Markovian quantum master equation with Poincaré symmetry

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We investigate what kind of Markovian quantum master equation (QME) in the Gorini-Kossakowski-Sudarshan-Lindblad form is realized under Poincaré symmetry. The solution of the Markovian QME is given by a quantum dynamical semigroup, for which we introduce invariance under Poincaré transformations. Using the invariance of the dynamical semigroup and applying the unitary representation of the Poincaré group, we derive the Markovian QME for a relativistic massive spin-0 particle. Introducing the field operator of the massive particle and examining its evolution, we find that the field follows a dissipative Klein-Gordon equation. In addition, we show that any two local operators for spacelike separated regions commute with each other. This means that the microcausality condition is satisfied for the dissipative model of the massive particle.

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I. INTRODUCTION

The Markovian quantum master equation (QME) is of extensive importance in modern physics. A typical study using Markovian QME is on the dynamics of a quantum system coupled to its surrounding environment [1,2]. Such a quantum system is called an open quantum system, and its Markovian process is governed by the Markovian QME. In the theory of open quantum systems, the celebrated Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) form of the Markovian QME [3,4] is crucial for describing the decoherence and dissipative phenomena of an open quantum system. The description based on the GKSL form has been applied in quantum information science [1,5–8].

Another study applying Markovian QME is on the interplay of quantum and gravity physics. In modern physics it has not been elucidated whether gravity follows quantum mechanics. This stimulates two theoretical standpoints: one is that "gravity is quantum" [9] and the other is that "gravity is classical" [10–16]. In the former standpoint, perturbative quantum gravity [9] was established as an effective theory of quantum gravity. In this effective theory we can describe the perturbative dynamics of canonically quantized matter and gravitational fields. From the latter standpoint, the Kafri-Taylor-Milburn model [10] and the Diósi-Penrose or the Diósi-Tilloy model [11–14] were proposed. In these models, gravity does not obey the principle of quantum superposition and is assumed to be in a classically definite state. Such gravity causes the collapse of the wave function of matter, which is described by a nonrelativistic model based on a Markovian QME.

As mentioned previously, Markovian QME has been broadly used for the description of dissipative and collapse dynamics. In these contexts, it is intriguing to discuss the Markovian QME for the dissipative and collapse dynamics of relativistic quantum systems. For example, the decay process of an unstable particle, such as pion decay, is considered to be a relativistic dissipative phenomenon. This phenomenon would be governed by a Markovian QME for a long timescale at which the Markovian approximation is valid. Such an unstable particle might be described by relativistic theories of open quantum fields [16–18]. Within the gravity-induced collapse of a wave function, a relativistic model using a Markovian (or possibly non-Markovian) QME should be preferred if the collapse occurs at a fundamental level and for a relativistic quantum matter. In Ref. [19], for formulating such a model the covariant theory of genuinely classical gravity coupled to quantum matter was discussed. Also, regarding collapse dynamics not directly related to gravity, relativistic collapse models have been discussed in the literature [20–25].

Towards understanding the dissipative and collapse dynamics of relativistic quantum systems, in this paper we discuss a relativistic Markovian QME in the GKSL form. This equation is developed by adopting Poincaré symmetry as a guiding principle, which is commonly employed in relativistic theories, such as quantum field theories and perturbative quantum gravity in the Minkowski background. To respect the symmetry, we define the Poincaré invariance of the quantum dynamical semigroup that gives the solution of Markovian QME. In the previous work [26], the Poincaré invariant reduced dynamics of a massive spin-0 particle was discussed. However, its time-local evolution equation based on Markovian QME was not fully understood. In the present paper, by leveraging the Poincaré invariance of the dynamical semigroup and the unitary representation of the Poincaré group, we construct the Markovian QME of a relativistic massive spin-0 particle. We further investigate the evolution of the particle field and assess whether the evolution is consistent with microcausality. We find that the field obeys a Klein-Gordon (KG) equation with a dissipative term and the microcausality is satisfied. This suggests that our approach has the potential to provide dissipative quantum field theories.

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The structure of this paper is as follows. In Sec. II we explain the basics of Markovian QME in the GKSL form. In Sec. III we introduce the Poincaré group, its unitary representation, and the Poincaré algebra. We also define the condition that a Markovian QME has Poincaré symmetry, which is crucial for formulating the relativistic theory of Markovian QME. In Sec. IV we present the concrete model of a massive particle of spin 0 and investigate the field dynamics of the particle. We then find the evolution equation of the field and that the microcausality is satisfied. In Sec. V we explain the derivation of the Markovian QME of the massive particle. In Sec. VI the conclusion and future prospects of this paper are devoted. Throughout this paper we use the natural unit $\hbar = c = 1$ and adopt the convention of the Minkowski metric as $\eta_{\mu\nu} = \mathrm{diag}[-1, 1, 1, 1]$ used for lowering and raising indices. Also, the commutator and the anticommutator are defined as $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ and $\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$, respectively.

II. MARKOVIAN QUANTUM MASTER EQUATION

Our main purpose is to formulate Markovian QMEs applicable to relativistic quantum systems. To this end, in this section we introduce the Markovian QME in a general GKSL form [1–4], which can describe the dissipative dynamics of the open quantum system and the collapse of the wave function of the quantum system. The Markovian QME for the density operator $\rho(t)$ of a quantum system at time t is

$$\frac{d}{dt}\rho(t) = \mathcal{L}[\rho(t)], \tag{1}$$

where the GKSL generator $\mathcal{L}$ is defined as

$$\mathcal{L}[\rho(t)] = -i[\hat{M}, \rho(t)] + \sum_{\lambda}\left[\hat{L}_\lambda \rho(t)\hat{L}_\lambda^\dagger - \frac{1}{2}\{\hat{L}_\lambda^\dagger \hat{L}_\lambda, \rho(t)\}\right], \tag{2}$$

with a Hermitian operator $\hat{M}$ and noise operators $\hat{L}_\lambda$ called Lindblad operators. In Eq. (2) the first term describes the unitary evolution of the system and the second term leads to the nonunitary evolution of the system, such as dissipation and wave function collapse. The GKSL generator $\mathcal{L}$ does not change under the following transformations [1,27]:

$$\hat{L}_\lambda \longrightarrow \hat{L}'_\lambda = \sum_{\lambda'} \mathcal{U}_{\lambda\lambda'}\hat{L}_{\lambda'} + \alpha_\lambda \hat{\mathbb{I}}, \tag{3}$$

$$\hat{M} \longrightarrow \hat{M}' = \hat{M} + \frac{1}{2i}\sum_{\lambda,\lambda'}[\alpha_\lambda^* \mathcal{U}_{\lambda\lambda'}\hat{L}_{\lambda'} - \alpha_\lambda \mathcal{U}_{\lambda\lambda'}^* \hat{L}_{\lambda'}^\dagger] + \beta\hat{\mathbb{I}}, \tag{4}$$

where α_λ are complex numbers, β is real, and $\mathcal{U}_{\lambda\lambda'}$ is a unitary matrix satisfying $\sum_\lambda \mathcal{U}_{\lambda_1\lambda}^* \mathcal{U}_{\lambda_2\lambda} = \sum_\lambda \mathcal{U}_{\lambda\lambda_1}^* \mathcal{U}_{\lambda\lambda_2} = \delta_{\lambda_1,\lambda_2}$. This invariance is important in discussing Poincaré symmetry in the later sections. The formal solution of the QME Eq. (1) is

$$\rho(t) = e^{\mathcal{L}t}[\rho(0)], \tag{5}$$

where $e^{\mathcal{L}t}$ maps the initial state $\rho(0)$ onto the evolved state $\rho(t)$. The map $e^{\mathcal{L}t}$ has two properties, complete positivity and trace-preserving, and the one parameter family $\{e^{\mathcal{L}t} : t \geqslant 0\}$ is called a quantum dynamical semigroup [1]. The QME Eq. (1) can describe Markovian dynamics of a quantum system under the following three approximations [1]: The first one is a Born approximation that the open system is coupled to the environment weakly enough. The second one is a Markovian approximation that the correlation function of the environment system decays sufficiently fast compared to the timescale of the open system. The final one is the rotating wave approximation that the typical timescale of the open quantum system is small enough compared to the relaxation time of the system. Alternatively, as mentioned in the Introduction, in theories from the standpoint that "gravity is classical," the dynamics of quantum matter coupled to gravity can be often described as Markovian dynamics. Even in such cases, the description by the QME may still be adopted.

In the next section we will discuss Poincaré symmetry and introduce the Poincaré invariance for the quantum dynamical semigroup considered here. The invariance condition will be used to identify the Markovian QME of a massive spin-0 particle in Sec. IV.

III. DYNAMICAL SEMIGROUP WITH POINCARÉ SYMMETRY

To make our formulation clear, in this section we first explain the Poincaré symmetry in quantum theory [28] and then define the Poincaré invariance of the quantum dynamical semigroup. The Poincaré symmetry is respected for relativistic theories and requires invariance under the coordinate transformation $x^\mu \rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$, where $\Lambda^\mu{}_\nu$ gives the Lorentz transformation matrix satisfying $\eta_{\mu\nu} = \eta_{\rho\sigma}\Lambda^\rho{}_\mu\Lambda^\sigma{}_\nu$, and a^μ are the parameters of space-time translation. This transformation is called a Poincaré transformation and preserves the line element of the Minkowski space-time, $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu$. In the following we only consider the continuous transformation, that is, the proper orthochronous Lorentz transformation with $\Lambda^0{}_0 \geqslant 1$ and $\det\Lambda = 1$.

Denoting the Poincaré transformation as the map $g(\Lambda, a) : x^\mu \rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$, this map satisfies the multiplication rule

$$g(\Lambda', a')g(\Lambda, a) = g(\Lambda'\Lambda, \Lambda' a + a'). \tag{6}$$

From Eq. (6) the inverse transformation and the identical transformation can be given as $g(I, 0)$ and $g(\Lambda^{-1}, -\Lambda^{-1}a)$, respectively, where I is the identity matrix and Λ^{-1} is the inverse of the Lorentz transformation matrix Λ. Therefore the whole of Poincaré transformation $g(\Lambda, a)$ forms a group, which is called a Poincaré group.

In quantum theory, the Poincaré transformation $g(\Lambda, a)$ is represented on a Hilbert space as a unitary operator $\hat{U}(\Lambda, a)$ satisfying

$$\hat{U}(\Lambda', a')\hat{U}(\Lambda, a) = \hat{U}(\Lambda'\Lambda, \Lambda' a + a'). \tag{7}$$

This unitary operator $\hat{U}(\Lambda, a)$ is called the unitary representation of a Poincaré group. Considering the infinitesimal Poincaré transformation $g(I + \omega, \epsilon)$ with

$$\Lambda^\mu{}_\nu = \delta^\mu_\nu + \omega^\mu{}_\nu, \quad a^\mu = \epsilon^\mu, \tag{8}$$

where ω^μ_ν with $\omega_{\mu\nu} = -\omega_{\nu\mu}$ and ϵ^μ are infinitesimal parameters, and δ^μ_ν is Kronecker's symbol, we can expand the unitary operator $\hat{U}(I+\omega,\epsilon)$ as

$$\begin{aligned}\hat{U}&(I+\omega,\epsilon)\\&=\hat{I}+\frac{i}{2}\omega_{\mu\nu}\hat{J}^{\mu\nu}-i\epsilon_\mu\hat{P}^\mu\\&\quad+\text{(higher-order term with respect to } \omega \text{ and } \epsilon),\end{aligned}\tag{9}$$

where the Hermitian operators $\hat{J}^{\mu\nu}$ with $\hat{J}^{\mu\nu}=-\hat{J}^{\nu\mu}$ and $\hat{P}^\mu$ in the Schrödinger picture are the generators of Lorentz transformation and space-time translation, respectively [28]. These generators satisfy the following commutation relations:

$$[\hat{J}_k,\hat{J}_\ell]=i\epsilon_{k\ell m}\hat{J}^m,\tag{10}$$

$$[\hat{J}_k,\hat{K}_\ell]=i\epsilon_{k\ell m}\hat{K}^m,\tag{11}$$

$$[\hat{K}_i,\hat{K}_j]=-i\epsilon_{ijk}\hat{J}^k,\tag{12}$$

$$[\hat{J}_k,\hat{P}_\ell]=i\epsilon_{k\ell m}\hat{P}^m,\tag{13}$$

$$[\hat{K}_j,\hat{H}]=i\hat{P}_j,\tag{14}$$

$$[\hat{K}_i,\hat{P}_j]=i\delta_{ij}\hat{H},\tag{15}$$

$$[\hat{P}_i,\hat{P}_j]=0,\tag{16}$$

$$[\hat{P}_i,\hat{H}]=0=[\hat{J}_i,\hat{H}],\tag{17}$$

where we used the notations $\hat{P}^\mu=(\hat{H},\hat{P}^1,\hat{P}^2,\hat{P}^3)$, $\hat{J}^k=\frac{1}{2}\epsilon^{ijk}\hat{J}_{ij}=(\hat{J}^{23},\hat{J}^{31},\hat{J}^{12})$, and $\hat{K}^k=\hat{J}^{k0}=(\hat{J}^{10},\hat{J}^{20},\hat{J}^{30})$. The above commutation relations are called the Poincaré algebra. From this Poincaré algebra, it turns out that the boost operators $\hat{K}^k$ do not commute with the Hamiltonian $\hat{H}$. In relativistic unitary theories such as quantum field theories, the boost operator in the Heisenberg picture, $\hat{K}^k_{\rm H}=e^{i\hat{H}t}\hat{K}^i e^{-i\hat{H}t}$, is conserved by the Noether theorem. This leads to that $\hat{K}^k$ in the Schrödinger picture is $\hat{K}^i_0-t\hat{P}^i$, where $\hat{K}^i_0$ is $\hat{K}^i$ at $t=0$. Hence the unitary $\hat{U}(\Lambda,a)$ has the explicit time dependence in the Schrödinger picture. In the following we denote $\hat{K}^i$ and $\hat{U}(\Lambda,a)$ as $\hat{K}^i_t$ and $\hat{U}_t(\Lambda,a)$, respectively, to specify the time dependence in the Schrödinger picture.

