Letter

Deterministic single-photon source in the ultrastrong-coupling regime

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We propose a high-quality deterministic single-photon source which can not only works in the ultrastrong light-matter coupling regime, but also emits two single photons with an arbitrary time separation, in one excitation process. We find that the special solutions of the two-qubit quantum Rabi and Jaynes-Cummings models which have at most one photon and constant eigenenergies can be used to implement this proposal through two consecutive adiabatic evolutions, and the system goes back to the initial state of the next period automatically after photon emission. Due to their peculiarities and reach of the ultrastrong coupling, the adiabatic evolution can be quite fast and further accelerated with the Stark shift. This Letter paves the way to fast quantum information protocols which take advantage of the ultrastrong coupling and avoid its dynamical complexity simultaneously.

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I. INTRODUCTION

Single-photon sources are fundamental building blocks in quantum information, with applications ranging from quantum computation [1,2] to quantum communication [3–5] and sensing [6]. Recently developed technologies for single-photon sources can be classified into two families [7]. The first one considers a nonlinear material process such as spontaneous parametric downconversion [8,9], which is probabilistic [7,10]. The second approach is based on single quantum emitters, which deterministically emit one photon at a time. This has been demonstrated in atoms [11], molecules [12], ions [13], color centers in diamonds [14], Rydberg atoms [15], cavity QED [16,17], and quantum dots [18,19]. However, the spontaneous emission in all directions makes photon collection difficult. One way to solve this problem is to couple the emitters to a cavity [20,21] to enhance the radiation into the cavity mode, so that the photon can be emitted from the cavity through a certain direction [22]. Meanwhile, the emission rate can be increased by Purcell effect [23]. Using a quantum dot coupled to cavities, near-unity indistinguishability and purity are realized simultaneously, with an extraction efficiency of 66% [24], a polarized single-photon efficiency of 60% [25], and an overall efficiency of 57% [26], respectively. Besides high efficiency [27], purity [28], and indistinguishability [29], a microwave single-photon source realized in circuit QED can also be tunable [27,30–32].

The Purcell effect, which enhances the emission rate, is proportional to the qubit-cavity coupling strength [33]. When we enter the strong-coupling regime, the swap between the qubit excited state and the single-photon state in the cavity will be further accelerated. Now the ultrastrong [34–41] and even deep strong coupling [42] have been realized in circuit QED, where Sánchez-Burillo *et al.* [43] and Huang and coworkers [44,45] proposed delicate schemes to generate photons. Hence it is natural to consider implementing a deterministic single-photon source using these stronger coupling strengths, which most likely will accelerate the photon generation speed. However, to the best of our knowledge, this has not been realized yet. Because the ultrastrong coupling will invoke counter-rotating terms, which excite the qubit and photon simultaneously, all photon number states become connected, and the system, described by the quantum Rabi model [46–48], normally involves an infinite number of photons [49–51].

Recently we have found special dark states of the two-qubit quantum Rabi model [52–54], which contain at most one photon, and have constant eigenenergy in the whole

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coupling regime. Here, we propose to implement a deterministic single-photon source that takes advantage of the ultrastrong coupling using these dark states. Our scheme begins with the two-qubit and resonator ground states, and then the qubits are excited. Next, the system undergoes an adiabatic evolution along the aforementioned dark state, and ends up with a product state of a single-photon state and a two-qubit singlet state. Because of the peculiarities of the dark state [55], the target state can be fast generated with fidelity 99.7% in a time of $12 \times 2\pi\omega^{-1}$, where ω is the cavity frequency. This time can be further reduced to $1.9 \times 2\pi\omega^{-1}$, with the addition of Stark shift terms [56–58], which are used to ultrafast generate the single-photon multimode W states [59]. After the photon is emitted, the system will be in an eigenstate of the two-qubit Jaynes-Cummings (JC) model with the excitation number $C = 1$, which has also constant eigenenergy in the whole coupling regime. So through another adiabatic process, another single photon can be generated and emitted through dissipation, leaving the system in its ground state, which is also the initial state of the next period. This adiabatic evolution can also be quite fast. Unlike the typical stimulated Raman adiabatic passage, no recycling of the qubit between photon generations is required here, which will increase the repetition rate [60]. Here one excitation process emits two single photons with efficiencies over 98% and an arbitrary time separation. Their purities approach unity and indistinguishabilities are over 95%. We also design a circuit to simulate the two-qubit quantum Rabi model and JC model.

