Modification of stimulated transition processes of a uniformly moving atom by the presence of a dielectric medium in blackbody radiation

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We investigate the effect of the presence of a dielectric medium on the stimulated transition processes of a uniformly moving atom in a thermal radiation field. The cases of atomic subluminal and superluminal motions are considered. The calculated results indicate that the atomic stimulated transition rates depend crucially on the atomic velocity, the temperature of the thermal radiation, the refractive index of medium, and the atomic polarizability. The transition rates are expanded by series in some limiting cases, such as the extreme temperature of thermal radiation, the extreme motion of the atom, and the extremely large refractive index. Our analytical and numerical analyses show that the presence of the dielectric medium has profoundly changed the behaviors and characteristics of the atomic stimulated transition rates, especially when the atom moves faster than the speed of light in the medium. Moreover, the behavior of the transition rates for an atom polarized parallel to the direction of atomic motion is quite distinct from that of an atom polarized perpendicularly to the direction of atomic motion.

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I. INTRODUCTION

Since Purcell discovered the enhancement of spontaneous emission rate of atoms when they are incorporated into a resonant cavity, there has been an increasing concern with investigations of the atomic radiative processes in different environments [1,2]. Some work has considered the effect of atomic center-of-mass motion on the spontaneous emission rate of an excited atom in vacuum [3–7]. By including the Röntgen contribution in the atom-field interaction Hamiltonian, it is found that the decay rate of an excited atom with a constant velocity is different from that of an atom at rest only by a factor γ^{-1}, where γ is the well-known Lorentz factor. This is consistent with the requirement of time dilation effect in special relativity. In fact, in most cases atoms are not always in the free space. When the space is filled with a continuous medium, the normal modes of electromagnetic field (whether fluctuating vacuum field or external radiation field) can be modified due to the polarization and magnetization of materials. For an atom embedded in a continuous medium and coupled to fluctuating vacuum fields, the atomic transition processes are thereby affected [8–16]. Meanwhile, many works have been devoted to the quantization of electromagnetic field in the presence of different types of background media from different perspectives [17–30], which is necessary for the investigation of radiative properties of atoms in the medium. These studies found that the presence of a continuous dielectric medium when the dispersion and the local field effect are neglected enhances the decay rate of an excited atom only by the refractive index n of the medium. If considering the dispersion and dissipation of the dielectric medium, the atomic decay rate is given by $\Gamma = \eta(\omega_A)\Gamma_0$, where $\eta(\omega_A)$ is the real part of the complex refractive index $n(\omega)$.

Further, Matloob showed that the decay rate of an excited atom is modified by the uniform motion of a nondispersive medium [31–35] when the velocity of the medium doesn't exceed the phase velocity of the light in the stationary medium [36,37]. Specifically, the atomic decay rate depends on the orientation of the atomic dipole moment and the velocity of the moving medium. This is physically understandable since a moving isotropic medium seems like an anisotropic material. Shafieiyan *et al.* gave the expression of the spontaneous emission rate and the energy level shift of a moving atom in the presence of absorbing and dispersive media in terms of the imaginary part of the classical Green tensor and the center-of-mass velocity of the atom [38]. By considering the special case of nondispersive and nonabsorbing background media, they obtain the result of decay rate for an excited atom moving with relativistic velocity $v < c/n$. After expressing the physical quantities in the results from the laboratory frame to the frame of the atom, the obtained decay rate is different from the result of [36,37] only by the Lorentz factor. Deservedly, the existence of a medium does not hinder the applicability of the principle of relativity. In the laboratory frame, the presence of the medium distinguishes two ranges of velocity for the moving atom, corresponding to the cases when the atomic velocity is greater or less than the phase velocity of light in the stationary medium, respectively. For the case of the atomic subluminal motion ($v < c/n$), the atomic decay rate has been well investigated in the aforementioned literatures. However, for the case of the atomic superluminal motion ($c/n < v < c$), there are some important and unusual physical implications. In analogy with the Cherenkov radiation for a charged particle in classical electrodynamics [39–41], even an inertial particle

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detector with internal levels (such as atoms) moving through a medium can be excited to a higher energy level if its velocity exceeds the velocity of light in the medium [41–48]. Meanwhile, the excitation of the detector is accompanied by the emission of quantum particles from the fluctuating vacuum field. This phenomenon can be viewed as the quantum version of the Cherenkov effect and was named the Ginzburg effect. Some works try to comprehend the strange physical phenomenon from different perspectives, for example, the anomalous Doppler effect [43,45], the spacelike character of the Minkowski four-momentum [49], the negative frequency modes [50,51], etc.

As is well known, when atoms are exposed to external radiation fields, the external fields can both drive transitions between atomic states and induce a Stark shift of the atomic energy levels. Some works found the temperature dependence of the atomic radiative properties when an atom is bathed in blackbody radiation (BBR) [52–62]. Due to the relativistic Doppler effect, BBR seen by an observer in a moving reference frame is found to be non-Planckian and anisotropic [63–67]. This character is closely related to the highly controversial subject of relativistic thermodynamics, and for a detailed history readers can see comments about this topic [68–73] and the references cited therein. The influence of this character on the transition properties of a uniformly moving quantum detector or atom has been carefully investigated [74–80]. The results show that BBR-induced transitions are crucially dependent on the velocity of detectors or atoms. Actually, the background medium can also modify the properties of the external thermal radiation field and then affect the atomic stimulated emission and absorption processes [8,10,22,81–88]. In this paper we further consider the stimulated transition processes of an atom with a constant velocity bathed in a thermal radiation field in the presence of a homogeneous and isotropic nondispersive dielectric medium. On the one hand, our aim is to explore how the presence of the dielectric medium modifies the stimulated transition rates of the uniformly moving atom, as compared with the case when the medium is absent [78]. On the other hand, inspired by the adequate investigations on the spontaneous transition processes of a moving atom or Unruh-DeWitt detector [36–38,44,46,48], we are curious if the stimulated transition properties for atoms with superluminal motion are thoroughly distinct from those of atomic subluminal motion.

The natural units $\hbar = c = 1$ and $k_B = 1$ are adopted throughout the paper.

II. GENERAL EXPRESSIONS FOR ATOMIC TRANSITION RATES IN THE FRAMEWORK OF PERTURBATION THEORY

The interacting system of a multilevel atom and a quantum electromagnetic field can be described with respect to the laboratory coordinate time t by the total Hamiltonian $H(t) = H_A(t) + H_F(t) + H_I(t)$. Thereinto, H_A is the Hamiltonian operator that governs the evolution of a multilevel atom,

$$H_A(\tau) = E_k + \sum_m \omega_m \sigma_{mm}(t), \tag{1}$$

in which E_k is the kinetic energy of the atomic center-of-mass motion, the internal dynamics of the atom $\sigma_{mm} = |m\rangle\langle m|$, and ω_m gives the energy corresponding to the inner stationary state $|m\rangle$. H_F is the Hamiltonian operator of the electromagnetic field, given by

$$H_F(t) = \sum_{\mathbf{k}\xi} \omega a^\dagger_{\mathbf{k}\xi}(t) a_{\mathbf{k}\xi}(t), \tag{2}$$

where $a_{\mathbf{k}\xi}$ ($a^\dagger_{\mathbf{k}\xi}$) is the annihilation (creation) operator for a photon with the wave vector $\mathbf{k}$ and the polarization ξ, and ω corresponds to the photon's energy. The moving atom is coupled to the electromagnetic field through the interaction Hamiltonian [3–7]

$$H_I(t) = -\mathbf{d}(t)\cdot[\mathbf{E}(\boldsymbol{x}_A(t)) + \boldsymbol{v}(t)\times\mathbf{B}(\boldsymbol{x}_A(t))], \tag{3}$$

where $\mathbf{d}$ denotes the atomic electric dipole moment operator, $\mathbf{E}$ and $\mathbf{B}$ represent the electric and magnetic field operators, $\boldsymbol{x}_A(t)$ denotes the atomic center-of-mass position, and $\boldsymbol{v}(t)$ is the associated velocity. The first term in Eq. (3) is the usual electric dipole interaction, and the second term is identified with the so-called Röntgen term that a magnetic-dipole-like interaction arises due to the uniform motion of the atom. It should be noted that all the operators and quantities above are expressed in the laboratory frame.

