Incompatibility of local measurements providing an advantage in local quantum state discrimination

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The uncertainty principle may be considered as giving rise to the notion of incompatibility of observables, a property that has been carefully analyzed in the literature for single systems. A pack of quantum measurements that cannot be measured simultaneously is said to form a set of incompatible measurements. Every set of incompatible measurements has an advantage over the compatible ones in a quantum state discrimination task where one prepares a state from an ensemble and sends it to another party, and the latter tries to detect the state using available measurements. Comparison between global and local quantum state discriminations is known to lead to a phenomenon of "nonlocality." In this work, we seal a connection between the domains of local quantum state discrimination and incompatible quantum measurements. We consider the local quantum state discrimination task where a sender prepares a bipartite state and sends the subsystems to two receivers. The receivers try to detect the sent state using locally incompatible measurements. We analyze the ratio of the probability of successfully guessing the state using incompatible measurements and the maximum probability of successfully guessing the state using compatible measurements. We find that this ratio is upper bounded by a simple function of robustnesses of incompatibilities of the local measurements. Interestingly, corresponding to every pair of sets of incompatible measurements, there exists at least one local state discrimination task where this bound can be achieved. We argue that the optimal local quantum state discrimination task does not present any "nonlocality," where the term is used in the sense of a difference between the ratios, of probabilities of successful detection via incompatible and compatible measurements, in global and local state discriminations. The results can be generalized to the regime of multipartite local quantum state distinguishing tasks.

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I. INTRODUCTION

The uncertainty principle is one of the fundamental pillars that influenced the formation of quantum mechanics by introducing us to the concept of incompatibility of observables. Given two observables, if the operators corresponding to those observables (within the quantum formalism) can be jointly measured using a "parent" measurement, then the observables are called "compatible." Otherwise, they are "incompatible" [1–8]. Incompatibility is entirely a single-system property, i.e., a system considered as a whole even if it possesses multiple constituents. Incompatibility of observables is a signature quantum mechanical property, absent in classical systems, and plays an important role in many quantum tasks and phenomena, like quantum key distribution [9–12], quantum steering [2,3,13–16], and so on.

In a few recent works, connections between minimum-error quantum state discrimination and incompatibility of measurements have been explored [1,5,17–20]. The state discrimination task involves a sender, Alice, and a receiver, Bob. Alice prepares a quantum system in a particular state, taken from a particular ensemble and then sends the system to Bob, who is possibly at a distance. The ensemble appears at Alice with a certain given probability, and along with the quantum system, Alice may also send the information regarding the ensemble to Bob. The set of possible ensembles and their constituents are known to both parties. After receiving the system, Bob tries to identify the state of the system through measurements, i.e., Bob tries to distinguish between the states of the ensemble. It is possible that Bob has access to a fixed set of measurements. Depending on the available set of measurements, Bob may not be able to identify the state of the system perfectly. In such a state discrimination problem, Bob can try to identify the state of the system through what is known as the minimum-error quantum state discrimination strategy [21–24] by minimizing the overall probability of error in guessing the state of the system.

In Ref. [17], the authors considered a particular type of state discrimination task where the sender may provide some information about the state before the receiver performs any measurement. The optimal guessing probability when the premeasurement information is provided is equal to the optimal guessing probability when the information is given after the measurement if the available set of measurements are compatible. This implies that premeasurement information can improve the situation if the measurements are incompatible. The maximum advantage one can get from incompatibility increases linearly with the robustness of incompatibility [18]. There exists certain state discrimination tasks where incompatibility provides an advantage, and thus the incompatibility of measurements can be regarded as a resource [19]. The collection of the compatible set of measurements forms a closed and convex set, and in Ref. [5], a witness operator was formulated to detect incompatible measurements. We mention here that a relation between quantum state discrimination and channel incompatibility has also been established [25,26]. Hitherto, in research works where incompatible measurements were examined in the context of their ability to

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discriminate quantum states, a global state discrimination task was considered. The receiver was allowed to do measurements on the entire state considering it as a single entity. We want to explore the situation where the sender sends each part of the system to different receivers, so that the receivers, situating at distant locations, are not able to perform measurement on the entire system, but are only allowed to do local measurements.

There exists unique and interesting properties of distributed quantum systems which can provide advantages in many quantum devices over the corresponding classical ones. The difference between the ability to distinguish shared quantum states using global and local operations provides evidence of the nonlocality present in the considered situation, which itself is an interesting quantum phenomenon, but is also of crucial importance in several quantum tasks.

In this work, we try to combine these two fundamental notions of quantum mechanics, viz. the incompatibility of quantum measurements and nonlocality in the state discrimination of shared quantum systems. Specifically, we want to examine if the single-system property of measurement incompatibility can influence the quantum state discrimination protocol of a shared system. Therefore, we consider quantum state discrimination tasks where more than two parties are involved. Specifically, a sender, Alice, prepares a quantum system of *more than one subsystem* in a particular state and then sends the subsystems to two or more spatially separated parties. These parties try to identify the state of the system but they are not allowed to employ quantum communication between the spatially separated locations. In this situation, the allowed class of operations can be categorized into two groups, depending on the resources available: (i) local quantum operations (LO) without classical communication [27–29] and (ii) local quantum operations and classical communication (LOCC) [30–74]. In Fig. 1, we schematically present a comparison between the state discrimination tasks considered in previous literature in the context of determining the advantage of incompatible measurements with our discrimination task.

