Erratum: Hybrid completely positive Markovian quantum-classical dynamics [Phys. Rev. A 107, 062206 (2023)]

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Errors in the above-named paper are hereby corrected. Contrary to the claim made in Sec. IV, preceding Eq. (36), this HME is not yet covariant under general coordinate transformations but under the linear ones. For the general covariant form, the scalar hybrid density and the co(ntra)variant drift vector should be used via two respective replacements:

$$\hat{\rho} \to \frac{1}{\sqrt{g}}\hat{\rho},$$

$$V^n \to V^n + \frac{1}{2}\partial_m\big(\sqrt{g}D_{\mathrm{C}}^{nm}\big),$$

where $g = 1/\det D_{\mathrm{C}}$. With these replacements the covariant (but not general covariant) Fokker-Planck structure in Eq. (36) becomes identical to the general covariant one:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H},\hat{\rho}] + D_{\mathrm{Q}}^{\beta\alpha}(\hat{L}_\alpha\hat{\rho}\hat{L}_\beta^\dagger - \mathbb{H}\hat{L}_\beta^\dagger\hat{L}_\alpha\hat{\rho}) + \frac{1}{2\sqrt{g}}\partial_n(\sqrt{g}D_{\mathrm{C}}^{nm}\partial_m\hat{\rho}) + \frac{1}{\sqrt{g}}\partial_n\big[\sqrt{g}\big(\overline{G}_{\mathrm{CQ}}^{n\alpha}\hat{L}_\alpha\hat{\rho} + \mathrm{H.c.} - V^n\big)\hat{\rho}\big], \tag{36a}$$

where we recognize the first- and second-order covariant derivations in a Riemann space of contravariant metric tensor D_{C}^{nm}. The general covariant unraveling of this HME could be obtained from the covariant unraveling (49) and (50) of the covariant HME (36). The two Ito type SDEs should be changed for the equivalent Stratonovich ones and the additional constraint $\partial_n(\sqrt{g}dW^n) = 0$ will have to be imposed [1]. All references to the covariant HME concern invariably Eq. (36) except the reference to the "explicitly covariant" HME in Sec. V, which should be understood for the general covariant (36a) and not for the covariant (36).

The following typographic mistakes of the original paper have to be corrected.

The expression $\delta(x-y,\epsilon) + \delta(y-x,\epsilon)$ in Eq. (27) should be $\delta(x^n-y^n,\epsilon) + \delta(y^n-x^n,\epsilon)$.

Equation (47) should be the trace of Eq. (36) and not of Eq. (35). Hence the correct diffusion term in Eq. (47) is $\frac{1}{2}\partial_n\partial_m[D_{\mathrm{C}}^{nm}(x)\rho(x)]$.

The frictional Hamiltonian in Eq. (48) is called anti-Hermitian incorrectly; in fact it has a Hermitian part as well. The operators $\hat{L}_\alpha, \hat{L}_\beta$ were mistakenly interchanged; the correct form reads

$$-i\hat{H}_{\mathrm{fr}}(x) = -\tfrac{1}{2}D_{\mathrm{Q}}^{\beta\alpha}\{[\hat{L}_\beta^\dagger(x) - \langle\hat{L}_\beta^\dagger(x)\rangle][\hat{L}_\alpha(x) - \langle\hat{L}_a(x)\rangle] + [\langle\hat{L}_\beta^\dagger(x)\rangle\hat{L}_\alpha(x) - \mathrm{H.c.}]\}. \tag{48}$$

There is a pair of correlated sign errors in Eqs. (49) and (56). The plus sign of $dW^n(x)$ in Eq. (49) has to be a minus sign. The minus sign of the right-hand side in the last line of Eq. (56) has to be a plus. Equation (56), with the correct diffusion term and all z dependences made explicit, should be

$$\begin{aligned}
\mathbb{M}d\hat{P}\delta(z-x) &= -i[\hat{H}(z),\hat{\rho}(z)]dt + D_{\mathrm{Q}}^{\beta\alpha}(z)[\hat{L}_\alpha(z)\hat{\rho}(z)\hat{L}_\beta^\dagger(z) - \mathbb{H}\hat{L}_\beta^\dagger(z)\hat{L}_\alpha(z)\hat{\rho}(z)]dt,\\
\mathbb{M}\hat{P}d\delta(z-x) &= \tfrac{1}{2}\partial_n\partial_m\big[D_{\mathrm{C}}^{nm}(z)\hat{\rho}(z)\big]dt - \partial_n\big[V^n(z)\hat{\rho}(z) - 2\mathbb{R}\mathrm{e}\overline{G}_{\mathrm{CQ}}^{n\alpha}(z)\langle\hat{L}_\alpha(z)\rangle\hat{\rho}(z)\big]dt,\\
\mathbb{M}d\hat{P}d\delta(z-x) &= \partial_n\big\{\overline{G}_{\mathrm{CQ}}^{n\alpha}(z)[\hat{L}_\alpha(z) - \langle\hat{L}_\alpha(z)\rangle]\hat{\rho}(z) + \mathrm{H.c.}\big\}dt.
\end{aligned} \tag{56}$$

Below Eq. (56) the original paper states that $\mathbb{M}\hat{P}\delta(z-x)$ satisfies the HME (35) but the correct number is (36).

The expressions $[\hat{L}_\alpha(x) - \langle\hat{L}_\alpha(x)\rangle]\hat{P}\,d\overline{\xi}^\alpha(x) + \mathrm{H.c.}$ in Eq. (54), $[\hat{L}_\alpha(x) - \langle\hat{L}_\alpha(x)\rangle]\hat{\sigma}\,d\overline{\xi}^\alpha(x) + \mathrm{H.c.}$ in Eq. (59), and $[\hat{L}_\alpha(x) - \langle\hat{L}_\alpha(x)\rangle]\hat{\sigma}\,d\overline{\xi}^\alpha(x) + \mathrm{H.c.}$ in Eq. (63) should have a factor dt each.

Apart from the sporadic typographic errors, only Sec. IV required a limited change of content, which makes the original results and conclusions of the paper valid as written.

[1] L. Diósi, The covariant Langevin equation, arXiv:2310.17314.