In the weak coupling regime, the evolution operator of the whole system can be expanded in the interaction representation as

$$U(t_f, t_i) = 1 - i\int_{t_i}^{t_f} dt' H_I(t') + \cdots. \tag{4}$$

So the probability amplitude of the transition from the initial state $|\omega_b\Phi_i\rangle$ at t_i to the final state $|\omega_d\Phi_f\rangle$ at t_f is given in the first-order approximation by

$$\begin{aligned}\langle\omega_d\Phi_f|U(t_f,t_i)|\omega_b\Phi_i\rangle = i\int_{t_i}^{t_f}\langle\omega_d\Phi_f|(\mathbf{d}(t')\cdot[\mathbf{E}(\boldsymbol{x}_A(t'))\\ +\ \boldsymbol{v}(t')\mathbf{B}(\boldsymbol{x}_A(t'))])|\omega_b\Phi_i\rangle\, dt'.\end{aligned} \tag{5}$$

Since we focus on the atomic transition from the initial state $|\omega_b\rangle$ to the final state $|\omega_d\rangle$, we should sum over all the possible final states of the field $|\Phi_f\rangle$, and then the atomic transition probability is found to be

$$P(\omega_{bd}, t_f, t_i) = \sum_{i,j=1}^{3}\langle\omega_b|d_i(0)|\omega_d\rangle\langle\omega_d|d_j(0)|\omega_b\rangle F_{ij}(\omega_{bd}, t_f, t_i), \tag{6}$$

where we have defined the function

$$\begin{aligned}F_{ij}(\omega_{bd}, t_f, t_i) = \int_{t_i}^{t_f} dt'\int_{t_i}^{t_f} dt'' e^{i\omega_{bd}(t'-t'')}\\ \times\ \langle\Phi_i|(\mathbf{E}(\boldsymbol{x}_A(t')) + \boldsymbol{v}(t')\times\mathbf{B}(\boldsymbol{x}_A(t')))_i\\ \times\ (\mathbf{E}(\boldsymbol{x}_A(t'')) + \boldsymbol{v}(t'')\times\mathbf{B}(\boldsymbol{x}_A(t'')))_j|\Phi_i\rangle\end{aligned} \tag{7}$$

with the notation $\omega_{bd} = \omega_b - \omega_d$. When the interaction time between the moving atom and the quantum electromagnetic field is sufficient, $t_f - t_i \to \infty$, the equilibrium transition rate is given by

$$R_{\omega_b\to\omega_d} = \left(\frac{dP}{dt_f}\right)_{t_f-t_i\to\infty} = \sum_{i,j=1}^{3}\langle\omega_b|d_i(0)|\omega_d\rangle\langle\omega_d|d_j(0)|\omega_b\rangle\dot{F}_{ij}(\omega_{bd}), \tag{8}$$

where the response function is given by

$$\dot{F}_{ij}(\omega_{bd}) = \int_{-\infty}^{\infty} du\, e^{i\omega_{bd}u}G_{ij}^{+}(u) \tag{9}$$

with the notations

$$G_{ij}^{+}(u) = \langle\Phi_i|(\mathbf{E}(\boldsymbol{x}_A(t)) + \boldsymbol{v}(t)\times\mathbf{B}(\boldsymbol{x}_A(t)))_i \times (\mathbf{E}(\boldsymbol{x}_A(t')) + \boldsymbol{v}(t')\times\mathbf{B}(\boldsymbol{x}_A(t')))_j|\Phi_i\rangle \tag{10}$$

and $u = t - t'$. It is evident that the correlation functions of the electromagnetic field in the initial state $|\Phi_i\rangle$ along the atomic trajectory, $G_{ij}^{+}(u)$ are crucial to the calculation of the atomic transition rate.

III. BBR-INDUCED STIMULATED TRANSITION FOR A UNIFORMLY MOVING ATOM IN A LINEAR, HOMOGENEOUS, AND ISOTROPIC MEDIUM

Now we apply the above method to calculate the transition rates of a uniformly moving atom bathed in a BBR field at a finite temperature T in the presence of a homogeneous and isotropic dielectric medium. The dielectric medium is characterized by a permittivity ϵ, and here we assume that dispersion is negligible. The atom moves with a constant velocity v along a certain direction (for example, in the x direction), and its spatial position is given in Cartesian coordinates by

$$\boldsymbol{x}_A(t) = (x_0 + vt, y_0, z_0), \tag{11}$$

where (x_0, y_0, z_0) denotes the atomic initial spatial coordinates. Then the atomic velocity $\boldsymbol{v}(t) = d\boldsymbol{x}_A(t)/dt = (v, 0, 0)$.

According to the quantization of the electromagnetic field in a dielectric medium [17–30], the electric and magnetic field operators are given by

$$\mathbf{E}(\boldsymbol{x}, t) = i\sum_{\xi}\int d^3\mathbf{k}\left(\frac{\omega}{2n^2(2\pi)^3}\right)^{1/2} \times [a_{\mathbf{k}\xi}e^{-i\omega t+i\mathbf{k}\cdot\boldsymbol{x}} - a_{\mathbf{k}\xi}^{\dagger}e^{i\omega t-i\mathbf{k}\cdot\boldsymbol{x}}]\mathbf{e}^{(\xi)}(\mathbf{k}), \tag{12}$$

$$\mathbf{B}(\boldsymbol{x}, t) = i\sum_{\xi}\int d^3\mathbf{k}\left(\frac{1}{2n^2\omega(2\pi)^3}\right)^{1/2} \times [a_{\mathbf{k}\xi}e^{-i\omega t+i\mathbf{k}\cdot\boldsymbol{x}} - a_{\mathbf{k}\xi}^{\dagger}e^{i\omega t-i\mathbf{k}\cdot\boldsymbol{x}}]\mathbf{k}\times\mathbf{e}^{(\xi)}(\mathbf{k}), \tag{13}$$

where n is the refractive index of the medium, and $a_{\mathbf{k}\xi}$ and $a_{\mathbf{k}\xi}^{\dagger}$ are the annihilation and creation operators for the transverse photons with momentum $\mathbf{k}$ and polarization unit vector $\mathbf{e}^{(\xi)}(\mathbf{k})$ ($\xi = 1, 2$). Here one has the relations $n = \sqrt{\epsilon}$, $|\mathbf{k}| = k = n\omega$, and $\mathbf{k}\cdot\mathbf{e}^{(\xi)}(\mathbf{k}) = 0$. Assume that initially the quantum field is in the thermodynamic equilibrium state, which is described by the density operator $\rho = e^{-\beta H_F}$ with the inverse temperature $\beta = 1/T$. We can defined the notations of some correlation functions of the electromagnetic field in the thermal state $|\beta\rangle$ as follows:

$$\mathcal{E}_{ij}(X_1, X_2) = \langle\beta|E_i(X_1)E_j(X_2)|\beta\rangle, \tag{14}$$

$$\mathcal{B}_{ij}(X_1, X_2) = \langle\beta|B_i(X_1)B_j(X_2)|\beta\rangle, \tag{15}$$

$$\mathcal{M}_{ij}(X_1, X_2) = \langle\beta|E_i(X_1)B_j(X_2)|\beta\rangle, \tag{16}$$

$$\mathcal{N}_{ij}(X_1, X_2) = \langle\beta|B_i(X_1)E_j(X_2)|\beta\rangle. \tag{17}$$

Inserting Eqs. (12) and (13) into the above expressions and using the formula $\langle\beta|G|\beta\rangle = \mathrm{tr}(\rho G)/\mathrm{tr}(\rho)$ with G being an arbitrary field operator, we obtain

$$\mathcal{E}_{ij}(X_1, X_2) = \frac{1}{16\pi^3}\int d^3\mathbf{k}\,\frac{\omega}{n^2}(\delta_{ij} - k_ik_j/k^2) \times\left[\left(1 + \frac{1}{e^{\omega/T}-1}\right)e^{-i\omega\Delta t+i\mathbf{k}\cdot\Delta\boldsymbol{x}} + \frac{1}{e^{\omega/T}-1}e^{i\omega\Delta t-i\mathbf{k}\cdot\Delta\boldsymbol{x}}\right], \tag{18}$$

$$\mathcal{B}_{ij}(X_1, X_2) = n^2\mathcal{E}_{ij}(X_1, X_2), \tag{19}$$

$$\mathcal{M}_{ij}(X_1, X_2) = \frac{1}{16\pi^3}\int d^3\mathbf{k}\,\frac{1}{n^2}\epsilon_{ijl}k_l \times\left[\left(1 + \frac{1}{e^{\omega/T}-1}\right)e^{-i\omega\Delta t+i\mathbf{k}\cdot\Delta\boldsymbol{x}} + \frac{1}{e^{\omega/T}-1}e^{i\omega\Delta t-i\mathbf{k}\cdot\Delta\boldsymbol{x}}\right], \tag{20}$$

$$\mathcal{N}_{ij}(X_1, X_2) = -\mathcal{M}_{ij}(X_1, X_2). \tag{21}$$

In the process of obtaining the above results, we have used the relations

$$\sum_{\xi=1}^{2}\mathbf{e}_i^{(\xi)}(\mathbf{k})\mathbf{e}_j^{(\xi)}(\mathbf{k}) = \delta_{ij} - k_ik_j/k^2, \tag{22}$$

$$\sum_{\xi=1}^{2}[\mathbf{k}\times\mathbf{e}^{(\xi)}(\mathbf{k})]_i[\mathbf{k}\times\mathbf{e}^{(\xi)}(\mathbf{k})]_j = k^2\delta_{ij} - k_ik_j, \tag{23}$$

$$\sum_{\xi=1}^{2}\mathbf{e}_i^{(\xi)}(\mathbf{k})[\mathbf{k}\times\mathbf{e}^{(\xi)}(\mathbf{k})]_j = \epsilon_{ijl}k_l, \tag{24}$$

where ϵ_{ijl} is the Levi-Civita symbol. The correlation functions obey the symmetry relations

$$\mathcal{E}_{ij}(X_1, X_2)=\mathcal{E}_{ji}(X_1, X_2), \quad \mathcal{M}_{ij}(X_1, X_2) = -\mathcal{M}_{ji}(X_1, X_2), \tag{25}$$

$$\mathcal{B}_{ij}(X_1, X_2) = \mathcal{B}_{ji}(X_1, X_2), \quad \mathcal{N}_{ij}(X_1, X_2) = -\mathcal{N}_{ji}(X_1, X_2). \tag{26}$$

