Dynamic equation for evaporative cooling of trapped atoms in microgravity

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In this paper, to investigate how atoms evaporate and cool in microgravity environments of the space station, we have developed the dynamic equation for evaporative cooling of trapped atoms and obtained the analytical expression of atomic temperature with respect to trapping laser parameters and gravitational acceleration. In our model, the evaporation of atoms is equivalent to applying a damping force to the trapped atoms, and the evaporative cooling process of trapped atoms is comprehended as the damped oscillation of trapped atoms in the optical dipole traps. By introducing the gravity in our model, we obtained the analytical model of temperature variation with gravity after cooling, and the theoretical results agree well with the evaporation experiment of rubidium-87 atoms on the ground. Our theoretical results show that, compared with the atomic evaporative cooling experiment on the ground, the microgravity environment of the space station can achieve cooler atomic gases when the losses of atoms, such as the one-body loss caused by background-gas collisions and the three-body recombination loss caused by interatomic inelastic collisions, can be ignored.

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I. INTRODUCTION

To obtain quantum degenerate gases in the space station, like cooling atoms on the ground, we still use forced evaporative cooling to cool atoms in optical dipole traps (ODTs). Forced evaporative cooling [1–4] played a critical role in realizing atomic Bose-Einstein condensates (BECs) [5–18] and degenerate Fermi gases [19–23]. The physical mechanism of forced evaporation is to reduce the trap depth, allowing the hotter atoms to escape the trap while the remaining cooler atoms will tend toward a new equilibrium at a lower temperature due to interatomic elastic collisions in evaporative cooling. Evaporative cooling, as the initially proposed method to obtain a quantum degenerate gas, has attracted scientists to continue exploring the interesting dynamics mechanism for decades. On the theoretical side, apart from direct numerical simulations [24–27], several theoretical analyses based on a classical gas model [1,28–42] or a Bose gas model [43–47] have solved the complex dynamic problems of evaporative cooling. These theoretical analyses [24–47] are very useful in understanding the evaporation process for BEC experiments.

However, these theoretical analyses [24–47] of evaporative cooling for trapped atoms did not consider the influence of gravity. Some BEC experiments [18,48–53] have shown that gravity will have a significant impact on the temperature of trapped atoms during the evaporative cooling process. Due to the influence of gravity, a gravitational potential will be superimposed on the original harmonic distribution potential of the trap, which creates a gap at the bottom of the trap potential. Atoms would only be trapped under a larger potential case. When the potential decreases, atoms will leak out of the gap, preventing evaporative cooling. Therefore, gravity must be considered in the theoretical model of evaporative cooling at the final stage of evaporative cooling [44].

To investigate how atoms evaporate and cool in microgravity environments of the space station, in this paper, we have derived the dynamic equation of motion of trapped atoms during the evaporation process considering the effect of gravity. The evaporation of atoms is equivalent to applying a damping force to the trapped atoms in evaporative cooling. The motion of trapped atoms can be comprehended as a damped oscillation during the evaporation process. Furthermore, we introduce the gravity effect and then theoretically analyze the influence of gravity on temperature of the trapped atoms during the evaporation process. Our theory has been verified by the evaporation experiment of rubidium-87 atoms on the ground. Our theoretical results show that, compared with the atomic evaporative cooling experiment on the ground, the microgravity environment of the space station can achieve cooler atomic gases when the losses of atoms, such as the one-body loss caused by background-gas collisions and the three-body recombination loss caused by interatomic inelastic collisions, can be ignored.

This paper is organized as follows. Section I is the introduction of the paper. In Sec. II, we introduce our experimental setup. In Sec. III, we derive the equation of motion of trapped atoms during the evaporation process from the perspective of energy and obtain an analytical expression of the temperature of trapped atoms. We present the results and discussion in Sec.IV. Section V concludes the paper with a summary.

II. EXPERIMENTAL SETUP

As displayed in Fig. 1, our experimental setup mainly consists of a science chamber, two-dimensional (2D) magneto-optical trap (MOT) chamber, and two ion pumps.