II. SCHEME AND CIRCUIT QED IMPLEMENTATION

Our scheme of the deterministic single-photon source is based on the one-photon solutions to the two-qubit Rabi and JC models ($\hbar = 1$)

$$H_R = \omega a^\dagger a + g_1\sigma_{1x}(a + a^\dagger) + g_2\sigma_{2x}(a + a^\dagger) + \Delta_1\sigma_{1z} + \Delta_2\sigma_{2z}, \tag{1}$$

$$H_{\mathrm{JC}} = \omega a^\dagger a + g_1(\sigma_1^\dagger a + \sigma_1 a^\dagger) + g_2(\sigma_2^\dagger a + \sigma_2 a^\dagger) + \Delta_1\sigma_{1z} + \Delta_2\sigma_{2z}, \tag{2}$$

where $a^\dagger$ and a are the photon creation and annihilation operators with frequency ω, respectively. Also, $\sigma_{j\alpha}(\alpha = x, y, z)$ are the Pauli matrices corresponding to the jth qubit. $2\Delta_j$ is the energy level splitting of the jth qubit, and g_j is the qubit-photon coupling parameter between the resonator and jth qubit. Normally, there is no solution with finite photon numbers to H_R since all the photon number states are connected. However, we have found special dark states with at most one photon [52]:

$$|\psi_R\rangle = \frac{1}{\mathcal{N}}[(\Delta_1 - \Delta_2)|0\uparrow\uparrow\rangle + g|1(\downarrow\uparrow - \uparrow\downarrow)\rangle] \tag{3}$$

when $\Delta_1 + \Delta_2 = \omega$ and $g_1 = g_2 = g$, which has constant eigenenergy $E = \omega$ in the whole coupling regime. The coherent superposition of $|1\downarrow\uparrow\rangle$ and $|1\uparrow\downarrow\rangle$ could cancel the population of higher photon number states. Surprisingly, energy levels with constant eigenenergies $E/\omega = N\,(N = 0, 1, 2, \ldots)$ emerge in the spectrum of the two-qubit JC model under the same conditions [61].

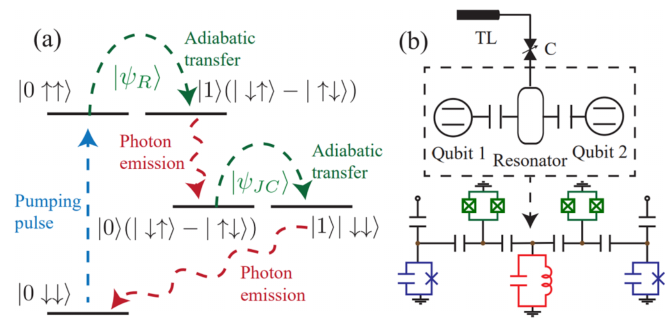

FIG. 1. (a) Relevant energy levels and transitions of our scheme. (b) Setup for the scheme: Two superconducting qubits are coupled to one resonator, whose photon emission rate into the TL is controlled by a variable coupler C. The lower part is a superconducting circuit design for the two-qubit Rabi and JC models with tunable couplings [55].

The corresponding eigenstates read [61]

$$|\psi_{\mathrm{JC}}\rangle = \frac{1}{\mathcal{N}'}[(\Delta_1 - \Delta_2)|1\downarrow\downarrow\rangle + g|0(\downarrow\uparrow - \uparrow\downarrow)\rangle] \tag{4}$$

for $N = 0$. Constant-energy solutions Eq. (3) with $E = \omega$ and Eq. (4) with $E = 0$ have the same structure if we simply reduce the excitation number of the former by one. We will show that $|\psi_R\rangle$ and $|\psi_{\mathrm{JC}}\rangle$ can be used to produce a special single-photon source through two consecutive adiabatic transfers.

