Mass of a weakly measured photon

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Bohmian mechanics has garnered significant attention as an interpretation of quantum theory since the paradigmatic experiments by [Kocsis *et al.*, Science **332**, 1170 (2011)] and [Mahler *et al.*, Sci. Adv. **2**, e1501466 (2016)], which inferred the average trajectories of photons in the nonrelativistic regime. These experiments were largely motivated by Wiseman's formulation of Bohmian mechanics, which grounded these trajectories in weak measurements. Recently, Wiseman's framework was extended to the relativistic regime by expressing the velocity field of single photons in terms of weak values of the photon energy and momentum. Here, we propose an operational, weak value-based definition for the Bohmian "local mass" of relativistic single particles. For relativistic wave functions satisfying the scalar Klein-Gordon equation, this mass coincides with the effective mass defined by de Broglie in his relativistic pilot-wave theory, a quantity closely connected with the quantum potential that is responsible for Bohmian trajectory self-bending and the anomalous photoelectric effect. We demonstrate the relationship between the photon trajectories and the mass in an interferometric setup.

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I. INTRODUCTION

Quantum mechanics is a useful framework for the prediction of measurement outcomes for systems delicate enough to be affected by the act of measurement [1]. While accurately predicting the probabilities of the possible measurement outcomes and their change over time [2], the theory is bound to uncertainty. Furthermore, it has been proven that a noncontextual theory that predicts observations with certainty cannot exist [3,4]. Although out of the ordinary theory's scope, the description of the individual quantum event has been the subject of intense research, the different aspects of which (ontological and epistemological) have been captured and scrutinized in the de Broglie–Bohm pilot wave theory and the weak measurement formalism, respectively. While the Bohmian theory suggested a hidden deterministic dynamics for the point-like corpuscle, accommodating a quantum description with the concept of particle trajectories therewith, weak measurement as an experimental technique allows for the probing of quantum systems with almost negligible perturbation to the wave function. Merging the two, Wiseman showed [5] how one can, in principle, obtain the "Bohmian" velocity field (with an additional assumption of determinism) and the trajectories of nonrelativistic particles using weak measurements. The utility of his framework was demonstrated in a few notable experiments by Kocsis *et al.* and Mahler *et al.* [6,7], where the average trajectories of nonrelativistic particles were constructed using the weak values of the transversely slow component of the photon wave vector via weak measurements. The interpretation of these experiments has been subject to interesting debate; for example, the authors of Ref. [8] argued that the observed effects were purely classical. More recently, Foo *et al.* [9] reformulated Wiseman's nonrelativistic weak value prescription to describe the relativistic Bohmian trajectories of photons using weak values. The authors defined the velocity of a Bohmian relativistic particle as the weak value of momentum, divided by the weak value of the Hamiltonian, yielding a Lorentz-covariant velocity field expressed in terms of the components of the conserved Klein-Gordon current vector. Grounding relativistic Bohmian mechanics in a measurement framework has opened the way for experimental verification of the resulting deterministic trajectories in this regime.

Here, we build on the weak-value framework of Foo *et al.* by operationally defining the "local mass" of relativistic single photons in terms of weak values of the photon's energy and momentum. Importantly, we demonstrate that this mass is related to the one conjectured by de Broglie in his early attempts at a relativistic pilot wave theory. A measurement of the local mass for scalar Klein-Gordon waves would therefore constitute an experimental observation of de Broglie's effective mass. We demonstrate how Bohmian trajectory bending in a single-photon interferometric setup can be understood in terms of the variable local mass that the particle acquires.

The rest of the paper is organized as follows. In Sec. II we review the theory of weak values and weak measurements. In Sec. III we review relativistic Bohmian mechanics. In Sec. IV we construct our weak-value-based operational definition for the local photon mass and apply this to trajectories within a Michelson-Sagnac-type interferometer, simulating the trajectories in various configurations. We conclude with some final remarks in Sec. V. For historical reasons, we keep factors of $\hbar$ and c in Secs. II and III and use natural units $\hbar = c = 1$ in Sec. IV for calculational simplicity.

