eigenvalue associated with the Hamiltonian in Eq. (1). In this paper, we present the time, length, and energy in units of $1/\tilde{\omega}$, $\sqrt{\hbar/m\tilde{\omega}}$, and $\hbar\tilde{\omega}$, respectively, where $\tilde{\omega}$ is a chosen characteristic frequency. Additionally, we quantify the atomic interactions using the dimensionless parameter $mNg/\sqrt{2\pi}\hbar^2 l_z$, which we set to 1.

To investigate the transition process from a dark soliton to a vortex under controlled configurations, we introduce a time-varying harmonic potential characterized by longitudinal and transverse confining frequencies, denoted as $\omega_x(t)$ and $\omega_y(t)$, respectively. In comparison to previous works [11–17], this time-varying potential not only allows us to easily identify the dark-soliton state and vortex state, but also ensures the generation of a single vortex in the appropriate parameter space. The experimentally feasible time-dependent variation in confining frequency can be achieved using an optical atomic trap through gradual modulation of applied beam waists over time [31].

Initially, we prepare a single dark soliton confined within a harmonic potential, which can be numerically continued from its underlying linear limit [32–34] based on the Newton-conjugate-gradient method [35,36]. Herein, the linear limit, as an initial guess for iterations, corresponds to its low-density limit, which can be approximately represented by the first excited state $|10\rangle$, where $|n_x n_y\rangle$ represents the linear quantum harmonic oscillator states in the basis of the complete set of Hermite polynomials with Gaussian weight. Subsequently, we proceed with the dynamical evolution of a dark soliton by fixing the longitudinal confining frequency as $\omega_x(t) = \omega_{x0}$ and gradually decreasing the transverse confining frequency as $\omega_y(t) = \omega_{y0} - \alpha t$, by employing the integrating-factor method with a fourth-order Runge-Kutta time-stepping scheme [35,37,38]. The ramp rate α is set to a small value to ensure the quasiadiabatic evolution (to avoid inducing additional excitations). Once ω_y matches ω_x, it is no longer reduced and maintained for a certain duration.

By comparing the quasiadiabatic evolution results with spontaneous evolution (where $\alpha = 0$), we identify three qualitatively different cases that govern the transition dynamics between a dark soliton and vortices. These cases include the spontaneous generation of vortices, adiabatic generation of vortices, and no vortex generation. It is well known that the transverse characteristic length $l_y = \sqrt{\hbar/m\omega_y}$ and the healing length $\xi = \hbar/\sqrt{2m\mu}$ play crucial roles in these transition processes [5,6]. With the quasiadiabatic condition maintained, we select the initial transverse frequency ω_{y0} and initial chemical potential μ_0 as the parameter space to demonstrate the regions corresponding to these three cases, referring to Fig. 1.

For the first case, represented by the red region above the line of $l_{y0} = 2\xi_0$ depicted in Fig. 1, the initial transverse characteristic length l_{y0} exceeds twice the initial healing length $2\xi_0$, where the dark soliton resides within the quasi-two-dimensional regime. Within this regime, the dark soliton can spontaneously generate one or more vortices through the well-established transverse instability mechanism [11–17], even without any adiabatic manipulations. When the initial transverse characteristic length $l_{y0} \gg 2\xi_0$, the quasi-two-dimensional dark soliton can generate the chains of vortex-antivortex pairs.

Within the region below the line of $l_{y0} = 2\xi_0$ depicted in Fig. 1, the dark soliton exhibits remarkable stability in both density and phase without any adiabatic manipulations, even in the presence of numerical noise. This region, known as the dark-soliton stability regime, extends beyond the conventional boundaries of the traditional quasi-one-dimensional regime, which is typically characterized by the conditions $\omega_y > \mu$ or $l_y < \xi$ [1,3,4]. Of particular interest is the observation that within the partial regime for dark-soliton stability, indicated by the green region in Fig. 1, the adiabatic manipulation induces a transition from a dark soliton to a vortex. This phenomenon will be investigated as the central focus of the present paper.

