Relations between group, energy, and phase velocities of surface electromagnetic waves in half-infinite bianisotropic homogeneous media

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This paper proves the equality of the group velocity and the energy velocity for surface electromagnetic waves on the interface between two bianisotropic homogeneous half-infinite nonabsorbing dielectrics. It is also shown that the projection of the energy velocity on the direction of propagation is equal to the phase velocity provided that the material constants do not depend on the frequency. Moreover, this equality holds true locally, i.e., the energy velocity along the direction of propagation coincides with the phase velocity at any distance from the interface. The obtained relations holds true in magneto-optically active media as well. It is assumed that the media are of generic crystallographic symmetry, so, when deriving the desired equalities, no simplifications due to a specific crystallographic symmetry are used.

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I. INTRODUCTION

It is known that surface electromagnetic waves (SEWs) exist on the interface between two homogeneous half-infinite dielectric media with positive dielectric permittivity and magnetic permeability provided that at least one of them is not optically isotropic. Examples of such waves were found by explicit analytic calculations and numerically [1–17]. In Refs. [18,19] by using the general properties of the surface impedance of dielectrics it was shown that at most one SEW emerges on the isotropic dielectric–anisotropic dielectric and superconductor–anisotropic dielectric.

Apart from the phase velocity, important characteristics of electromagnetic waves are the group and energy velocities. It was proved that the group velocity of bulk waves in anisotropic homogeneous infinite nonabsorbing media coincides with the energy velocity [20–23]. It was also shown by explicit calculations that these velocities are equal for plasmons on the metal–isotropic dielectric contact [24–26] and in graphene between two isotropic dielectrics [27,28]. In Ref. [29] the equality was proved by variational method for SEWs in nonbianisotropic stratified media bounded by a perfectly conducting plane. (Note that SEWs cannot exist in homogeneous substrates under such boundary conditions, which follows from the general properties of the surface impedance [19,30–33].) In addition, the equality of the two velocities was proved for Bloch waves in allowed zones of infinite anisotropic superlattices [34,35].

By the above-mentioned optical anisotropy, we refer to the case where the dielectric permittivity and/or magnetic permeability are not scalars but the electric displacement $\mathbf{D}$ does not depend on the magnetic field $\mathbf{H}$ and the magnetic induction $\mathbf{B}$ does not depend on the electric field $\mathbf{E}$. At the same time there exist media called bianisotropic where both $\mathbf{D}$ and $\mathbf{B}$ depend on both $\mathbf{E}$ and $\mathbf{H}$ [36–41]. Such a cross-dependence is attributed to the magnetoelectric effect and natural optical activity [42,43]. Bianisotropy can markedly affect the propagation of electromagnetic waves, including surface waves [10,30,31,44,45]. For instance, two SEWs may emerge on the interface between bianisotropic dielectrics whereas at most one SEW may exist in the absence of bianisotropy, and the velocities of SEWs propagating in mutually opposite directions do not need to be equal in bianisotropic media [30,31]. The velocities of bulk electromagnetic waves in bianisotropic media also can differ for mutually opposite directions [46–50]. On the other hand, the equality of the group and energy velocities of bulk waves in homogeneous bianisotropic media is preserved [51–53]. In Ref. [54] it was proved that this equality is valid for bulk Bloch waves in allowed zones of bianisotropic and/or magneto-optically active infinite superlattices. However, it is apparent that the already established equalities of the group and energy velocities do not imply that this equality is always valid, so it is necessary to investigate each particular situation since the general conditions under which these velocities coincide are unknown.

In this paper we prove the equality of the group and energy velocities of SEWs which propagate on the interface between two homogeneous half-infinite bianisotropic and/or magneto-optically active media. At this stage the frequency dispersion of material parameters is taken into account. Further, assuming that the frequency dispersion may be disregarded, we show that the projection of the group (energy) velocity on the direction of propagation is equal to the phase velocity. Note that in Ref. [55] an expression was given for the projection of the energy velocity of a single evanescent mode on the direction of its propagation in a bianisotropic medium but this mode is not a true SEW because it does not satisfy the boundary conditions. A true SEW is sought for as a linear combination of evanescent modes and usually it involves two modes in each medium unless all the media are isotropic or their orientation is tied to elements of the crystallographic symmetry [1–15].