We establish connections for the two categories of *local* state discrimination tasks with the incompatibility of available local measurements. See Fig. 2 to get a schematic understanding of the two phenomena which we are trying to bring in the same context. The spatially separated parties have access to sets of incompatible measurements, which by employing, they try to accomplish the given state discrimination task. We derive relations between the probability of successfully guessing (PSG) the state of the system using local incompatible measurements and incompatibility of the local measurements. We provide upper bounds, considering LO and LOCC separately, and analyzing a single round of measurements in the later case, on the PSG and these upper bounds are the functions of incompatibility of local measurements. Interestingly, corresponding to every set of incompatible local measurements there exists at least one local state discrimination task in which this upper bound can be reached. The optimal state discrimination task which achieves this bound does not exhibit any "nonlocality."

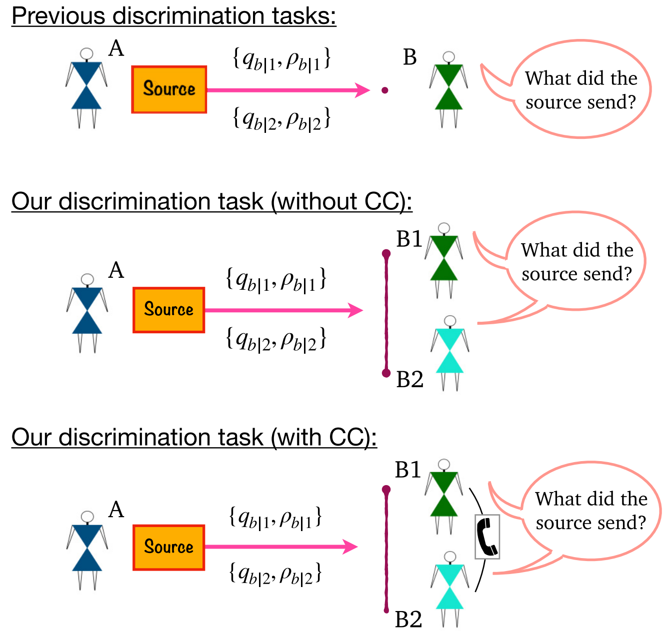

FIG. 1. Comparison between the discrimination task considered in previous literature and the one explored in this paper, with respect to incompatible measurements. Incompatible measurements are known to provide advantage over compatible ones in certain quantum state discrimination tasks. We schematically depict such a discrimination task where a girl, say Alice, randomly chooses a state from a randomly selected ensemble (among a given set of ensembles) and sends it to a boy, Bob (see upper panel). Bob has access to a set of measurements using which he tries to distinguish the state. In our protocol, Alice again randomly selects a state from a random ensemble with the only difference being that, in this case, the ensembles consist of bipartite states. Alice sends each party of the bipartite state to distant locations, say to Bob1 and Bob2. Bob1 and Bob2 being located at two different places are unable to perform any joint operation on the entire state consisting two parts. Thus Bob1 and Bob2 can either do only local operations without any classical communication (depicted in the middle panel) or local operation along with classical communication (depicted in the lower panel) on their part of the system. Whatever be the operations, be it global or local, the aim of the receivers, i.e., Bob or Bob1 and Bob2 is to distinguish the received state. For more details see the main text.

II. INCOMPATIBLE MEASUREMENTS AND ROBUSTNESS OF INCOMPATIBILITY

A set of measurements $\{M_x\}_x$ is called compatible or incompatible depending on their joint measurability. The suffix x, outside the braces in $\{M_x\}_x$, indicates the running variable that generates the set. A similar notation is used throughout the article. If $\{M_x\}_x$ can be measured simultaneously using a parent measurement G, we say that it is compatible. We denote the measurement operators, associated with different outcomes of a measurement M_x and G, by $\{M_{a|x}\}_a$ and $\{G_\lambda\}_\lambda$, respectively. The measurement operators corresponding to the measurements $\{M_x\}_x$ can be expressed in terms of G as

$$M_{a|x} = \sum_\lambda p(a|x, \lambda)G_\lambda, \tag{1}$$

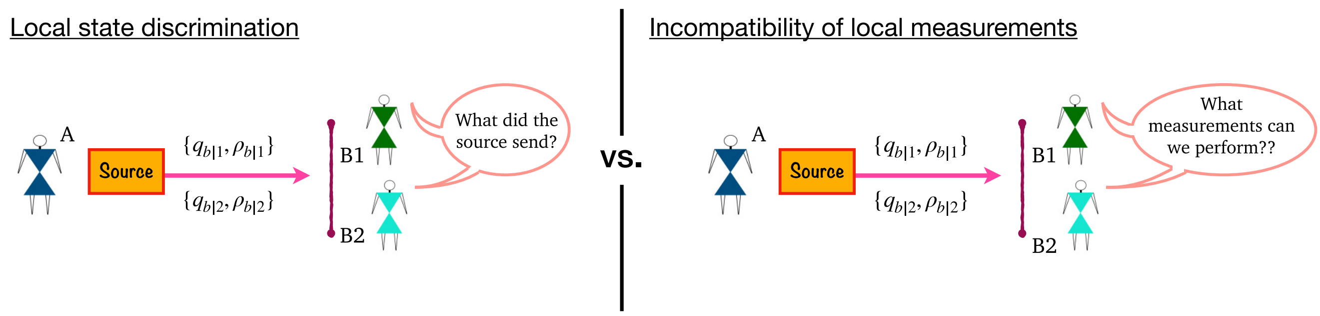

FIG. 2. Schematic presentation of the two physical phenomena that we want to relate in this paper. In the left panel, we show the local state discrimination task, where Alice sends each part of a randomly chosen bipartite state from a randomly selected ensemble (among a given set of ensembles) to Bob1 and Bob2. The receivers, Bob1 and Bob2, using local operations, want to discriminate the state. In the right panel, the situation is the same, but here Bob1 and Bob2 are deciding which measurements should be performed to maximize the probability of successful discrimination. Should the measurement be chosen from an incompatible set of measurements, or would a compatible set provide more advantage? Here, we do not consider classical communication. Similar figures can be drawn considering classical communication as well.