We are in a position to define the Poincaré invariance of a quantum dynamical semigroup. If the dynamical semigroup $\{e^{\mathcal{L}t}:t\geqslant 0\}$ given in Eq. (5) satisfies the following condition,

$$\hat{U}_t(\Lambda,a)e^{\mathcal{L}t}[\rho(0)]\hat{U}_t^\dagger(\Lambda,a)=e^{\mathcal{L}t}[\hat{U}_0(\Lambda,a)\rho(0)\hat{U}_0^\dagger(\Lambda,a)],\tag{18}$$

we regard that $\{e^{\mathcal{L}t}:t\geqslant 0\}$ has the Poincaré invariance (see also [26]). The similar invariance condition for the symmetry group has been discussed in Refs. [29–31]. The unitary operator $\hat{U}_t(\Lambda,a)$ is generated by $\hat{P}^\mu$, $\hat{J}^k$, and $\hat{K}^i_t=\hat{K}^i_0-t\hat{P}^i$, which satisfies the group multiplication rule $\hat{U}_t(\Lambda',a')\hat{U}_t(\Lambda,a)=\hat{U}_t(\Lambda'\Lambda,\Lambda'a+a')$. In unitary relativistic theories, the time evolution of a density operator $\rho(t)$ is given as

$$\rho(t)=\mathcal{U}_t[\rho(0)]=e^{-i\hat{H}t}\rho(0)e^{i\hat{H}t},\tag{19}$$

and this map $\mathcal{U}_t$ satisfies the condition Eq. (18) as

$$\hat{U}_t(\Lambda,a)\mathcal{U}_t[\rho(0)]\hat{U}_t^\dagger(\Lambda,a)=\mathcal{U}_t[\hat{U}_0(\Lambda,a)\rho(0)\hat{U}_0^\dagger(\Lambda,a)].\tag{20}$$

So, the invariance condition Eq. (18) for $\{e^{\mathcal{L}t}:t\geqslant 0\}$ is a generalization of that for $\{\mathcal{U}_t:t\geqslant 0\}$. We can get the derivative form of the invariance condition Eq. (18). Differentiating both sides of the condition Eq. (18) with respect to time t, we obtain

$$\frac{d}{dt}\rho'(t)=\mathcal{L}\rho'(t),\tag{21}$$

where $\rho'(t)=\hat{U}_t(\Lambda,a)\rho(t)\hat{U}_t^\dagger(\Lambda,a)$. Equation (21) means that the generator of the time evolution, that is, the GKSL generator $\mathcal{L}$, is invariant under the Poincaré transformation $\rho(t)\to\rho'(t)$.

Here, we explain our motivation why the Poincaré invariance is adopted. In the context of dissipative dynamics, the decay of an unstable particle is a relativistic dissipative phenomenon. In the context of collapse dynamics, possibly induced by gravity, if the collapse of a wave function is fundamental, its relativistic description should be preferred. These dissipative and collapse dynamics may be relativistically symmetric. As a step to exploring this possibility, in this paper we are considering a framework with the Poincaré symmetry as a relativistic symmetry.

To get a simple invariance condition, we reduce Eq. (18) for $\{e^{\mathcal{L}t}:t\geqslant 0\}$ to the condition for the GKSL generator $\mathcal{L}$. Evaluating Eq. (18) for a small t with the help of

$$\hat{K}^i_t=e^{-i\hat{H}t}\hat{K}^i_0e^{i\hat{H}t}\tag{22}$$

and

$$\hat{U}_t(\Lambda,a)=e^{-i\hat{H}t}\hat{U}_0(\Lambda,a)e^{i\hat{H}t},\tag{23}$$

we obtain

$$\begin{aligned}\hat{U}_0&(\Lambda,a)(\mathcal{L}-\mathcal{H})[\rho(0)]\hat{U}_0^\dagger(\Lambda,a)\\&=(\mathcal{L}-\mathcal{H})[\hat{U}_0(\Lambda,a)\rho(0)\hat{U}_0^\dagger(\Lambda,a)],\end{aligned}\tag{24}$$

where $\mathcal{H}[\rho(t)]$ means $\mathcal{H}[\rho(t)]=-i[\hat{H},\rho(t)]$. Our task is to get a Markovian QME for relativistic quantum systems from the GKSL generator $\mathcal{L}$ which satisfies Eq. (24). In the next section, as an example of the Markovian QME derived from Eq. (24), we present the Markovian QME of a massive relativistic particle of spin 0. After the next section, we derive the QME.

IV. MODEL OF A MASSIVE SPIN-0 PARTICLE AND ITS FIELD DYNAMICS

In this section we exemplify the Markovian QME of a massive spin-0 particle with a mass whose GKSL generator $\mathcal{L}$ satisfies Eq. (24). The Markovian QME is

$$\frac{d}{dt}\rho(t)=\mathcal{H}[\rho(t)]+\mathcal{D}[\rho(t)],\tag{25}$$

where the first term $\mathcal{H}[\rho(t)]=-i[\hat{H},\rho(t)]$ is given by the free Hamiltonian,

$$\hat{H}=\int d^3p\,E_{\boldsymbol{p}}\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}),\tag{26}$$

with the energy of the particle, $E_{\boldsymbol{p}}=\sqrt{\boldsymbol{p}^2+m^2}$, and the second dissipation term is

$$\mathcal{D}[\rho(t)]=\gamma\int d^3p\left[\hat{a}(\boldsymbol{p})\rho(t)\hat{a}^\dagger(\boldsymbol{p})-\frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}),\rho(t)\}\right] \tag{27}$$

with a non-negative dissipation rate γ. The superoperator $\mathcal{D}$ in Eq. (27) is called a dissipator. The annihilation and creation operators of the massive particle, $\hat{a}(\boldsymbol{p})$ and $\hat{a}^\dagger(\boldsymbol{p})$, satisfy the following commutation relations:

$$[\hat{a}(\boldsymbol{p}),\hat{a}^\dagger(\boldsymbol{p}')]=\delta^3(\boldsymbol{p}-\boldsymbol{p}'),\quad [\hat{a}(\boldsymbol{p}),\hat{a}(\boldsymbol{p}')]=0. \tag{28}$$

It is easy to check that the GKSL generator $\mathcal{L}=\mathcal{H}+\mathcal{D}$ of the Markovian QME Eq. (25) satisfies the invariance condition Eq. (24), the details of which are provided in Appendix A. Eq. (25) is explicitly derived in the next section.

In this section we investigate the properties of the model by introducing the field operator of the massive particle,

$$\hat{\Phi}(\boldsymbol{x})=\frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3p\,\frac{1}{\sqrt{2E_{\boldsymbol{p}}}}(e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{a}(\boldsymbol{p})+e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{a}^\dagger(\boldsymbol{p})), \tag{29}$$

where $\boldsymbol{x}$ is a spatial coordinate.[1] Let us first seek the time evolution equation of the massive spin-0 field in the Heisenberg picture. The time-evolved field $\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})$ is formally given as

$$\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})=e^{\mathcal{L}^\dagger t}[\hat{\Phi}(\boldsymbol{x})], \tag{30}$$

where the adjoint of the GKSL generator $\mathcal{L}=\mathcal{H}+\mathcal{D}$, denoted by $\mathcal{L}^\dagger$, is defined as

$$\mathcal{L}^\dagger[\hat{\Phi}_{\mathrm{H}}]=\mathcal{H}^\dagger[\hat{\Phi}_{\mathrm{H}}]+\mathcal{D}^\dagger[\hat{\Phi}_{\mathrm{H}}], \tag{31}$$

with

$$\begin{aligned}&\mathcal{H}^\dagger[\hat{\Phi}_{\mathrm{H}}]=i[\hat{H},\hat{\Phi}_{\mathrm{H}}],\\ &\mathcal{D}^\dagger[\hat{\Phi}_{\mathrm{H}}]=\gamma\int d^3p\left(\hat{a}^\dagger(\boldsymbol{p})\hat{\Phi}_{\mathrm{H}}\hat{a}(\boldsymbol{p})-\frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}),\hat{\Phi}_{\mathrm{H}}\}\right).\end{aligned} \tag{32}$$

For convenience, we consider the following operator $\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$, given as

$$\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})=e^{i\hat{H}t}\hat{\Phi}(\boldsymbol{x})e^{-i\hat{H}t}. \tag{33}$$

The relation between $\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})$ and $\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$ is

$$\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})=e^{\mathcal{D}^\dagger t}[\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})]. \tag{34}$$

Calculating the operation $\mathcal{D}^\dagger$ for $\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$, we get the time evolution $\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})$ from Eq. (34). Substituting Eq. (29) into (33), we obtain the operator $\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$ as

$$\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})=\frac{1}{(2\pi)^{\frac{3}{2}}}\times\int\frac{d^3p}{\sqrt{2E_{\boldsymbol{p}}}}(e^{i(\boldsymbol{p}\cdot\boldsymbol{x}-E_{\boldsymbol{p}}t)}\hat{a}(\boldsymbol{p})+e^{-i(\boldsymbol{p}\cdot\boldsymbol{x}-E_{\boldsymbol{p}}t)}\hat{a}^\dagger(\boldsymbol{p})), \tag{35}$$

where we used the Baker-Campbell-Hausdorff formula and the commutation relations $[\hat{H},\hat{a}^\dagger(\boldsymbol{p})]=E_{\boldsymbol{p}}\hat{a}^\dagger(\boldsymbol{p})$ and $[\hat{H},\hat{a}(\boldsymbol{p})]=-E_{\boldsymbol{p}}\hat{a}(\boldsymbol{p})$. We compute $\mathcal{D}^\dagger[\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})]$ as

$$\begin{aligned}\mathcal{D}^\dagger[\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})]&=\gamma\int d^3p'\bigg(\hat{a}^\dagger(\boldsymbol{p}')\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})\hat{a}(\boldsymbol{p}')\\&\quad-\frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p}')\hat{a}(\boldsymbol{p}'),\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})\}\bigg)\\&=\frac{\gamma}{2}\frac{1}{(2\pi)^{\frac{3}{2}}}\int\frac{d^3p}{\sqrt{2E_{\boldsymbol{p}}}}(e^{i(\boldsymbol{p}\cdot\boldsymbol{x}-E_{\boldsymbol{p}}t)}[\hat{N},\hat{a}(\boldsymbol{p})]\\&\quad+e^{-i(\boldsymbol{p}\cdot\boldsymbol{x}-E_{\boldsymbol{p}}t)}[\hat{a}^\dagger(\boldsymbol{p}),\hat{N}])\\&=-\frac{\gamma}{2}\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x}),\end{aligned} \tag{36}$$

where $\hat{N}$ is the number operator, defined as

$$\hat{N}=\int d^3p\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}). \tag{37}$$

Repeating the operation of $\mathcal{D}^\dagger$ for $\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$, we get $(\mathcal{D}^\dagger)^n[\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})]=(-\frac{\gamma}{2})^n\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$ and hence

$$\begin{aligned}\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})&=e^{\mathcal{D}^\dagger t}[\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})]=\sum_{n=0}^{\infty}\frac{(\mathcal{D}^\dagger t)^n}{n!}[\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})]\\&=\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{\gamma}{2}t\right)^n\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})=e^{-\frac{\gamma}{2}t}\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x}).\end{aligned} \tag{38}$$

Noticing that the field operator $\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$ follows the usual KG equation, $\partial_t^2\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})=(\nabla^2-m^2)\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})$, we find that $\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})$ obeys a dissipative KG equation,

$$\left[\frac{\partial^2}{\partial t^2}+\gamma\frac{\partial}{\partial t}+\frac{\gamma^2}{4}\right]\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})=(\nabla^2-m^2)\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x}). \tag{39}$$

If the dissipation rate vanishes, $\gamma=0$, Eq. (39) becomes the usual KG equation. In the case where $\gamma\neq 0$, the term $\gamma\partial_t\hat{\Phi}_{\mathrm{H}}$ represents the friction effect, which represents the dissipative dynamics of the field.

