Spontaneously generated structured light in a coherently driven five-level M-type atomic system

Muqaddar Abbas [1], Urgunoon Saleem [2], Rahmatullah [2,*], Yong-Chang Zhang [1,†], and Pei Zhang [1,‡]

[1] *Ministry of Education Key Laboratory for Nonequilibrium Synthesis and Modulation of Condensed Matter, Shaanxi Province Key Laboratory of Quantum Information and Quantum Optoelectronic Devices, School of Physics, Xi'an Jiaotong University, Xi'an 710049, China*

[2] *Quantum Optics Laboratory, Department of Physics, COMSATS University Islamabad, Islamabad, Pakistan*

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We propose a scheme to analyze the generation and detection of optical vortices via spontaneous emission spectra. We consider a five-level M-type atomic system that interacts with two structured coherent light sources and the vacuum radiation field. The photon emitted via spontaneous decay carries information about the orbital angular momentum (OAM) of the newly generated structured light. It is found that by varying the OAM of the driving fields, one can obtain various unique structured light fields via the interference of the two spontaneous decay channels. This research provides deep insights into how the manipulation of OAM of the driving fields coupled with multilevel atomic systems can enable control over the azimuthal aspects of the spontaneous emission. Considering the rich applications of structured light ranging from optical storage to optical communication, our proposal might be useful in quantum information processing techniques.

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I. INTRODUCTION

As a typical type of electromagnetic fields, light can carry both linear as well as angular momentum. While light's linear momentum is always directed in its propagation direction, its angular momentum is composed of an orbital part and a spin part. The orbital angular momentum is associated with the helical phase fronts (phase twist) irrespective of its radial distribution, namely, orbital angular momentum (OAM), while the spin part is associated with the polarization. One typical example is Laguerre-Gaussian (LG) mode light [1], which can host finite OAM. In the last two decades, LG-mode light has drawn plenty of attention because of its peculiar characteristics. By playing with the interference of multiple LG beams, it is also feasible to generate various structured light fields with rich profiles in the transverse direction. These shapes are determined not only by the OAM of the interfering beams, but also by the angle between them as well as the displacement between the beam centers. At the center of a single LG beam, the phase is indeterminate, while the amplitude is distributed radially around the axis of propagation and thus exhibits a doughnut-shaped structure. Thereby, the LG beam is often referred to as a structured light or vortex beam [2].

Owing to these special phase and intensity properties of light with OAM, it has potential applications in a wide range of areas. For example, the OAM degree of freedom permits one to transmit high-bandwidth data over long distances in optical communication [3,4]. Moreover, light beams carrying OAM have also been effectively employed in optical tweezers to control the rotation of particles. This has led to the development of microrotors, which have wide applications in different fields, such as microfluidics, micromechanics, and biological research [5,6]. Additionally, the transfer of OAM between light and matter has presented a feasible route to explore new physics in optomechanical systems [7] as well as cold atoms [8,9].

Because of the crucial role of structured light in light-matter interacting physics, plenty of effort has been paid to generate structured optical fields. For instance, the generation of structured light has been theoretically proposed in closed three- or four-level atomic gases using the electromagnetically induced transparency (EIT) [10–12], and then experimentally observed by Radwell *et al.* [13] in a cold rubidium atomic medium. Another theoretical scheme based on a combined Λ and tripod configuration of the light-atom coupling was reported as well [14]. To the best of our knowledge, the control fields in most of those proposals reported so far have no OAM, while the probe field is considered as a structured light [2,8,9,15,16]. The reason for such a setting is that the EIT condition is destroyed if the control light beam is a structured field whose central intensity is vanishing. Nevertheless, an additional control field without OAM can be applied to maintain transparency [17–20]. Recently, in contrast to the usual scheme by transferring OAM from the control beam to the probe beam, researchers have also studied the effectiveness of the OAM transformation among the weak probe fields [21,22].

The aforementioned schemes are all based on the coherent coupling between the atomic ensembles and the control beams as well as the probe beams. In addition to the coherent generation of structured light, the incoherent properties of atoms can be correspondingly modulated via the light fields as well. For example, the spontaneous emission, due to the atomic interaction with environmental modes, is possible to be manipulated via tuning the intensity as well as the frequency of external control fields [23–28]. Consequently, it may lead to a variety of unexpected phenomena, such as lasing without inversion

*rahmatktk@comsats.edu.pk
†zhangyc@xjtu.edu.cn
‡zhangpei@mail.ustc.edu.cn

[29,30], EIT [31], spontaneous correlated emission [32,33], and refractive index enhancement without absorption [34].

In addition, the spontaneous emission spectra can also be shaped utilizing phase control. For example, it has been proposed to modulate the spontaneous emission spectra with the assistance of a phase-dependent effect through designing the coupling schemes between atoms and driving fields [24,35,36]. In another study, a four-level atom driven by two lasers with the same frequency has been considered to manipulate the spontaneous emission [37], where the relative phase of the pump fields can lead to complete extinction of spontaneous emission.

Considering the rich possibility of shaping the spontaneous emission spectrum via control fields, here we present an alternative and simple method for the generation of structured light via spontaneous emission in sharp contrast to the probe absorption [10–12], four-wave mixing techniques [19,20,22], as well as other coherent process used in previous proposals.

In the following, we will discuss the details of our proposal and demonstrate that it is experimentally feasible and might be useful for quantum information technology. Our scheme is pivoted around a five-level M-type atomic system [38] as shown in Fig. 1, where the interference between the two decay channels from the excited states to the metastable state leads to the spontaneous generation of structured light. This emitted photon holds information about the new structured light's azimuthal quantum (winding) number ℓ.