Our scheme is depicted in Figs. 1(a) and 1(b). Two qubits are coupled to one resonator, which is connected to a transmission line (TL) through a variable coupler C, so that its dissipation rate κ_c is tunable. The qubits and the resonator are cooled down to the ground state $|0\downarrow\downarrow\rangle$ initially. Then pumping pulses

$$H = \frac{\Omega}{2}(\sigma_1^\dagger e^{-i\omega_{q1}t} + \sigma_1 e^{i\omega_{q1}t} + \sigma_2^\dagger e^{-i\omega_{q2}t} + \sigma_2 e^{i\omega_{q2}t}) \tag{5}$$

are applied to excite the qubits, where ω_{q1} and ω_{q2} are the frequencies of the two qubits, respectively. Initially, we set $\omega_{q1} = 2\Delta_1 \neq \omega_{q2} = 2\Delta_2$, and the coupling $g = g_1 = g_2 = 0$. After excitation, $|0\uparrow\uparrow\rangle$ just corresponds to $|\psi_R\rangle$ of Eq. (3) since $g = 0$ and $\Delta_1 \neq \Delta_2$. Next, we increase g to a nonzero value and decrease $\Delta_1 - \Delta_2$ to zero, so that the state $|0\uparrow\uparrow\rangle$ evolves adiabatically to $|1\psi_B\rangle$ through $|\psi_R\rangle$, where $|\psi_B\rangle = \frac{1}{\sqrt{2}}|\downarrow\uparrow - \uparrow\downarrow\rangle$, a two-qubit singlet Bell state, which is decoupled from the resonator, such that the changes of g and resonator dissipation rate will not affect the qubit state. Then, we activate C to increase the dissipation rate of the resonator into the TL to a very large value [62] to emit the single photon. During this process, we can decrease g to the JC regime simultaneously without affecting the system state, and the photon emission can be simply described by the decay of the resonator. Afterwards, the system state becomes $|0\psi_B\rangle$, corresponding to the one-photon solution $|\psi_{\mathrm{JC}}\rangle$ of Eq. (4) for the two-qubit JC model. After an arbitrary time separation, we begin another adiabatic evolution along $|\psi_{\mathrm{JC}}\rangle$ by decreasing g to zero and increasing $\Delta_1 - \Delta_2$ to a nonzero value, which generates $|1\downarrow\downarrow\rangle$, and the single photon can be emitted during this process. After that, the system returns to the initial state $|0\downarrow\downarrow\rangle$ which can be utilized for the next period automatically.

Usually, one excitation process can only emit one photon for deterministic single-photon sources, and the system has to be repumped afterwards, so there is always an unavoidable time separation between the photons emitted. However, with our protocol one pumping process can emit two single photons, since the system reaches the "excited state" of the second emission process automatically after the first photon is emitted, which also provides a controllable time separation of the photons.

The qubit frequencies [63–66] and couplings [67,68] are tunable in circuit QED [69,70], even independently [71]. Furthermore, we propose a superconducting circuit design to simulate the two-qubit Rabi and JC models by tuning the proportion between the rotating and counter-rotating terms, shown in the lower part of Fig. 1(d), with a detailed demonstration in Ref. [55]. First, we activate both the rotating and counter-rotating terms in our circuit design to simulate the two-qubit Rabi model. When $|1\psi_B\rangle$ is generated, we increase the dissipation rate of the resonator to release the photon. In this process, the two-qubit singlet $|\psi_B\rangle$ is decoupled from the resonator, so we can decrease the coupling strength $g = g_1 = g_2$ to the JC regime $g < 0.1$ and simulate the JC model, without affecting the system states. After that, another adiabatic evolution along $|\psi_{\rm JC}\rangle$ begins. The qubit frequencies will have better tunability if we use a tunable transmon [63], flux qubit, or fluxonium [64] in our circuit design.