Then the functions $G^{+}_{ij}(u)$ defined in Eq. (10) can be expressed as

$$G^{+}_{11}(u)=\mathcal{E}_{11}, \tag{27}$$

$$G^{+}_{22}(u)=\mathcal{E}_{22}-v\mathcal{M}_{23}-v\mathcal{N}_{32}+v^2\mathcal{B}_{33}, \tag{28}$$

$$G^{+}_{33}(u)=\mathcal{E}_{33}+v\mathcal{M}_{32}+v\mathcal{N}_{23}+v^2\mathcal{B}_{22}, \tag{29}$$

$$G^{+}_{12}(u)=\mathcal{E}_{12}-v\mathcal{M}_{13},\quad G^{+}_{21}(u)=\mathcal{E}_{21}-v\mathcal{N}_{31}, \tag{30}$$

$$G^{+}_{13}(u)=\mathcal{E}_{13}+v\mathcal{M}_{12},\quad G^{+}_{31}(u)=\mathcal{E}_{31}+v\mathcal{N}_{21}, \tag{31}$$

$$G^{+}_{23}(u)=\mathcal{E}_{23}-v\mathcal{N}_{33}+v\mathcal{M}_{22}-v^2\mathcal{B}_{32}, \tag{32}$$

$$G^{+}_{32}(u)=\mathcal{E}_{32}-v\mathcal{M}_{33}+v\mathcal{N}_{22}-v^2\mathcal{B}_{23}, \tag{33}$$

where we have used the relation $\boldsymbol{v}\times\mathbf{B}=(0,-vB_3,vB_2)$. Since the spontaneous transition processes for a uniformly moving atom due to ubiquitous vacuum fluctuations in a medium have been widely investigated in previous works [36–38,44,46,48], here we focus on the stimulated transition processes of atoms due to the external thermal radiation field. Using the results of Eqs. (18)–(21) and inserting into the atomic trajectory (11), we obtain the BBR-induced contributions on the functions $G^{+}_{ij}(u)$,

$$G^{+}_{11}(u)=\frac{1}{16\pi^3}\int d^3\mathbf{k}\,\frac{\omega}{n^2}\left(1-k_1^2/k^2\right)\times\left[\frac{1}{e^{\omega/T}-1}e^{-i(\omega-k_1v)u}+\frac{1}{e^{\omega/T}-1}e^{i(\omega-k_1v)u}\right], \tag{34}$$

$$G^{+}_{22}(u)=\frac{1}{16\pi^3}\int d^3\mathbf{k}\left\{\frac{\omega}{n^2}\left[\left(1-k_2^2/k^2\right)+n^2v^2\left(1-k_3^2/k^2\right)\right]-\frac{2v}{n^2}k_1\right\}\left(\frac{1}{e^{\omega/T}-1}e^{-i(\omega-k_1v)u}+\frac{1}{e^{\omega/T}-1}e^{i(\omega-k_1v)u}\right), \tag{35}$$

$$G^{+}_{33}(u)=\frac{1}{16\pi^3}\int d^3\mathbf{k}\left\{\frac{\omega}{n^2}\left[\left(1-k_3^2/k^2\right)+n^2v^2\left(1-k_2^2/k^2\right)\right]-\frac{2v}{n^2}k_1\right\}\left(\frac{1}{e^{\omega/T}-1}e^{-i(\omega-k_1v)u}+\frac{1}{e^{\omega/T}-1}e^{i(\omega-k_1v)u}\right), \tag{36}$$

and

$$G^{+}_{ij}(u)=0\quad\text{for } i\neq j. \tag{37}$$

Significantly, the atomic velocity can be greater or smaller than the speed of light in the dielectric medium $(1/n)$, i.e., there are two cases of subluminal motion $(0<v<1/n)$ and superluminal motion $(1/n<v<1)$. The two situations can give rise to quite different consequences in some physical processes [31–51]. In the following, we will separately examine them in detail.

IV. THE CASE OF SUBLUMINAL MOTION

Inserting Eq. (34) into Eq. (9), we obtain the response function of the transition rate

$$\begin{aligned}\dot{F}_{11}(\omega_{bd})&=\int_{-\infty}^{\infty}du\,e^{i\omega_{bd}u}G^{+}_{11}(u)\\&=\frac{1}{8\pi^2}\int_0^{\infty}dk\int_{-1}^{1}d(\cos\theta)\frac{k^3(1-\cos^2\theta)}{n^3}\frac{1}{e^{k/(nT)}-1}\left[\int_{-\infty}^{\infty}du(e^{-i(\omega-k_1v)u}+e^{i(\omega-k_1v)u})e^{i\omega_{bd}u}\right]\\&=\frac{1}{4\pi}\int_0^{\infty}dk\int_{-1}^{1}d(\cos\theta)\frac{k^3(1-\cos^2\theta)}{n^3}\frac{1}{e^{k/(nT)}-1}\{\delta[k(1/n-v\cos\theta)-\omega_{bd}]+\delta[k(1/n-v\cos\theta)+\omega_{bd}]\}.\end{aligned} \tag{38}$$

Due to $nv<1$, $\cos\theta<\frac{1}{nv}$ always holds for each θ, and then the photons with momentum k in all directions contribute to the above integral. Therefore, the above equation reduces to

$$\dot{F}_{11}(\omega_{bd})=\frac{n|\omega_{bd}|^3}{4\pi}\int_{-1}^{1}d(\cos\theta)\frac{(1-\cos^2\theta)}{(1-nv\cos\theta)^4}\frac{1}{e^{|\omega_{bd}|/[T(1-nv\cos\theta)]}-1}[\Theta(\omega_{bd})+\Theta(-\omega_{bd})], \tag{39}$$

where $\Theta(\omega_{bd})$ is the step function. Following the same steps, we obtain

$$\dot{F}_{22}(\omega_{bd})=\frac{n|\omega_{bd}|^3}{4\pi}\int_{-1}^{1}d(\cos\theta)\frac{(1+\cos^2\theta)(1+n^2v^2)/2-2nv\cos\theta}{(1-nv\cos\theta)^4}\frac{1}{e^{\omega_{bd}/[T(1-nv\cos\theta)]}-1}[\Theta(\omega_{bd})+\Theta(-\omega_{bd})] \tag{40}$$

and

$$\dot{F}_{33}(\omega_{bd})=\dot{F}_{22}(\omega_{bd}) \tag{41}$$

with other components $(i\neq j)$ being zero.

According to the time dilation effect in special relativity, the atomic transition rate in the laboratory frame is related to the transition rate in the rest frame of the atom by

$$R^{\text{sub}}=\gamma^{-1}R^{\text{sub}}_{\text{rest}}. \tag{42}$$

Substituting Eqs. (39)–(41) into expression (8), we can express the atomic stimulated transition rate as the form

$$R^{\rm sub} = \gamma^{-1}R_0 \sum_{i=\parallel,\perp} \alpha_i g_i^{\rm sub}(|\lambda|, v, n)[\Theta(\omega'_{bd}) + \Theta(-\omega'_{bd})], \tag{43}$$

where we have defined two dimensionless parameters involving the parallel and perpendicular components of the atomic polarizability,

$$\alpha_\parallel = \frac{|\langle\omega'_b|d'_1(0)|\omega'_d\rangle|^2}{|\langle\omega'_b|\mathbf{d}'(0)|\omega'_d\rangle|^2}, \tag{44}$$

$$\alpha_\perp = \frac{|\langle\omega'_b|d'_2(0)|\omega'_d\rangle|^2 + |\langle\omega'_b|d'_3(0)|\omega_{d'}\rangle|^2}{|\langle\omega'_b|\mathbf{d}'(0)|\omega'_d\rangle|^2}. \tag{45}$$

Here R_0 is the spontaneous emission rate for an atom at rest in vacuum,

$$R_0 = \frac{|\omega'_{bd}|^3}{3\pi}|\langle\omega'_b|\mathbf{d}'(0)|\omega'_d\rangle|^2. \tag{46}$$

We have replaced the parameters involving the atomic intrinsic quantities in the expression of atomic transition rates by the corresponding quantities in the atomic rest frame by using the relations $\omega_{bd} = \gamma^{-1}\omega'_{bd}$, $d_1 = \gamma^{-1}d'_1$, $d_2 = d'_2$, and $d_3 = d'_3$ due to time dilation and length contraction. Moreover, we also define the functions

$$\begin{aligned} g_\parallel^{\rm sub}(\lambda, v, n) &= \frac{3}{4}n\int_{-1}^{1} d(\cos\theta)\,\frac{(1-\cos^2\theta)}{\gamma^4(1-nv\cos\theta)^4}\frac{1}{e^{\lambda/[\gamma(1-nv\cos\theta)]}-1} \\ &= \frac{3}{4}n\sum_{m=1}^{\infty}\frac{2}{m^3\lambda^3n^3v^3\gamma}\big[(1-n^2v^2+m\lambda nv/\gamma)e^{-\frac{m\lambda}{\gamma(1-nv)}} - (1-n^2v^2-m\lambda nv/\gamma)e^{-\frac{m\lambda}{\gamma(1+nv)}}\big] \end{aligned} \tag{47}$$

and

$$\begin{aligned} g_\perp^{\rm sub}(\lambda, v, n) &= \frac{3}{4}n\int_{-1}^{1} d(\cos\theta)\frac{(1+\cos^2\theta)(1+n^2v^2)/2 - 2nv\cos\theta}{\gamma^2(1-nv\cos\theta)^4}\frac{1}{e^{\lambda/[\gamma(1-nv\cos\theta)]}-1} \\ &= -\frac{3}{4}n\sum_{m=1}^{\infty}\frac{\gamma}{m^3\lambda^3n^3v^3}\bigg\{\bigg[(1-n^2v^2)^2 + \frac{m\lambda nv}{\gamma}(1-n^2v^2) + m^2\lambda^2n^2v^2/\gamma^2\bigg]e^{-\frac{m\lambda}{\gamma(1-nv)}} \\ &\quad - \bigg[(1-n^2v^2)^2 - \frac{m\lambda nv}{\gamma}(1-n^2v^2) + m^2\lambda^2n^2v^2/\gamma^2\bigg]e^{-\frac{m\lambda}{\gamma(1+nv)}}\bigg\} \end{aligned} \tag{48}$$

with the notation $\lambda = \omega'_{bd}/T$. Clearly, the information of the atomic stimulated transition rates is mainly encoded in the functions $g_i^{\rm sub}(\lambda, v, n)$ with $i = \parallel, \perp$. The stimulated upward transition rate is exactly equal to the stimulated downward transition rate for the same level transition spacing $|\omega'_{bd}|$. It seems that the atomic stimulated transition rates depend on the temperature of thermal radiation, the atomic velocity, the refractive index of medium, and the atomic polarizability. In general, it is hard to handle the integral or sum in the above expressions of the functions $g_i^{\rm sub}(\lambda, v, n)$. However, approximate analytical results can be obtainable under some limit conditions.