The rest of this paper is organized as follows. We discuss the theoretical details of our scheme in Sec. II, and then analyze and demonstrate our results and discussions under different conditions for the generated light in Secs. III and IV. Finally, we give a conclusion in Sec. V.

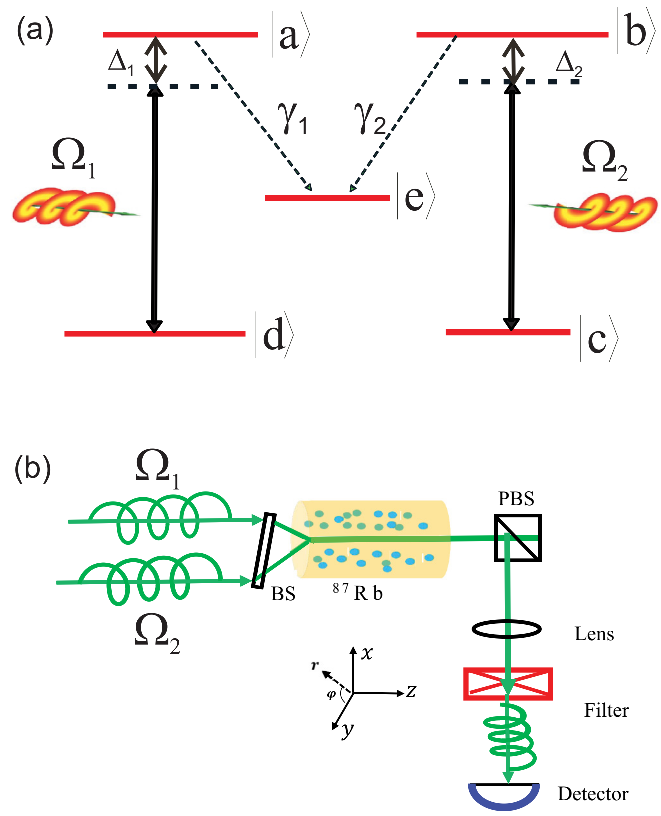

FIG. 1. (a) Schematic diagram of a five-level M-type atomic system interacting with two structured light fields. Here, Ω_1 and Ω_2 are the Rabi frequencies of the two driving fields, respectively. The model consists of two upper levels ($|a\rangle$ and $|b\rangle$) and three lower levels ($|c\rangle$, $|d\rangle$, and $|e\rangle$). The two structured light fields drive the transitions $|d\rangle \to |a\rangle$ and $|c\rangle \to |b\rangle$. Δ_1 and Δ_2 are the corresponding detunings, respectively. γ_1 and γ_2 are the decay rates from the two upper levels $|a\rangle$ and $|b\rangle$ to $|e\rangle$. (b) Schematic plot of the proposed setup. Control structure beams are incident upon the medium. The vapor cell is loaded with ^{87}Rb and the atoms can be cooled down by using the magneto-optical trap (MOT) technology. Here, a digital hologram is used to generate structure control beams, with lens, Rb cell, filter, beam splitter (BS), and polarizing beam splitter (PBS) used in the proposed setup.

II. THEORY AND DISCUSSION

A. Theoretical model

We consider a five-level M-type atomic configuration [38] consisting of three lower levels $|d\rangle$, $|e\rangle$, and $|c\rangle$ and two upper levels $|a\rangle$ and $|b\rangle$, as shown in Fig. 1(a). The upper levels $|a\rangle$ and $|b\rangle$ can spontaneously decay to the lower level $|e\rangle$ by emitting vacuum modes (ν_k) in free space with the respective decay rates γ_1 (γ_2), while two LG-mode structured light fields, E_1 and E_2, are applied to coherently drive the transitions $|d\rangle \leftrightarrow |a\rangle$ and $|c\rangle \leftrightarrow |b\rangle$, respectively. The corresponding Rabi frequencies are Ω_1 (Ω_2) and Δ_1 (Δ_2), denoting the associated detunings, respectively. Because of the nonhomogeneous profile of the driving beams, the spontaneous emitted light field would appear as a structured distribution as well. To observe such spontaneously generated structured light fields, a schematic plot of the experimental setting is presented in Fig. 1(b). By employing dipole and rotating-wave approximation (RWA), the Hamiltonian of such an M-type light-atom coupling system reads

$$\mathcal{V}(t) = -\hbar\Big[\Omega_1 e^{-i\Delta_1 t}|a\rangle\langle d| + \Omega_2 e^{-i\Delta_2 t}|b\rangle\langle c| + \sum_k g_{k_a}^{a,e} e^{-i\delta_a t}\widehat{b}_{k_a}|a\rangle\langle e| + \sum_k g_{k_b}^{b,e} e^{-i\delta_b t}\widehat{b}_{k_b}|b\rangle\langle e| + \text{H.c.}\Big], \quad (1)$$

where H.c. is the abbreviation for the Hermitian conjugate. $\Delta_1 = \nu_1 - \omega_{ad}$ and $\Delta_2 = \nu_2 - \omega_{bc}$ are the frequency detunings between the control fields and the atom resonance for transitions $|d\rangle \to |a\rangle$ and $|c\rangle \to |b\rangle$, respectively. $g_{k_a}^{a,e}$ and $g_{k_b}^{b,e}$ represent the coupling strengths between the vacuum mode k and atomic transitions $|a\rangle \to |e\rangle$ and $|b\rangle \to |e\rangle$ with the corresponding resonant transition frequency ω_{ae} and ω_{be}. $\widehat{b}_{k_{a,b}}$ ($\widehat{b}^{\dagger}_{k_{a,b}}$) is the annihilation (creation) operator of the reservoir modes with the frequency $\nu_k = ck$. $\delta_a = \nu_k - \omega_{ae}$ and $\delta_b = \nu_k - \omega_{be}$ denote the frequency detunings between the vacuum mode and atom resonance. For simplicity, we shall substitute $\hbar = 1$ hereafter. The dynamics of this system can be described by the probability amplitude equations through deploying the state vector as