Surprisingly, the adiabatic evolution can be quite fast because the peculiarities of $|\psi_R\rangle$ (see Supplemental Material [55] for a brief description, which includes Refs. [72–74]) and $|\psi_{\rm JC}\rangle$. If $\Delta_{1,2}$ and g evolve linearly as in Ref. [55], then $|1\psi_B\rangle$ can be generated from $|0\uparrow\uparrow\rangle$ along $|\psi_R\rangle$ in $12 \times 2\pi\omega^{-1}$ with fidelity 99.7%. After photon emission, $|1\downarrow\downarrow\rangle$ can be generated from $|0\psi_B\rangle$ along $|\psi_{\rm JC}\rangle$ in $12 \times 2\pi\omega^{-1}$ with fidelity 99.8%. If we choose $\omega = 2\pi \times 3$GHz, then the generation time is 4 ns.

However, the parameters cannot evolve that fast in current circuit QED experiments. E.g., the transmon frequency can be tuned by 1 GHz in 10–20 ns [70], so we consider this limit in our numerical simulation, although in principle the flux qubit can be faster tuned [64]. We simulate the process described above using master equations [55] according to real circuit QED experiment, with the resonator frequency $\omega = 2\pi \times 3$ GHz and intrinsic lifetime $T_1 = 5$ μs [62]. The dissipation rate into the TL can be tuned from zero to $\kappa_c = 1/(5\text{ ns})$ in 2 ns [62]. The tunable transmon or flux-qubit frequency is tuned from $2\pi \times 4.41(2\pi \times 1.59)$ to $2\pi \times 3$ GHz [64], with relaxation time $T_1 = 17$ μs [65] and pure dephasing time $\tau_\phi = 17$ μs [63]. We find the Lindbladian master equation and dressed master equation for ultrastrong coupling [75,76] give almost the same result in this specific case [55]. The evolution of different states in a period is shown in Fig. 2(a), with corresponding parameters depicted in Fig. 2(b). The absolute values of the detunings of both qubits with respect to the resonator $|2\Delta_{1,2} - \omega|$ are always the same during the evolution, making its implementation easier. The photon emission rate into the TL is shown in Fig. 2(c), where the time separation between two emitted photons is zero. The first photon has a typical single-sided exponential waveform decaying with the lifetime of the resonator, because the qubit singlet is decoupled from the resonator although the coupling strength

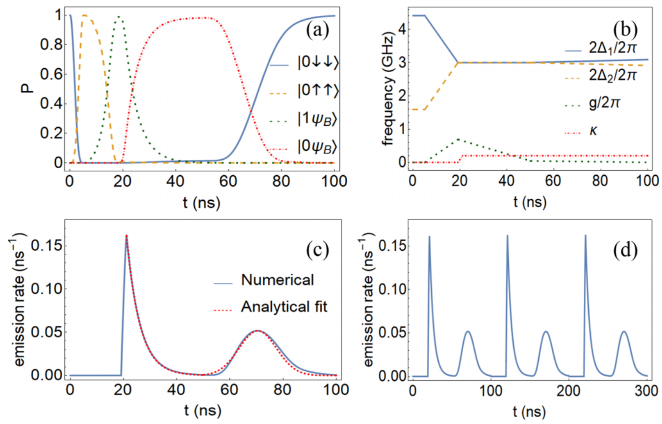

FIG. 2. (a) The evolution of system states during one period, where $|\psi_B\rangle = \frac{1}{\sqrt{2}}|\downarrow\uparrow - \uparrow\downarrow\rangle$. (b) The corresponding evolution of the parameters. (c) The waveforms of the emitted photons. The solid line corresponds to the numerical simulation, fitted by an exponential decaying function $\exp(-\kappa t)$ and a Gaussian function, represented by the dashed line. The emission of the first photon ends at $t = 50$ ns, and the second photon just begins to emit. (d) A sequence of single photons is generated when we repeat the above processes.

is nonzero. The second photon has a Gaussian shape which is optimal for tolerance to mode mismatch [77], because the dissipation rate is a constant and has the same magnitude of the coupling, shown by numerical test. We use an exponential function $\exp(-\kappa t)$ and a Gaussian function to fit the simulated data and find good consistency, as depicted in Fig. 2(c). The system is reset to $|0\downarrow\downarrow\rangle$ automatically after the second photon emission, so we can repeat the above process to obtain a sequence of single photons, as shown in Fig. 2(d).