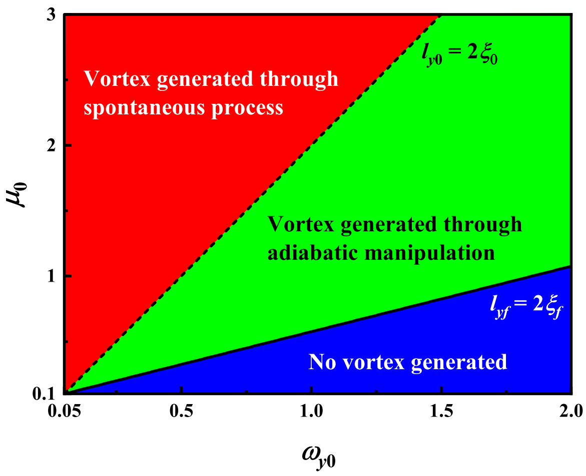


FIG. 1. The paths of transition from a dark soliton to vortices under the various initial transverse confining frequency ω_{y0} and initial chemical potential of a dark soliton μ_0. These transitions occur within the framework of a quasiadiabatic dynamic evolution process, wherein a dark soliton is confined within a time-varying harmonic potential possessing a gradually decreasing transverse confining frequency $\omega_y(t) = \omega_{y0} - \alpha t$ with a small ramp rate $\alpha = 10^{-4}$. The weak longitudinal confining frequency is fixed as $\omega_x(t) = \omega_{x0} = 0.05$. These paths include the spontaneous generation of a vortex, adiabatic generation of a vortex, and no vortex generation, which are depicted in red, green, and blue, respectively. The parameters l_{y0}, l_{yf}, ξ_0, and ξ_f respectively denote the initial and final transverse characteristic length and the initial and final healing length, corresponding to the initial and final transverse confining frequency and chemical potential, respectively. All quantities are dimensionless in corresponding units.

Lastly, the dark soliton residing within the blue region depicted in Fig. 1 remains stable, even after undergoing the adiabatic manipulation. This stability arises from the fact that the final transverse characteristic length l_{yf} remains smaller than twice the final healing length $2\xi_f$, preventing the transition from a dark soliton to vortices.

III. TRANSITION FROM A DARK SOLITON TO A VORTEX

We primarily focus on the transition process from a dark soliton to a vortex through the adiabatic path, as demonstrated in Fig. 2. Specifically, our observations commence with an initial stationary dark soliton, as depicted in Fig. 2(a). As the transverse confining frequency ω_y decreases, the initial dark soliton gradually evolves into a single solitonic vortex [13,14,39–42], as demonstrated in Fig. 2(b) (see Supplemental Material [43]). Finally, as ω_y approaches and equals the longitudinal confining frequency ω_x, the solitonic vortex gradually transforms into a standard vortex, as shown in Fig. 2(c). Note that the generated vortex admits linear stability and dynamic stability.

Additionally, we note a significant competition in scale between the transverse characteristic length l_y and twice the healing length 2ξ, as depicted in Fig. 2(d). In the dark-soliton state, l_y is smaller than 2ξ. However, through the adiabatic manipulation of transverse confining frequency ω_y, once l_y exceeds 2ξ, the dark soliton rapidly evolves into a solitonic