We examine whether microcausality holds in the dissipative dynamics. The microcausality condition suggests that any two local operators at spacelike distance commute with each other. In special relativity there is no transmission beyond the speed of light, and this fact is reflected in the notion of microcausality. Using the solution of the field $\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x})$, we can check that the field satisfies the microcausality as

$$[\hat{\Phi}_{\mathrm{H}}(x^0,\boldsymbol{x}),\hat{\Phi}_{\mathrm{H}}(y^0,\boldsymbol{y})]=0, \tag{40}$$

[1]One sometimes uses another annihilation operator such as $\hat{\alpha}(\boldsymbol{p})=\sqrt{2E_{\boldsymbol{p}}}\hat{a}(\boldsymbol{p})$, which satisfies the commutation relations $[\hat{\alpha}(\boldsymbol{p}),\hat{\alpha}^\dagger(\boldsymbol{p}')]=2E_{\boldsymbol{p}}\delta^3(\boldsymbol{p}-\boldsymbol{p}')$, $[\hat{\alpha}(\boldsymbol{p}),\hat{\alpha}(\boldsymbol{p}')]=0$. Using this notation, we have the field operator as $\hat{\Phi}(\boldsymbol{x})=(2\pi)^{-\frac{3}{2}}\int\frac{d^3p}{2E_{\boldsymbol{p}}}[e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{\alpha}(\boldsymbol{p})+e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{\alpha}^\dagger(\boldsymbol{p})]$.

for any spacelike distance $(x^0-y^0)^2-(\boldsymbol{x}-\boldsymbol{y})^2<0$. Here, we introduce a local operator

$$\hat{\Pi}(\boldsymbol{x})=-\frac{i}{(2\pi)^{\frac{3}{2}}}\int d^3p\sqrt{\frac{E_{\boldsymbol{p}}}{2}}[e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{a}(\boldsymbol{p})-e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{a}^\dagger(\boldsymbol{p})], \quad (41)$$

and the evolved operator $\hat{\Pi}_{\mathrm{H}}(t,\boldsymbol{x})=e^{\mathcal{L}^\dagger t}[\hat{\Pi}(\boldsymbol{x})]$ is given as

$$\hat{\Pi}_{\mathrm{H}}(t,\boldsymbol{x})=e^{-\frac{\gamma}{2}t}\hat{\Pi}_{\mathrm{I}}(t,\boldsymbol{x}) \quad (42)$$

with

$$\hat{\Pi}_{\mathrm{I}}(t,\boldsymbol{x})=\frac{\partial}{\partial t}\hat{\Phi}_{\mathrm{I}}(t,\boldsymbol{x})=-\frac{i}{(2\pi)^{\frac{3}{2}}}\times\int d^3p\sqrt{\frac{E_{\boldsymbol{p}}}{2}}[e^{i(\boldsymbol{p}\cdot\boldsymbol{x}-E_{\boldsymbol{p}}t)}\hat{a}(\boldsymbol{p})-e^{-i(\boldsymbol{p}\cdot\boldsymbol{x}-E_{\boldsymbol{p}}t)}\hat{a}^\dagger(\boldsymbol{p})]. \quad (43)$$

For the field $\hat{\Phi}_{\mathrm{H}}$ and the operator $\hat{\Pi}_{\mathrm{H}}$, the simultaneous time commutation relation $[\hat{\Phi}_{\mathrm{H}}(t,\boldsymbol{x}),\hat{\Pi}_{\mathrm{H}}(t,\boldsymbol{y})]=ie^{-\gamma t}\delta^3(\boldsymbol{x}-\boldsymbol{y})$ holds. In this paper the local operator $\hat{\Pi}_{\mathrm{H}}$ is just called the conjugate momentum of $\hat{\Phi}_{\mathrm{H}}$ even though these operators do not follow the usual canonical commutation relations. We then find that the field $\hat{\Phi}_{\mathrm{H}}$ and its conjugate momentum $\hat{\Pi}_{\mathrm{H}}$ satisfy

$$[\hat{\Pi}_{\mathrm{H}}(x^0,\boldsymbol{x}),\hat{\Pi}_{\mathrm{H}}(y^0,\boldsymbol{y})]=0=[\hat{\Phi}_{\mathrm{H}}(x^0,\boldsymbol{x}),\hat{\Pi}_{\mathrm{H}}(y^0,\boldsymbol{y})] \quad (44)$$

for $(x^0-y^0)^2-(\boldsymbol{x}-\boldsymbol{y})^2<0$. The detailed proof of the above three commutation relations are provided in Appendix B. Furthermore, we can show that Eqs. (40) and (44) hold for more general local operators. As shown in Ref. [28], the arbitrary operator of a particle is expanded by the creation and annihilation operators of the particle. This leads to that arbitrary local operator $\hat{O}$ at time $t=0$ for a finite spatial region O is expanded by $\hat{\Phi}(\boldsymbol{x})$ and $\hat{\Pi}(\boldsymbol{x})$, and the time evolution of $\hat{O}$ is given as

$$\hat{O}_{\mathrm{H}}(t)=\sum_{N,M}\int d^3x'_1\cdots d^3x'_N\cdots d^3x_1\cdots d^3x_M\sum_{i'_1,\dots,i'_N=1,2}\sum_{i_1,\dots,i_M=1,2}O_{NM}^{i'_1\cdots i'_N i_1\cdots i_M}(\boldsymbol{x}'_1,\dots,\boldsymbol{x}'_N,\boldsymbol{x}_1,\dots,\boldsymbol{x}_M)\times e^{\mathcal{L}^\dagger t}[\hat{\xi}^{i'_1}(\boldsymbol{x}'_1)\cdots\hat{\xi}^{i'_N}(\boldsymbol{x}'_N)\hat{\xi}^{i_1}(\boldsymbol{x}_1)\cdots\hat{\xi}^{i_M}(\boldsymbol{x}_M)], \quad (45)$$

where the expansion coefficients $O_{NM}^{i'_1\cdots i'_N i_1\cdots i_M}(\boldsymbol{x}'_1,\dots,\boldsymbol{x}'_N,\boldsymbol{x}_1,\dots,\boldsymbol{x}_M)$ only have nonzero values for the region O, and $\hat{\xi}^j(\boldsymbol{x})=(\hat{\Phi}(\boldsymbol{x}),\hat{\Pi}(\boldsymbol{x}))$. Note that in the nonunitary theory considered here, $e^{\mathcal{L}t}[\hat{\xi}^{i'_1}(\boldsymbol{x}'_1)\cdots\hat{\xi}^{i'_N}(\boldsymbol{x}'_N)\hat{\xi}^{i_1}(\boldsymbol{x}_1)\cdots\hat{\xi}^{i_M}(\boldsymbol{x}_M)]\neq\hat{\xi}_{\mathrm{H}}^{i'_1}(t,\boldsymbol{x}'_1)\cdots\hat{\xi}_{\mathrm{H}}^{i'_N}(t,\boldsymbol{x}'_N)\hat{\xi}_{\mathrm{H}}^{i_1}(t,\boldsymbol{x}_1)\cdots\hat{\xi}_{\mathrm{H}}^{i_M}(t,\boldsymbol{x}_M)$. In Appendix C we show that the time evolution of the product $\hat{\xi}^{i'_1}(\boldsymbol{x}'_1)\cdots\hat{\xi}^{i'_N}(\boldsymbol{x}'_N)\hat{\xi}^{i_1}(\boldsymbol{x}_1)\cdots\hat{\xi}^{i_M}(\boldsymbol{x}_M)$ is given as the product of operators chosen from $\hat{\xi}_{\mathrm{H}}^{i'_1}(t,\boldsymbol{x}'_1),\dots,\hat{\xi}_{\mathrm{H}}^{i'_N}(t,\boldsymbol{x}'_N),\hat{\xi}_{\mathrm{H}}^{i_1}(t,\boldsymbol{x}_1),\dots,\hat{\xi}_{\mathrm{H}}^{i_M}(t,\boldsymbol{x}_M)$. This implies that the commutation relation of arbitrary local operators is computed from the commutation relations $[\hat{\xi}_{\mathrm{H}}^i(x^0,\boldsymbol{x}),\hat{\xi}_{\mathrm{H}}^j(y^0,\boldsymbol{y})]$, which vanish for $(x^0-y^0)^2-(\boldsymbol{x}-\boldsymbol{y})^2<0$ as shown in Eqs. (40) and (44). Hence, the time-evolved operators $\hat{A}_{\mathrm{H}}(t)$ and $\hat{B}_{\mathrm{H}}(t')$ of any two local operators $\hat{A}$ at time $t=0$ for a spatial region A and $\hat{B}$ at time $t'=0$ for a spatial region B commutes if $\hat{A}_{\mathrm{H}}(t)$ and $\hat{B}_{\mathrm{H}}(t')$ are spacelike separated. Therefore the microcausality holds for the present model of the massive particle.

In the following section we will derive the Markovian QME of the massive particle considered here.

V. DERIVATION OF THE MARKOVIAN QUANTUM MASTER EQUATION

In this section we derive the Markovian QME of the massive spin-0 particle, which was presented in the previous section. For the derivation, let us first return to the condition Eq. (24) of the Poincaré invariance, $\hat{U}_0(\Lambda,a)(\mathcal{L}-\mathcal{H})[\rho(0)]\hat{U}_0^\dagger(\Lambda,a)=(\mathcal{L}-\mathcal{H})[\hat{U}_0(\Lambda,a)\rho(0)\hat{U}_0^\dagger(\Lambda,a)]$. Because the superoperator $\mathcal{L}-\mathcal{H}$ is given as

$$(\mathcal{L}-\mathcal{H})[\rho]=-i[\hat{M}-\hat{H},\rho]+\sum_\lambda\left[\hat{L}_\lambda\rho\hat{L}_\lambda^\dagger-\frac{1}{2}\{\hat{L}_\lambda^\dagger\hat{L}_\lambda,\rho\}\right], \quad (46)$$

the condition, Eq. (24), is rewritten as

$$\begin{aligned}(\mathcal{L}-\mathcal{H})[\rho]&=\hat{U}_0^\dagger(\Lambda,a)(\mathcal{L}-\mathcal{H})[\hat{U}_0(\Lambda,a)\rho(0)\hat{U}_0^\dagger(\Lambda,a)]\hat{U}_0(\Lambda,a)\\&=-i[\hat{U}_0^\dagger(\Lambda,a)(\hat{M}-\hat{H})\hat{U}_0(\Lambda,a),\rho(0)]+\sum_\lambda[(\hat{U}_0^\dagger(\Lambda,a)\hat{L}_\lambda\hat{U}_0(\Lambda,a))\rho(0)(\hat{U}_0^\dagger(\Lambda,a)\hat{L}_\lambda^\dagger\hat{U}_0(\Lambda,a))\\&\quad-\frac{1}{2}\{(\hat{U}_0^\dagger(\Lambda,a)\hat{L}_\lambda\hat{U}_0(\Lambda,a))(\hat{U}_0^\dagger(\Lambda,a)\hat{L}_\lambda^\dagger\hat{U}_0(\Lambda,a)),\rho(0)\}].\end{aligned} \quad (47)$$

Equation (47) means that the Markovian QME has invariance for the transformations $\hat{M}-\hat{H} \longrightarrow \hat{U}_0^\dagger(\Lambda,a)(\hat{M}-\hat{H})\hat{U}_0(\Lambda,a)$ and $\hat{L}_\lambda \longrightarrow \hat{U}_0^\dagger(\Lambda,a)\hat{L}_\lambda\hat{U}_0(\Lambda,a)$. Recalling that the form of Markovian QME does not change under the transformations Eqs. (3) and (4), we get the following rules:

$$\hat{U}_0^\dagger(\Lambda,a)\hat{L}_\lambda(\Lambda,a)\hat{U}_0(\Lambda,a)=\sum_{\lambda'}\mathcal{U}_{\lambda\lambda'}(\Lambda,a)\hat{L}_{\lambda'}+\alpha_\lambda(\Lambda,a)\hat{\mathbb{I}}, \tag{48}$$

$$\hat{U}_0^\dagger(\Lambda,a)(\hat{M}-\hat{H})\hat{U}_0(\Lambda,a)=\hat{M}-\hat{H}+\frac{1}{2i}\sum_{\lambda,\lambda'}[\alpha_\lambda^*\mathcal{U}_{\lambda\lambda'}(\Lambda,a)\hat{L}_{\lambda'}-\alpha_\lambda(\Lambda,a)\mathcal{U}_{\lambda\lambda'}^*(\Lambda,a)\hat{L}_{\lambda'}^\dagger]+\beta(\Lambda,a)\hat{\mathbb{I}}. \tag{49}$$

