Reply to "Comment on 'Multitime quantum communication: Interesting but not counterfactual' "

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This is a response to comments and criticisms found in the preceding Comment [Phys. Rev. A **108**, 056201 (2023)] by Vaidman on the paper in Phys. Rev. A **107**, 062219 (2023).

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A significant part of Vaidman's Comment [1] is devoted to a discussion of *counterfactuals*, starting with a quotation from Penrose. The use of counterfactuals in discussions of quantum foundations in fact goes back much earlier (see, e.g., [2]). The basic idea involved in a counterfactual is a comparison between two (or more) situations: one the "actual" world and the other the "counterfactual" world, which differs from the former in certain specified ways. One then considers various consequences of these differences. In quantum theory this can lead to difficulties and paradoxes when the physical properties of interest in the two worlds are represented by incompatible observables or noncommuting projectors. The consistent histories (CH) approach to quantum theory avoids such paradoxes by refusing to compare incompatibles (see [3] and Chap. 19 of [4]). For the way in which CH resolves the (supposedly) interaction-free measurement paradox mentioned by Vaidman, see Chap. 21 of [4].

In [5], the paper addressed by Vaidman's Comment, it is argued that the claim of counterfactual communication by Salih *et al.* in [6] fails in that it incorrectly assigns a probability for a photon to be in the communication channel connecting Alice and Bob at intermediate times when quantum interference effects are important, as well as incorrectly counting the number of times it passes through the channel. While both errors are significant, the first is more interesting in that it raises the question of what can properly be said about a quantum particle's location at an intermediate time given a wave function evolving unitarily from an initial state on its way to a later measurement.

In Hilbert space quantum theory, which is to say, the basic principles set forth by von Neumann [7] (see in particular Sec. III.5 therein), a physical property is represented by a projector P on a Hilbert subspace. In particular, if the property is that the particle is in some spatial region R, the projector P applied to the position wave function $\psi(\mathbf{r})$ leaves it unchanged for all $\mathbf{r} \in R$, but sets it equal to zero elsewhere. Consequently, if $\psi(\mathbf{r})$ is nonzero for $\mathbf{r}$ in both some region inside R and also some other region outside R, the projector $|\psi\rangle\langle\psi|$ corresponding to the particle's wave function (assumed normalized) does not commute with P, and when projectors do not commute—this is the essence of quantum uncertainty principles—there is no meaningful way to discuss whether or not the particle is in R. A well-known example is the double-slit paradox where, in the presence of interference, one cannot meaningfully say which slit the photon passed through. For this reason the CH interpretation of quantum mechanics considers the conjunction of two properties represented by noncommuting projectors to be meaningless: To say the particle is in or outside R makes no more sense than to discuss whether S_x is $+1/2$ of $-1/2\hbar$ for a spin-1/2 particle when S_z is $+1/2\hbar$.

Vaidman tries to get around this difficulty by asking whether a quantum particle leaves a trace of its presence at a particular location via a weak interaction with some other physical system at this location. That such a weak measurement does not resolve the problem but simply generates more paradoxes was known to Feynman; see his discussion in [8] of a weak light source following the double slit, in his case the two holes with a coherent electron wave passing through them. For an analysis of this situation based on consistent quantum principles, see Sec. 13.5 of [4]. Vaidman's nested Mach-Zehnder paradox [9] has the same general character. Its resolution when weak measurements are analyzed using consistent quantum principles [10] was not discussed in Vaidman's Comment [11] on that paper. The objections to the nested Mach-Zehnder paradox by Englert *et al.* in [12] are similar: They argue that one cannot assign a probability to a particle's following a particular path when it is in a coherent superposition of amplitudes on different paths.

Towards the end of [1] Vaidman discusses the use of a quantity called *Cost*, used in [5] as a measure of channel usage. In response it may be noted that Cost was introduced as a replacement for the misleading use of "probability" in [6], as in much of the succeeding literature. In its favor is the fact that Cost is a well-defined mathematical quantity in situations where probabilities cannot be consistently assigned, and its use leads to the rigorous bound in Sec. III D of [5], probably the most interesting technical result in that paper. However, as with any novel idea, only the future will show whether Cost is really useful or needs to be replaced by something else. Vaidman's concern that the analysis using Cost includes both cases in which a communication protocol succeeds as well as

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when it fails is not relevant to the situation considered in [6], where the protocol always succeeds with high probability, an instance of what is called a *full* protocol in Sec. III C of [5]. The contrasting case of *partial* protocols, such as when by convention the nonarrival of a photon in Alice's apparatus at a particular time signals that Bob has transmitted the bit 1, requires a separate discussion, which might be a useful subject for some future paper.

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