Cooper pairing and tripling in one-dimensional spinless fermions with attractive two- and three-body forces

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We theoretically investigate three-body correlations on the top of a Fermi sea in one-dimensional spinless fermions with antisymmetrized two- and three-body attractive interactions. By investigating the variational problem of three-body states above the Fermi sea, we illuminate the fate of the in-medium three-body cluster states (namely, Cooper triples in the presence of Fermi sea) in the special case with pure attractive three-body interaction as well as in the case with the coexistence of two- and three-body interactions. Our results testify that the fermion-dimer repulsion is canceled by including the three-body interactions, and stable three-body clusters can be formed. We further feature a phase diagram consisting of the p-wave Cooper pairing and Cooper tripling phases in a plane of p-wave two- and three-body coupling strengths.

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I. INTRODUCTION

A clean and controllable cold-atomic Fermi gas is one of the best candidates to investigate the unconventional states in quantum many-body systems in a systematic way. A remarkable feature of this system is the adjustable s-wave interaction via the Feshbach resonance [1]. It is well known that in a three-dimensional s-wave superfluid Fermi gas, the pairing superfluid undergoes a crossover from a Bardeen-Cooper-Schrieffer (BCS) regime with weak-coupling Cooper pairs to a Bose-Einstein condensation (BEC) regime of tightly bound molecules [2–4].

The p-wave interaction is also tunable near the p-wave Feshbach resonance [1], and a large number of related experiments have been performed towards the realization of p-wave Fermi superfluids [5–8]. In this regard, an ultracold Fermi gas near the p-wave Feshbach resonance may pave a promising way to systematically investigate the role of strong p-wave interactions in unconventional superfluids [9]. However, the p-wave superfluid has not been achieved experimentally yet due to the difficulty associated with strong atomic losses [7,10–14]. The p-wave Fermi superfluid state is shown to be unstable against three-body clustering in the three-dimensional system even without the Fermi degeneracy [15], which leads to the three-body recombination accompanying a strong particle loss. Indeed, strong three-body losses in three-dimensional Fermi gases near the p-wave Feshbach resonance have been observed in experiments [13,16]. In contrast, the suppression of the three-body loss in the one-dimensional p-wave system has been predicted theoretically [17], and a similar result has also been found in our previous work [18]. In this regard, the systems under the low-dimensional confinement [19] and the lattice geometry [20] have been studied experimentally to suppress the atomic loss. On the other hand, such a suppression of the atomic loss in the one-dimensional system is still under experimental investigation [19,21,22]. Indeed, the quasi-one-dimensional geometry may induce the s-wave interaction between identical fermions using the orbital degrees of freedom [22]. Moreover, the three-body clustering with coexistence of s- and p-wave interactions in one dimension has also been found theoretically [23].

The stability against the three-body clustering is deeply related to the properties of the interactions and quantum statistics. While the s-wave superconductivity and superfluidity involve the formation of spin-singlet Cooper pairs consisting of two fermions with antiparallel spins due to the fermionic antisymmetrization [24], the p-wave counterpart can induce the Cooper pairs consisting of two identical fermions in spite of the Pauli exclusion principle. In such a case, one cannot exclude the possibilities of more-than-two-body clustering correlations such as three- and four-body clusters, in contrast with a spin-1/2 Fermi gas with strong s-wave attractive interaction where the BCS-BEC crossover is realized without any larger clusters due to the Pauli exclusion principle. To investigate larger clusters, the generalized Cooper problem has been further applied to in-medium cluster states such as Cooper triples [18,23,25–29] and even Cooper quartets [30–34], which can be regarded as three- and four-body counterparts of a Cooper pair.