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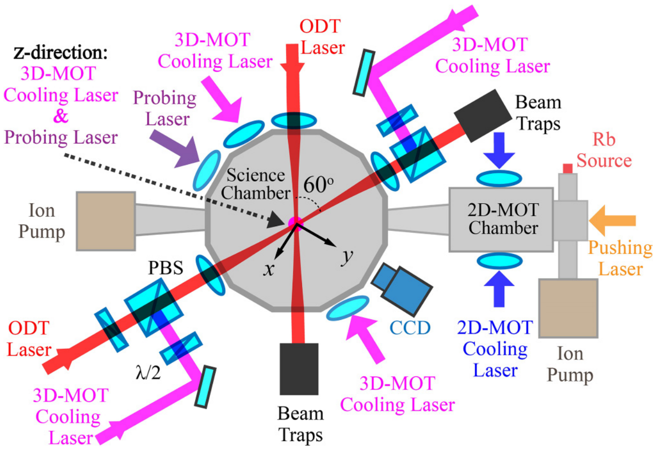

FIG. 1. A schematic illustration of the experimental setup for the rubidium-87 atom Bose-Einstein condensate (BEC) experiment. The optical dipole traps consist of two crossed laser beams with an angle of 60°. The left and right ion pumps prepare the vacuum pressure of the science chamber and two-dimensional magneto-optical trap (2D-MOT) chamber at 3×10^{-9} and 1×10^{-7}Pa, respectively.

The other part of the setup mainly includes a camera, various lasers [1060 nm ODT laser, 2D-MOT cooling laser, three-dimensional (3D) MOT cooling laser, pushing laser, repumping laser, and probing laser] and two beam traps. In this 2D-MOT chamber, a cold atomic beam is collected and pushed into the science chamber with a pushing laser. In the science chamber, we apply a 3D-MOT to collect and cool atoms from the atomic beam and then further cool atoms by molasses and transfer it to ODTs. We finally evaporatively cool the atoms to the degenerate in the ODTs.

III. THEORETICAL MODEL

Based on the total energy of trapped atoms, we derive the dynamic equation for evaporative cooling of trapped atoms. The total energy $\varepsilon_{\rm trap}$ of trapped atoms can be expressed as

$$\varepsilon_{\rm trap} = \sum (E_k + \mathrm{U} + E_g + E_I). \tag{1}$$

Here, E_k is the kinetic energy, U is the trap potential energy, E_g is the gravitational potential energy, and E_I is the interaction energy.

When not considering gravity, the trap potential U is symmetrical (see the red dashed line in Fig. 2). In the presence of gravity (here, gravity pointing along the z axis), the gravitational potential breaks the symmetry of the trap potential. The trap potential becomes U+E_g (here, $E_g = -mgz$ is the pure gravitational potential). The trap potential is tilted along the direction of gravity (see the green solid line in Fig. 2). This will lead to an effective lower trap depth $\mathrm{U}_{\rm eff} < \mathrm{U}_0$ in the direction of gravity. Therefore, in the presence of gravity, trapped atoms tend to escape along this direction.

If the average thermal energy k_BT of trapped atoms is much lower than the trap depth U_0, the trap potential U can be approximated harmonically [20,54]. Thus,

$$\mathrm{U} \approx \tfrac{1}{2}m\omega_x^2x^2 + \tfrac{1}{2}m\omega_y^2y^2 + \tfrac{1}{2}m\omega_z^2z^2. \tag{2}$$

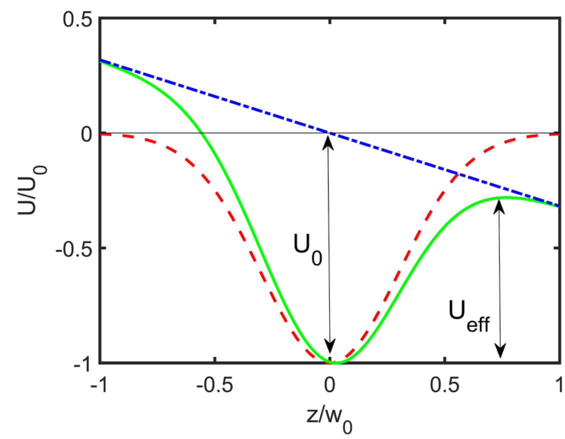

FIG. 2. Here, U_0 is the trap depth without gravity, w_0 is the beam waist of the trapping laser. The trap potential U without gravity (red, dashed), including gravity (green solid), and the pure gravitational potential (blue, dash-dotted line) at $x = y = 0$.