where $p(a|x,\lambda)$ is a conditional probability distribution, $M_{a|x}, G_\lambda \geqslant 0$, $\sum_a M_{a|x} = \mathbb{1}$, and $\sum_\lambda G_\lambda = \mathbb{I}\ \forall a, x, \lambda$ with $\mathbb{1}$ being the identity operator.

Measurements which are not compatible (i.e., not jointly measurable) are called incompatible measurements. To quantify the incompatibility of a set of measurements $\{M_k\}_k$, the robustness of incompatibility (ROI), denoted I_M, was introduced in the literature (for example, see Ref. [18]). ROI can be defined by the minimal amount of *noise* that is required to be mixed with a set of incompatible measurements $\{M_k\}_k$ to make it compatible, i.e.,

$$I_M = \min r, \tag{2}$$

$$\text{such that} \quad \frac{M_{c|k} + r\Lambda_{c|k}}{1+r} = \sum_\lambda p(c|k,\lambda) G_\lambda, \tag{3}$$

$$\Lambda_{c|k} \geqslant 0,\ \sum_c \Lambda_{c|k} = \mathbb{1}, \tag{4}$$

$$G_\lambda \geqslant 0,\ \sum_\lambda G_\lambda = \mathbb{1}, \tag{5}$$

$$0 \leqslant p(c|k,\lambda) \leqslant 1,\ \sum_c p(c|k,\lambda) = 1, \tag{6}$$

where $M_k = \{M_{c|k}\}_c$, i.e., $\{M_{c|k}\}_c$ are the outcomes of the measurement M_k. I_M denotes the amount of incompatibility present in the set of measurements $\{M_k\}_k$. Equation (2) provides a generic definition for the quantification of incompatibility. Specifically, the definition does not depend explicitly on the nature of $\{M_k\}_k$, i.e., if it is a set of projective measurements (PM) or positive operator-valued measurements (POVM). Thus each element of the set $\{M_k\}_k$ just satisfies the usual properties of a measurement, $M_k \geqslant 0$ and $\sum_c M_{c|k} = \mathbb{I}$.. By *noise*, we mean an arbitrary set of measurements $\{\Lambda_k\}_k$, $\Lambda_k = \{\Lambda_{c|k}\}_c$, which is mixed with the set of measurements $\{M_k\}_k$ so that after mixing, the final set of measurements $\frac{M_{c|k}+r\Lambda_{c|k}}{1+r}$ become compatible. r is the amount of noise that has to be mixed with $\{M_k\}_k$ to make the final measurement compatible. The minimization is over r, $\Lambda_{c|k}$, G_λ, and probability distributions $p(c|k,\lambda)$. By minimization over conditional probability distributions we mean minimization over any set of real numbers $\{p(c|k,\lambda)\}$, which satisfies $p(c|k,\lambda) \geqslant 0$ for all c, k, and λ, and $\sum_c p(c|k,\lambda) = 1$ for all k and λ. Whenever we do any optimization over conditional probabilities, we use this same concept. $\{\Lambda_k\}_k$ and $\{G\}$ represent measurements with outcomes $\{\Lambda_{c|k}\}_c$ and $\{G_\lambda\}_\lambda$, respectively. These measurements can also be POVM as well as PM. The constraints on these measurement operators are mentioned in the expressions (3) to (6).

III. CONNECTION OF INCOMPATIBILITY WITH STATE DISCRIMINATION PROBLEM

In several articles [5,17–19], it was shown that incompatible measurements can provide an advantage over the compatible ones in certain state discrimination tasks. In Ref. [18], the authors considered a task involving two people where one randomly selects a state from an ensemble $\mathcal{E}_y$, which was randomly chosen from a given set of ensembles $\{\mathcal{E}_y\}_y$, and sends the state to the other. The latter, after receiving the state, tries to discriminate it using a set of measurements. Let $\mathcal{P}^C(\{\mathcal{E}_y\})$ and $\mathcal{P}^I(\{\mathcal{E}_y\}, \{Q_x\})$ be the maximum probability of successfully guessing the state maximized over all sets of compatible measurements and the same for a fixed set of measurements $\{Q_x\}$ for the optimal strategy. In Ref. [18], a precise mathematical expression is provided to represent the advantage achievable through incompatible measurements which goes as follows:

$$\frac{\mathcal{P}^I(\{\mathcal{E}_y\}, \{Q_x\})}{\mathcal{P}^C(\{\mathcal{E}_y\})} \leqslant 1 + I_Q.$$

Here I_Q is the ROI of $\{Q_x\}$. The authors also showed that, for any set of measurements, there exists a corresponding state discrimination task for which the bound is achievable.

In this work, we restrict ourselves to a smaller set of operations, i.e., we consider state discrimination using only local operations with or without classical communication instead of global measurements, and try to determine how the above expression gets modified in the new situation. The considered state discrimination tasks are discussed in detail below.