Moreover, the three-body force gives a significant influence on the properties of one-dimensional systems. An important example is the emergence of a quantum anomaly in one-dimensional fermions with the three-body interaction [35], where even an infinitesimally small three-body attraction can cause the three-body bound state. In such a case, the three-body clusters may survive even in the high-density regime

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where fermions are quantum degenerate [29,36]. Another example is the broken integrability [37], which is recently discussed in terms of the transport coefficient [38]. Also, the three-body interaction is needed to maintain the Bose-Fermi mapping in one dimension [39]. For the cases of one-dimensional bosonic systems, the three-body interactions lead to the formations of quantum droplet states and excited few-body bound states [40–43]. Based on the Bose-Fermi mapping [39,44–46], the counterpart of such unconventional states in bosonic systems are also expected to exist in one-dimensional fermions. In bosonic systems, the controllable three-body force has been proposed theoretically [47] and demonstrated experimentally [48].

While the present authors showed the absence of both in-medium (in the presence of Fermi sea) and in-vacuum three-body bound states in the one-dimensional system with only two-body p-wave interaction due to the in-medium fermion-dimer repulsion [18], if some additional three-body forces exist, the in-vacuum three-body bound state can be induced as found in Ref. [39]. Indeed, by including the dimensionless three-body coupling in the atom-dimer scattering, we found the solution of the binding energy for the in-medium three-body bound state in the previous paper [18].

In this way, it is expected that the three-body ground state can be found by further introducing the three-body interactions. However, the detailed investigation of the three-body force is still lacking because a specific form of the three-body interaction was phenomenologically introduced in Refs. [18,39]. In this regard, we start with an antisymmetrized three-body interaction with minimal momentum dependence which can be a leading-order contribution at the low-energy limit and investigate the three-body clustering in one-dimensional spinless fermions with the coexistence of two- and three-body interactions. Our results can be further testified in future cold-atomic experiments via three-body loss measurements. Such an antisymmetrized three-body interaction might be achieved through the quasi-one-dimensionality [38] or Rabi coupling [48]. In addition, it might also be possible via the medium-induced interaction as we proposed recently by preparing an additional medium [49]. While the above approaches were performed in the bosonic systems, they are also expected to be realized in the fermionic ones. The systematic studies of the effects of two- and three-body forces in fermionic systems would also be useful for understanding the role of three-body forces in nuclear systems [50], which have been examined in recent experiments [51].

This paper is organized as follows: In Sec. II, we first introduce the Hamiltonian for the one-dimensional spinless fermions with the coexistence of two- and three-body forces. After that, we calculate the expectation value of the energy and derive the corresponding variational equation. The results and discussion will be given in Sec. III. In detail, we first investigate the in-medium three-body problem in one-dimensional spinless p-wave fermions with pure three-body interaction in Sec. III A. As a step further, by also including the two-body interaction, we study the in-medium three-body clustering in the general case with the coexistence of two- and three-body interactions in Sec. III B. Finally, we summarize this paper in Sec. IV. In the following, we take $\hbar = c = k_{\rm B} = 1$. The system size is taken to be unity.

II. THEORETICAL FRAMEWORK

We consider one-dimensional spinless fermions with two- and three-body interactions described by the Hamiltonian

$$H = K + V_2 + V_3, \tag{1}$$

where the kinetic term K and two-body interaction V_2 are given as

$$K = \sum_k \xi_k c_k^\dagger c_k, \tag{2}$$

$$V_2 = \frac{U_2}{2} \sum_{k_1,k_2,k_1',k_2'} \left(\frac{k_1 - k_2}{2}\right)\left(\frac{k_1' - k_2'}{2}\right) \times B^\dagger_{k_1,k_2} B_{k_2',k_1'} \delta_{k_1+k_2,k_1'+k_2'}, \tag{3}$$

respectively. Here, $\xi_k = k^2/(2m) - \mu$ is the single-particle energy with momentum k, atomic mass m, and chemical potential μ. The two-body interaction adopted here corresponds to the short-range p-wave type with a coupling constant U_2, which is related to the zero-range limit of the two-channel model for the Feshbach resonance [52,53]. The relation between U_2 and the p-wave scattering length a is obtained from the two-body T matrix as [45,52]