II. WEAK VALUES

A well-understood and experimentally verified feature of quantum mechanics is that a given system, prepared in a certain initial state, might end up in several different final states after measurement [10]. At best, by repeatedly preparing and measuring the system, one can infer the probabilities of the possible outcomes. Therefore, ordinary quantum theory restricts itself to the prediction of these probabilities and the expression of their change over time. However, as noted by Aharanov, Bergmann, and Lebowitz [11], the uncertainty inherent in the outcomes of quantum measurements is inconsistent with the principle of time symmetry. While initial states of quantum systems are definite and predetermined, their final state is uncertain and probabilistic. Following this observation, the authors developed in their paper a time-symmetrized formulation of quantum mechanics. In the theory, conditions are arranged such that time symmetry with respect to preparation and measurement of a quantum system is established. Accomplished in ensembles defined by postselection as well as preselection (where both the initial and final states of the system are predetermined), time symmetrization entails fascinating opportunities for the research of extremely delicate quantum phenomena often publicized as "paradoxes" [12–18] and has also been useful in the analysis of counterfactual communication protocols [19], among other applications. An interesting property of time-symmetrized systems is the weak value [20], which is a measurable quantity uniquely assigned to such systems as they evolve from their initial to their final state. Expressed as

$$\langle \hat{A}_w \rangle = \frac{\langle \phi|\hat{A}|\psi\rangle}{\langle \phi|\psi\rangle}, \tag{1}$$

where $|\psi\rangle$ is the initial and $\langle\phi|$ the final states of the system and $\hat{A}$ the quantum observable, weak values characterize a quantum system without collapsing its wave function. In particular, weak values generalize the concept of an expectation value to pre and postselected ensembles, are not restricted to the operator's spectrum, and may, in general, be complex. Moreover, weak values can, in principle, be calculated and measured using standard quantum mechanics. In such a framework, they are interpreted as properties of subensembles of identically prepared systems devoid of deeper physical meaning. The association of the weak value in the work of Aharanov, Albert, and Vaidman [20,21] with a measurement procedure known as weak measurement has allowed to view them as a feature of a single quantum particle in the period of time between two strong (ordinary) quantum measurements. Particularly useful in sensing applications [22,23] where it enables the amplification of minuscule quantum effects, weak measurement is achieved by weakening the coupling between the pointer and system, preventing significant perturbation of the system and the subsequent collapse of the wave function. The final state of the pointer is the result of a weak measurement, corresponding to the real part of a weak value, collecting information about the measured system without destroying coherence. Interestingly, weak values have recently been applied to advocate an ontological model for the existence of negative-mass particles [24], facilitating positive-negative pair "counterparticles" at the vertices of a connected graph. While the mass we analyze in this paper has its origin elsewhere (relativistic Bohmian mechanics, which we present in the next section), it might be interesting to ask whether a connection between the two notions exists. Let us now introduce the theory of relativistic Bohmian mechanics.

III. RELATIVISTIC BOHMIAN MECHANICS

Another theory accounting for individual quantum events is Bohmian mechanics [25–28]. The theory uses the wave function to construct a velocity field for the quantum particle in a way that is consistent with the predictions of standard quantum theory. This construction allows quantum mechanics to accommodate the notion of trajectory, implying that all dynamics can be described deterministically. To construct the velocity field, the particle's "Bohmian momentum," that is, the time derivative of its deterministic position multiplied by mass, is assumed to be proportional to the phase gradient of the quantum wave, known in wave optics as the local wave vector [29,30], defined as

$$\vec{k}_{\text{local}} \equiv \text{Im}\{\nabla \ln \psi\}. \tag{2}$$

The identification of the local wave vector with Bohmian momentum leads to the so-called Bohmian guidance equation

$$m\frac{d\vec{x}}{dt} = \hbar \text{Im}\{\nabla \ln \psi\}. \tag{3}$$

Since the initial position of the particle is hidden (in the sense that it is always ontologically well defined but cannot be empirically determined without introducing a significant perturbation to the wave function, which, in turn, affects the local velocity field), the equation yields a family of possible solutions (a set of trajectories). The guidance equation requires the specification of wave-function dynamics. In the nonrelativistic quantum case it is given by the usual Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi, \tag{4}$$

where m is the mass and V the potential term to-be-distinguished from the Bohmian "quantum potential" (see below). The dynamics of the Bohmian momentum can be extracted from the Schrödinger equation by means of a polar transformation of the wave function. Written as

$$\psi = Re^{iS/\hbar}, \tag{5}$$

this ansatz transforms the single complex-valued equation into a coupled pair of real-valued equations, a continuity equation,

$$\frac{\partial(R^2)}{\partial t} + \nabla\left(R^2\frac{\nabla S}{m}\right) = 0, \tag{6}$$

and a Hamilton-Jacobi-like equation,

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = 0. \tag{7}$$

According to the identification in (3), the two equations specify the dynamics of the local momentum

$$\vec{p} = \nabla S. \tag{8}$$

The second equation is called the quantum Hamilton-Jacobi equation. It differs from its classical counterpart by an extra term called the quantum potential [31]

