

Spontaneous-emission-induced ratchet in atom-optics kicked-rotor quantum walks

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Quantum walks have gained significant attention over the past decades, mainly because of their variety of implementations and applications. Atomic quantum walks are typically subject to spontaneous emissions arising from the control fields. We investigate spontaneous emission in an atom-optics kicked-rotor quantum walk. Here, spontaneous emission occurs naturally due to the driving by the kicks, and it is generally viewed as a nuisance in the experiment. We find, however, that spontaneous emission may induce asymmetries in an otherwise symmetric quantum walk. Our results underscore the utility of spontaneous emission and the application of the asymmetric evolution in the walker's space, i.e., for the construction of a quantum walk ratchet or for Parrondo-like quantum games. This highlights the potential for reinterpreting seemingly adverse effects as beneficial under certain conditions, thus broadening the scope of quantum walks and their applications.

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I. INTRODUCTION

Over the past two decades, quantum walks (QWs) have gained considerable attention owing to their potential to outperform classical algorithms in a range of computational tasks [1–3]. The distinct characteristics of QWs stem from quantum interference phenomena. For instance, QWs typically have characteristic peaks that exhibit ballistic expansion [1,2], as opposed to the classical random walks that are governed by stochastic processes normally showing a diffusive expansion with a central maximum in the distribution [4]. A quantum-to-classical transition, as will become relevant below, would correspond in this framework to a transition from a distribution with ballistic peaks toward a more normally distributed shape with diffusive expansion. QWs can be broadly divided into two primary types: discrete-time and continuous-time QWs [1]. Notably, the discrete-time variant introduces an additional coin degree of freedom, which dictates the direction of the walker in the subsequent step, entangling its evolution with the coin degree of freedom.

In preceding studies, we have investigated a distinctive model, describing a discrete-time QW, employing a Bose-Einstein condensate (BEC), consisting of ⁸⁷Rb atoms, with an additional spin-1/2 degree of freedom [5–9]. This approach markedly differs from various other experimental investigations [10–26], distinguishing itself by realizing the QW to occur within quantized momentum space, where the steps of the walk are implemented by periodic pulses applied to the condensate. This process is meticulously described by the atom-optics kicked-rotor (AOKR) model [27–29], governed by the following Hamiltonian:

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2} + k \cos(\hat{\theta}) \sum_{j=-\infty}^{\infty} \delta(t - j\tau). \quad (1)$$

\hat{p} and $\hat{\theta}$ are the momentum and angular position operators. The angle corresponds to $\theta = x \bmod 2\pi$ and typically corresponds to the position operator in a ring geometry. In the experimental realization [6,7,9], this geometry is effectively implemented by a periodic standing-wave potential. The parameters k and τ represent the laser kick strength and time interval between successive pulses. Considering that experiments occur in a periodic potential, Bloch's theorem may be applied [28]. In our dimensionless variables, the momentum separates into $p = n + \beta$, with an integer part n and a dimensionless quasimomentum $\beta \in [0, 1)$. The experimental system's quasi-momentum distribution, primarily determined by the BEC's initial temperature, has typically a width of a few percent in the Brillouin zone.

The evolution over one time period, τ , can be described by the Floquet operator,

$$\hat{U} = e^{-i\tau\hat{p}^2/2} e^{-ik\sigma_z \cos(\hat{\theta})}, \quad (2)$$

that consists of a free evolution part (left factor) and a kick operator (right factor). For AOKR QWs, the free evolution between subsequent kicks has a duration of a full Talbot time [6,7,9,30], which corresponds to $\tau = \tau_T = 4\pi$ in our dimensionless variables [28,31]. Without further nuisance, the free evolution operator is thus equal to unity.

A kick typically possesses a finite pulse duration τ_p itself. Numerically, this is handled by dividing the kick into h subkicks [31]. Each subkick is characterized by $\frac{1}{h}$ of the full kicking strength and is followed by a free evolution over $\frac{1}{h}$ of the total pulse duration. The parameter h , balancing computational time and approximation accuracy, is chosen based on numerical efficiency; $h = 10$ has been deemed sufficient for reaching convergence within our parameter regime.

$$\hat{U}_{\text{kick}} = \prod_h e^{-i\frac{\hat{p}^2}{2} \frac{\tau_p}{h}} e^{-i\frac{k}{h} \cos(\hat{\theta}) \sigma_z}. \quad (3)$$

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In this work, the primary concern is spontaneous emission (SE) occurring when the kick potential is on [7,27,28,31–44], rather than effects arising from the finite pulse width. Since a finite pulse width effectively introduces reflective walls in momentum space [27], we set a sufficiently small duration for τ_p to ensure minimal perturbation to momentum-space distributions from this effect.

In resonance conditions, i.e., $\tau = 4\pi$, atoms exhibit ballistic movement in momentum space. This means that momentum increases linearly with the number of kicks, revealing the typical signature of a QW [27,28,31,45]. Two internal (spin) states of the atoms are participating in the dynamics. One is moving to the left, while the second one is moving to the right in momentum space [7,9,30], mirrored by the Pauli matrix σ_z in Eqs. (2) and (3). A spin rotation by a so-called coin matrix mixes those two states in each step of the QW [7,9,30]. In the absence of external perturbations, both internal states would undergo identical evolution due to the kicks. To put it another way, their motion would be symmetric in the walker's space under the influence of the AOKR's evolution operator. This inherent symmetry is broken by an adequate choice of the initial state, achieved by setting specific initial conditions in the walker's space. To establish this initial ratchet state, we link at least two adjacent momentum classes with a relative phase of $e^{i\pi/2}$

$$|\psi_R\rangle = \frac{1}{\sqrt{J}} \sum_j e^{ij\pi/2} |n = j\rangle \otimes |l\rangle, \quad (4)$$

where $|l\rangle$ represents the preparation of the ratchet state in either of the two initial states $|l = 1\rangle$ or $|l = 2\rangle$, respectively. In what follows, we always assume an initialization in state $|l = 2\rangle$. A relative phase shift, as presented in Eq. (4), can be introduced experimentally by the application of conveniently timed Bragg pulses [7,9,46]. The mean momentum transfer is determined by the sign of the kick potential, which varies between the two internal states [30]. Notably, as the parameter J in Eq. (4) increases, the dispersion due to directed kicking decreases. With $J \geq 3$, we observe an evolution that most closely mirrors an ideal QW [8,46]. Unless specified differently, our analyses will be based on an initial state with $J = 3$:

$$|\psi_R\rangle = \frac{1}{\sqrt{3}}(-i|n = -1\rangle + |n = 0\rangle + i|n = 1\rangle) \otimes |2\rangle. \quad (5)$$

The final distribution then also depends heavily on the choice of the coin. This coin operation is executed using a Rabi coupling between the atom's two internal states, enabled by resonant microwave pulses, effecting a unitary rotation of these states [7,30,47]. The QWs presented in Refs. [6,7,9] are characterized by a series of unitary operations, as given by

$$\hat{U}'_{\text{step}} = [\hat{U}\hat{Q}_2]^\dagger \hat{U}\hat{Q}_1. \quad (6)$$

Here, $t \in \mathbb{N}$ represents the steps of the walk applied to the initial state. The final momentum distribution's key attributes, like symmetry and form within the walker space, hinge on the specific selection of the initial coin \hat{Q}_1 and the coin during the walk protocol \hat{Q}_2 . We find an asymmetry that emerges due to specific combinations of the coin matrix and a nonresonant evolution of the walk due to the effects of SE. Here, we are

especially interested in this asymmetry and in its use. SE is a quantum effect that naturally appears in driven systems. During the application of the laser pulse, the optical electron in one of the atoms is excited to the energetically higher state and spontaneously collapses into an energetically lower state. This effect is to some extent always present in AOKR experiments [27–29] and is typically treated as a nuisance. In this paper, however, we will demonstrate the utility of SE-induced asymmetry for the construction of a QW ratchet [48].