$$\frac{1}{U_2} - \sum_p \frac{mp^2}{k^2 + i\delta - p^2} = \frac{m}{2}\left(\frac{1}{a} - \frac{1}{2} r_{\rm eff} k^2 + ik\right), \tag{4}$$

where $r_{\rm eff}$ is the effective range and δ is an infinitesimally small number. $r_{\rm eff}$ is associated with the momentum cutoff $\Lambda = 4/\pi r_{\rm eff}$. Taking the consideration of the antisymmetry and parity for the form factor, we introduce an antisymmetrized attractive three-body interaction as

$$V_3 = \sum_{k_1,k_2,k_3} \sum_{k_1',k_2',k_3'} U_3 \times \left(\frac{k_1 - k_2}{2}\right)\left(\frac{k_2 - k_3}{2}\right)\left(\frac{k_3 - k_1}{2}\right) \times \left(\frac{k_1' - k_2'}{2}\right)\left(\frac{k_2' - k_3'}{2}\right)\left(\frac{k_3' - k_1'}{2}\right) \times c^\dagger_{k_1} c^\dagger_{k_2} c^\dagger_{k_3} c_{k_3'} c_{k_2'} c_{k_1'} \delta_{k_1+k_2+k_3,k_1'+k_2'+k_3'}, \tag{5}$$

where U_3 is the coupling constant of the three-body interaction. Moreover, by introducing the pair operator

$$B^\dagger_{k_1,k_2} = c^\dagger_{k_1} c^\dagger_{k_2}, \quad B_{k_1,k_2} = c_{k_2} c_{k_1}, \tag{6}$$

and the trimer operator

$$F^\dagger_{k_1,k_2,k_3} = c^\dagger_{k_1} c^\dagger_{k_2} c^\dagger_{k_3}, \quad F_{k_1,k_2,k_3} = c_{k_3} c_{k_2} c_{k_1}, \tag{7}$$

one can rewrite V_3 as

$$V_3 = \sum_{k_1,k_2,k_3} \sum_{k_1',k_2',k_3'} U_3 \times \left(\frac{k_1 - k_2}{2}\right)\left(\frac{k_2 - k_3}{2}\right)\left(\frac{k_3 - k_1}{2}\right)$$

$$\times\left(\frac{k_1'-k_2'}{2}\right)\left(\frac{k_2'-k_3'}{2}\right)\left(\frac{k_3'-k_1'}{2}\right)$$
$$\times B^\dagger_{k_1,k_2}c^\dagger_{k_3}c_{k_3'}B_{k_1',k_2'}\delta_{k_1+k_2+k_3,k_1'+k_2'+k_3'}$$
$$=\sum_{k_1,k_2,k_3}\sum_{k_1',k_2',k_3'}U_3$$
$$\times\left(\frac{k_1-k_2}{2}\right)\left(\frac{k_2-k_3}{2}\right)\left(\frac{k_3-k_1}{2}\right)$$
$$\times\left(\frac{k_1'-k_2'}{2}\right)\left(\frac{k_2'-k_3'}{2}\right)\left(\frac{k_3'-k_1'}{2}\right)$$
$$\times F^\dagger_{k_1,k_2,k_3}F_{k_1',k_2',k_3'}\delta_{k_1+k_2+k_3,k_1'+k_2'+k_3'}. \tag{8}$$

Such a kind of three-body interaction would be the leading order of the antisymmetrized attractive ones at low energy in the sense of the derivative expansion.

We note that the two- and three-body interactions considered in the Hamiltonian (1) can also be related to the scattering hypervolume D_F [54], which is a three-body analog of the two-body scattering length. In general, D_F can be extracted by solving the three-body Schrödinger equation numerically at zero energy and matching the resultant wave function with the asymptotic expansions of wave function [55]. In this regard, since the calculation of D_F for given U_2 and U_3 is not so straightforward and moreover D_F has not been experimentally measured yet in contrast to the two-body scattering length, we measure U_3 by using the critical coupling strength U_c where the three-body bound state appears only with the three-body attraction in vacuum. Accordingly, U_c may be regarded as the resonant coupling where D_F diverges [55].