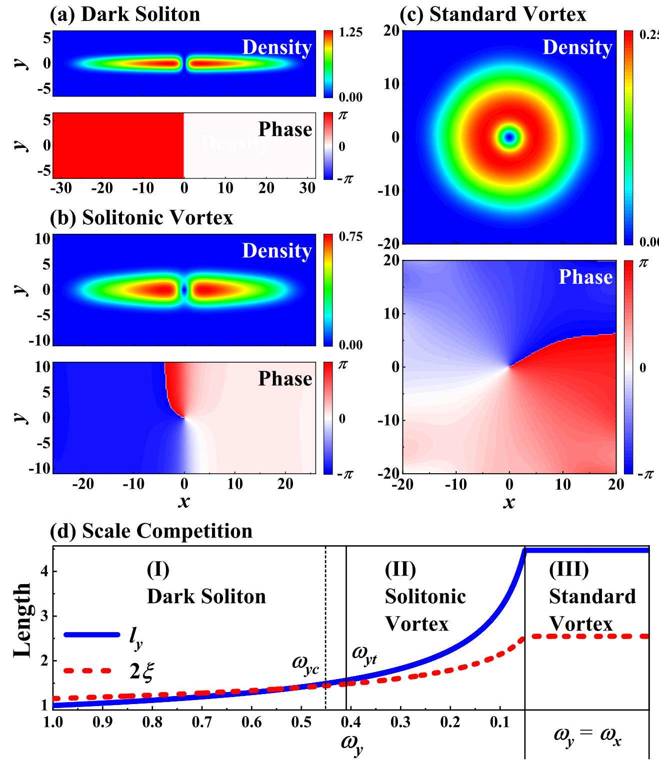


FIG. 2. The dynamical evolution process of transition from a dark soliton to a vortex through the adiabatic path. (a–c) Respective states of a dark soliton, solitonic vortex, and standard vortex under different transverse confining frequency ω_y. (d) Scale competition between the transverse characteristic length l_y and twice the healing length 2ξ, respectively. The initial values of the weak longitudinal confining frequency, strong transverse confining frequency, chemical potential of a dark soliton, and ramp rate are set to $\omega_{x0} = 0.05$, $\omega_{y0} = 1$, $\mu_0 = 1.5$, and $\alpha = 10^{-4}$, respectively. All quantities are dimensionless in corresponding units.

vortex state at the transition frequency ω_{yt}. Of course, if ω_y surpasses the critical transition frequency ω_{yc}, the dark soliton can also evolve into a vortex without requiring adiabatic manipulation. Therefore, the key to facilitating the transition from a dark soliton to a vortex lies in achieving a condition where l_y exceeds 2ξ through the adiabatic manipulation.

Moreover, we analyze the excitation kinetic energy, which can be defined as $E_k = \int \frac{\hbar^2}{2m}(|\nabla\psi|^2 - |\nabla\psi_{gn}|^2)dxdy$, where ψ_{gn} represents the ground state with the same particle number as the initial dark soliton [44]. The ground state can be numerically obtained by the imaginary-time evolution method [35,37,45,46]. As demonstrated in Fig. 3(a), we observe a decrease in the longitudinal excitation kinetic energy E_{kx} accompanied by a sudden increase in the transverse excitation kinetic energy E_{ky} as ω_y surpasses the transition frequency ω_{yt}. This indicates the occurrence of the transition from a dark soliton state to a vortex state.

In addition, we investigate several nonlinear excited states in relation to the transition process, aiming to provide profound insights into the transition from a dark soliton to a vortex. All nonlinear excited states (with the same particle number as the initial dark-soliton state) can also be numerically continued from their respective underlying