When the atomic velocity is low compared with the light velocity in the medium ($v \ll 1/n$), one has

$$g_\parallel^{\rm sub}(\lambda, v, n) \approx \frac{n}{e^\lambda - 1} + \mathrm{g}_\parallel(\lambda, n)v^2, \tag{49}$$

$$g_\perp^{\rm sub}(\lambda, v, n) \approx \frac{n}{e^\lambda - 1} + \mathrm{g}_\perp(\lambda, n)v^2, \tag{50}$$

where the functions $\mathrm{g}_i(\lambda, n)$ are defined as

$$\mathrm{g}_\parallel(\lambda, n) = n\bigg[\frac{2(n^2-1)}{e^\lambda - 1} - \frac{\lambda e^\lambda(10n^2-5)}{10(e^\lambda-1)^2} + \frac{\lambda^2n^2e^\lambda(e^\lambda+1)}{10(e^\lambda-1)^3}\bigg], \tag{51}$$

$$\mathrm{g}_\perp(\lambda, n) = n\bigg[\frac{n^2-1}{e^\lambda - 1} - \frac{\lambda e^\lambda(10n^2-5)}{10(e^\lambda-1)^2} + \frac{2\lambda^2n^2e^\lambda(e^\lambda+1)}{10(e^\lambda-1)^3}\bigg]. \tag{52}$$

The first terms in Eqs. (49) and (50) are exactly identical, and they refer to the results of an atom at rest in BBR within the dielectric medium [8,10,22,81,85–88]. The second terms are the modifying contributions due to the slow motion of the atom, which is always proportional to v^2. The effect of the atomic slow motion is encoded in the functions $\mathrm{g}_i(\lambda, n)$, and it is easy to prove that they can be positive, negative, or zero when λ and n take different values. This means that the presence of the medium can enhance or weaken the effect of the atomic slow motion on the stimulated transition processes.

When the temperature of the thermal radiation is high enough, $\hbar|\omega'_{bd}| \ll k_BT$ ($\lambda \ll 1$), one has

$$g_\parallel^{\rm sub}(\lambda, v, n) \approx \frac{6nv - 3(1-n^2v^2)\ln\frac{1+nv}{1-nv}}{4n^2v^3\gamma^3(1-n^2v^2)\lambda}, \tag{53}$$

$$g_\perp^{\rm sub}(\lambda, v, n) \approx \frac{-6nv + 3(1+n^2v^2)\ln\frac{1+nv}{1-nv}}{8n^2v^3\gamma\lambda}. \tag{54}$$

Compared with the case in the absence of the medium ($n = 1$) [78], the stimulated transition rates are still proportional

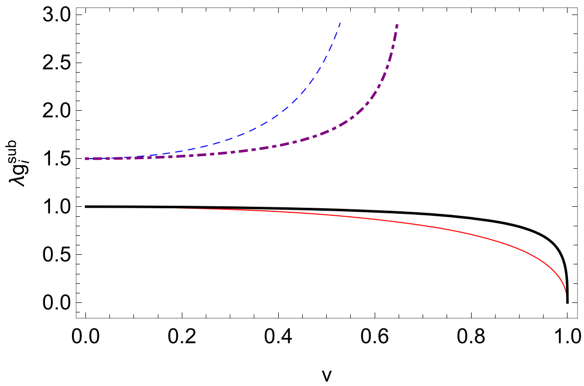

FIG. 1. Behavior of $\lambda g_i^{\text{sub}}(\lambda, v, n)$ in the limit of high temperature, as a function of the velocity of atomic subluminal motion. The two solid lines refer to the case in the absence of the dielectric medium ($n = 1$), and the dashed and dot-dashed lines refer to the case in the presence of the dielectric medium ($n > 1$). We choose the parameter $n = 1.5$. The thin and thick lines represent the cases for an atom polarizable parallel to the atomic velocity ($\alpha_\parallel = 1, \alpha_\perp = 0$) and perpendicular to the atomic velocity ($\alpha_\parallel = 0, \alpha_\perp = 1$), respectively.

to T. However, the above functions have been modified by the factor n. By Fig. 1 we find that the functions always rise with the increase of the atomic velocity, which is completely opposite to the case in the absence of the medium. Here we can simply define the effective temperatures for the thermal radiation perceived by the moving atom,

$$T_\parallel^{\text{eff}} = \left[\frac{6nv - 3(1-n^2v^2)\ln\frac{1+nv}{1-nv}}{4n^2v^3\gamma^3(1-n^2v^2)}\right]T, \tag{55}$$

$$T_\perp^{\text{eff}} = \left[\frac{-6nv + 3(1+n^2v^2)\ln\frac{1+nv}{1-nv}}{8n^2v^3\gamma}\right]T. \tag{56}$$

So the effective temperatures are always greater than T. Further, in the limit of both low velocity and high temperature, we have

$$g_\parallel^{\text{sub}}(\lambda, v, n) \approx n\left(1 - \frac{15-12n^2}{10}v^2\right)\lambda^{-1}, \tag{57}$$

$$g_\perp^{\text{sub}}(\lambda, v, n) \approx n\left(1 - \frac{5-4n^2}{10}v^2\right)\lambda^{-1}. \tag{58}$$

The effective temperatures reduce to

$$T_\parallel^{\text{eff}} = n\left(1 - \frac{15-12n^2}{10}v^2\right)T, \tag{59}$$

$$T_\perp^{\text{eff}} = n\left(1 - \frac{5-4n^2}{10}v^2\right)T. \tag{60}$$

The modifying term of v^2 in the above brackets due to the atomic slow motion can be negative, zero, or positive when $n < \frac{\sqrt{5}}{2}$, $n = \frac{\sqrt{5}}{2}$, or $n > \frac{\sqrt{5}}{2}$ and thus can enhance or weaken the effective temperatures. Moreover, when $n > \frac{\sqrt{5}}{2}$, $T_\parallel^{\text{eff}}$ begins to become greater than $T_\perp^{\text{eff}}$. These characteristics are in contrast sharp with the case in the absence of the medium ($n = 1$) [78], where $T_\parallel^{\text{eff}} < T_\perp^{\text{eff}}$, the two effective temperatures are always smaller than T and the atomic slow motion always weakens the effective temperatures. This means that the presence of the dielectric medium can reverse the effects of the atomic uniform motion.

When the temperature of the thermal radiation is low enough, i.e., $\hbar|\omega'_{bd}| \gg k_B T$ ($\lambda \gg 1$), one has

$$g_\parallel^{\text{sub}}(\lambda, v, n) \approx \frac{3}{2\lambda^2 nv^2\gamma^2}\left(e^{-\frac{\lambda}{\gamma(1-nv)}} + e^{-\frac{\lambda}{\gamma(1+nv)}}\right), \tag{61}$$

$$g_\perp^{\text{sub}}(\lambda, v, n) \approx -\frac{3}{4\lambda v\gamma}\left(e^{-\frac{\lambda}{\gamma(1-nv)}} - e^{-\frac{\lambda}{\gamma(1+nv)}}\right). \tag{62}$$

This result is quite different from the case when a static atom is bathed in the thermal radiation within the dielectric medium (i.e., $ne^{-\lambda}$). Here the effective temperatures are hard to be simply defined. In particular, in the limit of both low velocity and low temperature, we have

$$g_\parallel^{\text{sub}}(\lambda, v, n) \approx n\left(1 + \frac{\lambda^2 n^2}{10}v^2\right)e^{-\lambda}, \tag{63}$$

$$g_\perp^{\text{sub}}(\lambda, v, n) \approx n\left(1 + \frac{\lambda^2 n^2}{5}v^2\right)e^{-\lambda}. \tag{64}$$

Clearly, the modifying contributions of the atomic slow motion are always enhanced by the presence of the medium due to $n^2 > 1$.