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}. \tag{9}$$

In Bohmian mechanics, this term is interpreted as an extra potential that governs the dynamics of particles and marks the departure from classical theory. The resulting quantum force (gradient of the quantum potential) is seen, in Bohmian theory, as mediating the influence of the wave on the particle, guiding it along its otherwise classical trajectory. In the Bohmian interpretation, the quantum potential accounts for all quantum phenomena. Moreover, the amplitude of the quantum potential need not decay with distance, a property that accounts for the nonlocality of quantum phenomena in the Bohmian perspective, and is also a necessary feature of the theory *a la* Bell's theorem [32,33].

A suitable relativistic generalization of the local wave vector (2) is a simple replacement of the gradient with a covariant derivative, yielding a local four-frequency

$$k^\mu \equiv \mathrm{Im}\{\partial^\mu \ln\psi\}, \tag{10}$$

which is proportional to the relativistic four-momentum of the particle in the context of quantum mechanics. Since relativistic invariant mass is the norm of the energy-momentum four-vector, a definition of "local mass" for relativistic scalar waves was proposed in [34], where it was defined as

$$m_{\mathrm{local}}^2 \equiv \frac{\hbar^2}{c^2}k_\mu k^\mu. \tag{11}$$

Such a mass, proportional to the norm of the local four-frequency, is Lorentz invariant by construction.

In the Bohmian formulation of the massless scalar Klein-Gordon equation

$$\Box\psi = 0, \tag{12}$$

the polar decomposition of the wave function (5) uncovers the dynamics of the local four-momentum. The procedure leads to a relativistic conservation equation

$$\partial_\mu(R^2\partial^\mu S) = 0, \tag{13}$$

where the conserved four current is given by

$$j^\mu = R^2\partial^\mu S, \tag{14}$$

along with the relativistic quantum Hamilton-Jacobi equation

$$(\partial_\mu S)(\partial^\mu S) = -\hbar^2\frac{\Box R}{R}, \tag{15}$$

which is a relativistic Hamilton-Jacobi equation for a massless particle with an extra term which is a simple relativistic generalization of the quantum potential.

Historically, a similar treatment of the scalar Klein-Gordon equation led de Broglie to define an effective mass [35] for the relativistic Bohmian particle in which its (classical) mass squared and its quantum potential enter on an equal footing. For a massless particle, the effective mass squared gives rise to a relativistic generalization of the quantum potential

$$m_{\mathrm{eff}}^2 \equiv -\frac{\hbar^2}{c^2}\frac{\Box R}{R}. \tag{16}$$

With this definition, the quantum Hamilton-Jacobi equation stemming from the scalar Klein-Gordon equation (15) can be elegantly rewritten as

$$m_{\mathrm{local}}^2 = m_{\mathrm{eff}}^2. \tag{17}$$

The crucial difference between the two masses is that the local mass is defined for any relativistic scalar wave equation while the effective mass is specific to scalar Klein-Gordon waves as defined by de Broglie. De Broglie used the effective mass to explain some objectionable properties of the Klein-Gordon equation in the Bohmian perspective. First, the conserved four-current (14) might be negative in some regions of spacetime, even for positive frequency solutions [36–40]. Writing the relativistic guidance equation as the simplest relativistic generalization of its classical counterpart [41], we have

$$\frac{dx^\mu}{ds} \propto j^\mu, \tag{18}$$

where s is an affine parameter, x^μ the position four-vector, and j^μ the four current defined in (14). When the time component of the current is negative, the particle moves backwards in time. As a consequence, such particles might assume several positions simultaneously and move faster than c. Now, with the definition in (16), writing the local mass explicitly in terms of the local momentum and energy, we arrive at the modified Einstein relation for photons

$$E^2 = p^2c^2 + m_{\mathrm{eff}}^2c^4. \tag{19}$$

Since the effective mass squared term can be negative, not only can the photon acquire a mass, but it might also be imaginary. This is compatible with the anticipated tachyonic behavior of relativistic particles in Bohmian mechanics. We note that this peculiarity is not in strict contradiction with standard quantum mechanical observations [36] since it is not concerned with the individual quantum event and its ontological status. However, the treatment and interpretation of tachyonic trajectories is unavoidable within a Bohmian ontology. Nevertheless, our proposed measurement scheme (as an extension to that proposed in Ref. [9]) provides a natural framework for understanding the unusual properties of these trajectories and their compatibility with relativity. Finally, it should be noted there exist experimental evidence for the existence of the effective photon mass and its effects. In Ref. [42], the relationship between the effective mass and the quantum potential was proposed as an explanation for the anomalous photoelectric effect, whereby anomalous photoelectric emission and gas photoionization by light was observed for single photons whose energy was lower than the work function of the material. The effect provided an experimental hint at the existence of the local mass and further motivated us to propose a framework for its direct measurement. Such a framework is outlined in the next section.