For convenience, we introduce the vectors $\hat{\boldsymbol{L}}$ and $\boldsymbol{\alpha}$ with the components $\hat{L}_\lambda$ and α_λ, respectively. Then the transformation rules Eqs. (48) and (49) are rewritten as

$$\hat{U}_0^\dagger(\Lambda,a)\hat{\boldsymbol{L}}\hat{U}_0(\Lambda,a)=\mathcal{U}(\Lambda,a)\hat{\boldsymbol{L}}+\boldsymbol{\alpha}(\Lambda,a)\mathbb{I}, \tag{50}$$

$$\hat{U}_0^\dagger(\Lambda,a)(\hat{M}-\hat{H})\hat{U}_0(\Lambda,a)=\hat{M}-\hat{H}+\frac{1}{2i}[\boldsymbol{\alpha}^\dagger(\Lambda,a)\mathcal{U}(\Lambda,a)\hat{\boldsymbol{L}}-\hat{\boldsymbol{L}}^\dagger\mathcal{U}^\dagger(\Lambda,a)\boldsymbol{\alpha}(\Lambda,a)]+\beta(\Lambda,a)\mathbb{I}. \tag{51}$$

Using that the operator $\hat{U}_0(\Lambda,a)$ is the unitary representation satisfying $\hat{U}_0(\Lambda'\Lambda,a'+\Lambda'a)=\hat{U}_0(\Lambda',a')\hat{U}_0(\Lambda,a)$, we can give the conditions of the unitary matrix $\mathcal{U}(\Lambda,a)$, the vector $\boldsymbol{\alpha}(\Lambda,a)$, and the real parameter $\beta(\Lambda,a)$:

$$\mathcal{U}(\Lambda'\Lambda,a'+\Lambda'a)=\mathcal{U}(\Lambda',a')\mathcal{U}(\Lambda,a), \tag{52}$$

$$\boldsymbol{\alpha}(\Lambda'\Lambda,a'+\Lambda'a)=\mathcal{U}(\Lambda',a')\boldsymbol{\alpha}(\Lambda,a)+\boldsymbol{\alpha}(\Lambda',a'), \tag{53}$$

$$\beta(\Lambda'\Lambda,a'+\Lambda'a)=\beta(\Lambda,a)+\beta(\Lambda',a')+\frac{1}{2i}[\boldsymbol{\alpha}^\dagger(\Lambda',a')\mathcal{U}(\Lambda',a')\boldsymbol{\alpha}(\Lambda,a)-\boldsymbol{\alpha}^\dagger(\Lambda,a)\mathcal{U}^\dagger(\Lambda',a)\boldsymbol{\alpha}(\Lambda',a')]. \tag{54}$$

It is easy to derive these conditions. Equations (52) and (53) are given by comparing both sides of $\hat{U}_0^\dagger(\Lambda'\Lambda,a'+\Lambda'a)\hat{\boldsymbol{L}}\hat{U}_0(\Lambda'\Lambda,a'+\Lambda'a)=\hat{U}_0^\dagger(\Lambda,a)\hat{U}_0^\dagger(\Lambda',a')\hat{\boldsymbol{L}}\hat{U}_0(\Lambda',a')\hat{U}_0(\Lambda,a)$. Equation (54) is obtained by comparing both sides of $\hat{U}_0^\dagger(\Lambda'\Lambda,a'+\Lambda'a)(\hat{M}-\hat{H})\hat{U}_0(\Lambda'\Lambda,a'+\Lambda'a)=\hat{U}_0^\dagger(\Lambda,a)\hat{U}_0^\dagger(\Lambda',a')(\hat{M}-\hat{H})\hat{U}_0(\Lambda',a')\hat{U}_0(\Lambda,a)$.

Our task is to derive the expression of the self-adjoint operator $\hat{M}-\hat{H}$ and the Lindblad operator $\hat{\boldsymbol{L}}$ satisfying Eqs. (50) and (51). In particular, Eq. (50) is decomposed into irreducible representation subspaces for ease of computation. Hence, the irreducible unitary representations of the Poincaré group are useful for our analysis. We introduce the standard momentum ℓ^μ and the Lorentz transformation $(S_q)^\mu{}_\nu$ with

$$q^\mu=(S_q)^\mu{}_\nu\ell^\nu. \tag{55}$$

From Eq. (52), the unitary matrix $\mathcal{U}(\Lambda,a)$ is written as

$$\mathcal{U}(\Lambda,a)=\mathcal{U}(I,a)\mathcal{U}(\Lambda,0)=\mathcal{T}(a)\mathcal{V}(\Lambda), \tag{56}$$

where I is the identity matrix, $\mathcal{U}(I,a)=\mathcal{T}(a)$, $\mathcal{U}(\Lambda,0)=\mathcal{V}(\Lambda)$. The unitary matrix $\mathcal{U}(\Lambda,a)$ is the unitary representation on a vector space, and then we define the following vector on the vector space:

$$R^\mu\boldsymbol{v}_{\ell,\xi}=\ell^\mu\boldsymbol{v}_{\ell,\xi}, \tag{57}$$

where R^μ is the generator of space-time translation, and the label ξ describes the degrees of freedom which cannot be specified by momentum ℓ^μ. Also, $\mathcal{T}(a)$ can be written as $e^{-iR_\mu a^\mu}$ by using the generator of space-time translation R^μ. We now define the eigenvector $\boldsymbol{v}_{q,\xi}$ that belongs to the eigenvalue q^μ of R^μ as

$$\boldsymbol{v}_{q,\xi}=N_q\mathcal{V}(S_q)\boldsymbol{v}_{\ell,\xi}, \tag{58}$$

where N_q is the normalization. Then we can obtain the following rules $\boldsymbol{v}_{q,\xi}$:

$$\mathcal{T}(a)\boldsymbol{v}_{q,\xi}=e^{-iq^\mu a_\mu}\boldsymbol{v}_{q,\xi}, \tag{59}$$

$$\mathcal{V}(\Lambda)\boldsymbol{v}_{q,\xi}=\frac{N_q}{N_{\Lambda q}}\sum_{\xi'}\mathcal{D}_{\xi\xi'}[W(\Lambda,q)]\boldsymbol{v}_{\Lambda q,\xi'}, \tag{60}$$

where $W(\Lambda,q)=S_{\Lambda q}^{-1}\Lambda S_q$ is an element of the little group and satisfies $W^\mu{}_\nu\ell^\nu=\ell^\mu$. For the derivations of Eqs. (59) and (60), see Ref. [28]. The matrix $\mathcal{D}(W)$ with the components $\mathcal{D}_{\xi\xi'}(W)$ forms the unitary representation of the little group. For the various four-momentum, we give the standard momenta, and the little groups correspond to each standard momentum in Table I. For simplicity, ξ is assumed to be the label of basis vectors of the irreducible representation

TABLE I. Classification of the standard momentum ℓ^μ and the little group associated with ℓ^μ, where M is mass and κ is arbitrary positive energy.

Standard momentum ℓ^μ	Little group
$\ell^\mu=[M,0,0,0],\ M>0$	SO(3)
$\ell^\mu=[-M,0,0,0],\ M>0$	SO(3)
$\ell^\mu=[\kappa,0,0,\kappa],\ \kappa>0$	ISO(2)
$\ell^\mu=[-\kappa,0,0,\kappa],\ \kappa>0$	ISO(2)
$\ell^\mu=[0,0,0,N],\ N^2>0$	SO(2,1)
$\ell^\mu=[0,0,0,0]$	SO(3,1)

subspaces of the little group. Next we consider the unitary operator $\hat{U}_0(\Lambda, a)$. As in the unitary matrix $\mathcal{U}(\Lambda, a)$, the operator $\hat{U}_0(\Lambda, a)$ is decomposed in the same way. Therefore it is written as

$$\hat{U}_0(\Lambda, a) = \hat{U}_0(I, a)\hat{U}_0(\Lambda, 0) = \hat{T}(a)\hat{V}(\Lambda), \tag{61}$$

where $\hat{U}_0(I, a) = \hat{T}(a) = e^{-i\hat{P}_\mu a^\mu}$, with $\hat{P}^\mu = [\hat{H}, \hat{P}^1, \hat{P}^2, \hat{P}^3]$ and $\hat{U}_0(\Lambda, 0) = \hat{V}(\Lambda)$ with the generators $\hat{J}^i$ and $\hat{K}_0^i$. Let us focus on the transformation rule Eq. (50). From the rule for $\Lambda = I$, we obtain

$$\hat{T}^\dagger(a)\hat{\boldsymbol{L}}\hat{T}(a) = \mathcal{T}(a)\hat{\boldsymbol{L}} + \boldsymbol{\alpha}(I, a)\hat{\mathbb{I}}. \tag{62}$$

In Eq. (50) for $a^\mu = 0$ we obtain

$$\hat{V}^\dagger(\Lambda)\hat{\boldsymbol{L}}\hat{V}(\Lambda) = \mathcal{V}(\Lambda)\hat{\boldsymbol{L}} + \boldsymbol{\alpha}(\Lambda, 0)\hat{\mathbb{I}}. \tag{63}$$

Introducing $\hat{L}_{q,\xi} = \boldsymbol{v}_{q,\xi}^\dagger \hat{\boldsymbol{L}}$ and $\alpha_{q,\xi} = \boldsymbol{v}_{q,\xi}^\dagger \boldsymbol{\alpha}$, we can rewrite Eqs. (62) and (63) as follows:

$$\hat{T}^\dagger(a)\hat{L}_{q,\xi}\hat{T}(a) = e^{-iq_\mu a^\mu}\hat{L}_{q,\xi} + \alpha_{q,\xi}(I, a)\hat{\mathbb{I}}, \tag{64}$$

$$\hat{V}^\dagger(\Lambda)\hat{L}_{q,\xi}\hat{V}(\Lambda) = \frac{N_q^*}{N_{\Lambda^{-1}q}^*}\sum_{\xi'}\mathcal{D}_{\xi'\xi}^*[W(\Lambda^{-1}, q)]\hat{L}_{\Lambda^{-1}q,\xi'} + \alpha_{q,\xi}(\Lambda, 0)\hat{\mathbb{I}}. \tag{65}$$

Because of these transformation rules, we get the expression of the complex number $\alpha_{q,\xi}(\Lambda, a)$. Since (50) can be rewritten as

$$\hat{U}_0^\dagger(\Lambda, a)\hat{L}_{q,\xi}\hat{U}_0(\Lambda, a) = \boldsymbol{v}_{q,\xi}^\dagger\mathcal{U}(\Lambda, a)\hat{\boldsymbol{L}} + \alpha_{q,\xi}(\Lambda, a)\hat{\mathbb{I}}, \tag{66}$$

we get $\alpha_{q,\xi}(\Lambda, a)$ as

$$\alpha_{q,\xi}(\Lambda, a) = e^{-iq_\mu a^\mu}\alpha_{q,\xi}(\Lambda, 0) + \alpha_{q,\xi}(I, a) \tag{67}$$

by using Eqs. (64) and (65) and comparing both sides of Eq. (66). In Eq. (65) for $\Lambda = S_q$ we obtain

$$\hat{V}^\dagger(S_q)\hat{L}_{q,\xi}\hat{V}(S_q) = N_q^*\hat{L}_{\ell,\xi} + \alpha_{q,\xi}(S_q, 0)\hat{\mathbb{I}}, \tag{68}$$

where we used the fact that $N_\ell = 1$ and $W(S_q^{-1}, q) = S_{S_q^{-1}q}^{-1}S_q^{-1}S_q = S_\ell^{-1} = I$ hold. These facts can be checked by using Eq. (58) and the definition of $W(\Lambda, q)$. Equation (68) means that the Lindblad operator $\hat{L}_{q,\xi}$ is determined from the Lindblad operator $\hat{L}_{\ell,\xi}$ with the standard momentum ℓ^μ. To discuss the form of the Lindblad operator $\hat{L}_{\ell,\xi}$, we focus on the following equations obtained from Eqs. (64) and (65) for $q^\mu = \ell^\mu$ and $\Lambda = Q$ with $Q^\mu{}_\nu\ell^\nu = \ell^\mu$, respectively:

$$\hat{T}^\dagger(a)\hat{L}_{\ell,\xi}\hat{T}(a) = e^{-il_\mu a^\mu}\hat{L}_{\ell,\xi} + \alpha_{\ell,\xi}(I, a)\hat{\mathbb{I}}, \tag{69}$$

$$\hat{V}^\dagger(Q)\hat{L}_{\ell,\xi}\hat{V}(Q) = \sum_{\xi'}\mathcal{D}_{\xi'\xi}^*(Q^{-1})\hat{L}_{\ell,\xi'} + \alpha_{\ell,\xi}(Q, 0)\hat{\mathbb{I}}, \tag{70}$$

where note that $N_\ell = N_{Q^{-1}\ell} = 1$.