For the evaporation process, we mainly focus on the atomic gases far from degeneration and neglect the interaction energy E_I. Then according to Eq. (1), we have

$$\varepsilon_{\rm trap} = \sum_{i=1}^{N}\left(\frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2 + \frac{1}{2}mv_{iz}^2 + \frac{1}{2}m\omega_x^2x_i^2 + \frac{1}{2}m\omega_y^2y_i^2 + \frac{1}{2}m\omega_z^2z_i^2 - mgz_i\right). \tag{3}$$

Here, N is the total number of atoms in the trap; m is the atom mass; ω_x, ω_y, and ω_z are harmonic trap frequencies in three directions x, y, and z, respectively; and g is the gravitational acceleration. The evaporating gas can be accurately described by a truncated Boltzmann distribution during the evaporation process. The energy $\varepsilon_{\rm trap}$ of the trapped gas is given by [30]

$$\varepsilon_{\rm trap} = \left(\tfrac{3}{2} + \delta\right)R\left(\tfrac{3}{2} + \delta, \eta\right)Nk_BT. \tag{4}$$

Here, $\delta = \frac{3}{2}$ corresponds to the harmonic trap, η is the truncation parameter that represents the trap depth in units of k_BT, T is the temperature of trapped atoms, k_B is the Boltzmann's constant, and $R(a,\eta) = \frac{P(a+1,\eta)}{P(a,\eta)}$, $P(a,\eta) = \frac{1}{\Gamma(a)}\int_0^\eta t^{a-1}e^{-t}dt$ is the incomplete gamma function. According to Eq. (4), we obtain

$$\frac{d\varepsilon_{\rm trap}}{dt} = \left(\frac{3}{2} + \delta\right)R\left(\frac{3}{2} + \delta, \eta\right)k_B\left(T\frac{dN}{dt} + N\frac{dT}{dt}\right). \tag{5}$$

The characteristic quantities for the evaporation process can be described by logarithmic derivatives such as $\alpha = \frac{d(\ln T)}{d(\ln N)}$, that is, $\frac{dT}{dt} = \frac{\alpha T}{N}\frac{dN}{dt}$ [42]. Furthermore, according to $\frac{dN}{dt} = -NK_ef(\varepsilon > \eta k_BT)$ [42], Eq. (5) can be written as

$$\frac{d\varepsilon_{\rm trap}}{dt} = -N\left(\frac{3}{2} + \delta\right)R\left(\frac{3}{2} + \delta, \eta\right)(\alpha + 1) \times K_ef(\varepsilon > \eta k_BT)k_BT. \tag{6}$$

Here, K_e is the elastic collision rate, and f is the truncated Boltzmann distribution $f(\varepsilon > \eta k_BT) = 2e^{-\eta}\sqrt{\frac{\eta}{\pi}}$ [42]. For

Boltzmann statistics, the root mean squared speed can be written as

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3k_BT}{m}}. \tag{7}$$

Thus, we can obtain

$$\frac{d\varepsilon_{\text{trap}}}{dt} = -Nm\frac{\left(\frac{3}{2}+\delta\right)R\left(\frac{3}{2}+\delta,\eta\right)(\alpha+1)K_e f(\varepsilon > \eta k_BT)}{3}\overline{v^2}. \tag{8}$$

To simplify Eq. (8), we let

$$Y = \frac{\left(\frac{3}{2}+\delta\right)R\left(\frac{3}{2}+\delta,\eta\right)(\alpha+1)K_e f(\varepsilon > \eta k_BT)}{3}. \tag{9}$$