IV. OPERATIONAL DEFINITION FOR LOCAL MASS

To give the local mass an operational definition, we use the language of weak values. For the remainder of this section we utilize natural units for simplicity, $\hbar = c = 1$. As suggested in Ref. [9], the Bohmian velocity field of the photons can be constructed in a Lorentz covariant manner by the identification of the local momentum (in the x direction) p_x and energy E, with weak values of the associated quantum operators $\hat{p}_x$ and $\hat{H}$:

$$v(t,x) = \frac{\mathrm{Re}\langle(\hat{p}_x)_w\rangle}{\mathrm{Re}\langle\hat{H}_w\rangle}, \tag{20}$$

where we specialize, without loss of generality, to one spatial dimension. We note that in Ref. [43], an alternative velocity field is proposed in terms of a weak value of the operator $\hat{v} = \hat{k}/(\hat{k}+1)^{1/2}$, where $E(k) = \sqrt{k^2+1}$ is the relativistic dispersion relation (with $m = 1$). However, a connection with a Bohmian interpretation of deterministic particle trajectories is not made.

We assume that the evolution of the state vector $|\psi(t)\rangle$ is described by the Hamiltonian $\hat{H}$ (we choose a specific form of the first when plotting trajectories in Sec. IV), while postselection of the photons occurs at the position x in the position eigenstate

$$|x\rangle = \int dk\, e^{-ikx}|k\rangle. \tag{21}$$

From a field-theoretic point of view, (21) should be understood as the limit of a highly resolved (in space) projection onto position in a particular reference frame. Inspired by the weak value prescription given in Eq. (20), we use the connection between local quantum properties and weak values to write the local mass in (11) as

$$m_{\mathrm{local}}^2 \equiv (\mathrm{Re}\langle\hat{H}_w\rangle)^2 - (\mathrm{Re}\langle\vec{p}_w\rangle)^2. \tag{22}$$

This statement is an operational definition for the local mass of a relativistic quantum particle. We stress that this expression does not depend on the dynamical equation satisfied by the wave function, nor does it contain a specific form of the operators that are to be weakly measured. In what follows we implement the definition for scalar Klein-Gordon waves, understanding that this is a simplification of a full vector-valued electrodynamical theory describing photons. Nevertheless, it is pertinent to study the scalar theory since the local mass coincides with de Broglie's effective mass. As suggested in Ref. [9], we can utilize the differential forms generated from the Klein-Gordon Hamiltonian and momentum to explicitly evaluate the weak values in Eq. (20):

$$\langle\hat{H}_w\rangle = \frac{\langle x|\hat{H}|\psi(t)\rangle}{\langle x|\psi(t)\rangle} = \frac{(-i\hbar)\langle x|(\partial/\partial t)|\psi(t)\rangle}{\langle x|\psi(t)\rangle}, \tag{23}$$

$$\langle(\hat{p}_x)_w\rangle = \frac{\langle x|\hat{p}|\psi(t)\rangle}{\langle x|\psi(t)\rangle} = \frac{i\hbar\langle x|(\partial/\partial x)|\psi(t)\rangle}{\langle x|\psi(t)\rangle}. \tag{24}$$

In analogy to the weak-value velocity field, our weak-value mass can be understood as a "classical" expectation constructed from repeated weak measurements performed on an ensemble of identically prepared particles. According to (17), when the particles are described by the scalar Klein-Gordon equation, the local mass (which is a general property of relativistic waves) is equal to the effective mass (which is unique to de Broglie's Bohmian treatment of Klein-Gordon waves). When the particle is massless, the effective mass squared is proportional to the relativistic quantum potential, the expression of which is given in (16). In what follows we restrict ourselves to the later case. Using the identifications in (23) and (24), we can express the effective mass in terms