A. State discrimination using only LO without CC

If the spatially separated parties are restricted to perform local operations only on their subsystems to accomplish the given state discrimination task, then it is known as state discrimination by LO. We note that, in this scenario, classical communication among the parties during the local operations within a round or between the rounds, is not allowed. However, after all the local operations are accomplished, the parties can discuss the measurement outcomes to identify the state of the system [27–29]. Since, in this scenario, classical communication is not allowed in between the measurements, we will denote the corresponding probability of a successful guess using the suffix LOCC.

B. State discrimination task with premeasurement information

In this task, Alice chooses an ensemble $\mathcal{E}_y$ of *bipartite states* with probability $q(y)$. Then she prepares a quantum system in a bipartite state $\rho_{b|y}$ taken from $\mathcal{E}_y$ with probability $q(b|y)$ and sends the subsystems to Bob1 and Bob2 along with the information of y. Bob1 and Bob2 have sets of the measurements $\{M_k\}_k$ and $\{N_l\}_l$, respectively, which, by using, Bob1 and Bob2 want to identify the state of the system. We refer to this as SD1.

C. State discrimination task with postmeasurement information

This task is the same as the preceding one except that, in this case, Bob1 and Bob2 do not have any information about y prior to the measurement. After the performance of measurements, Alice informs them about the particular ensemble $\{\mathcal{E}_y\}$, and then, depending on y and the measurement outcomes, Bob1 and Bob2 make a guess about $\rho_{b|y}$. This state discrimination task will be referred to as SD2.

Here we consider SD1, i.e., state discrimination with premeasurement information, and provide a relation between the probability of successfully guessing (PSG) the state of the system by using the local measurements $\{M_k\}_k$ and $\{N_l\}_l$, and the incompatibility of these measurements. We consider the case where only local operations (LO) are allowed.

IV. UPPER BOUND ON THE GUESSING PROBABILITY USING ONLY LOCAL OPERATIONS WITHOUT CLASSICAL COMMUNICATION

We consider here the case where Bob1 and Bob2 try to discriminate the state using LO without CC, and first the case where they have the knowledge of y prior to the measurement. We restrict Bob1 and Bob2 from using classical communication. The set of measurements available to Bob1 and Bob2 are locally incompatible. After receiving the state $\rho_{b|y}$, Bob1 (Bob2) chooses a measurement M_k (N_l) with probability $p(k|y)$ [$p(l|y)$]. The probability of guessing the state correctly using these measurements is given by

$$P^{\rm SD1}_{\rm LO\cancel{CC}} = \sum_{y,b,k,l,c,d} q(y)q(b|y)\mathrm{tr}[\rho_{b|y}M_{c|k} \times \otimes N_{d|l}]p(k|y)p(l|y)p(b|c,d,y), \tag{7}$$

where $M_{c|k}$ and $N_{d|l}$ are the measurement operators, corresponding to the outcomes c, d, associated with the measurements M_k and N_l, respectively. Note that here state discrimination with pre-measurement information is considered. After getting the outcomes c and d, Bob1 and Bob2 call for a guess regarding the value of b, and this is according to the probability $p(b|c,d,y)$. Then, the optimal PSG using measurement M_k and N_l is

$$\begin{aligned} &P^{I}_{\rm LO\cancel{CC}}(\{\mathcal{E}_y\},\{M_k\},\{N_l\}) \\ &= \max_{p(k|y),p(l|y),p(b|c,d,y)} \sum_{y,b,k,l,c,d} q(y)q(b|y) \\ &\quad \times \mathrm{tr}[\rho_{b|y}M_{c|k}\otimes N_{d|l}]p(k|y)p(l|y)p(b|c,d,y). \end{aligned} \tag{8}$$

Here, the maximization over conditional probabilities represent optimization over any set of real numbers which satisfies the usual properties of a conditional probability distribution. For example, for the set of probabilities $\{p(k|y)\}_k$ the conditions are $p(k|y)\geqslant 0$ for all k and y and $\sum_k p(k|y)=1$ for all y. The final set of conditional probabilities $\{p^{\max}(l|y), p^{\max}(l|y), p^{\max}(b|c,d,y)\}$, which maximize the function, can be used to strategize the task. Specifically, if Bob1 and Bob2 choose their measurement M_k and N_l with probabilities $p^{\max}(k|y)$ and $p^{\max}(l|y)$ and after doing the measurement, if they guess about the state depending on the probability distribution $p^{\max}(b|c,d,y)$, they will reach the maximum probability of success. However, if Bob1 and Bob2 had locally compatible measurements and obtained the information of y only after performing the measurement, then the PSG would be

$$P^{\rm SD2}_{\rm LO\cancel{CC}} = \sum_{y,b,k,l,c,d} q(y)q(b|y)\mathrm{tr}[\rho_{b|y}M_{c|k} \times \otimes N_{d|l}]p(k)p(l)p(b|c,d,y). \tag{9}$$

Here, the suffix SD2 represents that state discrimination with postmeasurement information considered. Then the maximum PSG using locally compatible measurements in SD2, i.e., state discrimination with postmeasurement information, can be written as

$$P^{C}_{\rm LO\cancel{CC}}(\mathcal{E}_y) = \max_{M_k,N_l\in CM,p(k),p(l),p(c,d,y)} P^{\rm SD2}_{\rm LO\cancel{CC}}, \tag{10}$$

where the maximization is taken over the set of locally compatible measurements CM and the probabilities $p(k)$, $p(l)$, and $p(c,d,y)$.