We consider that the media are of generic crystallographic symmetry and therefore we do not use any relations between material parameters which could be brought in by a particular crystallographic symmetry. In other words, it is assumed that the media have no elements of the crystallographic symmetry.

Our paper is organized as follows. In Sec. II a number of general relations are given. Section III proves the equality of the group and energy velocities. Section IV proves the equality of the phase velocity and the projection of the energy velocity on the direction of propagation and Sec. V summarizes the results. The Appendix contains some additional information.

II. GENERAL RELATIONS

The x and y components $E_{x,y}$ and $H_{x,y}$ of the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ field of a plane wave,

$$\begin{pmatrix}\mathbf{E}(\mathbf{r},t)\\ \mathbf{H}(\mathbf{r},t)\end{pmatrix}=\begin{pmatrix}\mathbf{E}(z)\\ \mathbf{H}(z)\end{pmatrix}e^{i(k_xx+k_yy-\omega t)}, \qquad (1)$$

may be found by solving a system of four ordinary differential equations,

$$\frac{1}{i}\frac{d\boldsymbol{\xi}}{dz}=\hat{\mathbb{N}}\boldsymbol{\xi}, \qquad (2)$$

where the vector column $\boldsymbol{\xi}$ involves E_x, E_y, H_x, and H_y which may be ordered arbitrarily [18,44,56–58]. Following Refs. [31,33,54] we put

$$\boldsymbol{\xi}(z)=\begin{pmatrix}\mathbf{u}\\ \mathbf{v}\end{pmatrix},\quad \mathbf{u}=\begin{pmatrix}-E_y\\ H_y\end{pmatrix},\quad \mathbf{v}=\begin{pmatrix}H_x\\ E_x\end{pmatrix}. \qquad (3)$$

Bianisotropic nonabsorbing media are described by constitutive connections [37–43] which we write in the form

$$\begin{pmatrix}\mathbf{D}\\ \mathbf{B}\end{pmatrix}=\hat{\boldsymbol{\Gamma}}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix},\quad \hat{\boldsymbol{\Gamma}}=\begin{pmatrix}\hat{\boldsymbol{\varepsilon}} & \hat{\boldsymbol{\kappa}}\\ \hat{\boldsymbol{\kappa}}^{\dagger} & \hat{\boldsymbol{\mu}}\end{pmatrix}, \qquad (4)$$

where $\mathbf{D}$ is the electric displacement, $\mathbf{B}$ is the magnetic induction, the tensors of dielectric permittivity $\hat{\boldsymbol{\varepsilon}}$ and magnetic permeability $\hat{\boldsymbol{\mu}}$ are assumed to be complex Hermitian in order to allow for the magneto-optical activity, and $\hat{\boldsymbol{\kappa}}$ is a complex nonsymmetric pseudotensor describing the bianisotropic coupling. This coupling is due to the magnetoelectric effect and natural optical activity. The symbol $\dagger$ denotes the Hermitian conjugation.

With vector $\boldsymbol{\xi}$ defined by (3) the matrix $\hat{\mathbb{N}}$ may be expressed in terms of the blocks $\hat{\boldsymbol{\Omega}}_J$, $J=1,2,4$, of the Hermitian matrix

$$\hat{\boldsymbol{\Omega}}=\hat{\boldsymbol{\Omega}}^{\dagger}=\hat{\boldsymbol{\Delta}}\hat{\boldsymbol{\Gamma}}\hat{\boldsymbol{\Delta}}^{-1}=\begin{pmatrix}\hat{\boldsymbol{\Omega}}_1 & \hat{\boldsymbol{\Omega}}_2\\ \hat{\boldsymbol{\Omega}}_2^{\dagger} & \hat{\boldsymbol{\Omega}}_4\end{pmatrix}, \qquad (5)$$

where $\hat{\boldsymbol{\Delta}}$ is a matrix which permutes the components of $\mathbf{E}$ and $\mathbf{H}$ as well as the components of $\mathbf{D}$ and $\mathbf{B}$ in such a way that