Quantum walks in momentum space are experimentally observed for up to about $T = 15$ steps; see Refs. [6,7,9]. Since the asymptotic behavior is only of little interest due to effective dissipation in the experiment [7,46], we will focus our analysis on the experimentally pertinent timescales of up to maximally 20 steps in the walk. An analytical approach for arbitrary step numbers but without SE is found in Ref. [49].

The shape of the momentum distribution itself will primarily depend on the choice of the coin. Since we intend to analyze the behavior of QWs under the effect of SE, we start in Sec. II with a quick analysis of the coin parameters. Section III is then devoted to the introduction of SE to the system and identifying a candidate coin for engineering a QW ratchet. The key section, Sec. IV, deals with the optimization of the QW ratchet. Experimental implications are discussed in Sec. V, while Sec. VI summarizes our results.

II. GLOBAL BEHAVIOR OF SYMMETRY

In a recent study, we analyzed discrepancies between experimental data on QWs in momentum space [6,7,9] and the corresponding theory. These discrepancies arise from light-shift-induced effects on energy levels, which effectively led to the use of a distinct, unintended coin during the walk. This effective coin matrix, when paired with a confined (only two momentum classes with $J = 2$) initial state in momentum space, revealed an unusual behavior, leaving a considerable portion of the probability distribution near the origin $n = 0$ that vanishes for an initial state with $J \geq 3$ [8]. Such observations suggest that the phase selections within a balanced coin matrix in the quantum-walk protocol can significantly influence key aspects of the final momentum distribution. In the following, we discuss the impact of the choice of the coin parameters on the symmetry of the walk.

A. Choice of coin

The most general form of the balanced coin matrix for the two-state system with a pseudospin degree of freedom of the atom is written with three Bloch angles χ , γ , and α and reads

$$\begin{aligned} \hat{M}\left(\chi = \frac{\pi}{4}, \gamma, \alpha\right) &= \begin{pmatrix} e^{i\alpha} \cos(\chi = \frac{\pi}{4}) & e^{-i\gamma} \sin(\chi = \frac{\pi}{4}) \\ -e^{i\gamma} \sin(\chi = \frac{\pi}{4}) & e^{-i\alpha} \cos(\chi = \frac{\pi}{4}) \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\alpha} & e^{-i\gamma} \\ -e^{i\gamma} & e^{-i\alpha} \end{pmatrix}. \end{aligned} \quad (7)$$

Here, $\chi = \frac{\pi}{4}$ determines the coin to be balanced, meaning that both internal states are weighted equally, even though

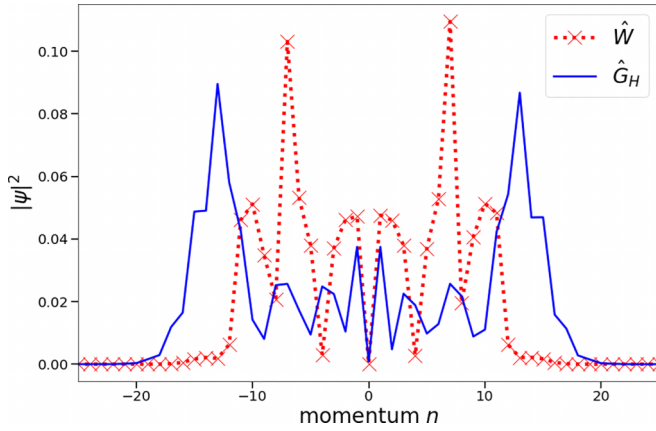


FIG. 1. Shown are the momentum distributions after $T = 20$ steps in the evolution. The walks are evolved with the \hat{W} coin and the \hat{G}_H coin, respectively.

with different phases. Since we are only interested in balanced QWs, we effectively have two free Bloch angles γ and α in the coin.

In the following analysis, all walks will be initialized by the so-called \hat{Y} coin [8], choosing in Eq. (6)

$$\hat{Q}_1 = \hat{Y} = \hat{M}\left(\frac{\pi}{4}, \frac{\pi}{2}, 0\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \quad (8)$$

The \hat{Q}_2 coin in the walk protocol is then executed by the \hat{W} coin or the \hat{G}_H coin [8]:

$$\hat{W} = \hat{M}\left(\frac{\pi}{4}, 0, 0\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad (9)$$

$$\hat{G}_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \hat{=} \frac{i}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (10)$$

$$= \hat{M}\left(\frac{\pi}{4}, \frac{3\pi}{2}, \frac{\pi}{2}\right). \quad (11)$$

Here, the equivalence in Eq. (10) is written, because both coins result in the same walk since a global phase does not induce any change. As discussed in detail in Ref. [8], a walk that evolves under the \hat{G}_H coin, without any additional distortions, displays markedly different characteristics than a walk evolved under the \hat{W} coin. Notably, there is a pronounced peak near zero momentum, as illustrated in Fig. 1.

In the following, we will introduce a quantity to characterize the different regimes of symmetry within the parameter regime spanned by the Bloch angles α and γ .

B. Mapping out parameter space

We are particularly interested in asymmetries evolving in the QW, necessitating the definition of an asymmetry observable S . Similarly to Ref. [50], the value S is determined by taking the probability to the right of the inversion point at momentum $n = 0$ and subtracting the probability to the left of it. In the context of Parrondo games, two losing strategies combined result in a winning strategy. Using the AOKR framework for Parrondo-like quantum games [50], the sign of the variable S defines the winning or the losing strategy, respectively. In our context, the same measure is subsequently

employed as a quantification of symmetry

$$S = \sum_{n>0, l=1,2} |\psi_n^l|^2 - \sum_{n<0, l=1,2} |\psi_n^l|^2. \quad (12)$$

Here, $|\psi_n^l|^2$ denotes the amount of probability associated with momentum class n summing over both internal levels $l = 1, 2$.

In what follows, we will present arguments to shape some intuition and provide qualitative insight into the overall behavior of the observable $S(\gamma, \alpha)$.

1. Symmetric coin configurations

For special relations between γ and α additional symmetries can be found in the coin itself. Empirically, this additional symmetry in the coin induces more symmetrical behavior in the walk. Let us discuss two cases.

For the first case consider $\gamma = \alpha$, $\gamma = \alpha \pm 2\pi$:

$$\begin{aligned} \hat{M}\left(\chi = \frac{\pi}{4}, \gamma, \alpha = \gamma\right) &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\alpha} & e^{-i\alpha} \\ -e^{i\alpha} & e^{-i\alpha} \end{pmatrix} \\ &\hat{=} \frac{1}{\sqrt{2}} \begin{pmatrix} \phi & \rho \\ -\phi & \rho \end{pmatrix}. \end{aligned} \quad (13)$$

For the second case consider $\gamma = \alpha \pm \pi$:

$$\begin{aligned} \hat{M}\left(\chi = \frac{\pi}{4}, \gamma, \alpha = \gamma \pm \pi\right) &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\alpha} & -e^{-i\alpha} \\ e^{i\alpha} & e^{-i\alpha} \end{pmatrix} \\ &\hat{=} \frac{1}{\sqrt{2}} \begin{pmatrix} \phi & -\rho \\ \phi & \rho \end{pmatrix}. \end{aligned} \quad (14)$$

In this setup, we consistently have two pairs of identical phases in the coin. Generally, this condition is not fulfilled. In subsequent computations for each time step, the internal states are mixed in a comparable manner. It appears plausible that when the states are mixed with similar phases along the protocol, they also evolve in a more balanced way, however systematically in opposite directions. This seems to induce overall a more balanced walk.