Specifically, Eq. (7) represents the PSG of a shared state $\rho_{b|y}$ when the parties Bob1 and Bob2 know about y before the performance of any measurement. Thus they choose their measurements M_k and N_l, respectively, randomly following the independent probability distributions $p(k|y)$ and $p(l|y)$, where these probability distributions depend on y. In the next equation, i.e., Eq. (8), we optimize the probability presented in Eq. (7) $P^{\rm SD1}_{\rm LO\cancel{CC}}$ over all possible conditional probability distributions $p(k|y)$, $p(l|y)$, and $p(b|c,d,y)$ to determine the best strategy for discrimination. In Eq. (9), we consider the case where Bob1 and Bob2 do not know about y before the performance of the measurements. Here also they randomly choose the measurements M_k and N_l, but these random distributions $p(k)$ and $p(l)$ do not depend on y. However, after doing the measurements, Alice tells Bob1 and Bob2 about y. Hence Bob1 and Bob2 can guess about b, depending on the information of y as well as the measurement outputs c and d. To guess the value of b they follow the probability distribution $p(b|c,d,y)$. In the final equation, Eq. (10), we optimize $P^{\rm SD2}_{\rm LO\not{C}\not{C}}$, which is expressed in Eq. (9) with respect to all possible strategies, $p(k)$, $p(l)$, $p(c,d,y)$, and all possible pairs of sets of measurements $\{M_k\}_k$ and $\{N_l\}_l$, which are locally compatible.

From now on, whenever a set of locally incompatible measurements will be used for state discrimination, we will consider that y is known to Bob1 and Bob2 prior to the measurement, and in state discrimination tasks, byusing compatible measurements, we will assume that the premeasurement information about y is not available [5,17–19].

Let the ROIs of the local measurements $\{M_k\}_k$ and $\{N_l\}_l$ be I_M and I_N, respectively. Moreover, let the optimization, shown in Eq. (2) be attained by $\Lambda^*_{c|k}$, $p^*(c|k,\lambda)$, G^*_λ for $\{M_k\}$ and $\Sigma^*_{d|l}$, $p^*(d|l,\lambda)$, H^*_λ for $\{N_l\}$. Then, using Eq. (2), one can find

$$M_{c|k} \leqslant (1+I_M)\sum_\lambda p^*(c|k,\lambda)G^*_\lambda,$$

$$\text{and}\qquad N_{d|l} \leqslant (1+I_N)\sum_\lambda p^*(d|l,\lambda)H^*_\lambda. \tag{11}$$

We take the tensor product of these two inequations, to find

$$M_{c|k}\otimes N_{d|l} \leqslant (1+I_M)(1+I_N) \sum_{\lambda,\nu} p^*(c|k,\lambda)p^*(d|l,\nu)G^*_\lambda\otimes H^*_\nu. \tag{12}$$

We now multiply both sides of the above inequality by $q(y)q(b|y)p(k|y)p(l|y)p(b|c,d,y)\rho_{b|y}$, and then sum over the parameters c, d, k, l, b, y. Thereafter, taking the trace, it becomes

$$\sum_{y,b,k,l,c,d} q(y)q(b|y)p(k|y)p(l|y)p(b|c,d,y)\mathrm{tr}[\rho_{b|y}M_{c|k}\otimes N_{d|l}] \leqslant (1+I_M)(1+I_N)\sum_{y,b,k,l,c,d,\lambda,\nu} q(y)q(b|y)p(k|y)p(l|y)p(b|c,d,y)$$
$$\times p^*(c|k,\lambda)p^*(d|l,\nu)\mathrm{tr}[\rho_{b|y}G^*_\lambda\otimes H^*_\nu]. \tag{13}$$

We substitute $\sum_{k,l,c,d} p(k|y)p(l|y)p(b|c,d,y)p^*(c|k,\lambda)p^*(d|l,\nu)$ by a new conditional probability distribution $p(b|\lambda,\nu,y)$. Thus we have

$$\sum_{y,b,k,l,c,d} q(y)q(b|y)\mathrm{tr}[\rho_{b|y}M_{c|k}\otimes N_{d|l}]p(k|y)p(l|y)p(b|c,d,y) \leqslant (1+I_M)(1+I_N)\sum_{y,b,\lambda,\nu} q(y)q(b|y)p(b|\lambda,\nu,y)\mathrm{tr}[\rho_{b|y}G^*_\lambda\otimes H^*_\nu]. \tag{14}$$

The expression $\sum_{y,b,\lambda,\nu} q(y)q(b|y)p(b|\lambda,\nu,y)\mathrm{tr}[\rho_{b|y}G^*_\lambda\otimes H^*_\nu]$ represents the PSG using a single pair of local measurements, viz., $\{G_\lambda\}_\lambda$ and $\{H_\nu\}_\nu$. Hence, it would be less than $P^C_{g,\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\})$, which is the PSG, optimized over all compatible measurements. So we can write

$$\sum_{c,d,k,l,b,y} q(y)q(b|y)\mathrm{tr}[\rho_{b|y}M_{c|k}\otimes N_{d|l}]p(k|y)p(l|y)p(b|c,d,y) \leqslant (1+I_M)(1+I_N)P^C_{\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\}). \tag{15}$$

The above relation holds for all probability distributions $p(k|y)$, $p(l|y)$, and $p(b|c,d,y)$. Hence it also holds if we maximize the left-hand side of the above inequality with respect to these probabilities. Therefore we obtain

$$P^I_{\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\},\{M_k\},\{N_l\}) \leqslant (1+I_M)(1+I_N)P^C_{\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\}),$$

so that

$$\frac{P^I_{\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\},\{M_k\},\{N_l\})}{P^C_{\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\})} \leqslant (1+I_M)(1+I_N). \tag{16}$$

Note that the numerator and the denominator of Eq. (16) are for local operations without classical communication.