We now can get a model of the Markovian QME for a massive spin-0 particle by assuming that the Lindblad operator $\hat{L}_{q,\xi}$ is given as

$$\hat{L}_{q,\xi} = \int d^3p\, f_{q,\xi}(\boldsymbol{p})\hat{a}(\boldsymbol{p}) \tag{71}$$

and that the self-adjoint operator $\hat{M} - \hat{H}$ is given as

$$\hat{M} - \hat{H} = \int d^3p\, d^3p'\, g(\boldsymbol{p}, \boldsymbol{p}')\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}'), \tag{72}$$

where $g(\boldsymbol{p}, \boldsymbol{p}') = g^*(\boldsymbol{p}', \boldsymbol{p})$ is satisfied because the $\hat{M} - \hat{H}$ is self-adjoint. The Markovian QME with the above operators gives an evolution from a Gaussian state to another Gaussian state. Furthermore, in the dynamics there are no particle creations because the GKSL generator $\mathcal{L}$ has no creation processes. The Poincaré transformation rules of $\hat{a}^\dagger(\boldsymbol{p})$ shown in Refs. [28,32],

$$\hat{T}(a)\hat{a}^\dagger(\boldsymbol{p})\hat{T}^\dagger(a) = e^{-ip^\mu a_\mu}\hat{a}^\dagger(\boldsymbol{p}), \tag{73}$$

$$\hat{V}(\Lambda)\hat{a}^\dagger(\boldsymbol{p})\hat{V}^\dagger(\Lambda) = \sqrt{\frac{E_{\boldsymbol{p}_\Lambda}}{E_{\boldsymbol{p}}}}\,\hat{a}^\dagger(\boldsymbol{p}_\Lambda), \tag{74}$$

are useful to obtain the model of the massive particle, where $E_{\boldsymbol{p}} = p^0$, $E_{\boldsymbol{p}_\Lambda} = (\Lambda p)^0$, and $(\boldsymbol{p}_\Lambda)^i$ is the vector with the elements written as $(\boldsymbol{p}_\Lambda)^i = (\Lambda p)^i$ [28]. Also, $W(\Lambda, p) = S_{\Lambda p}^{-1}\Lambda S_p$ with $(S_p)^\mu{}_\nu k^\nu = p^\mu$ is an element of the little group and satisfies $W^\mu{}_\nu k^\nu = k^\mu$, where k^μ is the standard momentum for a massive particle ($k^\mu = [m, 0, 0, 0]$, $m > 0$). Substituting the ansatz of $\hat{L}_{q,\xi}$ Eq. (71) into Eqs. (69) and (70) and using the transformation rules Eqs. (73) and (74), we obtain the equations

$$f_{\ell,\xi}(\boldsymbol{p})e^{-ip^\mu a_\mu} = f_{\ell,\xi}(\boldsymbol{p})e^{-i\ell^\mu a_\mu}, \tag{75}$$

$$\sqrt{\frac{E_{\boldsymbol{p}_Q}}{E_{\boldsymbol{p}}}}f_{\ell,\xi}(\boldsymbol{p}_Q) = \sum_{\xi'}\mathcal{D}_{\xi'\xi}^*(Q^{-1})f_{\ell,\xi'}(\boldsymbol{p}). \tag{76}$$

In addition, we get $\alpha_{q,\xi}(\Lambda, 0) = \alpha_{q,\xi}(I, a) = 0$, and then Eq. (67) leads to $\alpha_{q,\xi}(\Lambda, a) = 0$. Also, we can obtain the condition of the coefficient $g(\boldsymbol{p}, \boldsymbol{p}')$ by substituting the ansatz of $\hat{M} - \hat{H}$ Eq. (72) into Eq. (49) as

$$\sqrt{\frac{E_{\boldsymbol{p}_\Lambda}E_{\boldsymbol{p}'_\Lambda}}{E_{\boldsymbol{p}}E_{\boldsymbol{p}'}}}\, g(\boldsymbol{p}_\Lambda, \boldsymbol{p}'_\Lambda)e^{i[(\Lambda p)^\mu - (\Lambda p')^\mu]a_\mu} = g(\boldsymbol{p}, \boldsymbol{p}'). \tag{77}$$

From the fact that there is no term proportional to the identity operator $\hat{\mathbb{I}}$ in $\hat{M} - \hat{H}$ and the condition $\alpha_{q,\xi}(\Lambda, a) = 0$, the real number $\beta(\Lambda, a)$ vanishes.

Solving Eqs. (75), (76), and (77), we can derive the Markovian QME for a massive spin-0 particle as follows:

$$\frac{d}{dt}\rho(t) = -i[\hat{H} + g\hat{N}, \rho(t)] + \gamma\int d^3p\left[\hat{a}(\boldsymbol{p})\rho(t)\hat{a}^\dagger(\boldsymbol{p}) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}), \rho(t)\}\right], \tag{78}$$

with a non-negative parameter γ, a real parameter g, and $\hat{N}$ is the number operator Eq. (37). The Markovian QME, Eq. (25) presented in Sec. IV, is Eq. (78) for $g = 0$. The detailed derivation[2] of Eq. (78) is presented in Appendix D.

[2]As for whether the QME Eq. (78) can be derived from the explicit system-environment model, though it is an interesting issue, we do not mention it in this paper. Note that the QME Eq. (78) is derived

VI. DISCUSSION

In this section we discuss the previous works on relativistic Markovian QME [33–35] in comparison with the presented Markovian QME. In Refs. [33,34], the following master equation of a massive spin-0 particle is proposed:

$$\frac{d}{dt}\rho(t) = \mathcal{H}[\rho(t)] + \tilde{\mathcal{D}}[\rho(t)], \tag{79}$$

where $\mathcal{H}[\rho(t)] = -i[\hat{H}, \rho(t)]$ with the Hamiltonian $\hat{H}$ Eq. (26), and

$$\tilde{\mathcal{D}}[\rho(t)] = \kappa \int d^3p E_{\boldsymbol{p}}\bigg[\hat{a}(\boldsymbol{p})\rho(t)\hat{a}^\dagger(\boldsymbol{p}) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}), \rho(t)\}\bigg]. \tag{80}$$

The dissipation rate associated with the dissipator $\tilde{\mathcal{D}}$ is read out as $\kappa E_{\boldsymbol{p}}$, which depends on the energy of the particle. The master equation (79) was derived by respecting the Poincaré (or Lorentz) covariance [33,34].[3] From this master equation (79), we have the field equation of evolution,

$$\frac{\partial^2}{\partial t^2}\hat{\varphi}_{\mathrm{H}}(t,\boldsymbol{x}) + \kappa\sqrt{-\nabla^2 + m^2}\frac{\partial}{\partial t}\hat{\varphi}_{\mathrm{H}}(t,\boldsymbol{x}) = \left(1 + \frac{\kappa^2}{4}\right)(\nabla^2 - m^2)\hat{\varphi}_{\mathrm{H}}(t,\boldsymbol{x}), \tag{81}$$

where $\hat{\varphi}_{\mathrm{H}}(t,\boldsymbol{x}) = e^{\tilde{\mathcal{L}}^\dagger t}[\Phi(\boldsymbol{x})]$ with the adjoint $\tilde{\mathcal{L}}^\dagger$ of $\tilde{\mathcal{L}} = \mathcal{H} + \tilde{\mathcal{D}}$ and the initial condition $\hat{\Phi}(\boldsymbol{x})$ given in Eq. (29). We also find that the field $\hat{\varphi}_{\mathrm{H}}(t,\boldsymbol{x})$ does not satisfy the microcausality, that is, $[\hat{\varphi}_{\mathrm{H}}(x^0,\boldsymbol{x}), \hat{\varphi}_{\mathrm{H}}(y^0,\boldsymbol{y})] \neq 0$ even for $(x^0 - y^0)^2 - (\boldsymbol{x} - \boldsymbol{y})^2 < 0$, and the violation of microcausality typically appears in the range $1/m$ [33,35]. In contrast, our QME is based on not the Poincaré covariance but the invariance defined by Eq. (18). This results in the dissipator $\mathcal{D}$ in our QME, which gives the dissipation rate independent of the energy of the particle. Furthermore, as we checked, the microcausality holds in our model.

It is interesting to highlight the difference between the two dissipators $\mathcal{D}$ and $\tilde{\mathcal{D}}$ by following the discussion in Ref. [35]. We pick up two critiques given in [35] on the above master equation (79). One is that the master equation (79) does not fully respect the Lorentz symmetry. Concretely, this means that the dissipator $\tilde{\mathcal{D}}$ is not invariant under the Lorentz boost. On the contrary, our dissipator $\mathcal{D}$ in Eq. (25) is invariant under the Lorentz boost, since $\mathcal{D} = \mathcal{L} - \mathcal{H}$ satisfying Eq. (24) is invariant under any Poincaré transformations, including the Lorentz boost.

Another critique is that the field of the massive particle does not follow the covariant equation of evolution. Indeed, it seems that not only the above field equation (81) but also our field equation (39) are not Lorentz covariant. However, we guess that our field is not scalar, and this fact makes its equation covariant. To clarify this argument, it may be important to note that the field transformation in quantum theory is identified through unitary transformations. The scalar field of a massive spin-0 particle, $\phi(x)$, changes as $\phi(x) \to \phi'(x') = \phi(x)$ under Lorentz transformations $x'^\mu = \Lambda^\mu{}_\nu x^\nu$. In unitary relativistic quantum theories, such as quantum field theories, the corresponding transformation is

$$\hat{\phi}_{\mathrm{H}}(x) \to \hat{V}^\dagger(\Lambda)\hat{\phi}_{\mathrm{H}}(x')\hat{V}(\Lambda) = \hat{\phi}_{\mathrm{H}}(x), \tag{82}$$

where the unitary operator $\hat{V}(\Lambda) = \hat{U}_0(\Lambda, 0)$, defined around Eq. (61), represents the Lorentz transformation, and $\hat{\phi}_{\mathrm{H}}(t,\boldsymbol{x}) = e^{i\hat{H}t}\hat{\Phi}(\boldsymbol{x})e^{-i\hat{H}t}$. On the other hand, the field operator $\hat{\Phi}_{\mathrm{H}}$ obeying the dissipative KG equation (39) in our model is not transformed like Eq. (82). The field $\hat{\Phi}_{\mathrm{H}}$ satisfies $\hat{V}^\dagger(\Lambda)e^{\frac{\gamma}{2}x'^0}\hat{\Phi}_{\mathrm{H}}(x')\hat{V}(\Lambda) = e^{\frac{\gamma}{2}x^0}\hat{\Phi}_{\mathrm{H}}(x)$, because $\hat{\Phi}_{\mathrm{I}}(x) = e^{\frac{\gamma}{2}x^0}\hat{\Phi}_{\mathrm{H}}(x)$ is the solution of the usual free KG equation, which plays a role of Lorentz scalar field. Hence the field transformation of $\hat{\Phi}_{\mathrm{H}}$ is given as

$$\hat{\Phi}_{\mathrm{H}}(x) \to \hat{V}^\dagger(\Lambda)\hat{\Phi}_{\mathrm{H}}(x')\hat{V}(\Lambda) = e^{-\gamma(x'^0 - x^0)/2}\hat{\Phi}_{\mathrm{H}}(x). \tag{83}$$

This means that $\hat{\Phi}_{\mathrm{H}}(x)$ is not a Lorentz scalar field, which is crucial to discuss the Lorentz covariance of Eq. (39). If the field follows the transformation Eq. (83), the dissipative KG equation (39) is certainly Lorentz covariant, which is easily observed from the fact that $\hat{\Phi}_{\mathrm{I}}(x)$ is just a Lorentz scalar field satisfying the free KG equation. We think that the present discussion on the covariance of the field equation is very speculative. As a future issue, we need a clear understanding of this concern.

VII. CONCLUSION AND OUTLOOK

In this study we discussed the Markovian QME in the GKSL form, whose solution is given by a quantum dynamical semigroup with the Poincaré invariance. In particular, we derived the Markovian QME of a relativistic massive spin-0 particle and investigated its field dynamics. First, it turned out that the field of the massive particle follows the KG equation with a dissipation term. Besides, we also showed that not only field operators but also any local operators commute with each other when these operators are spacelike separated. This means that the present theory specified on the Markovian QME has the microcausality property. Our formulation for the Markovian QME respecting the Poincaré symmetry may give dissipative quantum field theories.

In this paper we just provided a theoretical framework for yielding the Markovian QME of relativistic particles. We expect that this framework may be valid for relativistic dissipative phenomena, such as the decay of unstable relativistic particles. Confirming this expectation is a future subject of this present paper, which would deepen the understanding of our framework.

Another future subject is to build a theoretical framework including the two standpoints mentioned in the Introduction: one is that "gravity is quantum," and the other is that "gravity is classical." For this purpose, we will expand the present the-

solely from the assumption that it has Poincaré symmetry and that the Lindblad operator $\hat{L}_{q,\xi}$ and self-adjoint operator $\hat{M} - \hat{H}$ are given by Eqs. (71) and (72).