In a way similar to the previous works [18,26,28], the trial wave function for the three-body cluster on the top of the Fermi sea is given by

$$|\Psi_3\rangle=\sum_{p_1,p_2,p_3}{}^{'}\delta_{p_1+p_2,-p_3}\Omega_{p_1,p_2}F^\dagger_{p_1,p_2,p_3}|\mathrm{FS}\rangle, \tag{9}$$

where Ω_{p_1,p_2} is the variational parameter and the three-body state with zero center-of-mass momentum ($p_1+p_2+p_3=0$) is considered. Hereafter, we introduce the momentum summation restricted by the Pauli blocking as

$$\sum_{k_1,k_2,\dots}{}^{'}\mathcal{F}(k_1,k_2,\dots)$$
$$=\sum_{k_1,k_2,\dots}\theta(|k_1|-k_\mathrm{F})\theta(|k_2|-k_\mathrm{F})\cdots F(k_1,k_2,\dots), \tag{10}$$

for an arbitrary function $\mathcal{F}(k_1,k_2,\dots)$, where $k_\mathrm{F}=\sqrt{2mE_\mathrm{F}}$ is the Fermi momentum.

From the variational principle with the in-medium three-body energy E_3, we obtain

$$\frac{\delta\langle\Psi_3|(H-E_3)|\Psi_3\rangle}{\delta\Omega^*_{p_1,p_2}}=\frac{\delta\langle\Psi_3|(K+V_2+V_3-E_3)|\Psi_3\rangle}{\delta\Omega^*_{p_1,p_2}}=0. \tag{11}$$

Detailed results for the expectation values of each term in the Hamiltonian are given in Appendix A. The resulting variational equation reads

$$\begin{aligned}
&2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)[\Omega_{p_1,p_2}+\Omega_{p_2,p_3}+\Omega_{p_3,p_1}]\\
&=-\frac{U_2}{2}\sum_q{}^{'}[(p_1-p_2)(2q+p_3)(2\Omega_{p_3,q}+\Omega_{q,-q-p_3})+(p_2-p_3)(2q+p_1)(2\Omega_{p_1,q}+\Omega_{q,-q-p_1})\\
&+(p_3-p_1)(2q+p_2)(2\Omega_{p_2,q}+\Omega_{q,-q-p_2})]\\
&-\frac{9U_3}{16}\sum_{p_1',p_2'}{}^{'}(p_1-p_2)(p_1'-p_2')(p_1+2p_2)(p_1'+2p_2')(2p_1+p_2)(2p_1'+p_2')\Omega_{p_1',p_2'}.
\end{aligned} \tag{12}$$

III. RESULTS AND DISCUSSION

A. Pure three-body interaction (without two-body interaction)

First, let us consider the case without two-body interaction ($U_2=0$). In this case, the variational equation (12) can be recast into

$$2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)[\Omega_{p_1,p_2}+\Omega_{p_2,p_3}+\Omega_{p_3,p_1}]$$
$$=\frac{9U_3}{16}\mathcal{X}_{p_1,p_2,p_3}\mathcal{C}, \tag{13}$$

where for convenience we introduced

$$\mathcal{X}_{p_1,p_2,p_3}=(p_1-p_2)(p_2-p_3)(p_3-p_1), \tag{14}$$

and

$$\mathcal{C}=-\sum_{p_1',p_2',p_3'}{}^{'}\delta_{p_1'+p_2'+p_3',0}\mathcal{X}_{p_1',p_2',p_3'}\Omega_{p_1',p_2'}. \tag{15}$$

Consequently, we can obtain the equation regarding the amplitude of the trial wave function $\Omega_{p,k}$ as

$$\mathcal{X}_{p_1,p_2,p_3}[\Omega_{p_1,p_2}+\Omega_{p_2,p_3}+\Omega_{p_3,p_1}]$$
$$=\frac{9U_3}{16}\frac{\mathcal{X}^2_{p_1,p_2,p_3}\mathcal{C}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{16}$$

