Erratum: "Hyperbits": The information quasiparticles [Phys. Rev. A 85, 022331 (2012)]

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The main result from our paper Theorem 1, is fatally flawed, as has been pointed out to the authors by Armin Tavakoli, Jef Pauwels, Erik Woodhead, and Stefano Pironio [1]. The claim is in the context of nonlocal games between two players with arbitrary inputs, limited communication from one player to the other, and a single-bit output by the receiver:

Theorem 1 (our paper). For tasks where Bob gives binary answers, sending one hyperbit from Alice to Bob is equivalent to sharing any amount of entanglement and sending (also from Alice to Bob) one classical bit.

Here, a *hyperbit* is the state of a system described by a generalized probabilistic theory whose state space is a hypersphere (of arbitrary dimension), which includes the possibility of a qubit (dimension 3) and a classical bit (dimension 1).

While it is true that every protocol for a task as considered in our paper involving sending a hyperbit, can be simulated by a setup with a suitable entangled state and sending one classical bit, the crucial converse direction is false. Indeed, Tavakoli *et al.* (Appendix C of [2]) provide a counterexample that leverages dimension witnesses, unequivocally demonstrating that there are tasks for which sending a hyperbit is strictly weaker than sending a classical bit assisted by entanglement. The numerical calculations demonstrating the counterexample are available in MATLAB code, appended as supplemental material to the arXiv version of [2].

The error can be traced to a specific step in the proof of Theorem 1 in Appendix A of our paper: the attempted simulation leads to scenarios where the numbers in Bob's stochastic matrix to post-process his input and the communication from Alice Appendix (A9) of our paper, may be negative, stripping them of operational probabilistic meaning.

In further examination, the present first author with Giovanni Scala and Seyed Arash Ghoreishi [3] have scrutinized the conditions under which the simulation proposed in Appendix A of our paper works, in particular when Eq. (A9) therein yields a bona fide hyperbit communication protocol. Their study yields further instances of the fact that the alleged equivalence between hyperbit theory and quantum mechanics is confined to a specific subset of the parameter space characterizing the quantum communication tasks discussed.

Unfortunately, this means that also the results in Secs. III and IV of our paper, which are crucially based on the claim of Theorem 1, have to be reevaluated and may not be correct.

[1] A. Tavakoli, J. Pauwels, E. Woodhead, and S. Pironio (private communication, 2021).

[2] A. Tavakoli, J. Pauwels, E. Woodhead, and S. Pironio, Correlations in entanglement-assisted prepare-and-measure scenarios, arXiv:2103.10748 [PRX Quantum **2**, 040357 (2021)].

[3] G. Scala, S. A. Ghoreishi, and M. Pawłowski, Advantages of quantum communication revealed by the reexamination of hyperbit theory limitations, Phys. Rev. A **109**, 022230 (2024).