$$|\Psi(t)\rangle = C_{a,0_k}(t)|a, 0_k\rangle + C_{b,0_k}(t)|b, 0_k\rangle + C_{c,0_k}(t)|c, 0_k\rangle + C_{d,0_k}(t)|d, 0_k\rangle + \sum_k C_{e,1_k}(t)|e, 1_k\rangle, \quad (2)$$

where the probability amplitude $C_{j,0_k}(t)(j=a-d)$ represents the temporary weight of each case without the photon emitted into the kth vacuum mode, i.e., labeled by $|0_k\rangle$, while $C_{e,1_k}(t)$ represents the probability of the atom on the metastable state $|e\rangle$ with one photon emitted in the kth vacuum mode. Here, we would like to point out that the measurement of the output field is conditioned on the detection of the photons (with frequency ν_k) spontaneously emitted through the two decay channels (i.e., $|a\rangle \rightarrow |e\rangle$ and $|b\rangle \rightarrow |e\rangle$). The detection of the photon that carries the information of the winding number ℓ at the time t in the vacuum mode k indicates that the atom is at the metastable state $|e\rangle$. To understand the properties of the spontaneously generated light field, we resort to the time-dependent Schrodinger equation $i\frac{\partial|\Psi(t)\rangle}{\partial t} = \mathcal{V}(t)|\Psi(t)\rangle$ with the interaction Hamiltonian given by Eq. (1) and the atomic state vector given by Eq. (2), which results in the following coupled equations for the atomic probability amplitudes:

$$i\frac{\partial C_{a,0_k}(t)}{\partial t} = \Omega_1 e^{-i\Delta_1 t}C_{d,0_k}(t) + \sum_k g_{k_a}^{a,e}C_{e,1_k}(t)e^{-i\delta_a t}, \quad (3)$$

$$i\frac{\partial C_{b,0_k}(t)}{\partial t} = \Omega_2 e^{-i\Delta_2 t}C_{c,0_k}(t) + \sum_k g_{k_b}^{b,e}C_{e,1_k}(t)e^{-i\delta_b t}, \quad (4)$$

$$i\frac{\partial C_{c,0_k}(t)}{\partial t} = \Omega_2^* e^{-i\Delta_2 t}C_{b,0_k}(t), \quad (5)$$

$$i\frac{\partial C_{d,0_k}(t)}{\partial t} = \Omega_1^* e^{-i\Delta_1 t}C_{a,0_k}(t), \quad (6)$$

$$i\frac{\partial C_{e,1_k}(t)}{\partial t} = \sum_k \left[g_{k_a}^{a,e}C_{a,0_k}(t)e^{i\delta_a t} + g_{k_b}^{b,e}C_{b,0_k}(t)e^{i\delta_b t}\right]. \quad (7)$$

To ascertain the spontaneous emission spectrum of the atom, we integrate Eq. (7) with respect to the variable $\bar{t}$, and then substitute it into Eqs. (3) and (4). After some standard deviations, eventually, one can obtain the following integro-differential equations:

$$\frac{\partial C_{a,0_k}(t)}{\partial t} = -i\Omega_1 e^{-i\Delta_1 t}C_{d,0_k}(t) - \int_0^t \mathcal{G}_{aa}(t-\bar{t})C_{a,0_k}(\bar{t})d\bar{t} - \int_0^t \mathcal{G}_{ab}(t-\bar{t})C_{a,0_k}(\bar{t})e^{-i\omega_{ba}t}d\bar{t} \quad (8)$$

and

$$\frac{\partial C_{b,0_k}(t)}{\partial t} = -i\Omega_1 e^{-i\Delta_2 t}C_{c,0_k}(t) - \int_0^t \mathcal{G}_{bb}(t-\bar{t})C_{b,0_k}(\bar{t})d\bar{t} - \int_0^t \mathcal{G}_{ba}(t-\bar{t})C_{a,0_k}(\bar{t})e^{-i\omega_{ba}t}d\bar{t}, \quad (9)$$

where $\mathcal{G}_{mn}(t-\bar{t})$ $(m,n=a,b)$ is defined as

$$\mathcal{G}_{mn}(t-\bar{t}) = \sum_k g_{k_m}^{m,e}g_{k_n}^{n,e}e^{-i\delta_{m,n}(t-\bar{t})}, \quad (10)$$

with $g_{k_m}^{m,e}g_{k_n}^{n,e}$ being the respective coupling strengths between the kth mode and the atoms. Here, we have assumed a vacuum initial state of the environment and thus there could be only one photon emitted from a single atom. Since the atom decays from the upper two levels (i.e., $|a\rangle$ and $|b\rangle$) spontaneously, the individual coupling between the upper two levels with the lower level $|e\rangle$ is by the reservoir of free vacuum modes k. We assume that the excited state amplitude $C_{a,0_k}(t)$ and $C_{b,0_k}(t)$ varies with a rate $\gamma_{1,2} < \nu_k$. Therefore, $C_{a,0_k}(t)$ and $C_{b,0_k}(t)$ changes little in the time interval over which the remaining part of the integrand has nonzero value ($\bar{t} \sim t$) and we can replace $C_{a,0_k}(\bar{t})$ $[C_{b,0_k}(\bar{t})]$ in the integrand with $C_{a,0_k}(t)$ $[C_{b,0_k}(t)]$ and take it out of integrand. This is called the Weisskopf-Wigner approximation, which can be recognized as a Markovian approximation: the dynamics of $C_{a,0_k}(\bar{t})$ $[C_{b,0_k}(\bar{t})]$ depends only on time t instead of $\bar{t} < t$, i.e., the system has no memory of the past. Conveniently, this Markovian process allows us to use the Weisskopf-Wigner theory [23,37,39], which yields

$$\mathcal{G}_{m,n} = \frac{\wp_{m,n}\sqrt{\gamma_1\gamma_2}}{2}\delta(t-\bar{t}). \quad (11)$$