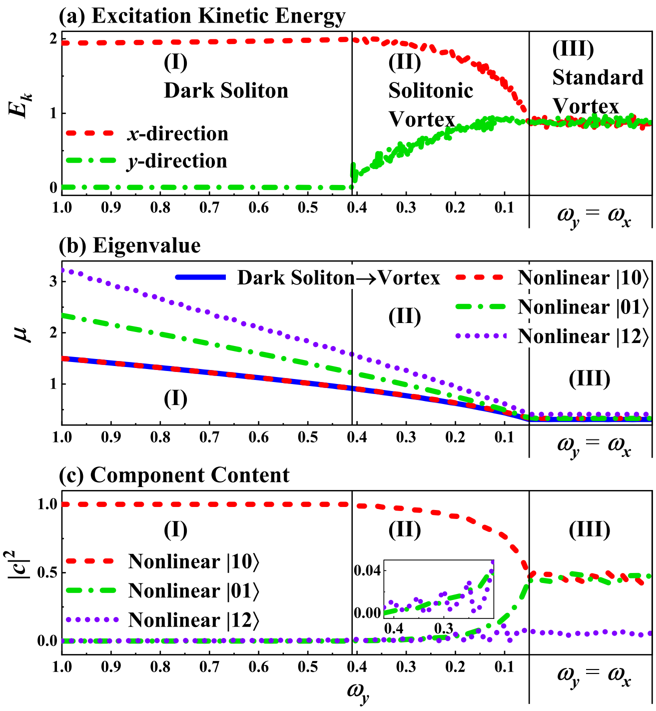


FIG. 3. The significant parameters involved in the transition process from a dark soliton to a vortex through the adiabatic path. (a) Excitation kinetic energy of a dark soliton (or vortex) during the transition process, considering both the transverse and longitudinal directions. (b) Eigenvalues of a dark soliton (or vortex) and several nonlinear states during the transition process. (c) Component content that characterizes these nonlinear states in relation to the dark soliton (or vortex) during the transition process. The initial values of various parameters are the same as those in Fig. 2. All quantities are dimensionless in corresponding units.

linear limit [32–34] based on the Newton-conjugate-gradient method [35,36]. As depicted in Fig. 3(b), the instantaneous eigenvalues of a dark soliton (or a vortex) closely align with the eigenvalues of the nonlinear $|10\rangle$ state during the transition process. This observation suggests that the transition process can be regarded as a quasiadiabatic process. Additionally, the differences between the eigenvalues of the nonlinear $|10\rangle$ state (or the dark-soliton state) and high nonlinear excited states reduce as ω_y decreases. This reduction allows for possible transitions between these different nonlinear excited states.

It is established that a single vortex with a topological charge of $\kappa = \pm 1$ can also be numerically continued from its underlying linear limit $(|10\rangle \pm i|01\rangle)/\sqrt{2}$ [33]. We calculate the component content of these nonlinear excited states during the transition process, which can be expressed as $|c|^2 = |\int \psi_{ns}^* \psi dxdy|^2$, where ψ_{ns}^* represents the complex conjugate of nonlinear excited states (with the same particle number as the initial dark-soliton state). As depicted in Fig. 3(c), it becomes evident that the dark soliton evolves into a vortex through a transition from a nonlinear $|10\rangle$ state to a superposition state of nonlinear $|10\rangle$ and $|01\rangle$ states. Notably, the emergence of the nonlinear $|12\rangle$ state is prominent during the transition process (the nonlinear $|11\rangle$ and $|02\rangle$ states do not appear throughout the entire transitional process). Considering

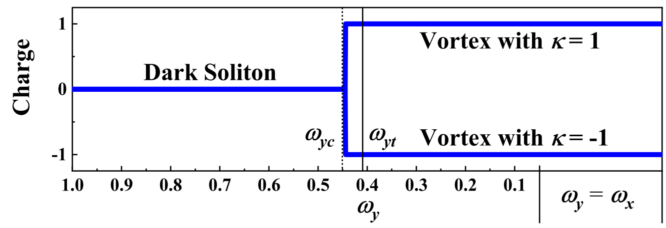


FIG. 4. The evolution process of topological charge during the transition from a dark soliton to a vortex through the adiabatic path. The initial values of various parameters are the same as those in Fig. 2. All quantities are dimensionless in corresponding units.

the parity symmetry of the system, we think that the nonlinear $|12\rangle$ state plays a crucial role in the occurrence of transition.