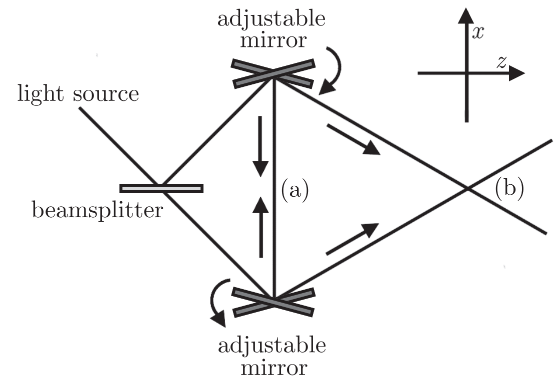

FIG. 1. Schematic diagram of the interferometric setup, reproduced from Ref. [9].

of the Klein-Gordon conserved current, $j(t,x)$ and conserved current density $\rho(t,x)$:

$$m_{\mathrm{eff}}^2 = \frac{1}{|\psi(t,x)|^4}[\rho^2(t,x) - j^2(t,x)], \tag{25}$$

where $\rho(t,x) \equiv j^0(t,x)$, $j(t,x) \equiv j^1(t,x)$ are the Klein-Gordon conserved current density and conserved current, respectively, with $j^\mu = 2\mathrm{Im}\psi^\star(t,x)\psi(t,x)$. By defining the "(squared) effective mass density"

$$\bar{m}_{\mathrm{eff}}^2 = m_{\mathrm{eff}}^2|\psi(t,x)|^4, \tag{26}$$

and utilizing the fact that $\rho(t,x)$ and $j(t,x)$ are components of a conserved two-vector, we find that $\bar{m}_{\mathrm{eff}}^2$ is an invariant quantity under Lorentz boosts, as desired.

Gaussian wavepackets

To plot the time and space dependence of the effective mass, we require an explicit form of the initial state. Let us consider a similar interferometric setup as studied in Ref. [9] displayed in Fig. 1. We consider the initial state to be a Gaussian wavepacket of momenta,

$$|\psi(t)\rangle = \int dk\, e^{-iE(k)t}f(k)|k\rangle, \tag{27}$$

where $E(k) = |k|$ corresponds to the relativistic dispersion for massless particles (noting, again, the standard simplification to the scalar Klein-Gordon theory) $m = 0$, while we assume that the wavepacket itself may be prepared in a superposition of left- and right-moving momenta

$$f(k) = \mathcal{N}\bigg(\sqrt{\alpha}\exp\bigg[-\frac{(k-k_0)^2}{4\sigma^2}\bigg] + \sqrt{1-\alpha}\exp\bigg[-\frac{(k+k_0)^2}{4\sigma^2}\bigg]\bigg), \tag{28}$$

where k_0 is the center frequency of the wavepacket(s) and σ their bandwidth, while $\mathcal{N}$ is a normalization constant and α is a superposition parameter such that $0 \leqslant \alpha \leqslant 1$. Using these ingredients, coupled with the optics approxiation employed in Ref. [9] (wherein the magnitude of the wave vector is much larger than its spread, $k_0 \gg \sigma$), it is possible to compute explicit forms for the conserved current and conserved current density

$$j(t,x) = \beta_R^2 - \beta_L^2 + 2\beta_R\beta_L\mathcal{S}_0, \tag{29}$$

$$\rho(t,x) = \beta_R^2 + \beta_L^2 + 2\beta_R\beta_L\mathcal{T}_0, \tag{30}$$

where we define

$$\mathcal{S}_0 = \frac{2\sigma^2}{k_0}\sin(2k_0x), \tag{31}$$

$$\mathcal{T}_0 = \cos(2k_0x) - \frac{2\sigma^2 x}{k_0}\sin(2k_0x), \tag{32}$$

and

$$\beta_R^2 = \alpha\sqrt{\frac{2}{\pi}}\sigma\exp[-2(t-x)^2\sigma^2], \tag{33}$$

$$\beta_L^2 = (1-\alpha)\sqrt{\frac{2}{\pi}}\sigma\exp[-2(t+x)^2\sigma^2]. \tag{34}$$

The effective mass density, (26), is given by

$$\begin{aligned}\bar{m}_{\text{eff}}^2 =& 4\beta_R^2\beta_L^2\left[1+\mathcal{T}_0^2-\mathcal{S}_0^2\right]\\ &+4\beta_L^3\beta_R\left[\cos(2k_0x)+\frac{2\sigma^2(t-x)}{k_0}\sin(2k_0x)\right]\\ &+4\beta_R^3\beta_L\left[\cos(2k_0x)-\frac{2\sigma^2(t+x)}{k_0}\sin(2k_0x)\right].\end{aligned} \tag{35}$$