Thus, Eq. (8) becomes

$$\frac{d\varepsilon_{\text{trap}}}{dt} = -NmY\overline{v^2} = -\sum_{i=1}^{N}\left(mYv_i^2\right) = -\sum_{i=1}^{N}\left[mY\left(v_{ix}^2+v_{iy}^2+v_{iz}^2\right)\right]. \tag{10}$$

The first derivative of Eq. (3) with respect to t is written as

$$\frac{d\varepsilon_{\text{trap}}}{dt} = \frac{d}{dt}\sum_{i=1}^{N}\left(\frac{1}{2}mv_{ix}^2+\frac{1}{2}mv_{iy}^2+\frac{1}{2}mv_{iz}^2+\frac{1}{2}m\omega_x^2x_i^2+\frac{1}{2}m\omega_y^2y_i^2+\frac{1}{2}m\omega_z^2z_i^2-mgz_i\right)$$
$$= \sum_{i=1}^{N}\left(mv_{ix}\frac{dv_{ix}}{dt}+mv_{iy}\frac{dv_{iy}}{dt}+mv_{iz}\frac{dv_{iz}}{dt}+m\omega_x^2x_i\frac{dx_i}{dt}+m\omega_y^2y_i\frac{dy_i}{dt}+m\omega_z^2z_i\frac{dz_i}{dt}-mg\frac{dz_i}{dt}\right). \tag{11}$$

Considering the system consists of dilute gas, one-body loss caused by background-gas collisions and three-body recombination loss caused by interatomic inelastic collisions can be ignored. Here, we only consider the process of atomic evaporation caused by continuously decreasing the trap depth as well as the phenomenological treatment of the process of atoms tending toward a new equilibrium due to interatomic elastic collisions. Thus, it can be assumed that the degrees of freedom of motion in each dimension are independent of each other. Combining Eq. (10) with Eq. (11), then using $v_{ix} = dx_i/dt$, $v_{iy} = dy_i/dt$, and $v_{iz} = dz_i/dt$, we obtain

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{d^2x_i}{dt^2}+Y\frac{dx_i}{dt}+\omega_x^2x_i\right) = 0, \tag{12}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{d^2y_i}{dt^2}+Y\frac{dy_i}{dt}+\omega_y^2y_i\right) = 0, \tag{13}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{d^2z_i}{dt^2}+Y\frac{dz_i}{dt}+\omega_z^2z_i-g\right) = 0. \tag{14}$$

Except for gravity, there is no difference between the other two directions (x and y directions) and the z direction, so here, we only consider the z direction. For Eq. (14), we can swap the derivative with respect to t and the sum. Thus, Eq. (14) can be written as

$$\frac{d^2}{dt^2}\left(\frac{1}{N}\sum_{i=1}^{N}z_i\right)+Y\frac{d}{dt}\left(\frac{1}{N}\sum_{i=1}^{N}z_i\right)+\omega_z^2\left(\frac{1}{N}\sum_{i=1}^{N}z_i\right)-g = 0. \tag{15}$$

Let $z = \frac{1}{N}\sum_{i=1}^{N}z_i$, where z denotes the average distance in the z direction from the atom to the center of the trap. We have

$$\frac{d^2z}{dt^2}+Y\frac{dz}{dt}+\omega_z^2z-g = 0. \tag{16}$$

Here, Y is the damping factor. This equation describes the quasidamped oscillation motion of trapped atoms during the evaporative cooling process. The evaporation of atoms is equivalent to applying a damping force to the trapped atoms, thereby causing the trapped atoms to undergo damping oscillation. In this sense, the evaporation process of trapped atoms can be comprehended as the process of atoms undergoing damping oscillations in the ODTs.