The same bound remains valid when Bob1 and Bob2 are allowed to use classical communication along with local operations. That is, if the maximum PSG using local operation and classical communication (LOCC) in the presence of the same set of incompatible measurements $\{M_k\}$ and $\{N_l\}$ are $P^I_{\rm LOCC}(\{\mathcal{E}_y\},\{M_k\},\{N_l\})$ and the maximum PSG using compatible measurements are $P^C_{\rm LO\not{C}\not{C}}(\{\mathcal{E}_y\})$, then

$$\frac{P^I_{\rm LOCC}(\{\mathcal{E}_y\},\{M_k\},\{N_l\})}{P^C_{\rm LOCC}(\{\mathcal{E}_y\})} \leqslant (1+I_M)(1+I_N), \tag{17}$$

See Sec. I of the Supplementary Material [75] for a proof. It should be noted that the numerator and the denominator of Eq. (17) are for local operations and classical communication.

Though the right-hand side (RHS) of the inequalities (16) and (17) are the same, the left-hand sides (LHS) of the same, by definition, differ significantly. Specifically, the LHS of Eq. (16) represents the ratio of PSGs in the state discrimination by using local operations without any classical communication, whereas the LHS of Eq. (17) describes the ratio of PSGs in the state discrimination when classical communication is allowed along with local operations. Certainly, if we individually compare the numerator and denominator of the LHS of Eq. (16) with Eq. (17), we see $P^I_{\rm LO\not{C}\not{C}} \leqslant P^I_{\rm LOCC}$ and $P^C_{\rm LO\not{C}\not{C}} \leqslant P^C_{\rm LOCC}$, but interestingly, the ratios are found to be upper bounded by the same quantity $(1+I_M)(1+I_N)$, which is the same as for LO without CC.

Instead of this bipartite state discrimination task, we can also consider an n-partite state discrimination task, where Alice prepares an n-partite system and then sends the subsystems to Bob1, Bob2, and so on. After receiving the subsystems, Bob1, Bob2, …, Bob$n$ tries to identify the state using a set of local measurements, say $\{O^1_{k_1}\}_{k_1}$, $\{O^2_{k_2}\}_{k_2}$, …, $\{O^n_{k_n}\}_{k_n}$, respectively. Let the ROI of $\{O^i_{k_i}\}_{k_i}$ be I_i. Using the same technique as described in the above, it is possible to show the following:

$$\frac{P^I_{\text{LOCC/LOCC}}(\{\mathcal{E}_y\},\{O^1_{k_1}\},\{O^2_{k_2}\},\ldots)}{P^C_{\text{LOCC/LOCC}}(\{\mathcal{E}_y\})} \leqslant \prod_{i=1}^{n}(1+I_i). \tag{18}$$

Let us revert back to the scenario of two Bobs. Corresponding to every pair of incompatible measurements $\{M_k\}_k$ and $\{N_l\}_l$, there exists at least one LO (which is a subset of LOCC) state discrimination task for which this upper bound can be achieved. Before discussing the actual scenario, let us first state the semi-definite program (SDP), through which ROI of a set of measurements, can be expressed.

The forms of the primal SDPs to determine the ROIs of the measurements $\{M_k\}_k$, and $\{N_l\}_l$ are given by

$$1+I_M = \min_{s,\{\tilde{G}_{\mathbf{c}}\}} s$$
$$\text{such that} \quad \sum_{\mathbf{c}} D_{\mathbf{c}}(c|k)\tilde{G}_{\mathbf{c}} \geqslant M_{c|k}$$
$$\sum_{\mathbf{c}} \tilde{G}_{\mathbf{c}} = s\mathbb{1}, \tilde{G}_{\mathbf{c}} \geqslant 0. \tag{19}$$

Here, $s = 1 + r$, where r is defined in Eq. (2). $\tilde{G}_{\mathbf{c}} = sG_{\mathbf{c}}$ and the positivity of $\Lambda_{c|k}$ in Eq. (2), leads to the inequality $\sum_{\mathbf{c}} D_{\mathbf{c}}(c|k)\tilde{G}_{\mathbf{c}} \geqslant M_{c|k}$, where $p(c|k,\lambda) = \sum_{\mathbf{c}} D_{\mathbf{c}}(c|k)p(\mathbf{c}|\lambda)$, $\mathbf{c} = \mathbf{c}_1\mathbf{c}_2\ldots\mathbf{c}_n$, a string of outcomes, and $D_{\mathbf{c}}(c|k) = \delta_{c,\mathbf{c}_k}$. The I_M defined in Eq. (19) quantifies the incompatibility of the set of measurements available on Bob1's side. In a similar manner, the incompatibility of the set of measurements $\{N_l\}_l$ accessible to Bob2 can also be defined. The corresponding SDP can be formulated as

$$\text{and} \quad 1+I_N = \min_{t,\{\tilde{H}_{\mathbf{d}}\}} t$$
$$\text{such that} \quad \sum_{\mathbf{d}} E_{\mathbf{d}}(d|l)\tilde{H}_{\mathbf{d}} \geqslant N_{d|l}$$
$$\sum_{\mathbf{d}} \tilde{H}_{\mathbf{d}} = t\mathbb{1}, \tilde{H}_{\mathbf{d}} \geqslant 0. \tag{20}$$

Mathematically, the parameters s and t carry the same meaning, with the only difference being that the optimal s and t are equal to the unity added with the ROI of measurements available on Bob1's side and Bob2's side, respectively.