In Fig. 2, the walk evolves for $T = 15$ time steps, and the symmetry indicator $S(\gamma, \alpha)$ is plotted as a function of the free Bloch angles. Dominant red or blue stripes within the plot mark regions where the QW skews left or right. Intersecting white lines between these areas with $S = 0$ align qualitatively with the anticipated lines in parameter space $\gamma = \alpha$ and $\gamma = \alpha \pm \pi$, which possess the coin's additional symmetry from Eqs. (13) and (14), respectively. Not all walks corresponding to $S = 0$ in Fig. 2 perfectly match predictions, seemingly distorted by high-asymmetry zones, indicating a more intricate dynamic than the rudimentary theory suggests. Still, the theory capably predicts the qualitative behavior of where the balanced walks are found in parameter space due to coin phase symmetry, aiding in understanding for the system. Also, the empirical lines with $S = 0$ get closer to the predicted lines for a higher number of time steps, as has been numerically verified.

In Fig. 2, the blue and red dots are coin types that are identical up to a global phase to \hat{W} and \hat{G}_H from Eqs. (9) and (10), respectively. Those points lie on the white lines predicted by our argument from above.

S indicates the direction of evolution for the walk, but not the walk's shape. This means that walks may be balanced, as

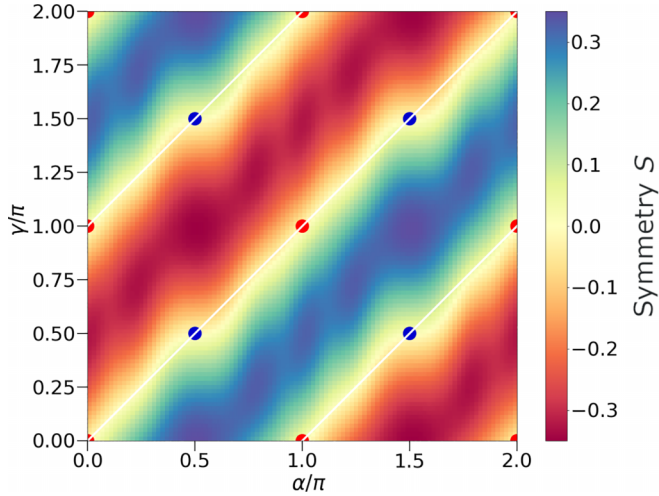


FIG. 2. Shown is a color map of the symmetry indicator $S(\gamma, \alpha)$ in dependency of the free Bloch angles α and γ , evaluated after $T = 15$ steps in the evolution. The blue and red dots reproduce the walk with the \hat{G}_H coin and the \hat{W} coin, respectively, as shown in Fig. 1. The white lines map out the coin configurations where an additional symmetry is found, investigated in detail in Sec. II B 1.

quantified by S , but their shape does not necessarily exhibit an inversion point, underscoring the rarity of true symmetry. Indeed, only two classes of walks exhibit a concrete inversion point.

One class is defined by the \hat{G}_H -coin walk, presented in Fig. 1. It is replicated across parameter combinations on the coin-symmetry lines, with half-integer multiples of π for γ and α , indicated by blue full dots in Fig. 2. Conversely, the \hat{W} -coin walk is echoed with integer Bloch angle multiples of π , shown by red full dots in Fig. 2. All these walks possess coins that are exclusively real or complex, respectively. When initialized by the \hat{Y} coin, they maintain the initial rotation of π between the internal states. All other cases introduce both real and imaginary phase components, thus lifting this initial rotation, which then appears unfavorable for a mirror-symmetric walk evolution. Thus, other configurations are either balanced without a mirror axis or are distinctly asymmetric.

2. Asymmetric coin configurations

In Fig. 2, the asymmetric regions manifest as broad diagonal stripes, echoing the observations from Sec. II B 1. To investigate the deep asymmetric regime, a broader relationship between α and γ can be expressed as $\gamma = \alpha + \zeta$, where ζ is arbitrary but constant, giving the coin matrix

$$\begin{aligned} \hat{M}\left(\chi = \frac{\pi}{4}, \gamma = \alpha + \zeta, \alpha\right) &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i(\alpha)} & e^{-i(\alpha+\zeta)} \\ -e^{i\alpha+\zeta} & e^{-i\alpha} \end{pmatrix} \\ &\equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi & \rho e^{-i\zeta} \\ -\phi e^{i\zeta} & \rho \end{pmatrix}. \end{aligned} \quad (15)$$

Empirically, we can conclude from Fig. 2 that the walk evolves asymmetrically to the left for $0 < \zeta < \pi$. Thus, $e^{i\zeta}$ lives on the right half of the complex unit circle with $\text{RE}(e^{i\zeta}) > 0$ and $e^{-i\zeta}$ lives on the left half of the complex unit

circle with $\text{RE}(e^{-i\zeta}) < 0$. In comparison, the walk evolves asymmetrically to the right for $-\pi < \zeta < 0$. In this case, again, $e^{i\zeta}$ lives on the left part of the complex unit circle and $e^{-i\zeta}$ on the respective right half. So, if the two ζ -related phase angles are effectively interchanged, then the direction in which the walk is skewed is also switched. If this additional ζ -related phase vanishes, then we recover the scenario from Sec. II B 1.

Looking at the parameter scan from Fig. 2, we also find spots in the asymmetric parameter areas, which are exceptionally asymmetric. In the $\mathcal{C} = (\alpha, \gamma)$ coordinate frame, we find that these spots are located at

$$\mathcal{C}_1^+ = (\alpha = 1/2 \pi, \gamma = 0) \rightarrow \zeta = -\pi/2 \quad (16)$$

$$\mathcal{C}_2^+ = (\alpha = 3/2 \pi, \gamma = \pi) \rightarrow \zeta = -\pi/2 \quad (17)$$

$$\mathcal{C}_1^- = (\alpha = 1/2\pi, \gamma = \pi) \rightarrow \zeta = +\pi/2 \quad (18)$$

$$\mathcal{C}_2^- = (\alpha = 3/2\pi, \gamma = 2\pi) \rightarrow \zeta = +\pi/2. \quad (19)$$

Here, the upper index notes that the asymmetry is to the right (+) or to the left (-), respectively. These four points have in common that they are fully complex on the diagonal and fully real on the off-diagonal elements. With a respective irrelevant phase factor of i , this characteristic is similar to the initialization by the \hat{Y} coin, which is the same for all walks. This effectively lifts the initial rotation of π between the internal states provided by the \hat{Y} coin and causes the walk to be highly asymmetric. Thus, the emergence of both highly symmetric and highly asymmetric behavior is determined by the rotation between the phases of the coin during the walk, \hat{Q}_2 , in reference to the phases of the initial coin, \hat{Q}_1 , see Eq. (6). Specifically, it depends on whether this relative phase rotation is maximal, i.e., π , in the symmetric case, or effectively lifted in the asymmetric case, on the off-diagonal of the coin matrix.