We note that, in the natural unit system adopted for our calculations, $\bar{m}_{\text{eff}}$ has units of inverse length squared. In the limits $\alpha \to 0, 1$ (i.e., a solely right- or left-moving Gaussian), the effective mass density vanishes. Likewise for $0 < \alpha < 1$, outside the interference region, $(t \pm x)\sigma \gg 1$, the effective mass is also negligible; the photons are lightlike in this regime. Notably, the effective mass density can become negative (imaginary, upon taking the square root) in certain regions, which corresponds exactly to points at which the tangent to the velocity curve becomes spacelike (the photons becomes tachyonic). This accords with the interpretation of $\bar{m}_{\text{eff}}^2$ as a proxy for the metric length $m_{\text{eff}}^2c^4 = E^2 - p^2c^2$ whose sign corresponds with spacelike, lightlike, and timelike spacetime intervals

$$\bar{m}_{\text{eff}}^2\begin{cases}<0 & \text{spacelike},\\ =0 & \text{lightlike},\\ >0 & \text{timelike}.\end{cases} \tag{36}$$

Indeed, the branch of solutions corresponding to imaginary effective (rest) masses ($\bar{m}_{\text{eff}}^2 < 0$) has long been associated with those describing relativistic tachyonic particles [44,45]. Crucially, our measurement-based construction of the Bohmian velocity field allows, as mentioned, for the existence of "anomalous weak values" through which an imaginary (negative) mass (squared mass) may be inferred from weak measurements of momentum and energy.

In Fig. 2, we plot the relativistic Bohmian trajectories on a spacetime diagram, with the effective mass density $\bar{m}_{\text{eff}}^2$ underlaid. The interference fringes in the effective mass density correspond with those observed in the actual probability density $\rho(t,x)$. Figure 2(a) displays the trajectories for a Gaussian distribution of initial conditions, weighted by the

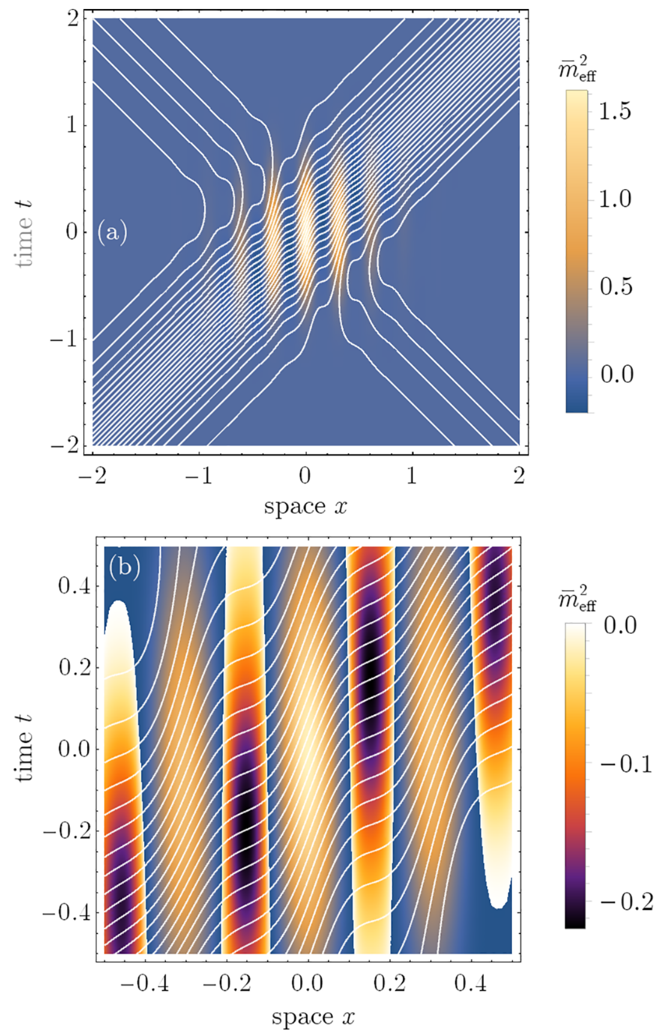

FIG. 2. (a) Plot of the photon trajectories overlaid on the effective mass density $\bar{m}_{\text{eff}}^2$. We use the settings $k_0/\sigma = 10$ and $\alpha = 0.83$. (b) A zoomed-in view of (a), where we distinguish regions of positive $\bar{m}_{\text{eff}}^2$ with negative $\bar{m}_{\text{eff}}^2$ using a different color scheme. Recall that $\bar{m}_{\text{eff}}$ has units of inverse length squared.