The corresponding dual SDPs can be expressed as

$$1+I_M = \max_{X,\{w_{ck}\}} \operatorname{tr}\left[\sum_{c,k} w_{ck}M_{c|k}\right]$$
$$\text{such that} \quad X \geqslant \sum_{c,k} w_{ck}D_{\mathbf{c}}(c|k), \tag{21}$$
$$w_{ck} \geqslant 0, \operatorname{tr}[X] = 1,$$

$$\text{and} \quad 1+I_N = \max_{Y,\{z_{dl}\}} \operatorname{tr}\left[\sum_{d,l} z_{dl}N_{d|l}\right]$$
$$\text{such that} \quad Y \geqslant \sum_{d,l} z_{dl}E_{\mathbf{d}}(d|l), \tag{22}$$
$$z_{dl} \geqslant 0, \operatorname{tr}[Y] = 1,$$

where w_{ck}, X, z_{dl}, and Y are the dual variables. See Ref. [18] for a more detailed treatment of these primal and dual problems.

We consider the dual variables w^*_{ck}, X^*, z^*_{dl}, and Y^* for which the optimizations in Eqs. (21) and (22) are achieved and write

$$1+I_M = \operatorname{tr}\left[\sum_{c,k} w^*_{ck}M_{c|k}\right] \text{ and } 1+I_N = \operatorname{tr}\left[\sum_{d,l} z^*_{dl}N_{d|l}\right]. \tag{23}$$

Let us now introduce some new variables, given by

$$M^* = \operatorname{tr}\left[\sum_{c,k} w^*_{ck}\right], \quad N^* = \operatorname{tr}\left[\sum_{d,l} z^*_{dl}\right],$$
$$q^*(cd,kl) = \frac{\operatorname{tr}[w^*_{ck}]\operatorname{tr}[z^*_{dl}]}{M^*N^*},$$
$$\text{and} \quad \rho^*_{cd|kl} = \frac{w^*_{ck}\otimes z^*_{dl}}{\operatorname{tr}[w^*_{ck}]\operatorname{tr}[z^*_{dl}]} = \frac{w^*_{ck}\otimes z^*_{dl}}{M^*N^*q^*(cd,kl)}. \tag{24}$$

The dual variables w^*_{ck} and z^*_{dl} are positive, hermitian operators. So, $\rho^*_{cd|kl}$ is a quantum state. We now state the corresponding state discrimination task: Alice can choose an ensemble $\mathcal{E}^*_{kl}$ with probability $q^*(kl)$ which consists of bipartite states $\rho^*_{cd|kl}$. The probability of choosing a state $\rho^*_{cd|kl}$ from $\mathcal{E}_{kl}$ is $q^*(cd|kl) = \frac{q^*(cd,kl)}{q^*(kl)}$, $q^*(kl) = \sum_{cd} q^*(cd,kl)$. Then, Alice prepares a quantum system in the state $\rho^*_{cd|kl}$ and the subsystems are sent to Bob1 and Bob2. The task of Bob1 and Bob2 is to identify cd. To complete the task successfully, Bob1 and Bob2 choose measurements from the sets of measurements $\{M_k\}_k$ and $\{N_l\}_l$. Since, in the case of SD1, Bob1 and Bob2 know the ensemble from which Alice has chosen the state, prior to their measurements, they can choose the measurement based on the information of kl. Let us assume that Bob1 and Bob2 choose the measurements $M_{k'}$ and $N_{l'}$ with probabilities $p(k'|kl)$ and $p(l'|kl)$, respectively. However, in the case of SD2, the information of kl is considered to be unknown before the performance of the measurements. Thus the measurements have to be chosen independently of the value of kl. We assume that, for SD2, the measurements $M_{k'}$ and $N_{l'}$ are chosen with probabilities $p(k')$ and $p(l')$, respectively.

The operators associated with the measurements $M_{k'}$ and $N_{l'}$ are given by $\{M_{c'|k'}\}_{c'}$ and $\{N_{d'|l'}\}_{d'}$. For SD1, i.e., state discrimination with premeasurement information, we consider a specific strategy, i.e., $p(k'|kl) = \delta_{kk'}$, $p(l'|kl) = \delta_{ll'}$, and $p(cd|c',d',kl) = \delta_{cc'}\delta_{dd'}$. It can be proved that this state discrimination task achieves the bound. The proof is presented in Sec. II of the Supplementary Material [75]. The state discrimination task can be generalized to n parties, and correspondingly, the bound given in inequality (18) can also be proved to be achievable.