More details about the derivation of the above equation can be found in the Appendix. Here, $\wp_{m,n} = \delta_{mn} + \wp(1-\delta_{mn})$, with δ_{mn} being the Kronecker delta, and $\wp = \frac{\vec{\mu}_{ae}\cdot\vec{\mu}_{be}}{|\vec{\mu}_{ae}||\vec{\mu}_{be}|}$ $(0 \leqslant \wp \leqslant 1)$ representing the quantum interference between the atomic transitions coupled to the free vacuum modes. $\vec{\mu}_{ae}$ $(\vec{\mu}_{be})$ denotes the dipole moment corresponding to the transition $|a\rangle \rightarrow |e\rangle$ $(|b\rangle \rightarrow |e\rangle)$. By this definition, it is clear to notice that $\wp\frac{\sqrt{\gamma_1\gamma_2}}{2}$, where $\gamma_j = \frac{1}{4\pi\epsilon_0}\frac{4\omega_k^3|\mu_{(m,n)e}|^2}{3\hbar c^3}$ $(j=1,2)$ is the decay rate from the upper levels $|a\rangle$ and $|b\rangle$ to $|e\rangle$, describes the correlation between the transitions $|a\rangle \rightarrow |e\rangle$ and $|b\rangle \rightarrow |e\rangle$. This correlation, in principle, is quantum interference between the two spontaneous decay channels. One can note that there is maximum quantum interference if the dipole moments are parallel (i.e., $\wp = 1$) and the interference between the decay pathways $|a\rangle \longrightarrow |e\rangle$ and $|b\rangle \longrightarrow |e\rangle$ vanishes if the dipole moments are orthogonal.

As a result of the coherent driving, the atom can oscillate between its ground state $|c\rangle$ $(|d\rangle)$ and the excited states $|a\rangle$ $(|b\rangle)$ in short-time evolution; however, it would eventually decay to the metastable state $|e\rangle$ due to the spontaneous emission in the long-time limit, i.e., $C_{a,0_k}(t\rightarrow\infty) = C_{b,0_k}(t\rightarrow\infty) = C_{c,0_k}(t\rightarrow\infty) = C_{d,0_k}(t\rightarrow\infty) = 0$. Hence, to obtain the spontaneous emission spectrum in the long-time limit [37],

$$\mathcal{S}(\delta_k) = |C_{e,1_k}(t\rightarrow\infty)|^2. \quad (12)$$

By applying the Laplace transform to Eqs. (7), we eventually obtain $C_{e,1_k}$ in the steady state as

$$C_{e,1_k}(t\rightarrow\infty) = -ig_{k_a}^{a,e}\tilde{C}_{a,0_k}(s=-i\delta_a) - ig_{k_b}^{b,e}\tilde{C}_{b,0_k}(s=-i\delta_b), \quad (13)$$

where $\tilde{C}_{a,0_k}(s)$ and $\tilde{C}_{b,0_k}(s)$ are the Laplace transforms of $C_{a,0_k}(t)$ and $C_{b,0_k}(t)$, respectively. Next, by carrying out the Laplace transforms of Eqs. (5), (6), (8), and (9), one can get

$$\tilde{C}_{a,0_k}(s=-i\delta_a) = \frac{BC - \wp\frac{\sqrt{\gamma_1\gamma_2}}{2}D}{AB - \wp^2\frac{\gamma_1\gamma_2}{2}}, \quad (14)$$

$$\tilde{C}_{b,0_k}(s=-i\delta_b) = \frac{AD - \wp\frac{\sqrt{\gamma_1\gamma_2}}{2}C}{AB - \wp^2\frac{\gamma_1\gamma_2}{2}}, \quad (15)$$

where

$$A = \frac{\gamma_1}{2} - i\delta_a + \frac{i\Omega_1^2}{\delta_a - \Delta_1}, \quad (16)$$

$$B = \frac{\gamma_2}{2} - i\delta_b + \frac{i\Omega_2^2}{\delta_b - \Delta_2}, \tag{17}$$

$$C = C_{a,0_k}(0) + \frac{\Omega_1}{\delta_a - \Delta_1} C_{d,0_k}(0), \tag{18}$$

and

$$D = C_{b,0_k}(0) + \frac{\Omega_2}{\delta_b - \Delta_2} C_{c,0_k}(0). \tag{19}$$

Finally, the probability for finding the atom in the state $|e\rangle$ with a spontaneously emitted photon ν_k in the vacuum mode k is given by

$$\begin{aligned}\mathcal{S}(\delta_k) &= \left|C_{e,1_k}(t\to\infty)\right|^2 \\ &= \left|g_{k_a}^{a,e}\tilde{C}_{a,0_k}(s=-i\delta_{k,a}) + g_{k_b}^{b,e}\tilde{C}_{b,0_k}(s=-i\delta_b)\right|^2 \\ &= \{\gamma_1|\tilde{C}_{a,0_k}(s=-i\delta_a)|^2 + \gamma_2|\tilde{C}_{b,0_k}(s=-i\delta_b)|^2 \\ &\quad + \wp\sqrt{\gamma_1\gamma_2}[\tilde{C}_{a,0_k}(s=-i\delta_a)\tilde{C}^*_{b,0_k}(s=-i\delta_b) \\ &\quad + \tilde{C}_{b,0_k}(s=-i\delta_b)\tilde{C}^*_{a,0_k}(s=-i\delta_a)]\},\end{aligned} \tag{20}$$

where $\delta_k = \nu_k - \frac{\omega_{ae}+\omega_{be}}{2} = \delta_a - 0.5\omega_{ab} = \delta_b + 0.5\omega_{ab}$, with $\omega_{ab} = \omega_{ae} - \omega_{be}$ being the frequency difference between the two upper levels. Equation (20) presents the dependence of the probability upon the detunings, the population of the atomic levels, and the spontaneously emitted photon frequency. This manifests that the spontaneous output field can inherit the characteristics of the control fields. Moreover, the interference effect between the two decay channels may lead to emergent phenomena of the emitted light, the details of which will be discussed in the following.