While the results of the section may be interesting for designing Parrondo-like games [50], we are in the following interested in how SE affects the symmetry of the QW.

III. SPONTANEOUS EMISSION

So far, our focus has been on asymmetries in the coherent QW stemming from the selection of the coin matrix. However, noise-induced asymmetries garner particular attention, given their potential utility in crafting a QW ratchet [48]. The QW ratchet described in this study is particularly noteworthy as it is induced by SE. Typically, SE is associated with merely causing decoherence [6], but in this case it contributes to the creation of a QW ratchet. This phenomenon represents a shift in perspective, where what is usually considered a nuisance becomes advantageous.

A. Spontaneous emission in the AOKR

In the context of AOKR dynamics a SE event can be interpreted as an event in which the atomic optical electron goes into an excited state during the application of the laser pulse and then spontaneously falls into either the energetically lower $|1\rangle$ or $|2\rangle$ state. All excitations are assumed to be far

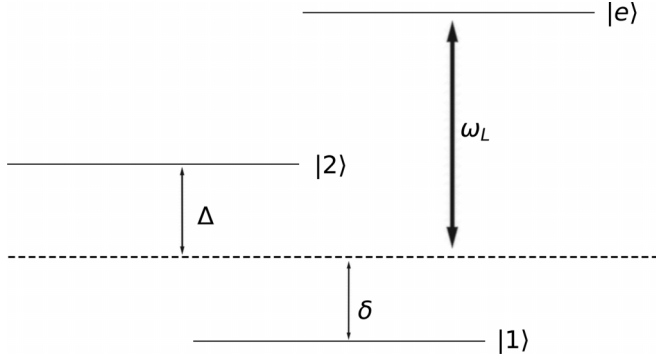


FIG. 3. Level scheme of the AOKR quantum walk with the two internal levels $|1\rangle$ and $|2\rangle$. The excited state $|e\rangle$ is used to impart the kicks to the atoms by an off-resonant standing wave of laser light. ω_L denotes the frequency of the kicking laser. δ and Δ denote the detunings of the two possible transitions with respect to the laser frequency.

away from an internal electronic resonance, in contrast to Refs. [44,51]. During the spontaneous de-excitation a photon is emitted, inducing also a shift of the atomic quasimomentum [28,31]. For the AOKR we effectively solve a quantum-optical Lindblad master equation [52] of the following form:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{\mathcal{H}}, \hat{\rho}] + \hat{\mathcal{D}}(\hat{\rho}), \quad (20)$$

where $\hat{\rho}$ is the reduced density operator including the walker's and the coin space. The master equation accounts for a Hamiltonian part and a jump part described by the dissipation operator $\hat{\mathcal{D}}(\hat{\rho})$. We arrive at the following effective Hamiltonian, for which a detailed derivation can be found in chapter 5 of Refs. [52] and [43] for nonresonant internal transitions:

$$\hat{\mathcal{H}} = \frac{1}{1 + \frac{\mu^2}{4\delta^2}} \frac{\Omega_1^2}{8\delta} [\cos(2k_L \hat{x}) + 1] |1\rangle\langle 1| - \frac{1}{1 + \frac{\mu^2}{4\Delta^2}} \frac{\Omega_2^2}{8\Delta} [\cos(2k_L \hat{x}) + 1] |2\rangle\langle 2|. \quad (21)$$

Let μ denote the SE event rate, defined as

$$\mu = \mu_1 + \mu_2 = \frac{k_1}{\tau_p \tau_{SE} |\delta|} + \frac{k_2}{\tau_p \tau_{SE} |\Delta|}, \quad (22)$$

where δ and Δ are the respective detunings from the resonance, shown in Fig. 3. Given that the detuning is small compared to the laser frequency, the approximation $k_1 = k_2 = k$ is valid [7,53]. Consequently, we have $\delta = -\Delta$, and thus we find $\mu_1 = \mu_2$ for the SE rates in Eq. (22). Also, τ_p is the pulse duration, and τ_{SE} represents the lifetime of the transition.

We solve the master equation (20) by the quantum trajectory method described, e.g., in Refs. [31,43,54]. A modified walk protocol is implemented that accounts for the Hamiltonian part and also for the dissipative part in Eq. (20) as a statistical event happening with probability p_{SE} . A picture of the overall dynamics of the system can be obtained by taking an average over a sufficiently large number of trajectories. Implementing the quantum trajectories, any SE event is effectively modeled by two actions, on the (i) internal and (ii) external degree of freedom [31], respectively: (i) a

projection onto one of the internal states by application of the projection operators $\hat{\mathcal{P}}_{|1\rangle}$ and $\hat{\mathcal{P}}_{|2\rangle}$, and (ii) accompanied by a shift in quasimomentum β_{SE} induced by the recoil of the emitted photon [31]. The projection operators on the internal states are

$$\hat{\mathcal{P}}_{|1\rangle} = \frac{1}{\mathcal{N}} (A|\psi_{|1\rangle}\rangle \otimes |1\rangle\langle 1| + B|\psi_{|2\rangle}\rangle \otimes |1\rangle\langle 2|), \quad (23)$$

$$\hat{\mathcal{P}}_{|2\rangle} = \frac{1}{\mathcal{N}} (C|\psi_{|1\rangle}\rangle \otimes |2\rangle\langle 1| + D|\psi_{|2\rangle}\rangle \otimes |2\rangle\langle 2|). \quad (24)$$

Here, \mathcal{N} serves as a normalization factor and A, B, C, D are detuning-dependent weights. For our analysis, A, B, C, D can be realistically approximated as $1/2$, see Ref. [43].

With the inclusion of SE, the kick's evaluation becomes more complex [28,31,34,37–39,43]. For each of the h subkicks from Eq. (3), we determine, based on an event probability $p_{\text{event}} = \frac{p_{SE}}{h}$, whether an SE event transpires. If it does, with a 50-50 chance we decide the projection onto either $\hat{\mathcal{P}}_{|1\rangle}$ or $\hat{\mathcal{P}}_{|2\rangle}$ internal states, and the quasimomentum shift is randomly selected from $\beta_{SE} \in \text{uniform}(-0.5, 0.5)$. In the event's presence, the partial kick is expressed as

$$\hat{\mathcal{U}}_{SE} = \hat{\mathcal{P}}_{|i\rangle} e^{-i\frac{(\hat{p} + \beta_{SE})^2}{2} \frac{\tau_p}{h}} e^{-i\frac{k}{h} \cos(\hat{\theta}) \sigma_z}, \quad (25)$$

with $|i\rangle$ being $|1\rangle$ or $|2\rangle$, with a probability of $1/2$. The Pauli matrix σ_z encodes that one internal state is moving left, the other one right due to the kick. Our approach concurrently accounts for the kick's finite duration and the potential of multiple events during a single pulse duration. For ^{87}Rb quantum walks [6,7,9], the Talbot time is $103 \mu\text{s}$, equivalent to 4π in our units. In AOKR experiments, the typical pulse width is of the order of 100 ns , but much shorter pulses should be possible, see, e.g., Refs. [55,56]. We use here throughout $\tau_p = 0.005$, corresponding to a pulse duration of a few tens of nanoseconds. This minimizes the impact of possible perturbations due to the finite pulse width [31,57–59] for longer walks of more than a few tens of steps.