V. THE CASE OF SUPERLUMINAL MOTION

When the atomic velocity is greater than the speed of light in the medium but is smaller than the speed of light in vacuum ($1/n < v < 1$), the response function $\dot{F}_{11}(\omega_{bd})$ also has the form of Eq. (38),

$$\dot{F}_{11}(\omega_{bd}) = \frac{1}{4\pi}\int_0^\infty dk \int_{-1}^1 d(\cos\theta)\,\frac{k^3(1-\cos^2\theta)}{n^3}\frac{1}{e^{k/(nT)}-1}\frac{1}{|1/n - v\cos\theta|}\left[\delta\left(k - \frac{n\omega_{bd}}{1-nv\cos\theta}\right) + \delta\left(k + \frac{n\omega_{bd}}{1-nv\cos\theta}\right)\right]. \tag{65}$$

We can handle first the integral of momentum k and then calculate the integral of azimuth angle θ. The middle process is given as follows:

$$\dot{F}_{11}(\omega_{bd}) = \left[\frac{n\omega_{bd}^3}{4\pi}\int_{-1}^{\frac{1}{nv}} d(\cos\theta)\frac{1-\cos^2\theta}{(1-nv\cos\theta)^4}\frac{1}{e^{\omega_{bd}/[T(1-nv\cos\theta)]}-1} + \frac{n\omega_{bd}^3}{4\pi}\int_{\frac{1}{nv}}^1 d(\cos\theta)\frac{1-\cos^2\theta}{(nv\cos\theta-1)^4}\frac{1}{e^{\omega_{bd}/[T(nv\cos\theta-1)]}-1}\right]\Theta(\omega_{bd})$$

$$+\left[\frac{n|\omega_{bd}|^3}{4\pi}\int_{\frac{1}{nv}}^{1} d(\cos\theta)\frac{1-\cos^2\theta}{(nv\cos\theta-1)^4}\frac{1}{e^{|\omega_{bd}|/[T(nv\cos\theta-1)]}-1}\right.$$
$$\left.+\frac{n|\omega_{bd}|^3}{4\pi}\int_{-1}^{\frac{1}{nv}} d(\cos\theta)\frac{1-\cos^2\theta}{(1-nv\cos\theta)^4}\frac{1}{e^{|\omega_{bd}|/[T(1-nv\cos\theta)]}-1}\right]\Theta(-\omega_{bd}). \tag{66}$$

Following the same steps, one has

$$\dot{F}_{22}(\omega_{bd}) = \frac{1}{4\pi}\int_0^\infty dk\int_{-1}^{1} d(\cos\theta)\,\frac{k^3}{n^3}\left[\frac{(1+n^2v^2)}{2}(1+\cos^2\theta)-2nv\cos\theta\right]\frac{1}{e^{k/(nT)}-1}$$
$$\times\frac{1}{|1/n-v\cos\theta|}\left\{\delta\left[k-\frac{n\omega_{bd}}{(1-nv\cos\theta)}\right]+\delta\left[k+\frac{n\omega_{bd}}{(1-nv\cos\theta)}\right]\right\}$$
$$=\left[\frac{n\omega_{bd}^3}{4\pi}\int_{-1}^{\frac{1}{nv}} d(\cos\theta)\frac{(1+n^2v^2)(1+\cos^2\theta)/2-2nv\cos\theta}{(1-nv\cos\theta)^4}\frac{1}{e^{\omega_{bd}/[T(1-nv\cos\theta)]}-1}\right.$$
$$\left.+\frac{n\omega_{bd}^3}{4\pi}\int_{\frac{1}{nv}}^{1} d(\cos\theta)\frac{(1+n^2v^2)(1+\cos^2\theta)/2-2nv\cos\theta}{(nv\cos\theta-1)^4}\frac{1}{e^{\omega_{bd}/[T(nv\cos\theta-1)]}-1}\right]\Theta(\omega_{bd})$$
$$+\left[\frac{n|\omega_{bd}|^3}{4\pi}\int_{\frac{1}{nv}}^{1} d(\cos\theta)\frac{(1+n^2v^2)(1+\cos^2\theta)/2-2nv\cos\theta}{(nv\cos\theta-1)^4}\frac{1}{e^{|\omega_{bd}|/[T(nv\cos\theta-1)]}-1}\right.$$
$$\left.+\frac{n|\omega_{bd}|^3}{4\pi}\int_{-1}^{\frac{1}{nv}} d(\cos\theta)\frac{(1+n^2v^2)(1+\cos^2\theta)/2-2nv\cos\theta}{(1-nv\cos\theta)^4}\frac{1}{e^{|\omega_{bd}|/[T(1-nv\cos\theta)]}-1}\right]\Theta(-\omega_{bd}) \tag{67}$$

and

$$\dot{F}_{33}(\omega_{bd}) = \dot{F}_{22}(\omega_{bd}) \tag{68}$$

with other components ($i \neq j$) being zero. We note that not all photons propagating in any direction contribute to the above integrals. When an atom with superluminal motion is bathed in the thermal radiation, there are two contributions on the response functions of stimulated transition rates corresponding to two different physical processes, respectively. For the atomic downward transition process ($\omega_{bd} > 0$), the first term in the above bracket represents the contribution of the emission of those photons outside the Cherenkov cone ($-1 < \cos\theta < \frac{1}{nv}$) and the second term represents the contribution of the absorption of those photons inside the Cherenkov cone ($\frac{1}{nv} < \cos\theta < 1$). For the atomic upward transition process ($\omega_{bd} < 0$), the first term represents the contribution of the emission of those photons inside the Cherenkov cone ($\frac{1}{nv} < \cos\theta < 1$), and the second term represents the contribution of the absorption of those photons outside the Cherenkov cone ($-1 < \cos\theta < \frac{1}{nv}$). This is quite different from the case of the atomic subluminal motion, where the atomic downward transition process is accompanied by only the emission of photons in any direction and the atomic upward transition process is accompanied by only the absorption of photons.

Substituting Eqs. (66)–(68) into expression (8), we get the atomic stimulated transition rate

$$R^{\rm sup} = \gamma^{-1}R^{\rm sup}_{\rm rest}, \tag{69}$$

where $R^{\rm sup}_{\rm rest}$ refers to the corresponding transition rate in the rest frame of the atom, given by

$$R^{\rm sup}_{\rm rest} = R_0\sum_{i=\parallel,\perp}\alpha_i g_i^{\rm sup}(|\lambda|, v, n)[\Theta(\omega'_{bd})+\Theta(-\omega'_{bd})]. \tag{70}$$

Here we have defined the functions

$$g_\parallel^{\rm sup}(\lambda, v, n) = \frac{3}{4}n\sum_{m=1}^{\infty}\frac{2}{m^3\lambda^3n^3v^3\gamma}(n^2v^2-1+m\lambda nv/\gamma)$$
$$\times\left(e^{-\frac{m\lambda}{\gamma(nv-1)}}+e^{-\frac{m\lambda}{\gamma(nv+1)}}\right), \tag{71}$$

$$g_\perp^{\rm sup}(\lambda, v, n) = \frac{3}{4}n\sum_{m=1}^{\infty}\frac{\gamma}{m^3\lambda^3n^3v^3}\left[(n^2v^2-1)^2\right.$$
$$\left.+\frac{m\lambda nv}{\gamma}(n^2v^2-1)+m^2\lambda^2n^2v^2/\gamma^2\right]$$
$$\times\left(e^{-\frac{m\lambda}{\gamma(nv-1)}}+e^{-\frac{m\lambda}{\gamma(nv+1)}}\right). \tag{72}$$

The discrepancy between the result here and that of the atomic subluminal motion [see Eqs. (47) and (48)] should be noted. It is easy to see that here the atomic stimulated upward transition rate is still equivalent to the stimulated downward transition rate as long as their level spacings of transition are identical.

When the temperature of the thermal radiation is high enough, $\hbar|\omega'_{bd}| \ll k_BT$ ($\lambda \ll 1$), one has

$$g_\parallel^{\rm sup}(\lambda, v, n) \approx \frac{3(n^2v^2-1)}{\lambda^3n^2v^3\gamma}\zeta(3), \tag{73}$$

$$g_\perp^{\rm sup}(\lambda, v, n) \approx \frac{3(n^2v^2-1)^2\gamma}{2\lambda^3n^2v^3}\zeta(3), \tag{74}$$

where ζ denotes the Riemann zeta function and $\zeta(3) \approx 1.2$. It is clear that the stimulated transition rates are always proportional to T^3. This is inconsistent with the case of atomic subluminal motion [see Eqs. (53) and (54)]. Furthermore, now the behavior of the function $g_\parallel^{\rm sup}$ with respect to the atomic velocity v is quite different from that of the function

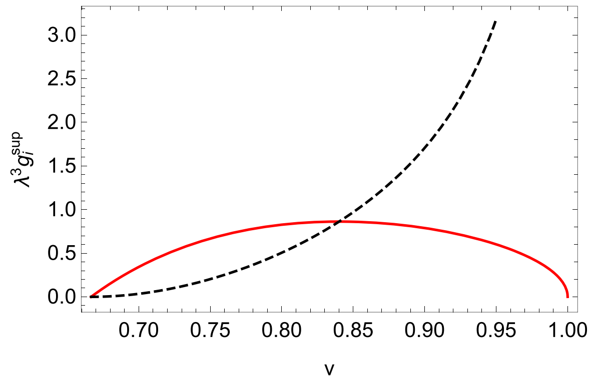

FIG. 2. Behavior of $\lambda^3 g_i^{\text{sub}}(\lambda, v, n)$ in the limit of high temperature, as a function of the velocity of atomic superluminal motion. We choose the parameter $n = 1.5$. The solid line represents the case $\alpha_\parallel = 1, \alpha_\perp = 0$, and the dashed line represent the case $\alpha_\parallel = 0$, $\alpha_\perp = 1$.

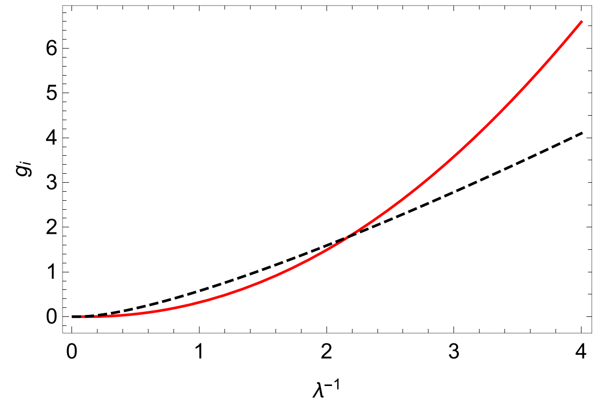

FIG. 3. Behavior of $g_i(\lambda, v, n)$ as a function of the temperature of the thermal radiation when $v = \frac{1}{n}$. We choose the parameter $n = 1.1$. The solid line represents the case $\alpha_\parallel = 1, \alpha_\perp = 0$, and the dashed line represents the case $\alpha_\parallel = 0, \alpha_\perp = 1$.