[3]Note that the normalization of the creation and annihilation operators is somewhat different in Refs. [33] and [34]. The former is the same as our normalization, but the latter uses the following normalization: $[\hat{\alpha}(\boldsymbol{p}), \hat{\alpha}^\dagger(\boldsymbol{p}')] = 2E_{\boldsymbol{p}}\delta^3(\boldsymbol{p} - \boldsymbol{p}')$, $[\hat{\alpha}(\boldsymbol{p}), \hat{\alpha}(\boldsymbol{p}')] = 0$, where $\hat{\alpha}(\boldsymbol{p})$ is defined as $\sqrt{2E_{\boldsymbol{p}}}\hat{a}(\boldsymbol{p})$.

ory to that it can take into account gravitational interactions. If such a framework is established, we can provide candidate theories that describe the interplay regime of quantum and gravity phenomena. This leads to an interesting theme of what candidates will be accepted from future quantum gravity experiments. In particular, we believe that recent experimental technologies [36–38] for testing quantum mechanics and gravity theory would explore the consistent theory unifying quantum and gravity physics. We hope that the present work will be a help for the research of quantum gravity.

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APPENDIX A: PROOF THAT THE GKSL GENERATOR IN EQ. (25) IS POINCARÉ INVARIANT

We prove the Poincaré invariance of the GKSL generator $\mathcal{L}$ in Eq. (25). Since the GKSL generator $\mathcal{L}$ has the form $\mathcal{L} = \mathcal{H} + \mathcal{D}$, the condition of Poincaré invariance Eq. (24) is nothing but the invariance condition for the dissipator $\mathcal{D}$,

$$\hat{U}_0(\Lambda, a)\mathcal{D}[\rho(0)]\hat{U}_0^\dagger(\Lambda, a) = \mathcal{D}[\hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)], \tag{A1}$$

where $\mathcal{D}$ is yielded as

$$\mathcal{D}[\rho(0)] = \gamma \int d^3p \left[\hat{a}(\boldsymbol{p})\rho(0)\hat{a}^\dagger(\boldsymbol{p}) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}), \rho(0)\}\right]. \tag{A2}$$

Substituting $\mathcal{D}[\rho(0)]$ into the left-hand side of condition Eq. (A1), the left-hand side can be rewritten as follows:

$$\begin{aligned}
\hat{U}_0(\Lambda, a)\mathcal{D}[\rho(0)]\hat{U}_0^\dagger(\Lambda, a) &= \gamma \int d^3p\, \hat{U}_0(\Lambda, a)\left[\hat{a}(\boldsymbol{p})\rho(0)\hat{a}^\dagger(\boldsymbol{p}) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}), \rho(0)\}\right]\hat{U}_0^\dagger(\Lambda, a)\\
&= \gamma \int d^3p \left[\hat{U}_0(\Lambda, a)\hat{a}(\boldsymbol{p})\rho(0)\hat{a}^\dagger(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a) - \frac{1}{2}\hat{U}_0(\Lambda, a)\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}), \rho(0)\}\hat{U}_0^\dagger(\Lambda, a)\right]\\
&= \gamma \int d^3p\, \hat{U}_0(\Lambda, a)\hat{a}(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a)[\hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)]\hat{U}_0(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a)\\
&\quad - \frac{\gamma}{2}\int d^3p\{\hat{U}_0(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a)\hat{U}_0(\Lambda, a)\hat{a}(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a), \hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)\}.
\end{aligned} \tag{A3}$$

Using the transformation rules Eqs. (73) and (74), which lead to

$$\hat{U}_0(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a) = \sqrt{\frac{E_{\boldsymbol{p}_\Lambda}}{E_{\boldsymbol{p}}}}e^{-i(\Lambda p)^\mu a_\mu}\hat{a}^\dagger(\boldsymbol{p}_\Lambda), \tag{A4}$$

we have

$$\begin{aligned}
&\hat{U}_0(\Lambda, a)\mathcal{D}[\rho(0)]\hat{U}_0^\dagger(\Lambda, a)\\
&= \gamma \int d^3p\, \hat{U}_0(\Lambda, a)\hat{a}(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a)[\hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)]\hat{U}_0(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a)\\
&\quad - \frac{\gamma}{2}\int d^3p\{\hat{U}_0(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a)\hat{U}_0(\Lambda, a)\hat{a}(\boldsymbol{p})\hat{U}_0^\dagger(\Lambda, a), \hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)\}\\
&= \gamma \int d^3p\, \frac{E_{\boldsymbol{p}_\Lambda}}{E_{\boldsymbol{p}}}\left[\hat{a}(\boldsymbol{p}_\Lambda)\hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p}_\Lambda) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p}_\Lambda)\hat{a}(\boldsymbol{p}_\Lambda), \hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)\}\right]\\
&= \gamma \int d^3p_\Lambda\left[\hat{a}(\boldsymbol{p}_\Lambda)\hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)\hat{a}^\dagger(\boldsymbol{p}_\Lambda) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p}_\Lambda)\hat{a}(\boldsymbol{p}_\Lambda), \hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)\}\right]\\
&= \mathcal{D}[\hat{U}_0(\Lambda, a)\rho(0)\hat{U}_0^\dagger(\Lambda, a)],
\end{aligned} \tag{A5}$$

where we used the fact that $d^3p/E_{\rm p}$ is the Lorentz invariant measure in the third equality. Certainly, it is confirmed that the GKSL generator $\mathcal{L}$ of the Markovian quantum master equation (25) satisfies the condition Eq. (24).

APPENDIX B: PROOF OF THE COMMUTATION RELATIONS EQS. (40) AND (44)

Before the proofs of them, we mention the Lorentz invariance of the measure $d^3p/2E_{\boldsymbol{p}}$. This measure can be rewritten as follows:

$$\frac{d^3p}{2E_{\boldsymbol{p}}} = d^4p\, \delta(p_\mu p^\mu + m^2)\theta(p^0), \tag{B1}$$

where $\theta(p^0)$ is step function defined as

$$\theta(p^0) = \begin{cases} 1\ (p^0 \geqslant 0) \\ 0\ (p^0 < 0) \end{cases}, \tag{B2}$$

and $\eta_{\mu\nu}$ is the Minkowski metric tensor. Under the Lorentz transformation $p^\mu \to p'^\mu = \Lambda^\mu{}_\nu p^\nu$, the right-hand side of Eq. (B1) is

$$d^4p'\,\delta(p'_\mu p'^\mu + m^2)\theta(p'^0) = d^4p\,\delta(p_\mu p^\mu + m^2)\theta(p'^0), \tag{B3}$$

where the matrix Λ^μ_ν is a proper orthochronous Lorentz transformation matrix. If p'^0 is positive, the step function is Lorentz invariant too. Looking at the δ function, the integration has a meaningful value when $p^\rho p_\rho + m^2 = 0$. The condition $p^\rho p_\rho + m^2 = 0$ means that p^ρ is a timelike vector. Since we consider a proper orthochronous Lorentz transformation, $\theta(p^0) = \theta(p'^0) = 1$ holds. Thus we obtain the fact that the measure $d^3p/2E_{\boldsymbol{p}}$ is Lorentz invariant.

We first prove Eq. (40). The commutator $[\hat{\Phi}_{\rm H}(x^0,\boldsymbol{x}), \hat{\Phi}_{\rm H}(y^0,\boldsymbol{y})]$ is computed as

$$\begin{aligned}[\hat{\Phi}_{\rm H}(x^0,\boldsymbol{x}), \hat{\Phi}_{\rm H}(y^0,\boldsymbol{y})] &= e^{-\frac{\gamma}{2}(x^0+y^0)}[\hat{\Phi}_{\rm I}(x^0,\boldsymbol{x}), \hat{\Phi}_{\rm I}(y^0,\boldsymbol{y})] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{\sqrt{2E_{\boldsymbol{p}}}}\int\frac{d^3p'}{\sqrt{2E_{\boldsymbol{p}'}}}[e^{i(p^\mu x_\mu - p'^\mu y_\mu)}[\hat{a}(\boldsymbol{p}),\hat{a}^\dagger(\boldsymbol{p}')] - e^{-i(p^\mu x_\mu - p'^\mu y_\mu)}[\hat{a}(\boldsymbol{p}),\hat{a}^\dagger(\boldsymbol{p}')]] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{2E_{\boldsymbol{p}}}[e^{ip^\mu(x_\mu-y_\mu)} - e^{-ip^\mu(x_\mu-y_\mu)}],\end{aligned} \tag{B4}$$

where we used the notations $p^\mu = [E_{\boldsymbol{p}}, \boldsymbol{p}]$, $x_\mu = [-x^0, \boldsymbol{x}]$, $y_\mu = [-y^0, \boldsymbol{y}]$, and Eqs. (28). For any spacelike distance $(x^0-y^0)^2 - (\boldsymbol{x}-\boldsymbol{y})^2 < 0$, we can introduce $x'_\rho - y'_\rho = \Lambda^\mu_\rho(x_\mu - y_\mu)$ and choose a proper orthochronous Lorentz transformation Λ such that $x'_0 - y'_0 = 0$. Thus, we obtain the following result:

$$\begin{aligned}[\hat{\Phi}_{\rm H}(x^0,\boldsymbol{x}), \hat{\Phi}_{\rm H}(y^0,\boldsymbol{y})] &= \frac{1}{(2\pi)^3}e^{-\frac{\gamma}{2}(x^0+y^0)}\int\frac{d^3p}{2E_{\boldsymbol{p}}}[e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')} - e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')}] \\ &= \frac{1}{(2\pi)^3}e^{-\frac{\gamma}{2}(x^0+y^0)}\int\frac{d^3p}{2E_{\boldsymbol{p}}}e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')} - \frac{1}{(2\pi)^3}e^{-\frac{\gamma}{2}(x^0+y^0)}\int\frac{d^3\tilde{p}}{2E_{\tilde{\boldsymbol{p}}}}e^{i\tilde{\boldsymbol{p}}\cdot(\boldsymbol{x}'-\boldsymbol{y}')} \\ &= 0,\end{aligned} \tag{B5}$$

where $\tilde{\boldsymbol{p}} \equiv -\boldsymbol{p}$ in the second line.

Next, we will prove Eq. (44). In the same manner performed to prove Eq. (40), we just use the Lorentz invariance of the measure $d^3p/2E_{\boldsymbol{p}}$ and choose the Lorentz transformation Λ such that $x'_0 - y'_0 = 0$ for $(x^0-y^0)^2 - (\boldsymbol{x}-\boldsymbol{y})^2 < 0$. Therefore, the following results will be obtained:

$$\begin{aligned}[\hat{\Pi}_{\rm H}(x^0,\boldsymbol{x}), \hat{\Pi}_{\rm H}(y^0,\boldsymbol{y})] &= e^{-\frac{\gamma}{2}(x^0+y^0)}[\hat{\Pi}_{\rm I}(x^0,\boldsymbol{x}), \hat{\Pi}_{\rm I}(y^0,\boldsymbol{y})] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{2E_{\boldsymbol{p}}}\cdot E_{\boldsymbol{p}}^2\big[e^{ip^\mu(x_\mu-y_\mu)} - e^{-ip^\mu(x_\mu-y_\mu)}\big] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int d^4p\,\delta(p_\mu p^\mu + m^2)\theta(p^0)\,(p^0)^2\big[e^{ip^\mu(x_\mu-y_\mu)} - e^{-ip^\mu(x_\mu-y_\mu)}\big] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int d^4p'\,\delta(p'_\mu p'^\mu + m^2)\theta(p'^0)(p'^0)^2\big[e^{ip'^\mu(x_\mu-y_\mu)} - e^{-ip'^\mu(x_\mu-y_\mu)}\big] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int d^4p\,\delta(p_\mu p^\mu + m^2)\theta(p^0)\,(\Lambda^0{}_\nu p^\nu)^2\big[e^{ip^\mu(x'_\mu-y'_\mu)} - e^{-ip^\mu(x'_\mu-y'_\mu)}\big] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{2E_{\boldsymbol{p}}}(\Lambda^0{}_0E_{\boldsymbol{p}} + \Lambda^0{}_jp^j)^2[e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')} - e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')}] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{2E_{\boldsymbol{p}}}\cdot 2\Lambda^0{}_0\Lambda^0{}_jE_{\boldsymbol{p}}p^j[e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')} - e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')}] \\ &= \frac{e^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\Lambda^0{}_0\Lambda^0{}_j\int d^3p\,(-i)\frac{\partial}{\partial x'_j}[e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')} + e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')}] \\ &= -2ie^{-\frac{\gamma}{2}(x^0+y^0)}\,\Lambda^0{}_0\Lambda^0{}_j\frac{\partial}{\partial x'_j}\delta^3(\boldsymbol{x}'-\boldsymbol{y}') = 0,\end{aligned} \tag{B6}$$

where we used the fact that the vectors $\boldsymbol{x}'$ and $\boldsymbol{y}'$ are spacelike separated, since the vectors x^μ and y^μ are spacelike separated. We further show that