The total kick is then again written as a product of h partial kicks, where for each partial kick we decide by probability p_{SE}/h whether an event occurs or not:

$$\hat{\mathcal{U}}_{SE-\text{kick}} = \prod_h \hat{\mathcal{U}}_{SE}. \quad (26)$$

The shift in quasimomentum enters in the free evolution between subsequent kicks with a duration of a full Talbot period in the same way,

$$\hat{\mathcal{U}}_{SE-\text{free}} = e^{-i\frac{(\hat{p} + \beta_{SE})^2}{2} \tau_T}. \quad (27)$$

Since the shift in quasimomentum comes from a uniform distribution and the Talbot period is with a duration of $\tau_T = 4\pi$ relatively large compared to the duration of the pulse itself, the shift in quasimomentum is of a much greater significance here than during the kick pulse in Eq. (25) [28,31]. Indeed, the free evolution between subsequent pulses with the long duration $\tau_T = 4\pi$ but off-resonant shift in quasimomentum due to SE can lead to strongly nonballistic (nonresonant) diffusive evolution [6,52] that effectively destroys the QW. Mainly, this effect limits the regime of event rates p_{SE} in which SE-induced effects will be observable, which will be discussed in more detail in the following analysis.

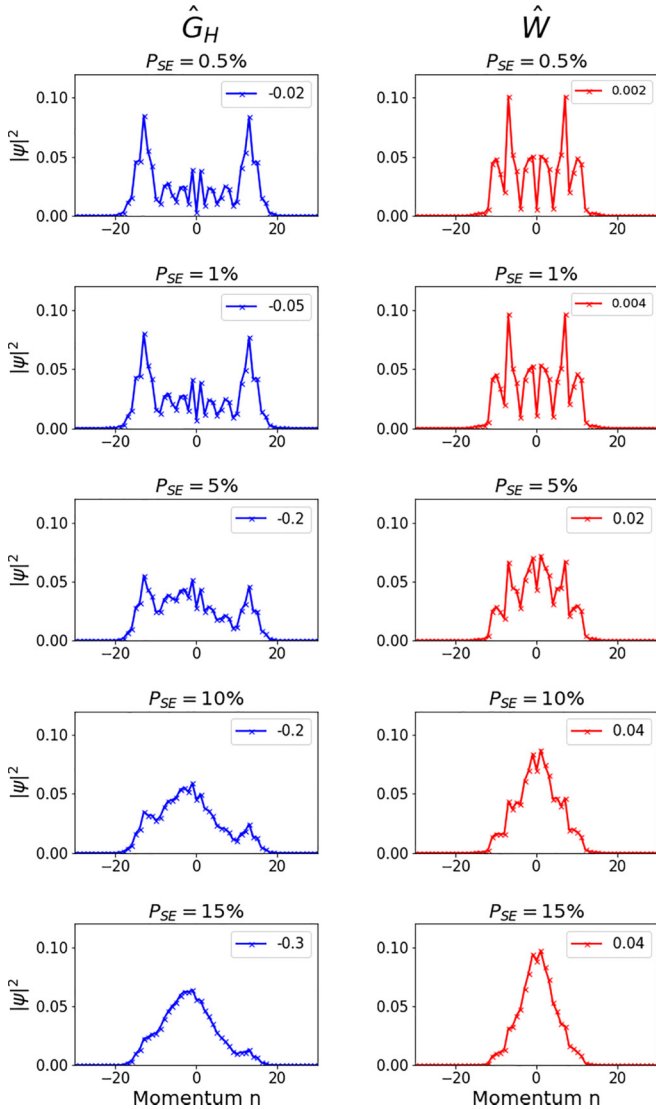


FIG. 4. Two different walk protocols evaluated after $T = 15$ steps in the evolution with increasing SE. Both protocols are initialized by the \hat{Y} coin. The left row is then evolved under the \hat{G}_H coin, while the right row is evolved under the \hat{W} coin. From top to bottom SE rates increase. The number in the upper right corner in each plot indicates the symmetry of the evolution S . The \hat{G}_H walk develops a strong asymmetry while the \hat{W} walk becomes only slightly asymmetric. It might be suspected that the weak asymmetry in the \hat{W} protocol is due to numerical fluctuations. However, it appears to be relatively robust. The distributions present an average over 10 000 realizations, with the given probabilities of p_{SE} per kick.

B. Spontaneous emissions during quantum walk

From Fig. 4, we observe that the walk under the \hat{W} coin, with increasing SE rates, becomes more classical and exhibits mild asymmetry. This can be read off S , which is shown in the top right corner of each subplot, where the distribution itself is calculated by the quantum trajectory method. Conversely, the walk under the \hat{G}_H coin displays a quite pronounced asymmetry, always a magnitude larger when compared to the \hat{W} walk. For high event probabilities, the ballistic peaks for positive momentum classes are noticeably damped by SE. However,

the peaks for negative momentum classes are less affected. As SE rates increase, the resonant peaks tend to vanish and a more Gaussian-like distribution is observed, indicating a quantum-to-classical transition. This transition will be analyzed in detail in Sec. IV B. Consequently, the asymmetry effect blurs out for too large p_{SE} .

With rising SE rates, the diagonal structures within the free Bloch angles parameter domain blur, as illustrated in Fig. 5. In this parameter landscape, the \hat{W} walk (red dot) maintains a position where even at high SE rates, there is only a small deviation from the initial parameter scan from Fig. 2, resulting in minor asymmetries, also consistent with Fig. 4. In contrast, the \hat{G}_H walk (blue dot) evolves strong asymmetries as it lies in a region dominated by the pronounced asymmetry. This is because, due to increasing SE rates, the asymmetric regions, governed by C_i^\pm , shift toward larger values of γ . This shift now situates the \hat{G}_H coin within the asymmetric region. This finding opens a route to design an open QW ratchet.

Within the parameter space of γ and α , SE introduces two key alterations: a transition from a strong stripe to a more faint pattern, followed by a drift of the asymmetric regions to larger values of γ . The walk exhibiting an asymmetry then depends on the coin's initial position within this parameter space. The effects are counteracting each other. The shift in γ situates the \hat{G}_H walk in a region of higher asymmetry vs the fainting amplitude of asymmetry as a consequence of the off-resonant walk. Thus, depending on the choice of coin, an asymmetric evolution of the walk can be constructed with a corresponding SE rate. In other words, for given windows of p_{SE} , the initially symmetric QW can be engineered to be maximally asymmetric.