$$\hat{\boldsymbol{\Delta}}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}=\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\phi}\end{pmatrix},\quad \hat{\boldsymbol{\Delta}}\begin{pmatrix}\mathbf{D}\\ \mathbf{B}\end{pmatrix}=\begin{pmatrix}\boldsymbol{\psi}\\ \boldsymbol{\nu}\end{pmatrix}, \qquad (6)$$

where $\boldsymbol{\phi}=(H_z\ E_z)^t$, $\boldsymbol{\psi}=(-D_y\ B_y\ B_x\ D_x)^t$, and $\boldsymbol{\nu}=(B_z\ D_z)^t$, and the superscript t denotes the transposition [31,54]. In (5) $\hat{\boldsymbol{\Omega}}_1$ and $\hat{\boldsymbol{\Omega}}_4$ are the upper 4×4 and lower 2×2 diagonal blocks of $\hat{\boldsymbol{\Omega}}$, respectively, and $\hat{\boldsymbol{\Omega}}_2$ is a 4×2 matrix with elements $(\hat{\boldsymbol{\Omega}}_2)_{ij}=(\hat{\boldsymbol{\Omega}})_{i,j+4}$, $i=1,\dots,4$, $j=1,2$.

Below we will use basically the matrix $\hat{\mathbb{N}}=\hat{\mathbf{T}}\hat{\mathbf{N}}$ rather than $\hat{\mathbf{N}}$, where

$$\hat{\mathbf{T}}=\begin{pmatrix}\hat{\mathbf{0}} & \hat{\mathbf{I}}\\ \hat{\mathbf{I}} & \hat{\mathbf{0}}\end{pmatrix}, \qquad (7)$$

and $\hat{\mathbf{0}}$ and $\hat{\mathbf{I}}$ are zero and identity 2×2 matrices. In our paper [54] an expression for the matrix $\hat{\mathbb{N}}$ has been derived by combining Maxwell's equations represented in the form (A1) and (A2) (see the Appendix), and relations (4)–(6), viz.,

$$\hat{\mathbb{N}}=\omega\hat{\mathbf{A}}-\hat{\mathbf{B}}-\omega^{-1}\hat{\mathbf{C}}, \qquad (8)$$

$$\hat{\mathbf{A}}=\hat{\boldsymbol{\Omega}}_1-\hat{\boldsymbol{\Omega}}_2\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\boldsymbol{\Omega}}_2^{\dagger}, \qquad (9)$$

$$\hat{\mathbf{B}}=\hat{\boldsymbol{\Omega}}_2\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\mathbf{J}}^t+\hat{\mathbf{J}}\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\boldsymbol{\Omega}}_2^{\dagger},\quad \hat{\mathbf{C}}=\hat{\mathbf{J}}\hat{\boldsymbol{\Omega}}_4^{-1}\hat{\mathbf{J}}^t, \qquad (10)$$

$$\hat{\mathbf{J}}=k_x\hat{\mathbf{J}}_x+k_y\hat{\mathbf{J}}_y, \qquad (11)$$

where $\hat{\mathbf{J}}_x$ and $\hat{\mathbf{J}}_y$ are 4×2 matrices,

$$\hat{\mathbf{J}}_x=\begin{pmatrix}\hat{\mathbf{I}}\\ \hat{\mathbf{0}}\end{pmatrix},\quad \hat{\mathbf{J}}_y=\begin{pmatrix}\hat{\mathbf{0}}\\ \hat{\mathbf{K}}\end{pmatrix},\quad \hat{\mathbf{K}}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}. \qquad (12)$$

The Hermitian matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ depend on the frequency only through material constants. In the general case the explicit expressions of the elements of $\hat{\mathbb{N}}$ in terms of material constants are very involved but they simplify significantly for certain geometries of propagation, especially in nonbianisotropic and magneto-optically inactive materials (see the Appendix).