III. FIGURES OF MERIT

There are commonly three important figures of merit for a single-photon source [78]: efficiency, purity, and indistinguishability. We will study them using numerical simulations. First, the generation and collection efficiency are defined as the photon generated and collected for one excitation pulse, respectively, which is equal to 1 for a perfect single-photon source. Here, two single photons are generated in one excitation process, with emission and collection probabilities (efficiencies), $\kappa_c \int a^\dagger(t)a(t)dt$, both larger than 98%, since the qubits are strongly coupled to the resonator, and their interaction is much faster than the decoherence rate of the qubit. The single photon is generated in the resonator and almost fully collected by the TL through the variable coupler because κ_c can be tuned to be 1000 times the intrinsic decay rate in 2 ns [62]. The emission efficiency is robust with respect to dissipation and decoherence of the resonator and qubits [55].

The second requirement for a single-photon source is high purity, which means exactly one photon is emitted at a time. It can be characterized by the second-order correlation functions in the Hanbury-Brown-Twiss (HBT) experiment:

$$G^{(2)}(t,\tau) = \langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle,$$
$$G^{(2)}(\tau) = \int G^{(2)}(t,\tau)dt, \tag{6}$$

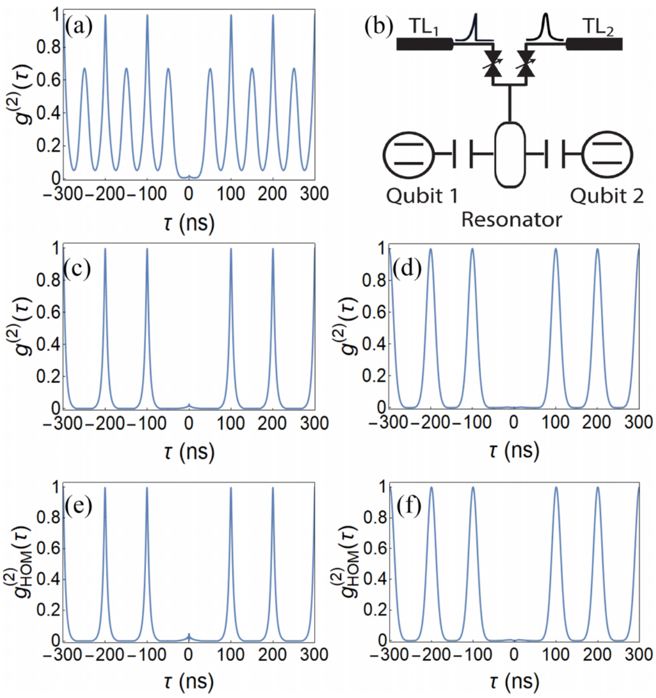

FIG. 3. (a) The second-order correlation function in HBT experiment for the single-photon source proposed above, which characterizes its purity. (b) Our scheme to collect two single photons separately. (c) The second-order correlation function in HBT experiment for the first single-photon source. (d) The second-order correlation function in HBT experiment for the second single-photon source. (e) The second-order correlation function in Hong-Ou-Mandel (HOM) experiment for the first single-photon source, which characterizes its indistinguishability. (f) The second-order correlation function in HOM experiment for the second single-photon source. All these correlation functions are obtained from numerical simulations and normalized to the highest peak.

where the former represents the probability of detecting a photon at time t and another one at time $t+\tau$, and the later is its integration over t, as shown in Fig. 3(a). We have normalized $G^{(2)}(\tau)$ to the highest peak at $\tau = T_{\mathrm{period}} = 100$ ns to obtain $g^{(2)}(\tau)$. There is a strongly suppressed $g^{(2)}(0) \approx 0.0225$, giving a clear evidence of the single-photon emission. $g^{(2)}(\tau)$ has the same period as the single-photon source, and two kinds of peaks other than $\tau = 0$, because the waveforms of photons in a period are not identical. The first peak arises at $\tau \approx 50$ ns, corresponding to the coincidence detection of the first and the second photons in a period, and the coincidence detection of the second photon and the first photon from the next period. The highest peak arises clearly at $\tau \approx 100$ ns $= T_{\mathrm{period}}$ because of the periodicity.