B. Control fields

From the above analysis, one can note that the emission spectrum is highly dependent on the control beam's properties. We consider that the control beams are a LG-mode light field, which appears as a doughnut-shaped intensity distribution, and explore the effect of the control field's spatial profile on the spontaneously emitted light field. In such a case, the Rabi frequencies of the control fields can be written as

$$\Omega_{(1,2)} = \epsilon_{(1,2)}\left(\frac{r_{(1,2)}}{w}\right)^{|\ell_{(1,2)}|} e^{-\frac{r^2_{(1,2)}}{w^2}} e^{i\ell_{(1,2)}\Phi_{(1,2)}}, \tag{21}$$

where $\Phi = \tan^{-1}(y/x)$, $r = \sqrt{x^2+y^2}$, w is the beam waist, and $\epsilon_{(1,2)}$ is the amplitude of the beam. Equation (20) clearly shows the dependence on the OAM number of the control beam's spatial profile. In the following, we are going to investigate a feasible way to manipulate the spontaneous emission spectrum via tuning the OAM number, and demonstrate that the angular momentum of the control fields has a significant impact on the spatial structure of the radiated field. This permits a promising measure to engineer the structured light via spontaneous emission.

III. RESULTS AND DISCUSSION

In this section, we investigate the behavior of the probability distribution given by Eq. (20) subject to different initial conditions. Based on this, we will present various unique structured light beam patterns by detecting the photon emitted in the kth vacuum mode. In the following, our discussion mainly focuses on two cases. We first analyze the scenario where only one of the control fields is a LG-mode beam. And then we proceed to consider the case where both of the two control fields are LG-mode beams.

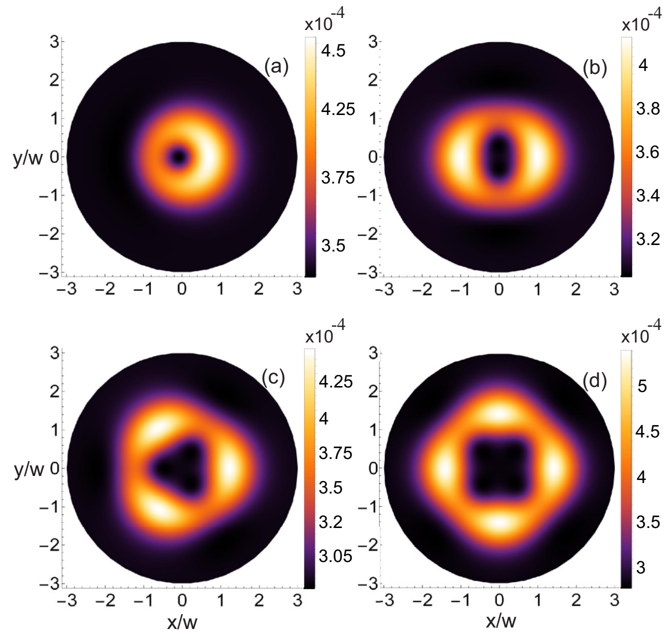

FIG. 2. Spontaneously emitted structure light vs the normalized positions x/w and y/w by considering one of the control beam is a LG-mode beam with OAM number (a) $\ell_1 = 1$, (b) $\ell_1 = 2$, (c) $\ell_1 = 3$, and (d) $\ell_1 = 4$. Initially, the atom is prepared in the superposition of the ground states, i.e., $C_{c,0_k} = 1/\sqrt{2}$ and $C_{d,0_k} = 1/\sqrt{2}$, and we assume a maximum quantum interference ($\wp = 1$). The other parameters are $\Omega_2 = 0.5\gamma$, $\gamma_1 = 0.1\gamma$, $\gamma_2 = 0.1\gamma$, $\omega_{ab} = 1\gamma$ $\epsilon_1 = 1\gamma$, $\Delta_1 = \Delta_2 = 0$, and $\gamma = 1$ MHz.

A. One LG-mode control beam

Let us start from the configuration where one of the control fields is taken as a LG beam and the other one is a plane-wave beam. In such a case, the spatially dependent spontaneous emission spectrum is shown in Fig. 2. Here, we examine the spontaneous emission spectrum in the transverse direction plotted against normalized positions x/w and y/w, where w is the beam waist. The atoms are initially prepared in the superposition states of the two lower levels, i.e., $C_{c,0_k} = C_{d,0_k} = 1/\sqrt{2}$. In Figs. 2(a)–2(d), the yellowish bright area within each petal represents the maximum spontaneous emission, indicating the tight localization of the atom at that point. This means that the atom is emitting light with a high probability in those regions. On the other hand, the black regions signify the quenching of spontaneous emission, indicating that little light is being emitted in those areas. Within the orange and purple regions, there is a moderate level of spontaneous emission compared to the yellowish bright region, which implies that the atom is emitting light at a lower intensity in those regions compared to the yellowish bright region. Moreover, from Fig. 2, one can also observe that the high-probability emission ring presents an ℓ-fold symmetry and its width increases as the winding number ℓ goes up.