In what follows we will need an expression of the time-averaged energy flux $\mathbf{P}(z)$ in terms of contractions of the vector $\boldsymbol{\xi}$ (3) with derivatives of $\hat{\mathbb{N}}$ with respect to $k_{x,y}$. Inserting in $\mathbf{P}=\mathrm{Re}[\mathbf{E}\times\mathbf{H}^*]/2$,

$$\begin{pmatrix}-E_a\\ H_a\end{pmatrix}=\hat{\mathbf{J}}_a\boldsymbol{\xi},\quad a=x,y, \qquad (13)$$

as well as

$$\begin{pmatrix}H_z\\ E_z\end{pmatrix}\equiv\boldsymbol{\phi}=-\hat{\boldsymbol{\Omega}}_4^{-1}(\hat{\boldsymbol{\Omega}}_2^{\dagger}+\omega^{-1}\hat{\mathbf{J}}^t)\boldsymbol{\xi} \qquad (14)$$

obtained by using (4)–(6) and Maxwell's equations (A2) for $\boldsymbol{\nu}=(B_z\ D_z)^t$, we find [54]

$$P_a(z)=-\frac{1}{4}\boldsymbol{\xi}^{\dagger}\frac{\partial\hat{\mathbb{N}}}{\partial k_a}\boldsymbol{\xi},\quad a=x,y. \qquad (15)$$

By (3) one has $P_z(z)=\boldsymbol{\xi}^{\dagger}\hat{\mathbf{T}}\boldsymbol{\xi}/4$.

In view of (5), (6), and (14) the time-averaged energy

$$W(z)=\frac{1}{4}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}^{\dagger}\frac{\partial(\omega\hat{\boldsymbol{\Gamma}})}{\partial\omega}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix} \qquad (16)$$

can be expressed in terms of $\boldsymbol{\xi}$ (3) and $\partial\hat{\mathbb{N}}/\partial\omega$ at $k_{x,y}=\mathrm{const}$ [31,54],

$$W(z)=\frac{1}{4}\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\phi}\end{pmatrix}^{\dagger}\frac{\partial(\omega\hat{\boldsymbol{\Omega}})}{\partial\omega}\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\phi}\end{pmatrix}=\frac{1}{4}\boldsymbol{\xi}^{\dagger}\frac{\partial\hat{\mathbb{N}}}{\partial\omega}\boldsymbol{\xi}. \qquad (17)$$

III. GROUP AND ENERGY VELOCITIES

Consider SEWs on the contact between two homogeneous media 1 and 2 occupying the half spaces $z>0$ and $z<0$, respectively. The tangential components $E_{x,y}$ and $H_{x,y}$ are continuous at $z = 0$.

The z dependence of the SEW field in media 1 and 2 is described by the vectors $\boldsymbol{\xi}^{(1)}(z)$ and $\boldsymbol{\xi}^{(2)}(z)$, respectively, where

$$\boldsymbol{\xi}^{(j)}(z) = \sum_{\alpha=1,2} b_\alpha^{(j)} \boldsymbol{\xi}_\alpha^{(j)} e^{ip_\alpha^{(j)} z}, \quad j = 1, 2, \tag{18}$$

$\boldsymbol{\xi}_\alpha^{(j)}$ and $p_\alpha^{(j)}$ are the eigenvectors and corresponding eigenvalues of the matrices $\hat{\mathbb{N}}^{(j)}$, $j = 1, 2$, which are the $\hat{\mathbb{N}}$ matrices of media $j = 1$ and $j = 2$, the eigenvalues $p_\alpha^{(j)}$ are such that $\mathrm{Im}(p_\alpha^{(1)}) > 0$ and $\mathrm{Im}(p_\alpha^{(2)}) < 0$, and the coefficient $b_\alpha^{(j)}$ is the amplitude of the mode α in the jth medium.

Due to $\hat{\mathbb{N}}^{(j)} = \hat{\mathbb{N}}^{(j)\dagger}$ both $p_\alpha^{(j)}$ and its complex conjugate $p_\alpha^{(j)*}$ are eigenvalues of $\hat{\mathbf{N}}^{(j)} = \hat{\mathbf{T}}\hat{\mathbb{N}}^{(j)}$ and

$$\boldsymbol{\xi}_\beta^{(j)\dagger} \hat{\mathbf{T}} \boldsymbol{\xi}_\alpha^{(j)} = 0, \tag{19}$$

when $p_\beta^{(j)} \neq p_\alpha^{(j)*}$. Hence, in view of the SEW field structure (18) and orthogonality (19), $P_z^{(j)}(z) = 0$.