Numerically, SE are implemented by two distinct effects, see previous Sec. III A: (i) the projection onto an individual internal state with a 50:50 probability, and (ii) the shift in quasimomentum. We studied each of these effects separately for a better understanding. Focusing solely on the projection with resonant quasimomentum, we observe a shift in the stripe pattern seen in Fig. 2 toward higher values of γ . This shift becomes pronounced at SE rates of approximately 20%, though it is also present at lower rates in a diminished form. The \hat{Y} coin is located at $(0,0)$, which, within this scenario, represents a particularity since the white line with $S = 0$ always extrapolates to the origin $(0,0)$. Hence, this point is not affected by the shift. The other coin, \hat{G}_H , is positioned at $(3/2\pi, 1/2\pi)$ and finds itself in a region with $S \neq 0$ due to the shifting of the stripes. Adding (ii) the shift in quasimomentum present in the numerical model of SE, the overall image becomes noticeably noisier. This is attributed to the new, shifted quasimomentum being drawn from a broad random distribution, hiding the effect of the projection. Unfortunately, with both SE-induced effects present, the clarity of the stripe shift in Fig. 5 is compromised. The combination of these effects leads to a washing out of the data for $p_{SE} \geq 15\%$ – 20% , which motivates the further study of $p_{SE} \leq 10\%$. In other words, the regime where we find a QW ratchet is for nonvanishing p_{SE} smaller than some threshold value of about 15%–20%, above which the QW becomes almost classical in shape and evolution. Overall, we can say that the SE-induced shift of quasimomentum, making it in particular nonresonant, together with the right

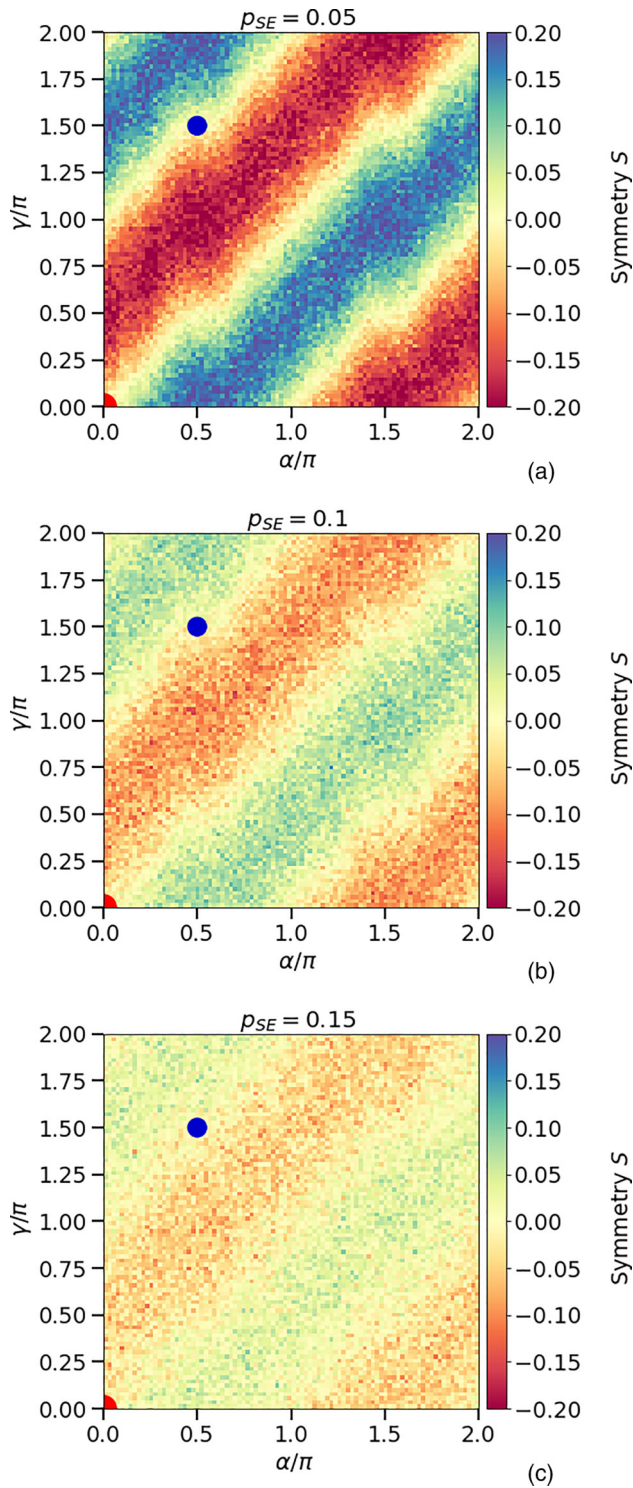


FIG. 5. Scan of the symmetry $S(\gamma, \alpha)$ of the QW, evaluated after $T = 15$ steps in the walk. The walk is initialized by the \hat{Y} coin and then evolved under $\hat{M}(\chi = \frac{\pi}{4}, \gamma, \alpha)$. From top to bottom SE rates are increased. The red dot at $(\gamma = 0, \alpha = 0)$ marks the more symmetric walk evolved under the \hat{W} coin, while the blue dot at $(\gamma = 1.5\pi, \alpha = 0.5\pi)$ marks the asymmetric walk evolved under the \hat{G}_H coin (compare to Fig. 2). It appears that the \hat{G}_H walk lives in a more unstable area in parameter space with respect to the symmetry of the evolution under the influence of SE, causing an initially symmetric walk to evolve asymmetrically under the introduction of SE. Each pixel is calculated as an average over 1000 event trajectories.

choice of the coin induces the asymmetry in the QW effected by SE.

IV. OPTIMIZING THE QW RATCHET

The SE-induced asymmetry observed for the \hat{G}_H walk is intriguing since it may be used for the construction of a ratchet. This section analyzes in detail such an engineering of an AOKR ratchet in the quantum as well as the classical regime.

Previous AOKR ratchets were so-called Hamiltonian ratchets [60–62], where the symmetry is only broken by the initial conditions, while the evolution remains coherent. This raises curiosity about the optimal parameter choice to maximize the asymmetry of the \hat{G}_H walk and thereby enhance the efficacy of an SE-induced ratchet. For our analysis we will only consider the walk of the \hat{G}_H coin, since unperturbed it has an inversion point. Therefore, it is a symmetric walk without SE that turns asymmetric only when perturbed by SE. Thus, our AOKR system is a suitable candidate for a purely SE-induced ratchet.

A. Initial state dependence

Any AOKR QW [6–9,30,45,46] is built on a coherent Hamiltonian ratchet and hence on the localization of the initial conditions within the kicking potential [28,46,60,61]. The width of the initial state in momentum space and thus the amount of momentum classes included in the initial state J , as in Eq. (4), holds significant relevance for the shape of the distribution [46]. As discussed in Ref. [8], a broader initial state considerably diminishes the portion of the distribution proximal to $n = 0$ for the \hat{G}_H walk. Consequently, a broader initial state implies diminished central probability, which is instead prevalent in the tails, amplifying the asymmetry.

The walk we are analyzing exhibits perfect symmetry for $p_{SE} = 0$. At elevated SE probabilities, the walk reverts to a classical random walk due to strong phase decoherence. This suggests the existence of an optimal SE probability that maximizes the asymmetric walk evolution. The argument holds within the region in which the walk is not yet dominated by classical dynamics, which would be the case around $p_{SE} = 10\%$ and below. Our hypothesis is supported by Fig. 6(a), which shows the dependency of the symmetry observable S on p_{SE} and on J . We find that the curve of the symmetry indicator $S(p_{SE}, J)$ improves with increasing momentum classes J in the initial state and maximizes for $T = 15$ at $p_{SE} \approx 0.1$. However, from Fig. 4 we know that the regime up to which the resonant tails of the distribution are still visible lies roughly around $p_{SE} \approx 0.05$, which would be the regime in which a ratchet that exhibits quantum resonance [27,28,31] might be well observable.