Finally, by further taking the momenta summations with $-\delta_{p_1+p_2+p_3,0}$, the three-body equation with only the

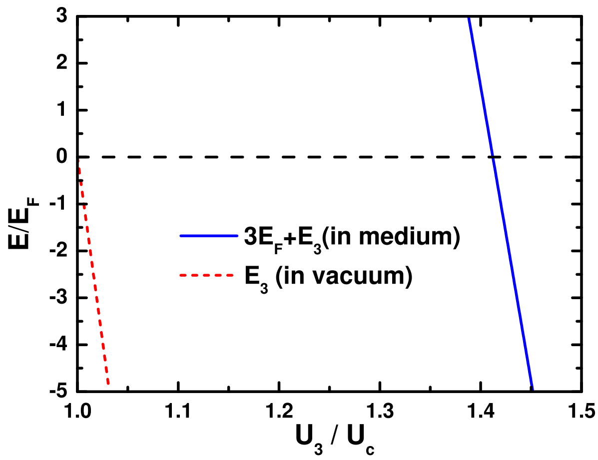

FIG. 1. The relation between three-body state energy E_3 and three-body interaction strength U_3. The critical coupling strength U_c is a coupling strength where the three-body bound state appears in vacuum. In this figure, Λ is taken as $10k_{\rm F}$.

three-body interaction reads

$$1 = -\frac{3U_3}{16}\sum_{p_1,p_2}{}^{\prime}\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}, \tag{17}$$

where the constraint $|p_3| = |-p_1-p_2| \geqslant k_{\rm F}$ is used in Eq. (17).

The solution of the in-medium three-body energy in the case with the pure three-body interaction (17) is shown in the solid line in Fig. 1 with a shift of $3E_{\rm F}$, where the ultraviolet momentum cutoff Λ is taken as $10k_{\rm F}$. For comparison, the solution of the in-vacuum three-body energy is also plotted in the dashed line. The three-body coupling constant U_3 is normalized by the in-vacuum critical coupling $U_{\rm c}$, where the three-body bound state starts to appear in vacuum. The results indicate that the in-medium three-body bound state assisted by the Fermi-surface effect does not exist in the absence of the in-vacuum counterpart, which is different from the three-component Fermi gas with two-body s-wave interactions in three dimensions [28]. However, there is still a regime (around $1.38 < U_3/U_c < 1.41$) with positive $E_3 + 3E_{\rm F}$ in medium, which corresponds to the so-called squeezed Cooper triple regime in the three-dimensional s-wave case [28]. In analogy with the pairing state in the BCS-BEC crossover [3,4], actually there is no noticeable structural change of Cooper triples. In this sense, the regime with positive $E_3 + 3E_{\rm F}$ in medium (around $1.38 < U_3/U_c < 1.41$) in Fig. 1 can be still regarded as the Cooper triple phase but may have a relatively smaller cluster size due to a strong attraction as to ensure a three-body cluster would be bound in vacuum. We note that the in-medium three-body bound states do not always require a stronger three-body coupling than the case in vacuum, and one can find the loosely bound Cooper triples due to the interplay between two- and three-body interactions, which is shown in the latter section.

Finally, the in-medium three-body binding energy $E_3 + 3E_{\rm F}$ turns out to negative when U_3/U_c is around 1.41. Such a regime corresponds to the formation of three-body bound states which are dominated by the strong three-body interaction. The coupling strength $U_3/U_c \simeq 1.41$, where $E_3 + 3E_{\rm F} = 0$, is analogous to the region where the chemical potential changes the sign in the BCS-BEC crossover [3,4]. Although in this work we consider a three-body system on top of the Fermi sea, our results can describe the qualitative features of both the weak- and strong-coupling limits appropriately by the variational equation in a unified manner [28].