$$\begin{aligned}
[\hat{\Phi}_{\mathrm{H}}(x^0,\boldsymbol{x}),\hat{\Pi}_{\mathrm{H}}(y^0,\boldsymbol{y})] &= e^{-\frac{\gamma}{2}(x^0+y^0)}[\hat{\Phi}_{\mathrm{I}}(x^0,\boldsymbol{x}),\hat{\Pi}_{\mathrm{I}}(y^0,\boldsymbol{y})]\\
&= \frac{ie^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{2E_{\boldsymbol{p}}}E_{\boldsymbol{p}}\big[e^{ip^\mu(x_\mu-y_\mu)}+e^{-ip^\mu(x_\mu-y_\mu)}\big]\\
&= \frac{ie^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int d^4p\,\delta(p_\mu p^\mu+m^2)\theta(p^0)\,p^0\big[e^{ip^\mu(x_\mu-y_\mu)}+e^{-ip^\mu(x_\mu-y_\mu)}\big]\\
&= \frac{ie^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int d^4p\,\delta(p_\mu p^\mu+m^2)\theta(p^0)\,\Lambda^0{}_\nu p^\nu\big[e^{ip^\rho(x'_\rho-y'_\rho)}+e^{-ip^\rho(x'_\rho-y'_\rho)}\big]\\
&= \frac{ie^{-\frac{\gamma}{2}(x^0+y^0)}}{(2\pi)^3}\int\frac{d^3p}{2E_{\boldsymbol{p}}}\Lambda^0{}_0E_{\boldsymbol{p}}[e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')}+e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}'-\boldsymbol{y}')}]\\
&= i\Lambda^0{}_0\,e^{-\frac{\gamma}{2}(x^0+y^0)}\delta^3(\boldsymbol{x}'-\boldsymbol{y}')=0,
\end{aligned}\tag{B7}$$

where we used the fact that the vectors x^μ and y^μ are spacelike separated. We conclude the proofs of the three commutation relations.

APPENDIX C: TIME EVOLUTION OF THE PRODUCT OF OPERATORS

In Sec. IV we stated that any operators commutate if they are spacelike separated under the model of the master equation derived in this study. We prove this statement in this section. To this end, we have to find the time evolution of the product $\hat{\xi}^{i'_1}(\boldsymbol{x}'_1)\cdots\hat{\xi}^{i'_N}(\boldsymbol{x}'_N)\hat{\xi}^{i_1}(\boldsymbol{x}_1)\cdots\hat{\xi}^{i_M}(\boldsymbol{x}_M)$. With reference to Eqs. (33) and (34), the following relation is satisfied:

$$e^{\mathcal{L}^\dagger t}[\hat{\xi}^{i'_1}(\boldsymbol{x}'_1)\cdots\hat{\xi}^{i'_N}(\boldsymbol{x}'_N)\hat{\xi}^{i_1}(\boldsymbol{x}_1)\cdots\hat{\xi}^{i_M}(\boldsymbol{x}_M)]=e^{\mathcal{D}^\dagger t}\big[\hat{\xi}_{\mathrm{I}}^{i'_1}(t,\boldsymbol{x}'_1)\cdots\hat{\xi}_{\mathrm{I}}^{i'_N}(t,\boldsymbol{x}'_N)\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\cdots\hat{\xi}_{\mathrm{I}}^{i_M}(t,\boldsymbol{x}_M)\big].\tag{C1}$$

If the action $(\mathcal{D}^\dagger)^k$ can be computed, the right-hand side of Eq. (C1) is obtained. We introduce the normal ordered product and use the Wick's theorem to calculate the right-hand side of Eq. (C1).

The operator $\hat{\xi}_{\mathrm{I}}^i(t,\boldsymbol{x})$ is decomposed into the term which contains annihilation operators $\hat{\zeta}_+^i(t,\boldsymbol{x})$ and the term which contains creation operators $\hat{\zeta}_-^i(t,\boldsymbol{x})$:

$$\hat{\xi}_{\mathrm{I}}^i(t,\boldsymbol{x})=\hat{\zeta}_+^i(t,\boldsymbol{x})+\hat{\zeta}_-^i(t,\boldsymbol{x}),\tag{C2}$$

where $\hat{\zeta}_+^i(t,\boldsymbol{x}_i)$ and $\hat{\zeta}_-^i(t,\boldsymbol{x}_i)$ are called the positive-frequency term and the negative-frequency term, respectively. For two field operators $\hat{\xi}_{\mathrm{I}}^i(t,\boldsymbol{x})$ and $\hat{\xi}_{\mathrm{I}}^j(t,\boldsymbol{y})$, the following product $\mathcal{N}[\hat{\xi}_{\mathrm{I}}^i(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^j(t,\boldsymbol{y})]$ is called the normal ordered product:

$$\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^i(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^j(t,\boldsymbol{y})\big]=\hat{\zeta}_+^i(t,\boldsymbol{x})\hat{\zeta}_+^j(t,\boldsymbol{y})+\hat{\zeta}_-^j(t,\boldsymbol{y})\hat{\zeta}_+^i(t,\boldsymbol{x})+\hat{\zeta}_-^i(t,\boldsymbol{x})\hat{\zeta}_+^j(t,\boldsymbol{y})+\hat{\zeta}_-^i(t,\boldsymbol{x})\hat{\zeta}_-^j(t,\boldsymbol{y}).\tag{C3}$$

The convenient point of the normal ordered product is that the calculation of $\mathcal{D}^\dagger$ is simple. The normal ordered product $\mathcal{N}[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]$ of the n fields is

$$\begin{aligned}
\mathcal{N}[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)] =&\hat{\zeta}_+^{i_1}(t,\boldsymbol{x}_1)\hat{\zeta}_+^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\zeta}_+^{i_n}(t,\boldsymbol{x}_n)\\
&+\hat{\zeta}_-^{i_1}(t,\boldsymbol{x}_1)\hat{\zeta}_+^{i_2}(t,\boldsymbol{x}_2)\hat{\zeta}_+^{i_3}(t,\boldsymbol{x}_3)\cdots\hat{\zeta}_+^{i_n}(t,\boldsymbol{x}_n)+\hat{\zeta}_-^{i_2}(t,\boldsymbol{x}_2)\hat{\zeta}_+^{i_1}(t,\boldsymbol{x}_1)\hat{\zeta}_+^{i_3}(t,\boldsymbol{x}_3)\cdots\hat{\zeta}_+^{i_n}(t,\boldsymbol{x}_n)+\cdots\\
&+\hat{\zeta}_-^{i_1}(t,\boldsymbol{x}_1)\hat{\zeta}_-^{i_2}(t,\boldsymbol{x}_2)\hat{\zeta}_+^{i_3}(t,\boldsymbol{x}_3)\cdots\hat{\zeta}_+^{i_n}(t,\boldsymbol{x}_n)+\hat{\zeta}_-^{i_1}(t,\boldsymbol{x}_1)\hat{\zeta}_-^{i_3}(t,\boldsymbol{x}_3)\hat{\zeta}_+^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\zeta}_+^{i_n}(t,\boldsymbol{x}_n)+\cdots\\
&\vdots\\
&+\hat{\zeta}_-^{i_1}(t,\boldsymbol{x}_1)\cdots\hat{\zeta}_-^{i_{n-1}}(t,\boldsymbol{x}_{n-1})\hat{\zeta}_+^{i_n}(t,\boldsymbol{x}_n)+\hat{\zeta}_-^{i_1}(t,\boldsymbol{x}_1)\cdots\hat{\zeta}_-^{i_n}(t,\boldsymbol{x}_n)\hat{\zeta}_+^{i_{n-1}}(t,\boldsymbol{x}_{n-1})+\cdots\\
&+\hat{\zeta}_-^{i_1}(t,\boldsymbol{x}_1)\hat{\zeta}_-^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\zeta}_-^{i_n}(t,\boldsymbol{x}_n).
\end{aligned}\tag{C4}$$

To make $\mathcal{D}^\dagger$ work for $\mathcal{N}[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]$, we can use the following relation: If the product with k pieces of $\hat{\zeta}_-$ lined up on the left, the action of $\mathcal{D}^\dagger$ on that is

$$\mathcal{D}^\dagger[\underbrace{\hat{\zeta}_-\cdots\hat{\zeta}_-}_{k\text{ pieces}}\underbrace{\hat{\zeta}_+\cdots\hat{\zeta}_+}_{n-k\text{ pieces}}]=-\frac{n}{2}\gamma\,\underbrace{\hat{\zeta}_-\cdots\hat{\zeta}_-}_{k\text{ pieces}}\underbrace{\hat{\zeta}_+\cdots\hat{\zeta}_+}_{n-k\text{ pieces}}.\tag{C5}$$

Equation (C5) holds for any k, so $\mathcal{D}^\dagger[\mathcal{N}[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]]$ is computed as

$$\mathcal{D}^\dagger\big[\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\big]=-\frac{n}{2}\gamma\,\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big].\tag{C6}$$

Since what we originally want to know is $\mathcal{D}^\dagger[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]$, the relation between $\mathcal{D}^\dagger[\mathcal{N}[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]]$ and $\mathcal{D}^\dagger[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]$ is needed. The relation is given by Wick's theorem as follows: the product $\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)$ is represented by a normal ordered product,

$$\begin{aligned}\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n) &= \mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] + \sum_{1-\mathrm{pair}}\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &+\sum_{2-\mathrm{paris}}\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] + (3\text{ pairs or more terms})\cdots,\end{aligned} \tag{C7}$$

where $\cdots\cdots$ is called the Wick contraction and is given as the expectation value taken in a vacuum for the product of two operators connected by the line. Considering, for example, the case of the product $\hat{\xi}_{\mathrm{I}}^{i}(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^{j}(t,\boldsymbol{y})$, the Wick's theorem gives

$$\hat{\xi}_{\mathrm{I}}^{i}(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^{j}(t,\boldsymbol{y}) = \mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i}(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^{j}(t,\boldsymbol{y})\big] + \hat{\xi}_{\mathrm{I}}^{i}(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^{j}(t,\boldsymbol{y}) = \mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i}(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^{j}(t,\boldsymbol{y})\big] + \langle 0|\hat{\xi}_{\mathrm{I}}^{i}(t,\boldsymbol{x})\hat{\xi}_{\mathrm{I}}^{j}(t,\boldsymbol{y})|0\rangle. \tag{C8}$$

Since $\mathcal{D}^\dagger$ is a linear superoperator, using Eqs. (C6) and (C7), $\mathcal{D}^\dagger[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)]$ is computed as

$$\begin{aligned}&\mathcal{D}^\dagger\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &= \mathcal{D}^\dagger\big[\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\big] + \sum_{1-\mathrm{pair}}\mathcal{D}^\dagger\big[\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\big]\\ &\quad+\sum_{2-\mathrm{pairs}}\mathcal{D}^\dagger\big[\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\big] + (3\text{ pairs or more terms})\\ &= -\frac{n}{2}\gamma\,\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] - \sum_{1-\mathrm{pair}}\frac{n-2}{2}\gamma\,\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &\quad-\sum_{2-\mathrm{pairs}}\frac{n-4}{2}\gamma\,\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] + (3\text{ pairs or more terms}).\end{aligned} \tag{C9}$$

Furthermore, $(\mathcal{D}^\dagger)^k$ operates as

$$\begin{aligned}&(\mathcal{D}^\dagger)^k\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &= \left(-\frac{n}{2}\gamma\right)^k\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] + \sum_{1-\mathrm{pair}}\left(-\frac{n-2}{2}\gamma\right)^k\mathcal{N}\left[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\right]\\ &\quad+\sum_{2-\mathrm{pairs}}\left(-\frac{n-4}{2}\gamma\right)^k\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] + (3\text{ pairs or more terms}).\end{aligned} \tag{C10}$$