Klein-Gordon density $\rho(t,x)$. As discussed, outside the interference region the effective mass density is approximately zero, corresponding to regions where the photon does not feel the influence of the quantum potential. Near the origin of coordinates (where the interference effects between the incident wavepackets become manifest) the trajectories exhibit features of single-particle interference, with the density of trajectories matching the quantum-mechanical probability density. In particular, we observe how, in the regions where the photon acquires a positive effective mass, it decelerates (leading to the bunching of trajectories in these regions), while in regions of destructive interference, the tangent vector to the trajectories become spacelike, Fig. 2(b). This behavior is consistent with the prescription given in (36) for the sign of the effective mass density. Likewise, when $j(t,x) = 0$ (i.e., the weak value of the momentum vanishes), one recovers the mass-energy equivalence for a particle in its rest frame

$$E = m_{\text{eff}} \tag{37}$$

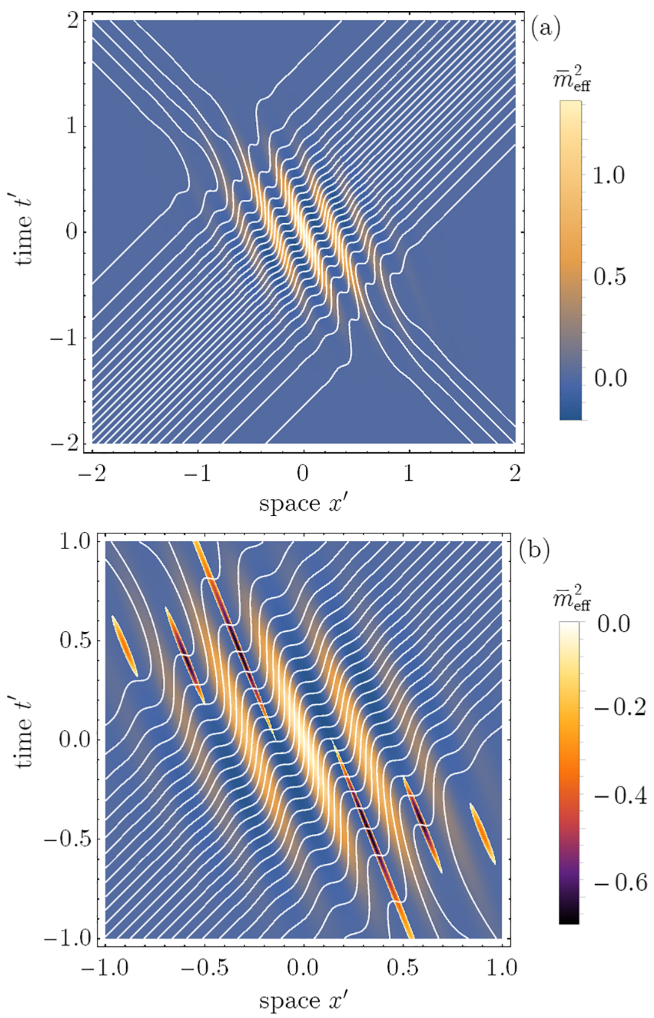

FIG. 3. (a) Plot of the photon trajectories in a boosted reference frame with coordinates (t', x'), overlaid on the effective mass density also plotted in these coordinates. We use the same settings as Fig. 2 with $\vartheta = 0.4$. (b) A zoomed-in view of (a), where the alternate color scheme denotes regions where both $\bar{m}^2_{\text{eff}} < 0$ and $\rho(t, x) < 0$, such that trajectories travel backwards in time.

(recalling that we work in the units where $\hbar = c = 1$). This accords with the association of $\rho(t, x)$ with the $\hat{T}^{00}$ component of the stress-energy tensor, commonly denoted the energy density of the (Klein-Gordon) wave function.