B. Coexistence of two- and three-body interactions

In this subsection, we further investigate the general case with the presence of both two- and three-body interactions. To simplify the expressions, besides $\mathcal{C}$ as given in Eq. (15), here we also introduce

$$\mathcal{A}(p_1,p_2) = \sum_{q}{}^{\prime}(p_1-p_2)(2q+p_3)(2\Omega_{p_3,q}+\Omega_{q,-q-p_3}), \tag{18}$$

$$\mathcal{B}(p_2) = \sum_{p_1,p_3}{}^{\prime}(\Omega_{p_1,p_2}+\Omega_{p_3,p_1}+\Omega_{p_2,p_3}) \times (p_3-p_1)\delta_{p_1+p_2+p_3,0}, \tag{19}$$

where

$$\mathcal{A}(p_1,p_2) = (p_1-p_2)\mathcal{B}(p_3). \tag{20}$$

With the help of above notations, we obtain the closed equation for $\mathcal{B}(p)$ and E_3 as

$$\mathcal{B}(p_2)\left[\frac{1}{U_2}+\sum_{p_1}{}^{\prime}\frac{(p_1+p_2/2)^2}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3}\right] = \sum_{p_1}{}^{\prime}\frac{(p_1+2p_2)(p_1+p_2/2)\mathcal{B}(p_1)}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3} + \frac{\frac{9U_3}{16}\sum_{p_1}^{\prime}\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}{1+\frac{3U_3}{16}\sum_{p_1,p_2}^{\prime}\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}\sum_{p_1,p_2}{}^{\prime}\frac{(p_2+p_1/2)\mathcal{X}_{p_1,p_2,p_3}\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)} \tag{21}$$

from the full variational equation (12). In Eq. (21), we take $p_3 = -p_1 - p_2$ because of the momentum conservation at the center-of-mass frame of the three-body system as in Eq. (17). The detailed derivations for Eq. (21) can be found in Appendix B. For convenience, we rewrite Eq. (21) as

$$\mathcal{B}(p_2)\left[\frac{1}{U_2}+I_2(p_2,E_3)\right] = I_3(p_2,E_3)+\frac{I_4(E_3)I_5(p_2,E_3)}{1+I_6(E_3)}, \tag{22}$$

where the integrals read

$$I_2(p_2,E_3)=\sum_{p_1}{}^{'}\frac{(p_1+p_2/2)^2}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3}, \tag{23}$$

$$I_3(p_2,E_3)=\sum_{p_1}{}^{'}\frac{(p_1+p_2/2)(p_1+2p_2)\mathcal{B}(p_1)}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3}, \tag{24}$$

$$I_4(E_3)=\sum_{p_1,p_2}{}^{'}\frac{(p_2+p_1/2)\mathcal{X}_{p_1,p_2,p_3}\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}, \tag{25}$$

$$I_5(p_2,E_3)=\frac{9U_3}{16}\sum_{p_1}{}^{'}\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}, \tag{26}$$

$$I_6(E_3)=\frac{3U_3}{16}\sum_{p_1,p_2}{}^{'}\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}, \tag{27}$$

respectively, where we apply the constraints $|p_3|\equiv|p_1+p_2|\geqslant k_{\rm F}$ in each momentum summation. We note that $1+I_6(E_3)=0$ corresponds to the three-body equation (17) for the case with pure three-body interaction discussed in Sec. III A. In addition, the right-hand side of Eq. (21) can be recast into