$g_\perp^{\text{sup}}$ (see Fig. 2). For $1/n < v < \sqrt{\frac{3}{n^2+2}}$, $g_\parallel^{\text{sup}} > g_\perp^{\text{sup}}$, and for $\sqrt{\frac{3}{n^2+2}} < v < 1$, $g_\parallel^{\text{sup}} < g_\perp^{\text{sup}}$ always holds. In particular, the two functions can be equal only when $v = \sqrt{\frac{3}{n^2+2}}$.

When the temperature of the thermal radiation is low enough, $\hbar|\omega'_{bd}| \gg k_B T$ ($\lambda \gg 1$), one has

$$g_\parallel^{\text{sup}}(\lambda, v, n) \approx \frac{3}{2\lambda^2 n v^2 \gamma^2}\left(e^{-\frac{\lambda}{\gamma(nv-1)}} + e^{-\frac{\lambda}{\gamma(nv+1)}}\right), \quad (75)$$

$$g_\perp^{\text{sup}}(\lambda, v, n) \approx \frac{3}{4\lambda v \gamma}\left(e^{-\frac{\lambda}{\gamma(nv-1)}} + e^{-\frac{\lambda}{\gamma(nv+1)}}\right). \quad (76)$$

This is similar to the case of subluminal motion [see Eqs. (61) and (62)].

In particular, when the atomic velocity is exactly equal to the light velocity in the medium ($v = \frac{1}{n}$), we have

$$\begin{aligned} g_\parallel^{\text{sup}}(\lambda, v, n) &= \frac{3n}{2}\sum_{m=1}^{\infty}\frac{1-\frac{1}{n^2}}{m^2\lambda^2}e^{-\frac{m\lambda}{2}\sqrt{1-\frac{1}{n^2}}} \\ &= \frac{3(n^2-1)}{2\lambda^2 n}\text{Ploylog}\left[2, e^{-\frac{1}{2}\lambda\sqrt{1-\frac{1}{n^2}}}\right], \end{aligned} \quad (77)$$

$$\begin{aligned} g_\perp^{\text{sup}}(\lambda, v, n) &= \frac{3n}{4}\sum_{m=1}^{\infty}\frac{\sqrt{1-\frac{1}{n^2}}}{m\lambda}e^{-\frac{m\lambda}{2}\sqrt{1-\frac{1}{n^2}}} \\ &= -\frac{3\sqrt{n^2-1}}{4\lambda}\ln\left(1-e^{-\frac{1}{2}\lambda\sqrt{1-\frac{1}{n^2}}}\right). \end{aligned} \quad (78)$$

In fact, in the limit of $v \to \frac{1}{n}$, the functions $g_\parallel^{\text{sub}}(\lambda, v, n)$ and $g_\perp^{\text{sub}}(\lambda, v, n)$ for the case of subluminal motion also tend to this result. In other words, the stimulated transition rates are continuous with respect to the critical velocity $1/n$. It should be noted that the previous approximate results in the limit of high temperature [see Eqs. (53), (54), (73), and (74)] are invalid for $v = 1/n$. So let us expand solely the above functions by series,

$$g_\parallel(\lambda, v, n) \approx \frac{(n^2-1)\pi^2}{4n\lambda^2}, \quad (79)$$

$$g_\perp(\lambda, v, n) \approx -\frac{3\sqrt{n^2-1}\ln\left(\frac{1}{2}\lambda\sqrt{1-\frac{1}{n^2}}\right)}{4\lambda}. \quad (80)$$

Similarly, in the limit of low temperature, we obtain

$$g_\parallel(\lambda, v, n) \approx \frac{3n(n^2-1)}{2n^2\lambda^2}e^{-\frac{\lambda}{2}\sqrt{1-\frac{1}{n^2}}}, \quad (81)$$

$$g_\perp(\lambda, v, n) \approx \frac{3\sqrt{n^2-1}}{4\lambda}e^{-\frac{\lambda}{2}\sqrt{1-\frac{1}{n^2}}}. \quad (82)$$

Surprisingly, the behaviors of the atomic transition rates for $v = 1/n$ in the limit of high or low temperature are different from both the cases of $v < 1/n$ and $v > 1/n$. By Fig. 3 we find that for a lower-temperature area the transition rate for an atom polarized perpendicularly to the atomic velocity ($\alpha_\parallel = 0, \alpha_\perp = 1$) is greater than the case for an atom polarized parallel to the atomic velocity ($\alpha_\parallel = 1, \alpha_\perp = 0$). This character will be inverse for a higher-temperature area. This clearly verifies the correctness of the above results of analytical analysis.

When the atomic velocity is close to the light velocity in vacuum ($v \to 1$), i.e., $\gamma \gg 1$, we have

$$g_\parallel^{\text{sup}}(\lambda, v, n) \approx \frac{3(n^2-1)}{n^2\lambda^3}\zeta(3)\gamma^{-1}, \quad (83)$$

$$g_\perp^{\text{sup}}(\lambda, v, n) \approx \frac{3(n^2-1)^2}{2n^2\lambda^3}\zeta(3)\gamma. \quad (84)$$

As a contrast, here we also give the missing analysis in [78] corresponding to the case in the absence of the medium ($n = 1$),

$$g_\parallel(\lambda, v) \approx \frac{\pi^2}{4\lambda^2}\gamma^{-2}, \quad (85)$$

$$g_\perp(\lambda, v) \approx \frac{3\ln\frac{2\gamma}{\lambda}}{4\lambda}\gamma^{-1}. \quad (86)$$

Notably, both are simply proportional to T^3 in the presence of the medium, unlike in the case when the medium is absent. More striking is that $g_\parallel^{\text{sup}}(\lambda, v, n)$ is proportional to γ^{-1} but $g_\perp^{\text{sup}}(\lambda, v, n)$ is proportional to γ, which means that after appending the Lorentz factor in Eq. (69) due to the time dilation effect the transition rate tends to be finite and nonzero for ultrarelativistic atoms polarizable perpendicular to the atomic velocity. This is in sharp contrast with the case in the absence

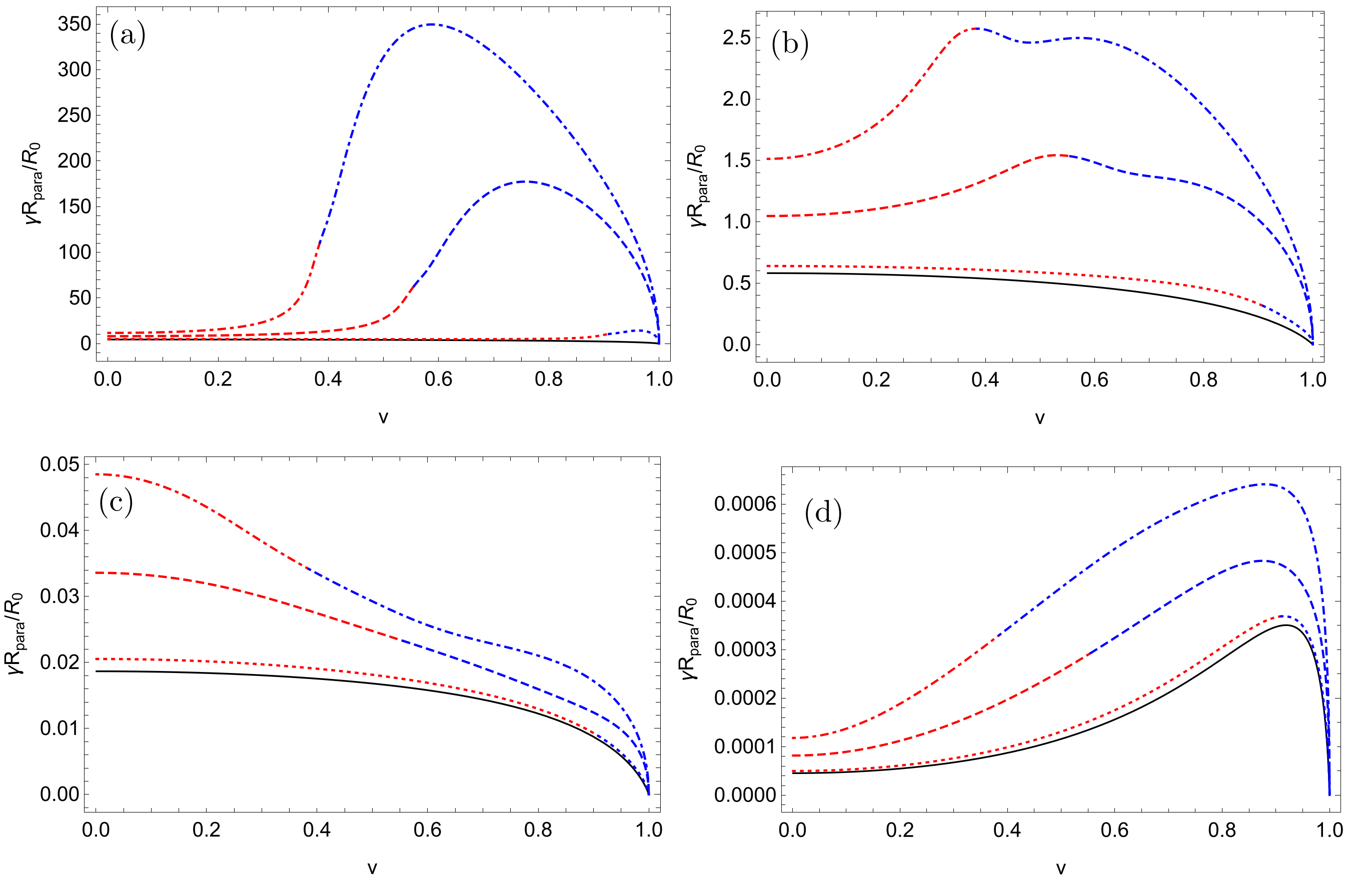

FIG. 4. Stimulated transition rate of a uniformly moving atom in a thermal radiation field within a dielectric medium as a function of the atomic velocity. The direction of the atomic dipole moment is parallel to the direction of atomic motion ($\alpha_\parallel = 1, \alpha_\perp = 0$). The atomic transition rate is depicted in units of $\gamma^{-1}R_0$. The dotted, dashed, and dot-dashed lines refer to the cases $n = 1.1$, $n = 1.8$, and $n = 2.6$, respectively. The red parts correspond to the result of atomic subluminal motion, and the blue parts correspond to that of atomic superluminal motion. As a contrast, the thin black solid lines denote the case in the absence of the medium ($n = 1$). (a) Case for $\lambda = 0.2$, (b) case for $\lambda = 1$, (c) case for $\lambda = 4$, (d) case for $\lambda = 10$.