We can also consider the trajectories and corresponding effective mass under a Lorentz boost. It was shown in Ref. [9] that the velocity field given by (20) satisfies the relativistic velocity addition rule

$$v'(t, x) = \frac{v(t, x) - \vartheta}{1 - v(t, x)\vartheta}, \tag{38}$$

where ϑ is the velocity of the boosted reference frame. In Fig. 3, we plot the trajectories overlaid upon the effective mass density for the same wavepacket properties as Fig. 2, in the coordinates of the boosted observer $(t', x') = \gamma(t - \vartheta x, x - \vartheta t)$. The Lorentz boost enacts a Doppler shift on the wavepackets, causing the width to broaden in the direction of the boost and narrow in the orthogonal direction, while the interference fringes become time dependent due to the coupling of the spatial and temporal variables. Importantly, and as highlighed in Fig. 3(b), the trajectories can retropropagate in the coordinates of the boosted observer (have a tangent vector pointing in the past lightcone). The segments of the trajectories that behave in this manner correspond exactly to those regions in which the above-mentioned condition, $\bar{m}^2_{\text{eff}} < 0$ and $\rho'(t', x') < 0$, is satisfied.

We note that such trajectories are not problematic given our interpretation of the velocity field as being a coordinate velocity obtained via measurements performed in a particular reference frame. This reference frame is originally the one in which the wave vectors and wavepacket bandwidths are equal. As shown in Ref. [9], it is always possible to find a frame in which the velocity field is globally forward-directed. On the other hand, it is inevitable that coordinate velocities with spacelike tangents will become backwards-directed upon a Lorentz boost, namely, when the denominator of (38) becomes zero. Finally, we note that, unlike the early works of Refs. [44,45] and more recent studies proposing superluminal extensions to special relativity [46,47], our framework is restricted to the single-particle sector. Hence in the present work, we cannot make the standard association of retropropagating trajectories with forward-propagating antiparticle trajectories. Nevertheless, such an interpretation motivates an extension of the weak-value framework to multiparticle interactions that incorporates particle production and annihilation effects.

V. CONCLUSION

In this paper, we provided an operational definition for the local mass of a photon using the language of weak values. We connected this mass to the relativistic Bohmian trajectories of photons in an interferometric setup and to the notion of a quantum potential arising in relativistic generalizations of Bohmian mechanics. We described an experiment for the observation of the mass and simulated the results. Since standard quantum mechanics does not concern itself with the individual quantum event or its ontological status, the description and observation of locally superluminal or locally massive photons is not relevant within this theory. However, such concepts can still be experimentally measured using weak values.

When employing theoretical frameworks that address individual quantum events, the aforementioned peculiar qualities of photons can be explicitly described and the result can be directly measured in the laboratory for a single photon. By appealing to Bohmian mechanics for the ontological description and interpretation of the individual quantum event and to weak measurement as a measurement scheme, our construction motivates novel opportunities for capturing phenomena that were, until now, out of the experimentalist's reach. Since the velocity field and the local mass are measurable, one must find solutions to the conceptual problems raised in the relativistic setting, which the standard quantum theory avoids by rejecting both the ontology of the single particle when it is not measured and the very possibility of direct observation of its properties.

Simulating an experiment in an optical setup, not only did we find locally massive particles of light, but they were also observed to exhibit local tachyonic behavior, which was perceived, in a boosted frame of reference, as time-traveling particles. Surprisingly, this did not pose a challenge to relativistic principles, as the velocity we operationally defined in a specific frame of reference was, in fact, a coordinate velocity which might exceed c. This was consistent with the fact that the locally spacelike trajectories cannot be revealed using strong measurements since no information, which the expectation values of quantum observables could reveal, traveled faster than c. Our results were also complementary to the "quantum metric" interpretation given in Ref. [9], where a geometric explanation of the photon trajectories was provided. There it was suggested that a natural generalization of the nonrelativistic quantum potential is that of a general relativistic metric that guides the photons along geodesics.

We hope future research utilizes this approach to demonstrate the more curious phenomena of this type by way of experiment and simulation. A possible suggestion for such an experiment could be the demonstration of "locally massless" particles of nonzero mass. Should the experiment presented in this paper be repeated with particles of nonzero charge, an emission of Cherenkov radiation in a vacuum, in regions where the particle becomes superluminal, should be expected [48]. Likewise, we encourage the investigation of the possibility of connection between the notion of mass explored herein and the negative mass particle ontology presented in Ref. [24]. Finally, since the definition used in this paper for the velocity field of relativistic quantum particles is not unique (a notable example of an alternative definition found in Ref. [43]), the different approaches should be compared both theoretically and experimentally. One could gain an intuition of the underlying physics by plotting the different trajectories side by side. We leave this for future work. The approach proposed herein should be applied to other relativistic equations (such as the Dirac equation) as well.

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