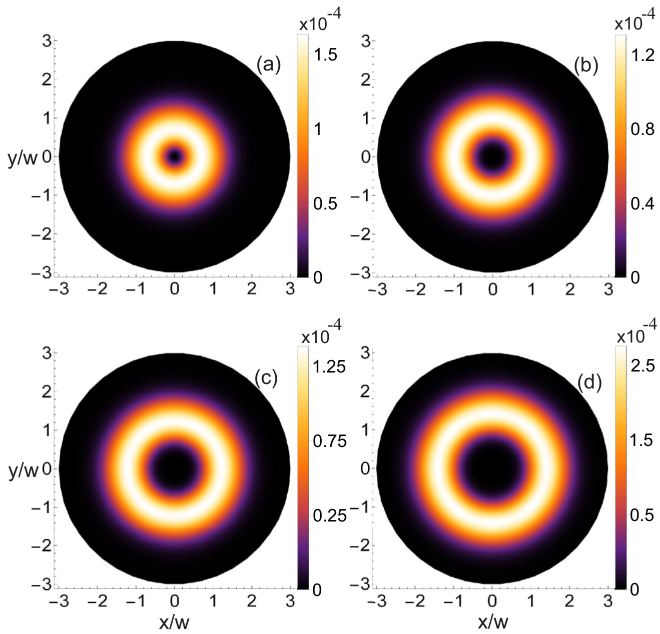

FIG. 3. Spontaneously emitted structure light vs the normalized positions x/w and y/w by considering that both control fields are LG-mode beams with the following OAM numbers: (a) $\ell_1 = \ell_2 = 1$, (b) $\ell_1 = \ell_2 = 2$, (c) $\ell_1 = \ell_2 = 3$, and (d) $\ell_1 = \ell_2 = 4$. The atom is initialized in the superposition of ground states, i.e., $C_{c,0_k} = 1/\sqrt{2}$ and $C_{d,0_k} = 1/\sqrt{2}$. The other parameters are $\wp = 1$, $\epsilon_1 = 0.7\gamma$, $\epsilon_2 = 0.4\gamma$, $\gamma_1 = 1\gamma$, $\gamma_2 = 1\gamma$, $\Delta_1 = \Delta_2 = 0$, $\delta_k = 5\gamma$, $\omega_{ab} = 1\gamma$, and $\gamma = 1$ MHz.

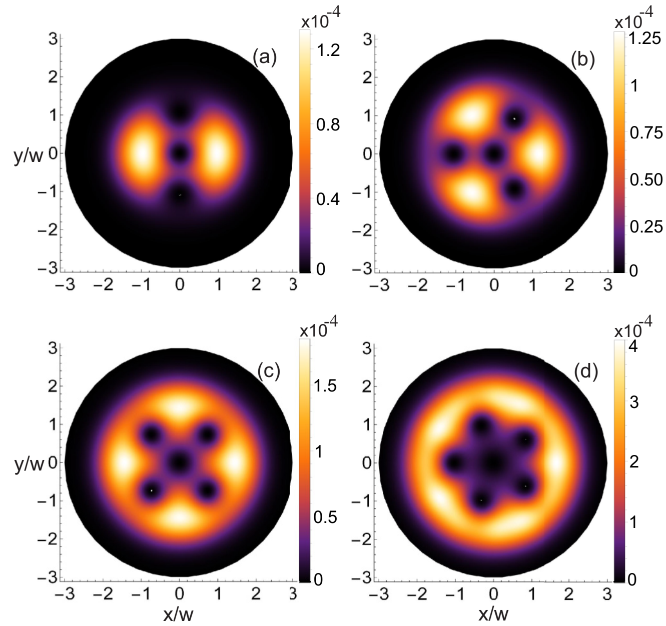

FIG. 4. Spontaneously generated structure light vs the normalized positions x/w and y/w by considering both control fields are structure beams with $\ell_1 < \ell_2$, i.e., $\ell_1 = 1$ and (a) $\ell_2 = 3$, (b) $\ell_2 = 4$, (c) $\ell_2 = 5$, and (d) $\ell_2 = 6$. The other parameters are same as that in Fig. 3.

B. Two LG-mode control beams

In this section, we turn to the scenario in which both of the control fields are LG-mode beams and show that emergent patterns of the spontaneous emission spectrum can be observed through tuning the value of $\ell_{1,2}$ as well as the ratio between them. Let us first consider the case of symmetric-helicity drivings (i.e., $\ell_1 = \ell_2$), the spatial distribution of the spontaneous emission spectrum of which is shown in Fig. 3. Figures 3(a)–3(d) exhibit several examples of the spatial dependence of the emitted light versus normalized position x/w and y/w for various OAM values, while the driving fields carry equal OAM, e.g., (a) $\ell_1 = \ell_2 = 1$, (b) $\ell_1 = \ell_2 = 2$, (c) $\ell_1 = \ell_2 = 3$, and (d) $\ell_1 = \ell_2 = 4$. As before, we assume the atom is initialized in the superposition state of the two lower levels, i.e., $C_{c,0_k} = C_{d,0_k} = 1/\sqrt{2}$. The pumping amplitudes are fixed at $\epsilon_1 = \epsilon_2 = \Gamma$, while the other parameters remain consistent with those used in Fig. 2. The intensity profiles present a regular doughnut shape with vanishing intensity at the center. Similarly to the case of a single LG driving beam, the size of the doughnut-shaped emission ring increases with the OAM number, which is inconsistent with the typical feature of a standard LG beam. Based on the preceding analysis, we can conclude that by shining LG-mode optical fields on such an M-type atomic medium, the vorticity of the control fields Ω_1 and Ω_2 can be transferred to the spontaneously emitted photons, and thus it is feasible to generate structured light via spontaneous emission. That is, the tunability of the OAM of the driving fields offers a potential avenue for tailoring the spatial characteristics of spontaneous emission and thus permits one to customize the emission properties.