Let us prove that the components $V_{g,a} = \partial\omega/\partial k_a$, $a = x, y$, of the group velocity of the SEW are equal to the x and y components of the energy velocity $V_{e,a} = \overline{P}_a/\overline{W}$, where $\overline{P}_a$ and $\overline{W}$ are the space-averaged $P_a(z)$ and $W(z)$. To this end we use the relation

$$\boldsymbol{\xi}^{(j)\dagger} \frac{d\hat{\mathbb{N}}^{(j)}}{dk_a} \boldsymbol{\xi}^{(j)} = -i \frac{d}{dz}\left(\boldsymbol{\xi}^{(j)\dagger} \hat{\mathbf{T}} \frac{d\boldsymbol{\xi}^{(j)}}{dk_a}\right), \tag{20}$$

where $\hat{\mathbb{N}}^{(j)} = \hat{\mathbf{T}}\hat{\mathbf{N}}^{(j)}$,

$$\frac{d}{dk_a} = \frac{\partial}{\partial k_a} + \frac{\partial\omega}{\partial k_a}\frac{\partial}{\partial\omega}, \tag{21}$$

with $k_b = \text{const}$, $a = x$, and $b = y$, or vice versa. Equality (20) follows from (2) and the fact that $\hat{\mathbb{N}}^{(j)} = \hat{\mathbb{N}}^{(j)\dagger}$.

In (21), $\frac{\partial\omega}{\partial k_a} = -\frac{\partial\Upsilon}{\partial k_a}/\frac{\partial\Upsilon}{\partial\omega}$, where $\Upsilon(\omega, k_a) = 0$ is the dispersion equation with $k_b = \text{const}$, so $\boldsymbol{\xi}^{(1)} = \boldsymbol{\xi}^{(2)}$ and $d\boldsymbol{\xi}^{(1)}/dk_a = d\boldsymbol{\xi}^{(2)}/dk_a$ at $z = 0$. Since $\boldsymbol{\xi}^{(1)} \to 0$ as $z \to +\infty$ and $\boldsymbol{\xi}^{(2)} \to 0$ as $z \to -\infty$, we obtain

$$\begin{aligned}\int_0^{+\infty} \boldsymbol{\xi}^{(1)\dagger} \frac{d\hat{\mathbb{N}}^{(1)}}{dk_a} \boldsymbol{\xi}^{(1)} dz &= i\boldsymbol{\xi}^{(1)\dagger}\hat{\mathbf{T}} \frac{d\boldsymbol{\xi}^{(1)}}{dk_a}\bigg|_{z=0} \\ &= i\boldsymbol{\xi}^{(2)\dagger}\hat{\mathbf{T}} \frac{d\boldsymbol{\xi}^{(2)}}{dk_a}\bigg|_{z=0} \\ &= i\int_{-\infty}^0 \frac{d}{dz}\left(\boldsymbol{\xi}^{(2)\dagger}\hat{\mathbf{T}}\frac{d\boldsymbol{\xi}^{(2)}}{dk_a}\right)dz \\ &= -\int_{-\infty}^0 \boldsymbol{\xi}^{(2)\dagger}\frac{d\hat{\mathbb{N}}^{(2)}}{dk_a}\boldsymbol{\xi}^{(2)} dz. \end{aligned} \tag{22}$$

Hence, due to (15) and (17) the substitution of (21) in

$$\int_0^{+\infty} \boldsymbol{\xi}^{(1)\dagger} \frac{d\hat{\mathbb{N}}^{(1)}}{dk_a} \boldsymbol{\xi}^{(1)} dz + \int_{-\infty}^0 \boldsymbol{\xi}^{(2)\dagger} \frac{d\hat{\mathbb{N}}^{(2)}}{dk_a} \boldsymbol{\xi}^{(2)} dz = 0 \tag{23}$$

yields

$$V_{g,a} = \frac{\overline{P}_a}{\overline{W}} = V_{e,a}, \quad a = x, y, \tag{24}$$

where $\overline{P}_a = \overline{P}_a^{(1)} + \overline{P}_a^{(2)}$ and $\overline{W} = \overline{W}^{(1)} + \overline{W}^{(2)}$,

$$\overline{P}_a^{(j)} = -\frac{1}{4}\int_{z_l^{(j)}}^{z_u^{(j)}} \boldsymbol{\xi}^{(j)\dagger} \frac{\partial\hat{\mathbb{N}}^{(j)}}{\partial k_a} \boldsymbol{\xi}^{(j)} dz, \tag{25}$$

$$\overline{W}^{(j)} = \frac{1}{4}\int_{z_l^{(j)}}^{z_u^{(j)}} \boldsymbol{\xi}^{(j)\dagger} \frac{\partial\hat{\mathbb{N}}^{(j)}}{\partial\omega} \boldsymbol{\xi}^{(j)} dz, \tag{26}$$

the integration limits are $z_l^{(1)} = 0$ and $z_u^{(1)} = +\infty$ whereas $z_l^{(2)} = -\infty$ and $z_u^{(2)} = 0$.