The best improvement on the asymmetry comes when increasing from an initial state constructed with $J = 2$ to $J = 3$, see Eq. (4). This observation seems to align with the results from Ref. [8], since for $J = 3$ the part of the momentum distribution located around $n = 0$ significantly decreases and is thus found in the tails, contributing more to the asymmetry. A similar argument can be made when increasing from $J = 3$

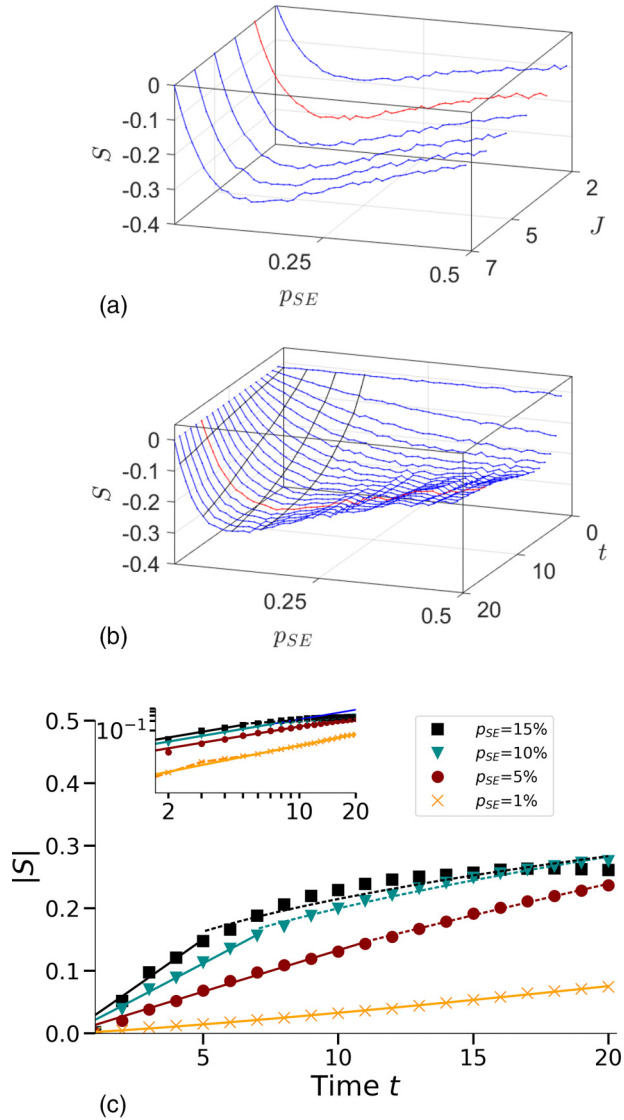


FIG. 6. Dependency of the symmetry observable $S(p_{SE})$ on the probability per kick p_{SE} for a SE event. (a) $S(p_{SE})$ for the \hat{G}_H walk observed after $T = 15$ steps with an initial state of increasing width, as described by Eq. (4) and denoted by J . (b) $S(p_{SE})$ with increasing numbers of time steps t . Here, the initial state is conducted by $J = 3$ momentum classes. The red curves show the system for three momentum classes in the initial state and observed at $t = 15$ steps in the evolution, as also discussed in Fig. 1. The black curves represent $S(p_{SE}, t)$ for fixed $p_{SE} = 1\%$, 5% , 10% , 15% . These curves are shown in (c) with their corresponding power-law fits with exponents $a = 1$ for $p_{SE} = 1\%$ for the entire time range and crossover from $a = 1$ toward $a = 0.5$ for $p_{SE} = 5\% \dots 15\%$. With increasing p_{SE} , the quantum regime with $a = 1$ (straight line fits) becomes shorter and shorter with respect to the crossover part (dotted lines). The curves and their respective fits are also shown in an inset on a double logarithmic scale. Data is obtained from averages over 10 000 trajectories modeling SE [43,52].

to $J = 4$. For broader initial states the improvement becomes less significant. The better the localization in angular θ space, the better the walks evolve with minimal dispersion. In our simulations, we have typically chosen $J = 3$ since it seems

a good tradeoff between experimental feasibility and the amplitude of asymmetry. Also note that the asymmetries do not fully revert to zero, even far in the classical regime.

B. Time dependence of the asymmetry

Figure 6(b) shows the time evolution of the asymmetry parameter $S(p_{SE}, t)$ over experimentally accessible QW timescales. Notably, the red curve therein represents the state at $t = 15$, as previously elaborated. The black curves delineate the asymmetry's temporal evolution, $S(p_{SE}, t)$, for the specific p_{SE} values where the walk still shows quantum features. The measurement of S is connected to the speed of the expansion of the walks, which clearly depends on p_{SE} . The signature of QWs is their ballistic expansion, which would correspond to an exponent of $a = 1$ for a power-law fit of $S(t) \propto t^a$. Conversely, a classical walk showcases a diffusive expansion, signified by $a = 1/2$. As Fig. 6(c) suggests, at a marginal $p_{SE} = 1\%$, the expansion remains entirely within the quantum regime, exhibiting $a = 1$. As p_{SE} increases, we discern a progression from a purely quantum expansion ($a = 1$) to one that is more classical with a diminished exponent a . Notably, at elevated p_{SE} values, the trend predominantly aligns with classical behavior, approaching an exponent of $1/2$. Also note that for progressing time steps t the maximal observable asymmetry is visible at smaller values of p_{SE} . This effect is mainly caused by the off-resonant quasimomenta in the free evolution between subsequent pulses, see the discussion at the end of Sec. III A.

For event probabilities far in the classical regime, the walk exhibits an asymmetry that initially grows fast and then reverts due to strong decoherence as time proceeds. This is especially present in the black curve in Fig. 6, corresponding to an event probability of $p_{SE} = 15\%$. In this case, not discussed in further detail, the off-resonant quasimomenta dominate the dynamics, and asymmetries effectively decline as time proceeds.

Essentially, all this demonstrates a transition from quantum to classical behavior. Augmented decoherence—either due to pronounced p_{SE} values or a combination of moderate p_{SE} with extended durations—renders the walk diffusive. Three distinct regimes emerge, identified by the best parameters from a power-law fitting: (1) the pure quantum regime, evident at $p_{SE} = 1\%$; (2) an intermediate regime exhibiting a quantum-to-classical transition as t increases, seen at $p_{SE} = 5\%$, 10% ; and (3) a predominantly classical regime, as evident for $p_{SE} = 15\%$ and above. This observation is in agreement with Fig. 4, where we identified the diminishing resonant tails in the momentum distribution for $p_{SE} = 15\%$.

V. EXPERIMENTAL CONSTRAINTS

Typical experiments start with a Bose-Einstein condensate centered around $n = 0$, but with a distribution in quasimomentum of finite width [7,9,55]. Between the pulses and during the pulse itself, this nonzero quasimomentum causes nonresonant phase effects [28,31]. Thus, similar to the influence of SE, quantum resonances are perturbed by nonresonant β 's. While up to this point the initial quasimomentum was chosen to be vanishing $\beta_{\text{init}} = 0$, now we consider for our simulations a