In the following, we proceed to other cases beyond the symmetric helicity (i.e., $\ell_1 \neq \ell_2$) and show that the beams created in this manner can appear in a variety of situations. For this purpose, let us turn to the scenario where the control structure light has different OAM as well as varied strength. Considering the symmetry of such an M-type configuration, without loss of generality, we restrict our discussion to the case of $\ell_1 < \ell_2$ for simplicity. As shown in Fig. 4, the spontaneous emission spectrum changes dramatically in comparison with the symmetric-helicity case discussed above. Figure 4(a) provides an example of the emission spectrum when $\ell_1 = 1$ and $\ell_2 = 3$. One can find that the regular ℓ-fold symmetry is disrupted, revealing a more intricate pattern with additional singularities. This disruption follows a $|\ell_1 - \ell_2|$-fold rotational symmetry. The resulting emission profile resembles a composite structured beam with a core charge of $+1$ surrounded by two vortices of charge $+1$ [refer to Fig. 4(a)]. This structural pattern appears because of the different intensity radii of the two driving fields, and is in accordance with recent experimental observations [40]. That is, the output mode carries both the characteristics of the $\ell_1 = 1$ and $\ell_1 = 3$ OAM. When the driving beams are merged coherently, a nontrivial superposition of vortices with a charge ℓ_1 core vortex surrounded by $|\ell_1 - \ell_2|$ single-charged peripheral vortices is predicted. Similar behaviors are also observed in other cases, as exhibited in Figs. 4(b)–4(d) where the $\ell_1 = 1$ is fixed while ℓ_2 is tuned from 4 to 6. The color bar shows the intensity ranges of the light beam. The white-yellow parts indicate the areas of maximum spontaneous emission intensity, while the purple-black areas correspond to the areas of vanishing intensity.

In a nutshell, by examining the spectrum of spontaneous emission, we have shown that the OAM of the driving fields may be transferred to the emitted Â photons. Moreover, remarkable spatial profiles of the emitted light can be obtained by tuning the OAM of the control fields, which offers a promising route to generate structured light. It is imperative to note that the phenomena discussed above are based on the assumption of a maximal dipole alignment, i.e., $\wp = 1$. Nonetheless, for smaller $\wp$ ranging from 0 to 1, similar results can be obtained as well. This method offers a different and straightforward solution that has prospective applications in quantum information technology. By capitalizing on the inherent coherence and precise control offered by atomic systems, we can achieve structured light patterns with high spatial and temporal coherence, enabling applications in quantum technologies that demand such properties. Owing to the simplicity of such a setting, our scheme is experimentally feasible within current techniques, which will be discussed in detail in the next section.

C. Experimental implementation

In the previous sections, we have proposed to generate structured light through spontaneous emission by employing LG optical fields coupled to a five-level atom. Now we would like to demonstrate that the experimental implementation of our scheme is feasible without demanding requirements on experiment conditions. Specifically, the LG-mode control beams can be generated via standard techniques, e.g., digitally developed holograms [41], which has been extensively used in light-matter interaction research [13,35]. Given that the structure of the generated light in our proposal significantly depends on the relative phase of the two pumping fields, an optical phase-locked loop approach may be employed to effectively fix the phases [42]. The appropriate separation of the two upper levels ensures the versatility of our scheme, making it applicable to a broad range of atomic systems. For instance, one can use the following hyperfine states of ^{87}Rb: $5P_{3/2}|F=2, m=+1\rangle$, $5P_{3/2}|F=1, m=-1\rangle$, $5S_{1/2}|F=1, m=0\rangle$, $5S_{1/2}|F=2, m=+2\rangle$, and $5S_{1/2}|F=2, m=-2\rangle$ functioning as $|a\rangle$, $|b\rangle$, $|c\rangle$, $|d\rangle$ and $|e\rangle$, respectively. Furthermore, the atoms can be loaded into a magneto-optical trap (MOT) and cooled down to suppress the Doppler broadening effect [43], which enhances the precision and control of the proposed scheme and thus paves the way for potential practical implementations.

In addition, by considering the correlation of such an atomic gas and their collective decay to a metastable state (i.e., superradiance) [44], it is possible to deterministically generate the structured light via superradiance instead of probabilistically. This superradiance results from the collective decay of the excited atoms, which can be realized in the case of strong coupling between the atoms and the pumping fields. In contrast to stimulated emission, this collective decay process allows for not only speeding up the emission, but also generating a coherent structured light. That is, by carefully controlling the driving fields as well as their coupling with the atoms, the desired outcome of a structured light field can be achieved via the collective emission.

IV. CONCLUSION

In conclusion, we have performed a thorough investigation of a spontaneously generated light field in an atomic system driven by LG-mode beams and proposed to generate structured light via spontaneous emission. Through considering the setting of an M-type atom pumped by LG beams, we demonstrated that there exists OAM transfer from pump fields to the spontaneously emitted light.

We first consider a scenario where one of the control beams is a LG-mode light, while the other one is a plane-wave field. By analyzing the consequent emission spectrum, we find that the emission intensity varies with a regular spatial profile, the maximum points of which indicate the tight localization of the atom. This implies that the atom emits light at the highest rate in those regions. In this case, the emission profile presents an l_1-fold rotational symmetry, with l_1 being the OAM number of the LG-mode driving fields.