Thus (24) proves the equality of the group and energy velocities of SEWs.

IV. ENERGY AND PHASE VELOCITIES

Let us show that the projection of the SEW group velocity on the direction of propagation equals the SEW phase velocity if the material constants do not depend on the frequency. Suppose that wave (1) propagates along the axis X. Letting $k_y = 0$ in the matrix $\hat{\mathbf{J}}$ (11), we change the notation $k_{||} = k_x$ and write $\hat{\mathbb{N}}$ (8) in the form

$$\hat{\mathbb{N}} = \omega\hat{\mathbf{A}} - k_{||}\hat{\mathbf{B}} - \frac{k_{||}^2}{\omega}\hat{\mathbf{C}}, \tag{27}$$

where $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ are still given by (10) but $\hat{\mathbf{J}}$ is replaced by the matrix $\hat{\mathbf{J}}_x$ (12). Correspondingly, the component $P_{||} = P_x$ (15), which now is the energy flux along the direction of propagation, reads as

$$P_{||} = -\frac{1}{4}\boldsymbol{\xi}^\dagger \frac{\partial\hat{\mathbb{N}}}{\partial k_{||}}\boldsymbol{\xi}^\dagger = \frac{1}{4}\boldsymbol{\xi}^\dagger\left(\hat{\mathbf{B}} + 2\frac{k_{||}}{\omega}\hat{\mathbf{C}}\right)\boldsymbol{\xi}. \tag{28}$$

If the material constants do not depend on the frequency, then

$$\frac{\partial\hat{\mathbb{N}}}{\partial\omega} = \hat{\mathbf{A}} + \frac{k_{||}^2}{\omega^2}\hat{\mathbf{C}}, \tag{29}$$

and one can notice that

$$\hat{\mathbf{B}} + 2\frac{k_{||}}{\omega}\hat{\mathbf{C}} = v_{\mathrm{ph}}\left(\hat{\mathbf{A}} + \frac{k_{||}^2}{\omega^2}\hat{\mathbf{C}} - \frac{1}{\omega}\hat{\mathbb{N}}\right), \tag{30}$$

where $v_{\mathrm{ph}} = \omega/k_{||}$ is the phase velocity, so due to (17)

$$\frac{P_{||}}{W} = v_{\mathrm{ph}} - \frac{1}{4k_{||}W}\boldsymbol{\xi}^\dagger\hat{\mathbb{N}}\boldsymbol{\xi}. \tag{31}$$

Assume that

$$\boldsymbol{\xi}(z) = \sum_{\alpha=1,2} b_\alpha \boldsymbol{\xi}_\alpha e^{ip_\alpha z}, \tag{32}$$

where $\boldsymbol{\xi}_\alpha$ and p_α are the eigenvectors and eigenvalues of the matrix $\hat{\mathbf{N}}$ and either $\mathrm{Im}(p_\alpha) > 0$, $\alpha = 1, 2$, or $\mathrm{Im}(p_\alpha) < 0$, $\alpha = 1, 2$. In this case $p_\alpha \neq p_\beta^*$, $\alpha, \beta = 1, 2$ in (32) and, by virtue of (19),

$$\boldsymbol{\xi}^\dagger\hat{\mathbb{N}}\boldsymbol{\xi} = \sum_{\alpha,\beta=1}^{2} b_\beta^* b_\alpha p_\alpha e^{i(p_\alpha - p_\beta^*)z}\boldsymbol{\xi}_\beta^\dagger\hat{\mathbf{T}}\boldsymbol{\xi}_\alpha = 0. \tag{33}$$

As a result, for the field (32)

$$\frac{P_{||}}{W} = v_{\mathrm{ph}} = \text{const} \tag{34}$$

over the depth of the medium.