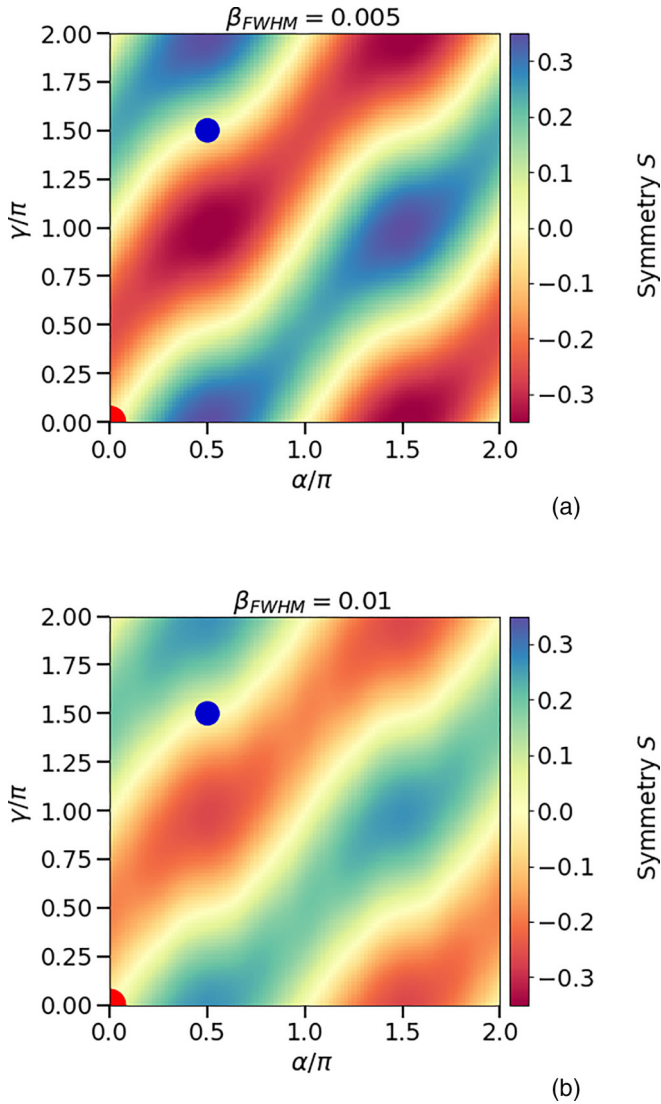


FIG. 7. S as a function of the free Bloch angles α and γ under increasing width of the initial quasimomentum distribution. S is evaluated after $T = 15$ steps in the evolution of the walks. The quasimomentum is drawn from a Gaussian distribution with width β_{FWHM} . Each point in the parameter scan is averaged over 1000 trajectories. The red dot at $(\gamma = 0, \alpha = 0)$ and the blue dot at $(\gamma = 1.5\pi, \alpha = 0.5\pi)$ mark the parameters corresponding to the \hat{W} walk and the \hat{G}_H walk, respectively.

Gaussian distribution of width β_{FWHM} modeling experimental reality [7,9,28,55].

Similarly to the introduction of SE from Sec. III B, a finite quasimomentum distribution causes the asymmetric areas to bulk around the points of high asymmetry C_i^\pm , as demonstrated in Figs. 7(a) and 7(b). A similar deviation from the original stripe pattern has been noted in Sec. III B. For broader quasimomentum distributions, see Fig. 7(b), the asymmetries become weaker and the walks become more diffusive. However, in contrast to the perturbations caused by SE, there is no additional shift of the asymmetric regions to higher γ . Thus, the walks calculated by the \hat{G}_H coin and the \hat{W} coin do not evolve new strong asymmetries. Overall they just become more “classical”-like in the sense that they show a diffusive

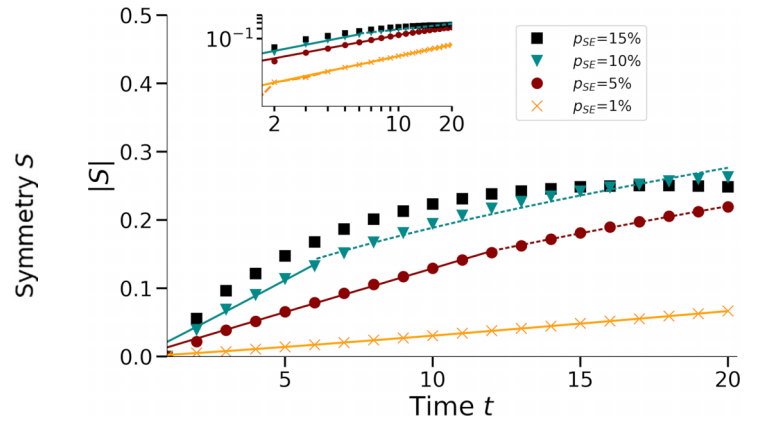


FIG. 8. Time dependence of the symmetry observable S . Considered are the combined effects of finite quasimomentum in the initial state with $\beta_{FWHM} = 0.01$ and SE. The individual curves showcase several different values of $p_{SE} = 1\%$, 5% , 10% , 15% . The straight lines mark out the regime that results in an exponent of $a = 1$ in a power-law fit, while the dotted areas mark the regime with $a < 1$. The curves are evaluated as an average of 200 initial quasimomenta, each realized with 1000 SE trajectories for the unraveling of the master equation.

instead of a ballistic evolution. Thus, the shift noticed in the asymmetric regions of Fig. 5 seems to be an effect unknown to other typical forms of perturbations in the AOKR framework.

However, the qualitative behavior of $S(\gamma, \alpha)$ under the influence of a nonresonant quasimomentum distribution is similar to the nonresonant evolution found under the influence of SE.

It has been numerically verified that the walk does indeed demonstrate no significant asymmetries over the range of 20 time steps under the influence of solely finite quasimomentum and without SE. In Fig. 8, we see the time dependence of S for several different values of p_{SE} , similar to Fig. 6(c), however with additional quasimomentum of $\beta_{FWHM} = 0.01$. The figure indicates that the effect survives also with an additional quasimomentum, which would be present in an experimental setup. Again, we observe, qualitatively similar to Fig. 6(c), the quantum-to-classical transition with three different regimes, corresponding to (1) a pure quantum mechanical regime, (2) a regime that starts ballistic and then evolves a diffusive motion as decoherence increases over time, and (3) the predominantly classical regime. For higher event probabilities $p_{SE} \leq 10\%$, we systematically find that the transition settles in for shorter times, which would also be expected due to higher dispersion in the system arising from the finite initial quasimomentum distribution. This additional dispersion slightly shifts the borders of the regime (2) with respect to the case of Fig. 6. It is worth mentioning that for a hypothetical future experiment, too-large pulse durations for τ_p would effectively induce barriers in momentum space [27,57,63]. However, a QW conducted with a ^{87}Rb BEC has already been found to be observable on the discussed timescales of up to 20 time steps [6–9].

VI. CONCLUSION

Our study sheds light on the intricate interplay between symmetry and the pivotal role of the choice of the coin and

the impact it has on the system's overall dynamics. The introduction of SE has proven instrumental as a mechanism that introduces asymmetry into an initially symmetric QW for the construction of a QW ratchet.

Typically, SE-induced effects are detrimental. However, SE manifests indeed beneficially under certain conditions [31,35,38,41,42,44]. Our observations on the temporal behavior of the symmetry observable S chart a distinct transition from quantum to classical behavior depending on the strength of the introduced SE. Evidence for this is found by a power-law fit to $S(t)$, thereby observing a transition from a ballistic to a sub-ballistic expansion, corresponding to a transition from a QW to a classical random walk. The observed asymmetry is an interference effect arising from specific choices of the coin matrix and the nonresonant free evolution due to the shifts in quasimomentum induced by SE.

Our findings have proven to be robust under the influence of finite initial quasimomentum, which would be present in any experimental setup due to finite temperature. The resilience of the observed effects under this perturbation

underscores the feasibility for the experimental realization of an SE-induced QW ratchet.

Overall, this investigation offers a more nuanced understanding of the dynamics of the AOKR QW and the influence of SE on its dynamics. By delineating the unexpected utility of SE for an AOKR ratchet and highlighting the robustness of our system against quasimomentum, our work provides stimuli for potential future experiments along the line of Refs. [6,7,9,50].

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