of the medium ($n = 1$), where the ultrarelativistic atoms are always insusceptible to the background thermal bath [74,78].

When the refractive index of the medium is extremely large ($n \gg 1$), one has

$$g_\parallel^{\rm sup}(\lambda, v, n) \approx \frac{3}{\lambda^3 v\gamma}\zeta(3), \tag{87}$$

$$g_\perp^{\rm sup}(\lambda, v, n) \approx \frac{3v\gamma}{2\lambda^3}\zeta(3)n^2. \tag{88}$$

The stimulated transition rate tends to a finite value independent of the refractive index for the atom polarized parallel to the atomic velocity. However, it is proportional to n^2 for the atom polarized perpendicularly to the atomic velocity and thus tends to be infinite.

VI. NUMERICAL ANALYSES OF THE RESULTS

For general values of v, λ, and n, some numerical results can be given. Figures 4 and 5 show the behavior of the atomic stimulated transition rate with the increase of the atomic velocity for the atoms with different polarizabilities. We note that for the atom polarized parallel to the atomic velocity, the presence of the medium can change the shape of the function, especially when temperature is high [Fig. 4(a)]. However, for the atom polarized perpendicularly to the atomic velocity, the behavior of the stimulated transition rates is completely reversed by the presence of the medium for an arbitrary temperature (Fig. 5). By comparing Fig. 4 and Fig. 5, we find that especially for areas of superluminal motion the behavior of the stimulated transition rates for the atom polarized parallel to the atomic velocity is quite different from the case for the atom polarized perpendicular to the atomic velocity.

We also present the behavior of the atomic stimulated transition rate with the increase of the refractive index of the medium (see Figs. 6 and 7). In general, the stimulated transition rates rise with the increase of the refractive index n. For the atom polarized parallel to the atomic velocity, the rise is steep for a small refractive index but then the rise gradually levels off for a large refractive index (Fig. 6). However, for the atom polarized perpendicularly to the atomic velocity, the rise is always steep and the transition rates keep rising even for large refractive index (Fig. 7). It should be pointed out that the numerical images here confirm the validity of the analytical results in the preceding sections.

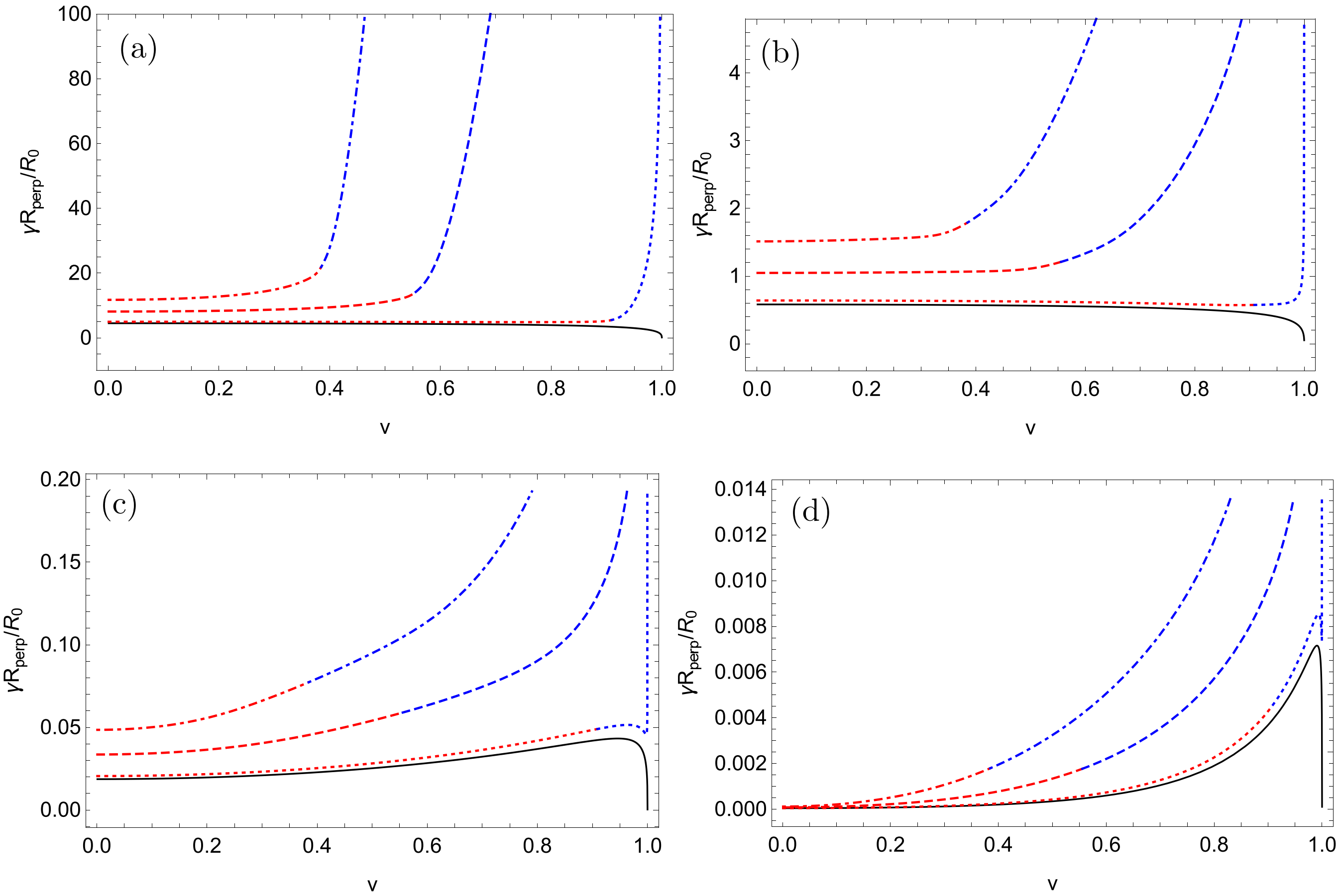

FIG. 5. Stimulated transition rate of a uniformly moving atom in a thermal radiation field within a dielectric medium, as a function of the atomic velocity. The direction of the dipole moment is perpendicular to the direction of atomic motion ($\alpha_\parallel = 0, \alpha_\perp = 1$). The atomic transition rate is depicted in the units of $\gamma^{-1}R_0$. The dotted, dashed, and dot-dashed lines refer to the cases $n = 1.1$, $n = 1.8$, and $n = 2.6$, respectively. The red parts correspond to the result of atomic subluminal motion, and the blue parts correspond to that of atomic superluminal motion. As a contrast, the thin black solid lines denote the case in the absence of the medium ($n = 1$). (a) Case for $\lambda = 0.2$, (b) case for $\lambda = 1$, (c) case for $\lambda = 4$, (d) case for $\lambda = 10$.

VII. CONCLUSIONS

Using the perturbation theory and including the Röntgen term in the atom-field interaction Hamiltonian, we have calculated the stimulated transition rate and analyzed its behaviors and properties for a uniformly moving atom in a thermal radiation field within a homogeneous isotropic dielectric medium. The cases of atomic subluminal motion ($0 < v < c/n$) and superlumial motion ($c/n < v < c$) are considered in detail, respectively. Notably, the stimulated transition rates are found to be finite when the atom moves at a superluminal motion in the nondispersive dielectric medium. This is opposite to the case of the spontaneous transition processes of an atom at a superluminal motion coupled to vacuum fluctuations of electromagnetic field in a medium, where the dispersion of the medium must be considered so as to avoid the divergence of the spontaneous transition rates [41–48]. Moreover, in our case the atomic stimulated transition rates are perfectly continuous on the critical speed c/n.

For the case of atomic subluminal motion, the atomic downward transition process is accompanied by only the emission of a photon in any direction and the atomic upward transition process is accompanied by the absorption of photons. However, for the case of atomic superluminal motion, the commonly called “stimulated emission process” (i.e., the atomic downward transition process) is accompanied by the two physical processes including the emission of the photon outside the Cherenkov cone and the absorption of the photon inside the Cherenkov cone. The commonly called “stimulated absorption process” (i.e., the atomic upward transition process) is accompanied by the absorption of the photon outside the Cherenkov cone and the emission of the photon inside the Cherenkov cone. The physical processes can be comprehensible in the atom’s rest frame, as the photons outside the Cherenkov cone have positive frequency (i.e., positive energy) but the photons inside the Cherenkov cone have negative frequency (i.e., negative energy) due to the anomalous doppler effect [42–45,49–51]. In the whole process, the conservation principle of energy is well satisfied.