In our evaporation experiment of rubidium-87 atomic gases, we obtain $\alpha = 2.23$, $\eta \approx 3(\alpha+1) = 9.69$, $\omega_z = 749 - 3300\text{s}^{-1}$, and $Y = 0.36 - 1.98\text{s}^{-1}$. Due to $Y \ll \omega_z$, the motion of trapped atoms in Eq. (16) is an underdamping oscillation. The solution of Eq. (16) can be approximately written as

$$z = Ae^{-Yt/2}\cos(\omega_zt+\varphi_0)-\frac{g}{\omega_z^2}, \tag{17}$$

Here, $A = \sqrt{\frac{2k_BT_0}{m\omega_0^2}+\frac{2g^2}{\omega_0^4}}$ depends on the initial temperature T_0 and trap frequency ω_0 in the z direction, and φ_0 is the initial phase in the z direction. Then using Eq. (17), we easily obtain the temperature of trapped atoms in the z direction:

$$T = \frac{m}{k_B}\left[\frac{A^2}{2}\omega_z^2e^{-Yt}-\frac{g^2}{\omega_z^2}\right]. \tag{18}$$

Applying $\omega_z = bP^{1/2}$ to Eq. (18), we obtain the temperature with trap laser power and gravity:

$$T = \frac{m}{k_B}\left[\frac{b^2A^2}{2}Pe^{-Yt}-\frac{1}{b^2}\frac{g^2}{P}\right]. \tag{19}$$

Here, P is the trap laser power, $b = \sqrt{\frac{16\alpha_{\text{eff}}}{\pi m \text{w}_0^4}}$, $\alpha_{\text{eff}} = \frac{3\pi c^2}{2\Omega_{\text{atom}}^3}\left(\frac{\Gamma_{\text{eff}}}{\Omega_{\text{atom}}-\Omega_{\text{laser}}}+\frac{\Gamma_{\text{eff}}}{\Omega_{\text{atom}}+\Omega_{\text{laser}}}\right)$, $\Omega_{\text{laser}} = 2\pi c/\lambda_{\text{laser}}$, and $\lambda_{\text{laser}} = 1064\,\text{nm}$ is the wavelength of trap laser. Also, $\Gamma_{\text{eff}} = \Gamma_1/3+2\Gamma_2/3$, $\Omega_{\text{atom}} = \Omega_1/3+2\Omega_2/3$, $\Omega_1 = 2\pi c/\lambda_1$, $\Omega_2 = 2\pi c/\lambda_2$, and c is the speed of light. Here, $\lambda_1 = 795\,\text{nm}$ and $\Gamma_1 = 2\pi \times 5.746\,\text{MHz}$ are the wavelength and natural linewidth of the $^{87}\text{Rb}\ \text{D}_1$ line ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$), respectively. Also, $\lambda_2 = 780\,\text{nm}$ and $\Gamma_2 = 2\pi \times 6.065\,\text{MHz}$ are the

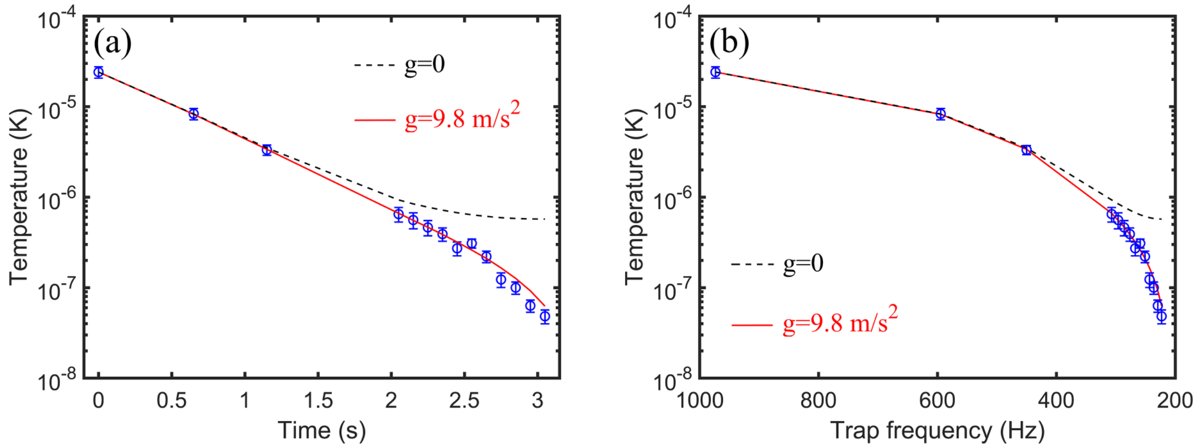

FIG. 3. (a) Temperature vs evaporation time t and (b) trap frequency during the evaporation process. Here, blue circles represent the experimental results on the ground ($\mathrm{g} = 9.8\mathrm{m/s^2}$), the black dashed line and red solid line represent the theoretical result of $\mathrm{g} = 0$ and $9.8\mathrm{m/s^2}$, respectively.

wavelength and natural line width of the $^{87}\mathrm{Rb\,D_2}$ line ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$), respectively.