V. LOCAL BOUNDS VERSUS THE GLOBAL ONE

In Ref. [18], the authors considered a state discrimination task that is different from the ones considered until now and where only two parties were involved, say Alice and Bob. In that protocol, Alice chose an ensemble $\mathcal{E}_y$ with probability $q(y)$. She then prepared a quantum system in a state $\rho_{b|y}$, taken from $\mathcal{E}_y$ with probability $q(b|y)$. After its preparation, she sent the entire quantum system to Bob. She also informed Bob about the value of y. Bob's task was to identify b. Since, in that situation, Bob was holding the complete state, i.e., was not sharing the state with any third party, he was able to do the measurement on the entire system. Thus the restriction of local operations and classical communication was not applicable. However, there Bob was also allowed to perform only a set of measurements, say $\{Q_x\}_x$. We remember that the suffix x, written outside the second bracket of the expression $\{Q_x\}_x$, indicates the running variable which generates the set. At this point, depending on the information of y, Bob chose a particular measurement Q_x from the set of measurements $\{Q_x\}_x$ with probability $p(x|y)$. The maximum PSG using the set of measurements $\{Q_x\}_x$ can be denoted by $\mathcal{P}^I(\{\mathcal{E}_y\}, \{Q_x\})$. The maximum PSG optimized over the set of compatible measurements, when no information is available about y until the measurement is performed, can be denoted as $\mathcal{P}^C(\{\mathcal{E}_y\})$. It was shown in Ref. [18] that

$$\frac{\mathcal{P}^I(\{\mathcal{E}_y\}, \{Q_x\})}{\mathcal{P}^C(\{\mathcal{E}_y\})} \leqslant 1 + I_Q, \tag{25}$$

where I_Q is the ROI of $\{Q_x\}_x$. This is certainly a "global" bound on the achievable advantage of incompatibility because here the entire state is available to Bob for measurements. In this paper, we considered the state to be shared between two distant parties, Bob1 and Bob2, who were only allowed to do local operations and classical communication on their parts of the system. Thus we determined a "local" bound on the achievable advantage of incompatibility. We now want to compare the global bound, expressed in Eq. (25), with the local ones, obtained in Eqs. (16) and (17).

Let the incompatibility of the set of global measurements $\{M_k \otimes N_l\}$ be $I_{M\otimes N}$. It can be shown that $1 + I_{M\otimes N} = (1 + I_M)(1 + I_N)$. (See Sec. III of the Supplementary Material [75] for a proof.) Since LO is a subset of LOCC and LOCC is a subset of separable operations [31,76], we have $P^I_{\cancel{\text{LOCC}}}(\{\mathcal{E}_y\}, \{M_k\}, \{N_l\}) \leqslant P^I_{\text{LOCC}}(\{\mathcal{E}_y\}, \{M_k\}, \{N_l\}) \leqslant \mathcal{P}^I(\{\mathcal{E}_y\}, \{M_k \otimes N_l\}) \leqslant (1 + I_{M\otimes N})\mathcal{P}^C(\{\mathcal{E}_y\}, \{M_k \otimes N_l\})$, for any set of ensembles $\{\mathcal{E}_y\}$. However, $P^C_{\cancel{\text{LOCC}}/\text{LOCC}}(\{\mathcal{E}_y\}, \{M_k\}, \{N_l\}) \leqslant \mathcal{P}^C(\{\mathcal{E}_y\}, \{M_k \otimes N_l\})$ is also true because there exist examples for which such an equality holds. For instance, let us consider that Alice has only one ensemble $\mathcal{E}_0$, i.e ., $q(y) = \delta_{y,0}$. The ensemble consists of equally probable two-qubit maximally entangled states. Since the states are orthogonal, they can be globally distinguished (by measuring onto the basis of the states). Thus we have $\mathcal{P}^C(\mathcal{E}_0) = 1$. But even if classical communication is allowed, they can never be deterministically distinguished using LOCC [35]. Thus $P^C(\mathcal{E}_0) < \mathcal{P}^C(\mathcal{E}_0)$. Hence, we can say the bounds given in Eqs. (16) and (17) restrict the PSG using incompatible measurements more than what would, in general, be possible via the previously known global bound.

VI. ABSENCE OF NONLOCALITY IN OPTIMAL LOCAL STATE DISCRIMINATION

Since $1 + I_{M\otimes N} = (1 + I_M)(1 + I_N)$, we see the bounds on the ratios of the probabilities, i.e., on $P^I_{\cancel{\text{LOCC}}/\text{LOCC}}/P^C_{\cancel{\text{LOCC}}/\text{LOCC}}$, given in Eqs. (16) and (17) are equal with the global bound on $\mathcal{P}^I/\mathcal{P}^C$ presented in Ref. [18]. For each set of local measurements, there exists a corresponding state discrimination task where the bounds are achievable. This indicates that there is no "nonlocality" present in the ratio of the success probabilities using incompatibility measures, in the optimal state discrimination process. Here, "nonlocality" is being used in the sense of a difference between the ratios of the probabilities, in global and local state distinguishability. Note, however, that nonlocality in the individual probabilities might still be present, which might have canceled out at the time of taking the ratios.

VII. CONCLUSION

Incompatibility of observables is a signature quantum mechanical property, which is active in arguably all quantum tasks. It was known that incompatibility can be used as a resource in quantum state discrimination protocols.

The behavior of shared systems is a widely researched topic which offers various fascinating results. These can then be used to develop quantum technologies. The difference between the ability to distinguish shared quantum states using global and local operations provides evidence of the nonlocality present in the considered situation.

In this paper, we tried to forge a bridge between the efficiency of local quantum state discrimination using incompatible measurements and the relevant quantum measurement incompatibility. We considered local quantum state discrimination tasks, where in one case, only local quantum operations were allowed, and in the other, unidirectional classical communication was allowed along with the local operations. We presented an upper bound on the ratio of the probability of successfully guessing the sent quantum state using incompatible measurements and the maximum probability of the same using any set of compatible ones. This upper bound is the same for both local operations and local operations assisted by unidirectional classical communication, and is an achievable bound in at least one local quantum state discrimination exercise. We compared the local bound with the existing global bound. We showed that the optimal local quantum state discriminations do not reveal any nonlocality in the ratios of the probabilities between incompatible and compatible measurements.

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