The third requirement is high indistinguishability, which means the photons must be identical in all degrees of freedom, to ensure the successful implementation of the two-photon gate through interference. The Hong-Ou-Mandel (HOM) experiment [79] is used to characterize this notion, where two photons are interfered on a $50:50$ beam splitter. If they are completely indistinguishable, they will coalesce and exit through the same beam splitter output. Here two single photons are emitted in one period with different temporal waveforms which may be made identical with error less than 1% by the photon waveform reshaping technology [80], to achieve high indistinguishability, so as to implement the high-fidelity two-photon controlled-NOT gate. Since the time separation between these two single photons is arbitrary, we have much freedom to choose the delay time between the control and target photons.

To make better use of this single-photon source, we can add another output channel to the resonator with a variable coupler to collect these two single photons separately, as shown in Fig. 3(b), so that the photons in each channel are indistinguishable, and with only one excitation process in a period we efficiently generate two sequences of indistinguishable single photons. The first one has a single-sided exponential waveform. Its simulated $g^{(2)}(\tau)$ is shown in Fig. 3(c), with $g^{(2)}(0) \approx 0.03$. The second one has a Gaussian shape, whose $g^{(2)}(\tau)$ is shown in Fig. 3(d), with $g^{(2)}(0) \approx 0.001$. They can be applied in different situations. We also simulated the normalized HOM experiment counts $g^{(2)}_{\mathrm{HOM}}(\tau)$ of photon detection in different outputs at time delay τ. Typical small peaks around $\tau = 0$ arise in Figs. 3(e) and 3(f), since the decoherence of the qubits causes random photon frequency differences. The indistinguishability is defined by dividing the area of the peaks around $\tau = 0$ by that of the uncorrelated peak around $\tau = T$, and subtracting this number from unity. This quantity can also be calculated as [33]

$$I = \frac{\int_0^\infty dt \int_0^\infty d\tau |\langle a^\dagger(t+\tau)a(t)\rangle|^2}{\int_0^\infty dt \int_0^\infty d\tau \langle a^\dagger(t+\tau)a(t+\tau)\rangle\langle a(t)^\dagger a(t)\rangle}. \quad (7)$$

Because of periodicity, we only need to consider the integration over one period [81]. The indistinguishability I obtained from both methods reaches 95.2% for the first photon and 99.2% for the second photon.

Another way to generate indistinguishable photons is to turn off the coupler C during the adiabatic evolution from $|0\psi_B\rangle$ to $|1\downarrow\downarrow\rangle$, so the second photon will be generated inside the resonator and then emitted into the TL when C is turned on, just like the first photon. These two single photons will clearly be indistinguishable. Tuning C will also not introduce too much error, because the qubits are either in a singlet Bell state or under the condition $g = 0$ when we are tuning C.

IV. ULTRAFAST GENERATION OF THE SINGLE PHOTON WITH THE ASSISTANCE OF THE STARK SHIFT

Another requirement for the single-photon source is high speed; meanwhile, the most prominent advantage of the ultrastrong coupling is the possibility of ultrafast state generation, which can be realized here by adding Stark shift terms [82–85] to the two-qubit quantum Rabi model

$$H_{\mathrm{RS}} = \omega a^\dagger a + g_1\sigma_{1x}(a+a^\dagger) + g_2\sigma_{2x}(a+a^\dagger) + \Delta_1\sigma_{1z} + \Delta_2\sigma_{2z} + U_1 a^\dagger a\sigma_{1z} + U_2 a^\dagger a\sigma_{2z}, \quad (8)$$

as proposed in Ref. [59] for the ultrafast generation of the single-photon multimode W states, where U_1 and U_2 are couplings of the Stark terms. The photon frequency is shifted by $U_1\sigma_{1z} + U_2\sigma_{2z}$, so the stability of the system requires $U_1 + U_2 \leqslant \omega$. H_{RS} has a $\mathbb{Z}_2$ symmetry generated by