IV. RESULTS AND DISCUSSION

To cool the atoms more efficiently, we optimized the power curve of the trap laser in the rubidium-87 atom evaporative cooling experiment on the ground. Here, $P(t) = P_0(1 + t/\tau)^{-\beta}$ is the optimized form of power, where $P_0 = 6.56\mathrm{W}$, $\tau = 1.3187\mathrm{s}$, and $\beta = 2.4598$. Based on this power curve, we observed the temperature variation of atomic ensembles with time and frequency in experiments, as shown by the blue circles in Fig. 3. As displayed in Fig. 3(a), to validate our model equations, applying the form $P(t)$ to Eq. (19), we theoretically calculated the time evolution of temperature of the trapped atoms during the evaporation process in Fig. 3(a). Then using the form $P(t)$, $\omega_z = bP(t)^{1/2}$, and Eq. (18), we theoretically analyzed the temperature of the trapped atoms vs trap frequency during the evaporation process, as shown in Fig. 3(b). It is shown that our theoretical results [red solid line in Figs. 3(a) and 3(b)] are in good agreement with the experimental results of rubidium-87 atoms on the ground [blue circles in Figs. 3(a) and 3(b)]. From Fig. 3, the evaporative cooling rate is faster with gravity (red solid line) than without gravity (black, dashed). Thus, gravity plays an important role during the evaporation process.

To further investigate the role of gravity during the evaporation process, Fig. 4 shows our theoretical analysis of atomic ensemble temperature in different gravitational acceleration conditions using Eqs. (19) and (18). As displayed in Fig. 4, at the early of the evaporative cooling $t < 2\mathrm{s}$ [Fig. 4(a)], where the trap frequency $f_z > 314\mathrm{Hz}$ [Fig. 4(b)], for the same evaporation time or trap frequency change, we can see that the temperature changes of trapped atoms are almost identical during the evaporative cooling process in different gravitational acceleration conditions. This is the reason that, due to higher trap frequency or trap laser power, the influence of gravity on temperature of the trapped atoms can be ignored at the early stage of evaporative cooling. In this case, the second term in the bracket on the right side of Eq. (18) or (19) has almost no contribution to temperature of the trapped atoms. At the final stage of evaporative cooling, as trap frequency or trap laser power decreases, the contribution of the second term in the bracket on the right side of Eq. (18) or (19) cannot be ignored. The influence of gravity on temperature of the trapped atoms becomes increasingly significant. At the final stage of the evaporative cooling $t > 2\mathrm{s}$ [Fig. 4(a)], where the trap frequency $f_z < 314\mathrm{Hz}$ [Fig. 4(b)], the greater the gravitational acceleration, the faster the trapped atoms will be cooled during the evaporation process. However, in the presence of gravity, according to Eq. (17), as the evaporation continues, the trap frequency also decreases, and the gravity term in Eq. (17) becomes larger, ultimately exceeding the first term in Eq. (17). This means that the atoms will escape the trap along the direction of gravity, at which point the atomic cooling is stopped and the mechanism of evaporative cooling fails. As shown in Fig. 4, our theoretical results show that, compared with the atomic evaporative cooling experiment on the ground, the microgravity environment of the space station can achieve cooler atomic gases when the losses of atoms, such as the one-body loss caused by background-gas collisions and the three-body recombination loss caused by interatomic inelastic collisions, can be ignored. However, as gravity decreases, obtaining atomic gases with lower temperatures comes at the cost of a longer evaporation time. Due to the presence of the loss of atoms, a longer evaporation time is a fatal disadvantage for atomic cooling. For actual evaporative cooling experiments, the trade-off between efficiency and cooling speed should be considered. The curves in Fig. 4 for $g > 0$ end precisely where the trapping depth becomes zero. At $g = 0$, the curves never end, allowing for indefinite cooling. It clearly shows that finite g helps to cool faster but provides a minimal temperature, whereas $g = 0$ has a slower rate of cooling, but $T = 0$ can be approached indefinitely given enough time.