Thus the sought equality follows from (24) and (34) and the fact that the SEW field (17) in each of the two media is structured similarly to $\boldsymbol{\xi}(z)$ (32),

$$\frac{\partial \omega}{\partial k_{||}} = \frac{\overline{P}_{||}}{\overline{W}} = v_{\text{ph}}, \tag{35}$$

where $\partial\omega/\partial k_{||}$ is the group velocity along the direction of propagation.

V. CONCLUDING REMARKS

We have shown that the group velocity $\mathbf{V}_g$ of SEWs occurring on the interface between two homogeneous half-infinite bianisotropic media is equal to the energy velocity $\mathbf{V}_e$. The media may be not only bianisotropic but also magneto-optically active. The equality remains valid if one of the dielectric is replaced by metal provided that the absorption of electromagnetic waves may be neglected.

The frequency dependence of the material constants does not affect the equality of the group and energy velocities but the absence of such a dependence yields a useful relation. Namely, we have proved that the projection of the energy velocity $V_{e||}$ (group velocity $V_{g||}$) on the direction of propagation is equal to the SEW phase velocity v_{ph} provided that the material constants do not depend on the frequency. The equality $V_{e||} = v_{\text{ph}}$ holds true locally, that is, the phase velocity equals the projection of the energy velocity onto the direction of propagation at any distance from the interface.

The equality of the group and energy velocities results in the planes of constant amplitudes of a monochromatic wave modulated by a smooth envelope function move with the energy velocity of this monochromatic carrier wave. In addition, the energy velocity of a monochromatic wave is directed along the normal to the surface of constant frequency [20,21]. Correspondingly, the direction of propagation of a smooth pulse will not coincide with the wave normal of the carrier wave but in the absence of frequency dispersion the pulse velocity along the wave normal of the carrier wave will be equal to the phase velocity of this wave.

In deriving the equalities in question, we did not employ any particular relations among material parameters, including those that could be due to a specific crystallographic symmetry, so variations of material properties do not affect the equalities established unless attenuationoccurs.

Our proof is applicable to SEWs in half-infinite superlattices and in this case the integrals in (22)–(26) should be viewed as sums of the integrals over individual layers. This proof also applies to waves in planar waveguides. Meanwhile, identity (33) does not hold true in layers since the wave field in them involves partial modes with $\text{Im}(p_\alpha) \lesseqqgtr 0$. Therefore $V_{e||} \neq v_{\text{ph}}$ for SEWs in superlattices and waves in planar waveguides when the dispersion is absent. However, if a planar waveguide is on a homogeneous half-infinite substrate then the wave field in such a substrate is similar to a SEW. Hence locally in the substrate, $P_{||}/W = v_{\text{ph}}$.

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APPENDIX

By using the notation introduced in Sec. II, we can write the result of the substitution of expression (1) in the Maxwell equations in the following form:

$$\frac{1}{i}\frac{d\boldsymbol{\xi}}{dz} = \hat{\mathbf{T}}(\omega\boldsymbol{\psi} + \hat{\mathbf{J}}\boldsymbol{\phi}), \tag{A1}$$

$$-\hat{\mathbf{J}}^t\boldsymbol{\xi} = \omega\boldsymbol{\nu}. \tag{A2}$$

The matrix $\hat{\mathbb{N}}$ (8) and hence $\hat{\mathbf{N}}$ diverge when $\omega \to 0$ because of the factor ω^{-1} at the matrix $\hat{\mathbf{C}}$ (10). However, Eq. (2) can be modified such that the new matrix $\hat{\mathbf{N}}^\omega$ will be finite when $\omega = 0$. Indeed, let us point the coordinate axis X along the direction of propagation of wave (1) and put $\mathbf{u} = \omega\mathbf{u}^\omega$, where $\mathbf{u} = (-E_y\ H_y)^t$ (3). Since $k_y = 0$ in (11), we find that in the limit $\omega = 0$ (A1) and (A2) reduce to

$$\frac{d\mathbf{u}^\omega}{dz} = i\boldsymbol{\psi}', \quad \frac{d\mathbf{v}}{dz} = ik_x\boldsymbol{\phi}, \quad -k_x\mathbf{u}^\omega = \boldsymbol{\nu}, \tag{A3}$$