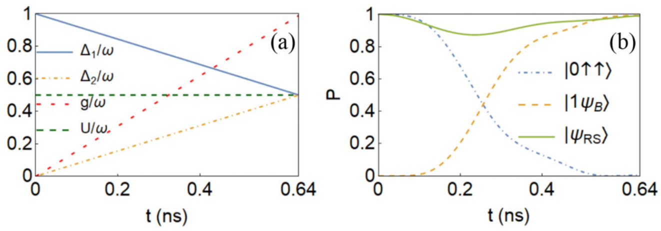

FIG. 4. (a) The change of parameters during the adiabatic evolution from $|0\uparrow\uparrow\rangle$ to $|1\psi_B\rangle$ along $|\psi_{\rm RS}\rangle$, where $U = U_1 = U_2 = 0.5\omega$. (b) The corresponding states population.

$\exp(i\pi a^\dagger a)\sigma_{1z}\sigma_{2z}$. We have found a similar special dark state [59] for even parity:

$$|\psi_{\rm RS}\rangle = \frac{1}{\mathcal{N}}[(\Delta_1 - \Delta_2 + U_1 - U_2)|0\uparrow\uparrow\rangle + g(|1\downarrow\uparrow\rangle - |1\uparrow\downarrow\rangle)] \tag{9}$$

with constant energy $E = \omega$ in the whole coupling regime, under the same condition $\Delta_1 + \Delta_2 = \omega$ and $g = g_1 = g_2$ as for H_R [Eq. (8)], which can be used to accelerate the adiabatic evolution [59], since the energy gap between $|\psi_{\rm RS}\rangle$ and its closest eigenstates is enlarged by the Stark terms. E.g., $|2\downarrow\downarrow\rangle$ is degenerate with $|\psi_R\rangle = |0\uparrow\uparrow\rangle$ at $g = 0$ for H_R, but the stark terms will reduce the energy of the former by $U_1 + U_2$, so these two energy levels are separated in the spectrum. If the parameters evolve as in Fig. 4(a), $|1\psi_B\rangle$ can be generated from $|0\uparrow\uparrow\rangle$ through an adiabatic evolution along $|\psi_{\rm RS}\rangle$ with fidelity 99.3%, operating at a time $t = 0.64$ ns $(1.9 \times 2\pi\omega^{-1})$, proportional to the inverse of the resonator frequency $\omega/2\pi = 3$ GHz, which is a sign of ultrafast generation [86], as shown in Fig. 4(b). This high adiabatic speed is due to the peculiarities of the dark state $|\psi_{\rm RS}\rangle$ [59]. Note that many other choices of U bring almost the same speed, e.g., $U_1 = U_2 = 0.45\omega$. If $U_1 = 0.66\omega$ and $U_2 = 0.33\omega$, the generation can be further accelerated. Following the same procedure detailed above, a faster single-photon source can be implemented.

V. CONCLUSION

We have proposed a scheme for a deterministic single-photon source that can work in the ultrastrong-coupling regime, that is, through two consecutive adiabatic transfers along the one-photon solutions of the two-qubit Rabi model and JC model, respectively. An important advantage of our scheme is that one pumping process can emit two deterministic single photons with an arbitrary time separation that is easily controlled. Furthermore, with our protocol, the system evolves naturally to the initial state of the next period after photon emission. We characterize our single-photon source by calculating efficiency, purity, and indistinguishability numerically, showing that all of them can reach near-unity values. Moreover, by introducing Stark shift terms, we can accelerate the speed of the single-photon generation to a degree proportional to the inverse of the resonator frequency. Our scheme paves the way for the application of ultrastrong coupling in fast computation and deterministic state generation.

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