$$\begin{aligned}
&\sum_{p_1}{}^{'}\frac{(p_1+2p_2)(p_1+p_2/2)\mathcal{B}(p_1)}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3}+\frac{\frac{9U_3}{16}\sum_{p_1}'\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}{1+\frac{3U_3}{16}\sum_{p_1,p_2}'\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}\sum_{p_1,p_2}{}^{'}\frac{(p_2+p_1/2)\mathcal{X}_{p_1,p_2,p_3}\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}\\
&=\sum_{p_1}{}^{'}\left[\frac{(p_1+2p_2)(p_1+p_2/2)}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3}+\frac{\frac{9U_3}{16}\sum_{p_1}'\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}{1+\frac{3U_3}{16}\sum_{p_1,p_2}'\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}\sum_{p_2}\frac{(p_2+p_1/2)\mathcal{X}_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}\right]\mathcal{B}(p_1)\\
&\equiv-\frac{m}{2}\sum_{p_1}{}^{'}[t_{\rm F}(p_1,p_2,E_3)+\mathcal{V}_3(p_1,p_2,E_3)]\mathcal{B}(p_1),
\end{aligned} \tag{28}$$

where

$$t_{\rm F}(p_1,p_2,E_3)=-\frac{2}{m}\frac{(p_1+2p_2)(p_1+p_2/2)}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3} \tag{29}$$

corresponds to Eq. (38) in Ref. [18] and

$$\mathcal{V}_3(p_1,p_2,E_3)\equiv-\frac{2}{m}\frac{\frac{9U_3}{16}\sum_{p_1}'\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}{1+\frac{3U_3}{16}\sum_{p_1,p_2}'\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}\sum_{p_2}{}^{'}\frac{(p_2+p_1/2)\mathcal{X}_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{30}$$

From the above discussion, one can find that $\mathcal{V}_3(p_1,p_2,E_3)$ corresponds to the dimensionless constant three-body (atom-dimer) coupling $\mathcal{V}_3=2$ introduced in Refs. [18,39], while instead $\mathcal{V}_3(p_1,p_2,E_3)$ has the momentum and energy dependence.

In the practical calculation, we solve Eq. (22) with an iteration method numerically evaluating $I_2(p_2,E_3)$, $I_3(p_2,E_3)$, $I_4(E_3)$, $I_5(p_2,E_3)$, and $I_6(E_3)$. We start the iteration from the initial value $\mathcal{B}(p)=1$ for a given value of E_3, since E_3 is unchanged by the scale transformation of the solution $\mathcal{B}(p)$ in Eq. (22). For the convergence of $\mathcal{B}(p)$, we have required

$$\sum_n\frac{[\mathcal{B}_{\rm in}(n)-\mathcal{B}_{\rm out}(n)]^2}{\mathcal{B}_{\rm in}(n)^2}\leqslant 10^{-8}, \tag{31}$$

where $\mathcal{B}_{\rm in}(n)$ [$\mathcal{B}_{\rm out}(n)$] is the input (output) during the iteration for Eq. (22). n is the number of the discretized momentum $p=n\Delta p+k_{\rm F}$ used in the Newton-Cotes integration with $\Delta p=(\Lambda-k_{\rm F})/N$. We have confirmed that $N=1000$ is sufficient for the convergence in the regime of interest here. For the momentum cutoff, as the same as that in the pure three-body interaction case, Λ is also taken as $10k_{\rm F}$ in the ensuing calculations.

Figure 2 shows the numerical solution of E_3 as a function of $1(k_{\rm F}a)$, where U_3 is taken as the in-medium critical coupling $U_c^{\rm med}$ for the formation of the in-medium three-body bound state $1/(k_{\rm F}a)=-1.0$. From the result shown in Fig. 2, E_3 monotonically decreases due to the increasing two-body coupling strength. In addition, as shown in the lower panel of Fig. 2, where $1/(k_{\rm F}a)=-0.6$ is adopted, while E_3 exhibits a cutoff dependence, the qualitative behavior of the binding energies (e.g., $E_3<E_{2,p}$) is unchanged. As a result, we conclude that $\Lambda/k_{\rm F}=10$ is sufficient for understanding physical properties of in-medium bound states in the present system.