Additionally, our model only describes the atomic evaporation process before reaching quantum degeneracy. At this stage, due to the low atomic density, only small background-gas collisional loss (one-body loss) and three-body recombination rates will occur. The three-body loss

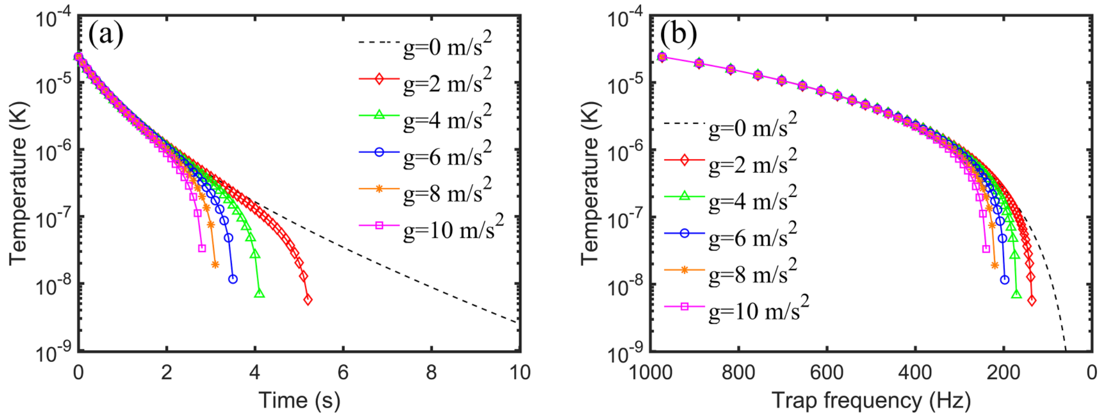

FIG. 4. The temperature T as a function of (a) evaporation time t and (b) trap frequency during the evaporative cooling process with different gravitational acceleration conditions.

is proportional to the cube of the atomic density and only increases sharply when the atoms tend to quantum degeneracy. Therefore, our model ignores one-body loss caused by background-gas collisions and three-body recombination loss caused by interatomic inelastic collisions. Our model only considers the process of atomic evaporation caused by continuously decreasing the trap depth as well as the phenomenological treatment of the process of atoms tending toward a new equilibrium due to interatomic elastic collisions.

V. CONCLUSIONS

We propose a dynamic equation and theoretically study the evaporative cooling process based on the dynamic equation of a classical gas, focusing on evaporation experiments with rubidium-87 atoms. Our calculated results agree well with the evaporative cooling experiment on the ground. Furthermore, we quantitatively analyze the influence of gravity on temperature of the trapped atoms during the evaporation process. Finally, we discuss the time evolution of atomic temperature in different gravitational acceleration conditions during the evaporation process. Although gravity can accelerate the evaporation of trapped atoms, it also hinders the continued evaporation of atoms, resulting in the failure of the mechanism of evaporative cooling at the final stage of evaporative cooling. Our theoretical results show that, compared with the atomic evaporative cooling experiment on the ground, the microgravity environment of the space station can achieve cooler atomic gases when the losses of atoms, such as the one-body loss caused by background-gas collisions and the three-body recombination loss caused by interatomic inelastic collisions, can be ignored.

In this paper, first, our model equation is only based on truncated Boltzmann distribution of classical gases, and future research should further extend it to the truncated Bose-Einstein distribution. Secondly, our theory neglects the interaction energy which will cause a contribution to temperature of the trapped atoms, especially when atoms tend toward quantum degeneracy or around the critical point of BEC transition. Finally, our model does not include any loss mechanisms, such as one- or three-body losses. In future research, our model will include a phenomenological description of the loss of atoms.

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