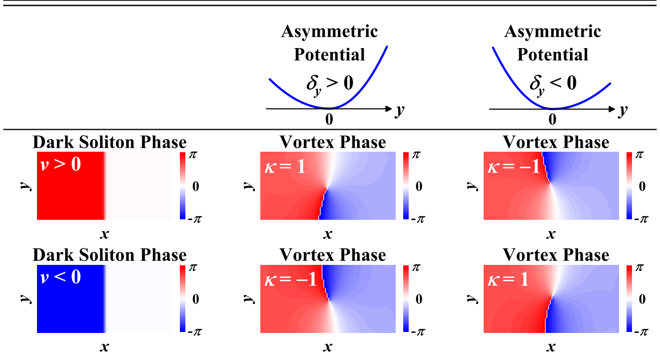
Furthermore, this superposition state of nonlinear $|10\rangle$ and $|01\rangle$ states manifests in two possible forms, which makes the vortex generated through the adiabatic path admit an equiprobable topological charge κ (either $+1$ or -1) in the symmetric configuration, as depicted in Fig. 4. This result is also corroborated by the numerical results obtained from applying random noise to the initial dark soliton. Note that, when ω_y surpasses ω_{yc}, the phase singularity appears (while the total angular momentum remains zero) and rapidly evolves into a vortex state at the transition frequency ω_{yt} (where the total angular momentum becomes nonzero). More importantly, the equiprobable charge in the symmetric configuration enables us to expect that one can control the charge of the generated vortex by introducing some weak asymmetric operations through the adiabatic manipulation starting from a dark soliton.

Additionally, we note that previous work [13,14] reported that a stationary solitonic vortex exists in a confinement regime characterized by $6 \lesssim L_t/\xi \lesssim 10$, where $L_t/\xi = \max[\int \sqrt{2|\psi|^2} dy]$ represents the transverse confinement of the healing length. In our paper, we identify the transition condition corresponding to transverse confinement to be approximately $L_t/\xi \approx 7$, which aligns with the previously discussed existence regime. In comparison to previous research [11–17], our investigation primarily focuses on the fundamental transition process from a dark soliton to a vortex, which uncovers that the vortex generated through an adiabatic path admits an equiprobable charge κ (either $+1$ or -1) in the symmetric configuration. This observation suggests that we can effectively control the final charge of the generated vortex by introducing some weak asymmetric operations.

IV. CONTROLLABLE GENERATION OF VORTICES

For the generation of a single vortex, we can introduce some weak asymmetric operations on the trapping potential or initial state to control its topological charge κ. For example, we introduce a weak transverse bias in the harmonic potential to demonstrate our expectations. The time-varying asymmetric harmonic potential in the transverse direction can be expressed as $V(y,t) = \{^{m[(1-\delta_y)\omega_y(t)]^2 y^2/2 \quad y<0}_{m[(1+\delta_y)\omega_y(t)]^2 y^2/2 \quad y \geqslant 0}$, where δ_y represents the asymmetric degree of introduced bias. In the case where $\delta_y > 0$ (or $\delta_y < 0$), the effective force exerted on the condensate by the potential in the positive half axis

TABLE I. The controllable generation of a single vortex from a dark soliton through adiabatic manipulation by introducing some weak asymmetric operations. The initial velocity and background density of a dark soliton are set to $|v| = 0.05$ and $n_g = 1$. The initial transverse confining frequency and asymmetric degree are set to $\omega_{y0} = 0.55$ and $|\delta_y| = 0.05$. The ramp rate and longitudinal wall boundary are set to $\alpha = 5 \times 10^{-4}$ and $L_x = 60$. All quantities are dimensionless in corresponding units.