We further consider both control fields as a structured field, including the cases of symmetric as well as unsymmetrical helicity, and investigate the spatial distributions of the spontaneous emitted spectrum. By assuming that the atoms are initially prepared in the superposition states of the two lower levels, we show that the interplay between the structured driving fields leads to a rich variety of spontaneous emission spectrum structures, e.g., the $|l_1 - l_2|$-fold rotational symmetry.

Moreover, due to the driving fields' interference and their space-dependent intensity distribution, the emitted light can appear in rich spatial profiles (e.g., disturbed rotational symmetry, emergent vortices) beyond the standard LG mode, which is highly tunable via controlling the phase or OAM of the driving fields. In addition, by adjusting the strength, frequency, and OAM of the control fields, it can lead to new physical phenomena in the spontaneous emission. This provides a promising approach to shape the distribution of the optical fields, which might be useful in the research related to structured light, for example, generating an off-axis optical beam pattern that is useful to store quantum information [3,45] as well as trap or rotate particles in the cases of superradiant structured light [44,46]. At the same time, this development in the creation and detection of structured light by spontaneous emission also offers a possible way to manipulate the atomic spontaneous spectrum through vortex pumping, which can be used to promote magnetometers or other precision measurement techniques [47,48].

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APPENDIX: WEISSKOPF-WIGNER THEORY

The Weisskopf-Wigner theory is a theoretical framework within quantum mechanics. It provides a description of the decay process of an unstable atom when it interacts with environmental modes, specifically, the vacuum modes. This theory allows for the calculation of decay rates and probabilities associated with the emission of particles or radiation by the unstable atom. By considering the interaction between the atom and its surrounding environment, the Weisskopf-Wigner theory provides valuable insights into the dynamics of decaying systems in quantum mechanics.

To ascertain the spontaneous decay of the atom, we undertake a formal integration of Eq. (7), performed with respect to the variable $\bar{t}$. The results obtained from this integration are subsequently substituted into Eqs. (3). The eventual derivation of deterministic conclusions necessitates the application of certain approximations. The resulting integro-differential Eqs. (8) and (9) define a new term $\mathcal{G}_{mn}(t-\bar{t})(m,n=a,b)$,

$$\mathcal{G}_{mn}(t-\bar{t}) = \sum_k g_{k_m}^{m,e} g_{k_n}^{n,e} e^{-i\delta_{m,n}(t-\bar{t})}, \tag{A1}$$

where $g_{k_m}^{m,e}$ ($g_{k_n}^{n,e}$) are the respective coupling terms between the kth mode and levels $|a\rangle$ ($|b\rangle$).

Assuming that the modes of the field are closely spaced in frequency, we can replace the summation over k by an integral [23],

$$\sum_k \rightarrow 2\frac{V}{(2\pi)^3}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dk k^2, \tag{A2}$$

where V is the quantization volume. Also, we define

$$g_{k_m}^{m,e} = g_{k_n}^{n,e} = \frac{-\wp_{m,n}.\hat{\epsilon}_k \mathcal{E}_k}{\hbar}, \tag{A3}$$

where $\mathcal{E}_k = (\hbar\nu_k/2\epsilon_0 V)^{1/2}$. After incorporating $\mathcal{E}_k$, we have

$$g_{k_m}^{m,e} g_{k_n}^{n,e} = \frac{\nu_k}{2\epsilon_0 V\hbar}\wp_{m,n}^2 \cos^2\theta, \tag{A4}$$

where θ is the angle between the atomic dipole moment $\wp$ and the electric-field polarization vector $\hat{\epsilon}_k$. Therefore, after incorporating all the terms, $\mathcal{G}_{mn}(t-\bar{t})$ can be written as

$$\mathcal{G}_{mn}(t-\bar{t}) = \frac{4\wp_{m,n}^2}{(2\pi)^2 6\epsilon_0\hbar c^3}\int_0^{\infty} d\nu_k \nu_k^3 \int_0^t e^{-i(\nu_k-\omega_{(m,n)e})(t-\bar{t})} d\bar{t}, \tag{A5}$$

where integrations over θ and ϕ have been carried out and we have used $k=\nu_k/c$. In the emission spectrum, the intensity of light associated with the emitted radiation is going to be centered about the atomic transition frequency ω. The quantity ν_k^3 varies little around $\nu_k=\omega_{(m,n)e}$ for which the time integral in Eq. (A5) is not negligible. We can therefore replace ν_k^3 by ω^3 and the lower limit in the ν_k integration by $-\infty$. The integral

$$\int_{-\infty}^{\infty} d\nu_k e^{i(\omega_{(m,n)e}-\nu_k)(t-\bar{t})} = 2\pi\delta(t-\bar{t}). \tag{A6}$$

In the proposed scheme, the measurement of the finally generated light is conditioned on the detection of the photon (with frequency ν_k), spontaneously emitted through the two decay channels (i.e., $|a\rangle \rightarrow |e\rangle$ and $|b\rangle \rightarrow |e\rangle$). The detection of the photon that carries the information on the OAM number $\ell_1(\ell_2)$ at any time t and vacuum mode k indicates that the atom, at this time, is in its internal state $|e\rangle$. Here, we have assumed a vacuum initial state of the environment and thus there could be only one photon emitted from a single atom. Since the atom decays from the upper two levels (namely, $|a\rangle$ and $|b\rangle$) spontaneously, the individual coupling between the upper two levels with lower level $|e\rangle$ is by the reservoir of free vacuum modes k, which means that the process is Markovian. Conveniently, this Markovian process allows us to use the Weisskopf-Wigner theory [25,37,39], which yields

$$\mathcal{G}_{m,n} = \frac{\wp_{m,n}\sqrt{\gamma_1\gamma_2}}{2}\delta(t-\bar{t}). \tag{A7}$$

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