where $\mathbf{v} = (H_x\ E_x)^t$, $\boldsymbol{\psi}' = (B_x\ D_x)^t$, $\boldsymbol{\phi} = (H_z\ E_z)^t$, and $\boldsymbol{\nu} = (B_z\ D_z)^t$. Thus $E_y = H_y = 0$ when $\omega = 0$ and, by using the constitutive connections written in terms of the matrix $\hat{\boldsymbol{\Omega}}$ (5), we transform (A3) into the system $d\boldsymbol{\xi}^\omega/dz = i\hat{\mathbf{N}}^\omega\boldsymbol{\xi}^\omega$, where $\boldsymbol{\xi}^\omega = (\mathbf{u}^\omega\ \mathbf{v})^t$ and $\hat{\mathbf{N}}^\omega$ is constructed from the elements of $\hat{\boldsymbol{\Omega}}$. Hence the matrix $\hat{\mathbf{N}}^\omega$ of system (2) for $\boldsymbol{\xi}^\omega$ does not involve terms divergent when $\omega \to 0$. Note that the equation $d\mathbf{v}/dz = ik_x\boldsymbol{\phi}$ allows one to introduce the magnetic and electric potentials $\boldsymbol{\Phi}(x, z) = (\varphi_H(z)\ \varphi_E(z))^t \exp(ik_x x)$ so that $\mathbf{v} = -d\boldsymbol{\Phi}/dx = -ik_x\boldsymbol{\Phi}$ and $\boldsymbol{\phi} = -d\boldsymbol{\Phi}/dz$.

In certain cases the explicit expression of $\hat{\mathbb{N}}$ in terms of material constants simplifies substantially. For instance, if a medium confined by the XZ plane is nonbianisotropic, biaxial, and magnetically isotropic ($\mu = 1$), the coordinate axes X, Y, and Z coincide with the principal axes of the dielectric permittivity and the direction of propagation is the X axis, then $\hat{\mathbb{N}}$ proves to be a diagonal matrix with elements $\mathbb{N}_{11} = \omega\varepsilon_{yy} - k_x^2/\omega\mu_0$, $\mathbb{N}_{22} = \omega\mu_0 - k_x^2/\omega\varepsilon_{zz}$, $\mathbb{N}_{33} = \omega\mu_0$, and $\mathbb{N}_{44} = \omega\varepsilon_{xx}$, where μ_0 is the magnetic constant. The matrix $\hat{\mathbf{N}}$ splits into two 2×2 blocks and system (2) decomposes into two systems of two equations describing transverse electric (TE) and transverse magnetic (TM) modes,

$$\frac{d\boldsymbol{\xi}'_{\text{TE}}}{dz} = i\begin{pmatrix} 0 & \omega\mu_0 \\ \omega\varepsilon_{yy} - \dfrac{k_x^2}{\omega\mu_0} & 0 \end{pmatrix}\boldsymbol{\xi}'_{\text{TE}}, \tag{A4}$$

$$\frac{d\boldsymbol{\xi}'_{\text{TM}}}{dz} = i\begin{pmatrix} 0 & \omega\varepsilon_{xx} \\ \omega\mu_0 - \dfrac{k_x^2}{\omega\varepsilon_{zz}} & 0 \end{pmatrix}\boldsymbol{\xi}'_{\text{TM}}, \tag{A5}$$

where $\boldsymbol{\xi}'_{\text{TE}} = (-E_y\ H_x)^t$ and $\boldsymbol{\xi}'_{\text{TM}} = (H_y\ E_x)^t$. The corresponding four component vectors (3) read as $\boldsymbol{\xi}_{\text{TE}} = (-E_y\ 0\ H_x\ 0)^t$ and $\boldsymbol{\xi}_{\text{TM}} = (0\ H_y\ 0\ E_x)^t$.

By (A4) and (A5) we obtain simple expressions for all the necessary characteristics of the modes, so one can straightforwardly check that, e.g., expressions (15) and $\mathbf{P} = \text{Re}[\mathbf{E}\times\mathbf{H}^*]/2$ yield the same result for the components $P_{x,y}$ of the energy flux.

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