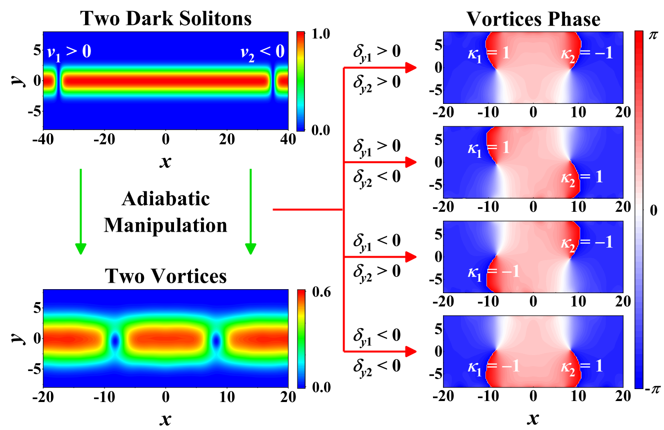


FIG. 5. The controllable generation of multiple vortices from multiple dark solitons through adiabatic manipulation by introducing some weak asymmetric operations. The initial velocities and background density of dark solitons are set to $v_1 = -v_2 = 0.05$ and $n_g = 1$. The initial transverse confining frequency and the asymmetric degree are set to $\omega_{y0} = 0.55$ and $|\delta_{y1}| = |\delta_{y2}| = 0.05$. The ramp rate and longitudinal wall boundary are set to $\alpha = 5 \times 10^{-4}$ and $L_x = 60$. All quantities are dimensionless in corresponding units.

is greater (or smaller) than that in the negative half axis. We suggest that the experimentally feasible weak transverse bias can be achieved by the application of a weak transverse magnetic field. In addition, considering that the phase shift of a stationary dark soliton can admit values of $\pm\pi$ (also making an equiprobable charge of either $+1$ or -1 for the generated vortex even introducing some weak asymmetric operations), we need to imprint a nonzero velocity (even at minimal velocity) on the dark soliton to ensure an ascertainable background current direction. Then, the transverse bias indeed influences this background current, which leads to the determination of generated vortex charge. To ensure the motion direction of the dark soliton free from the effects of nonuniform background along the longitudinal direction, we propose applying a fixed hard-wall potential [33], $V(x) = \tanh(|x| - L_x)$, as a uniform trap with an impenetrable wall, where $\pm L_x$ corresponds to the wall boundaries.

Specially, we prepare a moving dark soliton, which can be represented as $\psi = \psi_{\rm DS}\psi_g$ (where $\psi_{\rm DS}$ and ψ_g denote the dark-soliton and ground state, respectively). The dark soliton can be expressed as $\psi_{\rm DS} = iv + \sqrt{n_g - v^2}\tanh(\sqrt{n_g - v^2}x)$ (which is translationally invariant in the y direction), where v and n_g correspond to the soliton velocity and background density, and the ground state ψ_g can be numerically obtained by the imaginary-time evolution method [35,37,45,46]. The numerical simulation results are shown in Table I. Note that, although these initial dark-soliton states are approximate solutions, they do not affect the controllable generation processes and results, which indicates that this approach is robust enough to be realized in experiments.

In previous experiments, the controllable generation of a single vortex can be achieved by imprinting the vortex phase profile through adiabatically changing the strength and direction of an external magnetic field [20], controlling the atomic phase during the interconversion between different components [18], or transferring the angular momentum between the field and atoms during internal-state transition [19]. Alternatively, a single vortex can also be generated by rotating the condensate in a controlled direction about the trap axis [21–23], or by moving a pair of laser beams along specific trajectories through the condensate [26,27]. Compared to these methods for generating a single vortex, our approach appears to be simpler and reduces undesired excitation.