Therefore, we can calculate the Heisenberg operator $e^{\mathcal{L}^\dagger t}[\hat{\xi}^{i_1}(\boldsymbol{x}_1)\hat{\xi}^{i_2}(\boldsymbol{x}_2)\cdots\hat{\xi}^{i_n}(\boldsymbol{x}_n)]$ because of Eqs. (C1) and (C10):

$$\begin{aligned}&e^{\mathcal{L}^\dagger t}\big[\hat{\xi}^{i_1}(\boldsymbol{x}_1)\hat{\xi}^{i_2}(\boldsymbol{x}_2)\cdots\hat{\xi}^{i_n}(\boldsymbol{x}_n)\big]\\ &= \exp\left(-\frac{n}{2}\gamma t\right)\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &\quad+\exp\left(-\frac{n-2}{2}\gamma t\right)\sum_{1-\mathrm{pair}}\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &\quad+\exp\left(-\frac{n-4}{2}\gamma t\right)\sum_{2-\mathrm{pair}}\mathcal{N}\big[\hat{\xi}_{\mathrm{I}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{I}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{I}}^{i_n}(t,\boldsymbol{x}_n)\big] + (3\text{ pairs or more terms})\\ &= \mathcal{N}\big[\hat{\xi}_{\mathrm{H}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{H}}^{i_2}(t,\boldsymbol{x}_2)\cdots\hat{\xi}_{\mathrm{H}}^{i_n}(t,\boldsymbol{x}_n)\big] + \exp(\gamma t)\sum_{1-\mathrm{pair}}\mathcal{N}\big[\hat{\xi}_{\mathrm{H}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{H}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{H}}^{i_n}(t,\boldsymbol{x}_n)\big]\\ &\quad+\exp(2\gamma t)\sum_{2-\mathrm{pair}}\mathcal{N}\big[\hat{\xi}_{\mathrm{H}}^{i_1}(t,\boldsymbol{x}_1)\hat{\xi}_{\mathrm{H}}^{i_2}(t,\boldsymbol{x}_2)\cdots\cdots\hat{\xi}_{\mathrm{H}}^{i_n}(t,\boldsymbol{x}_n)\big] + (3\text{ pairs or more terms}),\end{aligned} \tag{C11}$$

where we used the relation Eq. (38) in the last equality. The factors such as $\exp(\gamma t)$ and $\exp(2\gamma t)$ appear since the Wick contraction of $\hat{\xi}^i_{\rm I}$ connects with that of $\hat{\xi}^i_{\rm H}$ as

$$\overbrace{\hat{\xi}^i_{\rm I}(t,\boldsymbol{x})\hat{\xi}^j_{\rm I}}(t,\boldsymbol{y}) = \exp(\gamma t)\overbrace{\hat{\xi}^i_{\rm H}(t,\boldsymbol{x})\hat{\xi}^j_{\rm H}}(t,\boldsymbol{y}). \tag{C12}$$

The result Eq. (C11) means that the time evolution $e^{\mathcal{L}^\dagger t}[\hat{\xi}^{i_1}(\boldsymbol{x}_1)\hat{\xi}^{i_2}(\boldsymbol{x}_2)\cdots\hat{\xi}^{i_n}(\boldsymbol{x}_n)]$ is written with the combinations of the products of the Heisenberg operators chosen from $\hat{\xi}^{i_1}_{\rm H}(t,\boldsymbol{x}_1)$, …., $\hat{\xi}^{i_n}_{\rm H}(t,\boldsymbol{x}_n)$ by repeatedly using Wick's theorem.

The above result implies that the commutation relation of the arbitrary two operators can be calculated from the commutation relations $[\hat{\xi}^{i_k}_{\rm H},\hat{\xi}^{j_l}_{\rm H}]$. Therefore, supposing that $\hat{A}$ is a operator defined for a spatial region A at time $t=0$ and $\hat{B}$ is a operator defined for a spatial region B at time $t'=0$, it turns out that the time-evolved operators $\hat{A}_{\rm H}(t)$ and $\hat{B}_{\rm H}(t')$ commute if they are spacelike separated.

APPENDIX D: DERIVATION OF THE MARKOVIAN QUANTUM MASTER EQUATION, EQ. (78)

We assume that the massive spectrum of $\hat{P}^\mu$ satisfies

$$\hat{P}^\mu\hat{P}_\mu = -m^2,\ \hat{P}^0 > 0. \tag{D1}$$

The Eq. (D1) leads to the Hamiltonian $\hat{H}=\hat{P}^0$ given as $\hat{H}=\sqrt{\hat{P}^k\hat{P}_k+m^2}$. In this Appendix, thinking about the cases in Table I, we derive the Lindblad operator $\hat{L}_{q,\xi}$ and the self-adjoint operator $\hat{M}$, which gives the Markovian QME, Eq. (25).

1. Lindblad operator for the case $\ell^\mu = [\pm M, 0, 0, 0]$, $M > 0$

Substituting $a^\mu=(0,\boldsymbol{a})$ into Eq. (75), this equation leads to

$$f_{\ell,\xi}(\boldsymbol{p})e^{-i\boldsymbol{p}\cdot\boldsymbol{a}} = f_{\ell,\xi}(\boldsymbol{p}) \ \therefore\ f_{\ell,\xi}(\boldsymbol{p}) = f_{\ell,\xi}\delta^3(\boldsymbol{p}). \tag{D2}$$

Equation (D2) can be checked by integrating both sides. Next, we think Eq. (75), in the case of $a^\mu=(a,\boldsymbol{0})$, leads to

$$f_{\ell,\xi}(\boldsymbol{p})e^{iE_{\boldsymbol{p}}a} = f_{\ell,\xi}(\boldsymbol{p})e^{\pm iMa}. \tag{D3}$$

This equation has a meaning only when $\boldsymbol{p}=\boldsymbol{0}$ because of Eq. (D2). So, we eventually get

$$f_{\ell,\xi}e^{ima} = f_{\ell,\xi}e^{\pm iMa}. \tag{D4}$$

Since the mass m is positive, to get a nontrivial solution as $f_{\ell,\xi}\neq 0$, we should choose $+M$ with $M=m$. The above analysis implies that the Lindblad operator $\hat{L}_{\ell,\xi}$ with $\ell^\mu = [m,0,0,0]$ has

$$\hat{L}_{\ell,\xi} = \int d^3p\, f_{\ell,\xi}\hat{a}(\boldsymbol{p})\delta^3(\boldsymbol{p}) = f_{\ell,\xi}\hat{a}(\boldsymbol{0}). \tag{D5}$$

Equation (68) tells us that

$$\hat{L}_{q,\xi} = N_q^*\hat{V}(S_q)\hat{L}_{\ell,\xi}\hat{V}^\dagger(S_q) = N_q^* f_{\ell,\xi}\sqrt{\frac{E_{\boldsymbol{q}}}{m}}\hat{a}(\boldsymbol{q}) = f_{\ell,\xi}\hat{a}(\boldsymbol{q}), \tag{D6}$$

where $E_{\boldsymbol{q}}=(S_q\ell)^0$, $q^i=(S_q\ell)^i$, and $N_q=\sqrt{m/E_{\boldsymbol{q}}}$. The normalization N_q is given by setting the inner product $\boldsymbol{v}^\dagger_{q',\xi'}\boldsymbol{v}_{q,\xi}$ as

$$\boldsymbol{v}^\dagger_{q',\xi'}\boldsymbol{v}_{q,\xi} = \delta^3(\boldsymbol{q}'-\boldsymbol{q})\delta_{\xi'\xi}. \tag{D7}$$

Under this inner product, we can get the following completeness condition:

$$\int d^3q\sum_\xi \boldsymbol{v}_{q,\xi}\boldsymbol{v}^\dagger_{q,\xi} = I. \tag{D8}$$

We then derive a part of $\mathcal{D}[\rho]$ as

$$\mathcal{D}[\rho] \supset \sum_\xi |f_{\ell,\xi}|^2\int d^3p\Big[\hat{a}(\boldsymbol{p})\rho(t)\hat{a}^\dagger(\boldsymbol{p}) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}),\rho(t)\}\Big]. \tag{D9}$$

2. Lindblad operator for the case $\ell^\mu = [\pm\kappa, 0, 0, \kappa]$, $\kappa > 0$

Substituting $a^\mu=[0,\boldsymbol{a}]$ into Eq. (75), we have

$$f_{\ell,\xi}(\boldsymbol{p})e^{-i\boldsymbol{p}\cdot\boldsymbol{a}} = f_{\ell,\xi}(\boldsymbol{p})e^{-i\boldsymbol{\ell}\cdot\boldsymbol{a}} \ \therefore\ f_{\ell,\xi}(\boldsymbol{p}) = f_{\ell,\xi}\delta^3(\boldsymbol{p}-\boldsymbol{\ell}), \tag{D10}$$

where $\boldsymbol{\ell}=[0,0,\kappa]^{\rm T}$. Also, substituting $a^\mu=[a,0,0,0]$ into Eq. (75), we obtain the following result:

$$f_{\ell,\xi}(\boldsymbol{p})e^{iE_{\boldsymbol{p}}a} = f_{\ell,\xi}(\boldsymbol{p})e^{\pm i\kappa a}. \tag{D11}$$

Because of $f_{\ell,\xi}(\boldsymbol{p}) = f_{\ell,\xi}\delta^3(\boldsymbol{p}-\boldsymbol{\ell})$, we get

$$f_{\ell,\xi}e^{i\sqrt{\kappa^2+m^2}a} = f_{\ell,\xi}e^{\pm i\kappa a} \ \therefore\ f_{\ell,\xi}=0, \tag{D12}$$

where $E_{\boldsymbol{\ell}}=\sqrt{\boldsymbol{\ell}^2+m^2}=\sqrt{\kappa^2+m^2}$, and hence $f_{\ell,\xi}(\boldsymbol{p})=0$. Combined with the above analysis, the Lindblad operator $\hat{L}_{\ell,\xi}$ vanishes, and then

$$\hat{L}_{q,\xi} = N_q^*\hat{V}(S_q)\hat{L}_{\ell,\xi}\hat{V}^\dagger(S_q) = 0. \tag{D13}$$

3. Lindblad operator for the case $\ell^\mu = [0, 0, 0, N]$, $N^2 > 0$

Equation (75) for all $a^\mu=[a,0,0,0]$ leads to

$$f_{\ell,\xi}(\boldsymbol{p})e^{iE_{\boldsymbol{p}}a} = f_{\ell,\xi}(\boldsymbol{p}) \ \therefore\ f_{\ell,\xi}(\boldsymbol{p}) = 0, \tag{D14}$$

where note that $E_{\boldsymbol{p}}=\sqrt{\boldsymbol{p}^2+m^2}\neq 0$. For this case, the Lindblad operator $\hat{L}_{\ell,\xi}$ vanishes and we have

$$\hat{L}_{q,\xi} = N_q^*\hat{V}(S_q)\hat{L}_{\ell,\xi}\hat{V}^\dagger(S_q) = 0. \tag{D15}$$

4. Lindblad operator for the case $\ell^\mu = [0, 0, 0, 0]$

For this case, Eq. (75) for all $a^\mu=[a,0,0,0]$ is written as

$$f_\xi(\boldsymbol{p})e^{iE_{\boldsymbol{p}}a} = f_\xi(\boldsymbol{p}) \ \therefore\ f_\xi(\boldsymbol{p}) = 0, \tag{D16}$$

where we dropped the label ℓ and note that $E_{\boldsymbol{p}}=\sqrt{\boldsymbol{p}^2+m^2}\neq 0$. Hence, the Lindblad operator vanishes:

$$\hat{L}_\xi = 0. \tag{D17}$$

5. Self-adjoint operator $\hat{M}$

Now that we can get the form of $\mathcal{D}[\rho]$, let us move to the analysis of the self-adjoint operator $\hat{M}-\hat{H}$. Adopting $\Lambda = I$ and $a^\mu=[0,\boldsymbol{a}]$ in Eq. (77), we can obtain

$$g(\boldsymbol{p},\boldsymbol{p}')e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{a}} = g(\boldsymbol{p},\boldsymbol{p}') \ \therefore\ g(\boldsymbol{p},\boldsymbol{p}') = g(\boldsymbol{p})\delta^3(\boldsymbol{p}-\boldsymbol{p}'). \tag{D18}$$

This result can be checked by integrating both sides. Also, adopting $a^\mu = 0$ and substituting into Eq. (77), we have

$$g(\boldsymbol{p}_\Lambda) = g(\boldsymbol{p}), \tag{D19}$$

where we used $E_{\boldsymbol{p}}\delta^3(\boldsymbol{p}-\boldsymbol{p}') = E_{\boldsymbol{p}_\Lambda}\delta^3(\boldsymbol{p}_\Lambda - \boldsymbol{p}'_\Lambda)$. For $\boldsymbol{p} = \boldsymbol{0}$ and $\Lambda = S_q$ with $(S_q)^\mu{}_\nu k^\nu = q^\mu$ for $k^\mu = [m, 0, 0, 0]$ in Eq. (D19), we obtain

$$g(\boldsymbol{q}) = g(\boldsymbol{0}). \tag{D20}$$

By defining $g(\boldsymbol{0})$ as g, the self-adjoint operator $\hat{M}$ is given as

$$\hat{M} = \hat{H} + g\hat{N}, \tag{D21}$$

where $\hat{N}$ is the number operator Eq. (37).

6. Summary

Combining the above results, Eqs. (D9), (D13), (D15), (D17), and (D21), we get the following form of the Markovian QME:

$$\frac{d}{dt}\rho(t) = -i\big[\hat{H} + g\hat{N}, \rho(t)\big] + \gamma \int d^3p \Big[\hat{a}(\boldsymbol{p})\rho(t)\hat{a}^\dagger(\boldsymbol{p}) - \frac{1}{2}\{\hat{a}^\dagger(\boldsymbol{p})\hat{a}(\boldsymbol{p}), \rho(t)\}\Big], \tag{D22}$$

where we defined γ as $\gamma = \sum_\xi |f_{\ell,\xi}|^2$.

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