In general, the stimulated upward or downward transition rates of the moving atom are always enhanced by the presence of the dielectric medium, as compared with the case in the absence of the medium [78]. The larger the refractive index of the medium is, the more obvious the effect is. In the limit of high temperature, the stimulated transition rates

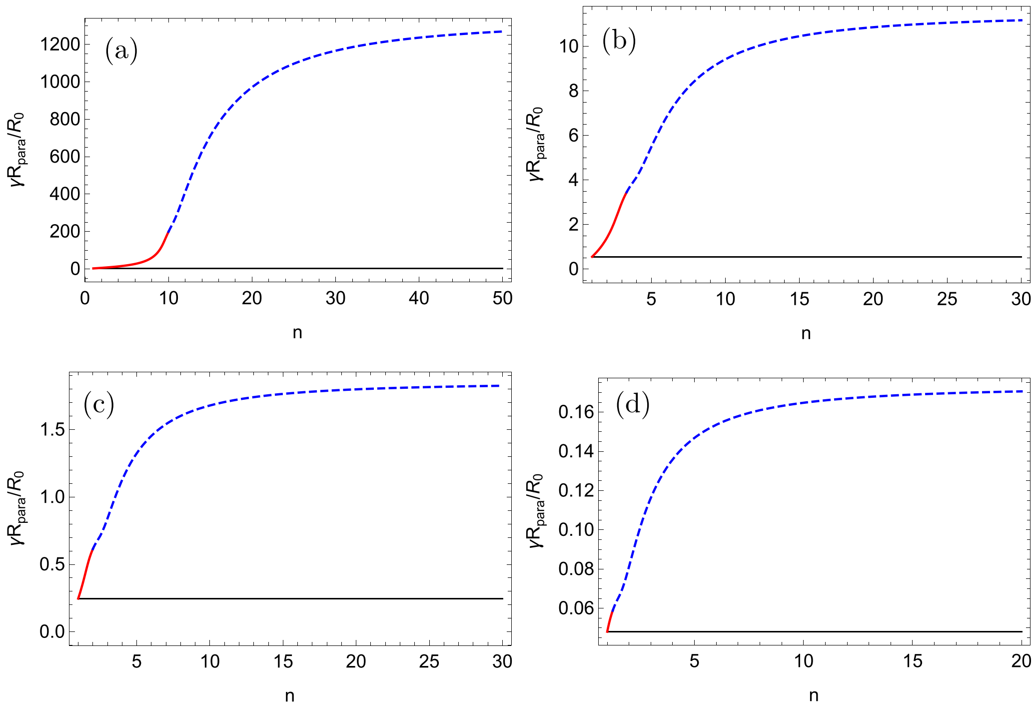

FIG. 6. Stimulated transition rate of a uniformly moving atom in a thermal radiation field within a dielectric medium, as a function of the refractive index of the medium. The direction of the dipole moment is parallel to the direction of atomic motion ($\alpha_{\parallel} = 1, \alpha_{\perp} = 0$). The atomic transition rate is depicted in the units of $\gamma^{-1}R_0$. The red parts correspond to the result of atomic subluminal motion, and the blue parts correspond to that of atomic superluminal motion. As a contrast, the thin black solid lines denote the corresponding result when $n = 1$. (a) Case for $v = 0.1$ and $\lambda = 0.3$, (b) case for $v = 0.3$ and $\lambda = 1$, (c) case for $v = 0.5$ and $\lambda = 1.5$, (d) case for $v = 0.8$ and $\lambda = 2.5$.

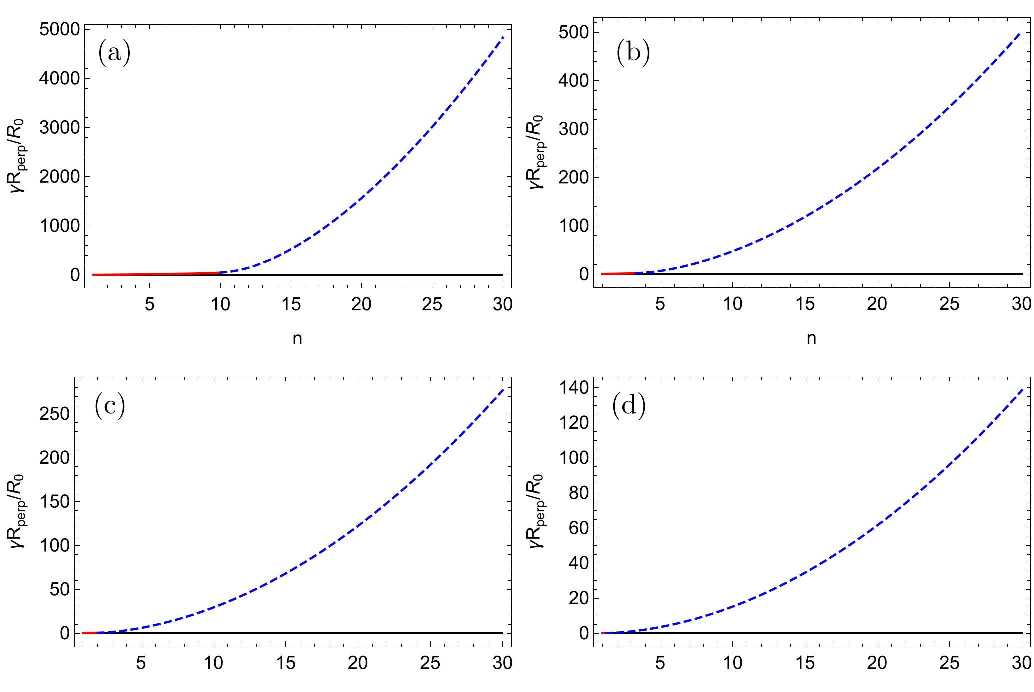

FIG. 7. Stimulated transition rate of a uniformly moving atom in a thermal radiation field within a dielectric medium, as a function of the refractive index of the medium. The direction of the dipole moment is perpendicular to the direction of atomic motion ($\alpha_{\parallel} = 0, \alpha_{\perp} = 1$). The atomic transition rate is depicted in the units of $\gamma^{-1}R_0$. The red parts correspond to the result of atomic subluminal motion, and the blue parts correspond to that of atomic superluminal motion. As a contrast, the thin black solid lines denote the corresponding result when $n = 1$. (a) Case for $v = 0.1$ and $\lambda = 0.3$, (b) case for $v = 0.3$ and $\lambda = 1$, (c) case for $v = 0.5$ and $\lambda = 1.5$, (d) case for $v = 0.8$ and $\lambda = 2.5$.

are still proportional to T for atoms with subluminal motion, but they are proportional to T^3 for atoms with superluminal motion. This indicates that the thermal radiation at high temperature in the medium has a more powerful impact on the stimulated transition processes of atoms with superluminal motion than that of atoms with subluminal motion. In the limit of low atomic velocity, the modifying contributions of atomic slow motion are always proportional to v^2, and they can be enhanced or weakened by the presence of the medium, contingent on the temperature and the refractive index. In particular, in the limit of both low velocity and high temperature, for different atomic polarizabilities we can simply define the effective temperatures $T_{\parallel}^{\text{eff}} = n(1 - \frac{15-12n^2}{10}v^2)T$ and $T_{\perp}^{\text{eff}} = n(1 - \frac{5-4n^2}{10}v^2)T$ for the thermal radiation perceived by the moving atom. The effective temperatures are always greater than T but into the modifying contributions of the atomic slow motion can be positive, zero, or negative corresponding to $n > \sqrt{5}/2$, $n = \sqrt{5}/2$, and $n < \sqrt{5}/2$, respectively. However, in the limit of both low velocity and low temperature, the modifying contributions of the atomic slow motion are always positive and are enhanced by the refractive index.

When the atomic velocity is close to the light speed in vacuum, the stimulated transition rate is simply proportional to T^3, just like in the limiting case of high temperature. Furthermore, the stimulated transition rate measured in the atom's frame is proportional to γ^{-1} for the atom polarized parallel to the atomic velocity, but it is proportional to the Lorentz factor γ for the atom polarized perpendicularly to the atomic velocity. This means that the presence of the medium has thoroughly changed the properties of the stimulated transition rates for ultrarelativistic atoms, as ultrarelativistic atoms are always insusceptible to the background thermal bath in the absence of the medium [74,78]. Finally, in the limit of a large refractive index, the behaviors of the stimulated transition rates are also quite different for different atomic polarizabilities. Our detailed analytical and numerical analyses clearly indicate that the stimulated transition rates for atoms with superluminal motion exhibit some unusual characteristics, which are quite distinct from the case of atomic subluminal motion. More extraordinary still, the behavior of the stimulated transition rates when the atomic velocity is equal to the velocity of light in the medium is different from the cases of both atomic subluminal motion and superluminal motion. These surprising physical consequences can actually be attributed to the combined action of the breaking of symmetry of the electromagnetic field by the dielectric medium and the normal or anomalous Doppler effect of the thermal radiation perceived by the uniformly moving atom. It is well known that the atomic transition rate is directly related to the density of states of the electromagnetic field. So our results imply that the thermal radiation spectrum in the dielectric medium as measured by an observer moving faster than the velocity of light in the stationary medium could have quite different behaviors and properties from the usual Planck spectrum. This needs to be further studied in future work. It is also necessary to further consider the effect of dispersion and dissipation of the medium on the atomic stimulated transition processes.

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