Furthermore, based on the controllable single vortex generation, we can systematically achieve the controllable generation of multiple vortices, each with varied topological charges, from multiple dark solitons. For the purpose of illustration, let us consider the two vortices' generation as an example. First, we propose a time-varying asymmetric harmonic potential in the transverse direction $V(y,t) = \begin{cases} m[(1-\delta_{y1})\omega_y(t)]^2y^2/2 & x<0\ \&\ y<0 \\ m[(1+\delta_{y1})\omega_y(t)]^2y^2/2 & x<0\ \&\ y\geqslant 0 \\ m[(1-\delta_{y2})\omega_y(t)]^2y^2/2 & x\geqslant 0\ \&\ y<0 \\ m[(1+\delta_{y2})\omega_y(t)]^2y^2/2 & x\geqslant 0\ \&\ y\geqslant 0 \end{cases}$ with a fixed hard-wall potential $V(x) = \tanh(|x| - L_x)$ along the longitudinal direction. Then, we prepare two moving dark solitons, which can be represented as $\psi_{\rm TDS} = \psi_{\rm DS1}\psi_{\rm DS2}\psi_g$. The jth dark soliton can be expressed as $\psi_{{\rm DS}j} = iv_j + \sqrt{n_g - v_j^2}\tanh[\sqrt{n_g - v_j^2}(x - x_{0j})]$, where x_{0j} corresponds to the initial location of the soliton. Note that the two dark solitons are independent and do not interact with each other (not being considered as bound states [34]). By appropriately selecting the asymmetric degree of the introduced biases δ_{y1} and δ_{y2}, we can generate all the different cases of two vortices in a controllable manner, as illustrated in Fig. 5. This result presents a controllable approach for generating multiple vortices with varied topological charges from multiple dark solitons.

In comparison to existing methods such as the rotating trapping potential [21–23] for generating multiple vortices with identical charges, or moving a laser beam through the condensate [24–28] or employing other methods to excite the condensate [29,30] for generating multiple vortices with different charges, our approach provides enhanced control over the topological charges of each individual vortex. Therefore,

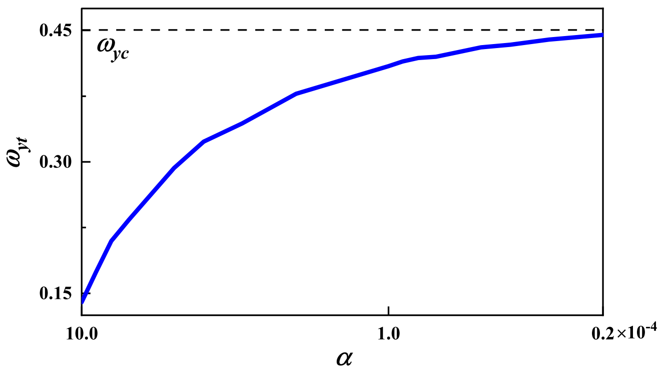


FIG. 6. The transition frequency ω_{yt} between the states of a dark soliton and a vortex for various ramp rates α. The initial values of various parameters are the same as those in Fig. 2. All quantities are dimensionless in corresponding units.

our approach also establishes a robust research platform for investigating the interactions among multiple vortices.

V. CONCLUSION AND DISCUSSION

In summary, we investigate the transition from a dark soliton to a vortex in a Bose-Einstein condensate. By adiabatically manipulating the external potentials, we observe this transition occurring near the critical condition where the transverse characteristic length exceeds twice the healing length of the condensate, wherein a dark soliton generates a vortex with a topological charge of ± 1. Notably, we observe the involvement of higher excited states in the fundamental transition process. In symmetric cases, the two charges of a single fundamental vortex are randomly generated with equal probabilities. By introducing weak asymmetric operations on the external potentials, we can effectively control the final charge of the generated vortex. These results significantly deepen our understanding of the transition process from dark solitons to fundamental vortices.

We also examine the influence of the ramp rate α on the transition from a dark soliton to a vortex, as depicted in Fig. 6. The results indicate that the actual adiabatic operation time decreases significantly as the ramp rate increases, which could be helpful for the experimental realization of our results. In real experiments, dark solitons are usually generated by phase-imprinting technique [3,4,47]. The phase-imprinting process always brings some other weak waves along with dark solitons. We investigate the nonlinear time evolution numerically of the whole process, from the initial state, to the phase imprinting, and up to the vortex generation and beyond. Our numerical simulation results indicate that these additional weak waves and noises do not affect our controllable generated vortex essentially. The robustness of our adiabatic manipulation also holds for multivortices cases. The generated multivortices are stable and they can interact with each other. The controllable generation of vortices, including both their numbers and individual charges, provides an excellent platform for investigating vortex interactions.

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