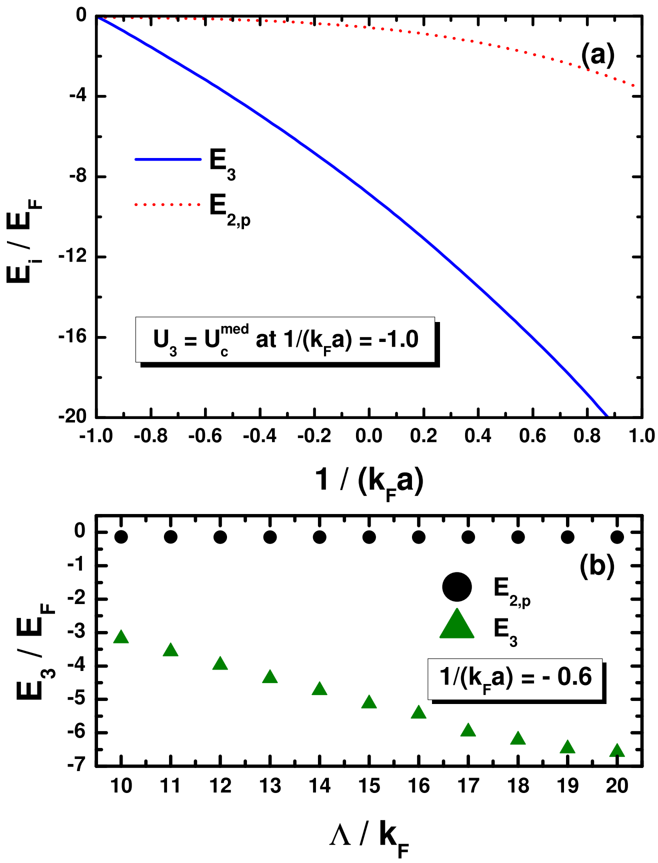

FIG. 2. (a) In-medium three-body energy E_3 as a function of $1/(k_{\rm F}a)$ solved from Eq. (22). The three-body coupling constant U_3 is taken as in-medium critical coupling $U_c^{\rm med} = 0.986U_c$ at $1/(k_F a) = -1.0$. (b) Momentum cutoff dependence of $E_{2,p}$ and E_3 at $1/(k_{\rm F}a) = -0.6$.

To see the effect of the renormalized three-body coupling, $t_{\rm F}(p_1, p_2, E_3) + \mathcal{V}_3(p_1, p_2, E_3)$ defined in Eq. (28) is shown as a function of p_1 for the cases with $\mathcal{V}_3(p_1, p_2, E_3) = 0$, $\mathcal{V}_3(p_1, p_2, E_3) = 2$ (i.e., the constant three-body coupling proposed in Ref. [39]), and the form of Eq. (30), respectively, in Fig. 3, where we take $p_2 = k_{\rm F}$ and $E_3 = -3.18E_{\rm F}$. In the case with $\mathcal{V}_3 = 2$, one can find the fermion-dimer repulsion induced by the three-body kernel $t_{\rm F}(p_1, p_2, E_3)$ is canceled by the constant coupling $\mathcal{V}_3 = 2$, when comparing it with the result of $\mathcal{V}_3 = 0$. On the other hand, the momentum-dependent coupling $\mathcal{V}_3(p_1, k_{\rm F}, E_3)$ generated by U_3 in Eq. (30) exhibits the low-energy attraction at $p_1 \lesssim 5k_{\rm F}$, leading to the in-medium three-body bound state. While it shows a cutoff dependence at large momenta due to the present form factor in the three-body interaction, such a high-momentum repulsion is weakened when the cutoff increases. This result is consistent with the cutoff dependence of E_3 shown in Fig. 3(b). Eventually, even though the fermion-dimer repulsion is present in the high-momentum regime, the low-momentum attraction near the Fermi surface (i.e., $p_1 \simeq k_{\rm F}$ and $p_2 \simeq k_{\rm F}$) associated with U_3 induces the in-medium three-body bound states.

To see the role of U_3 in detail, E_3 as a function of three-body coupling strength U_3 at $1/(k_{\rm F}a) = -1.0$ solved from

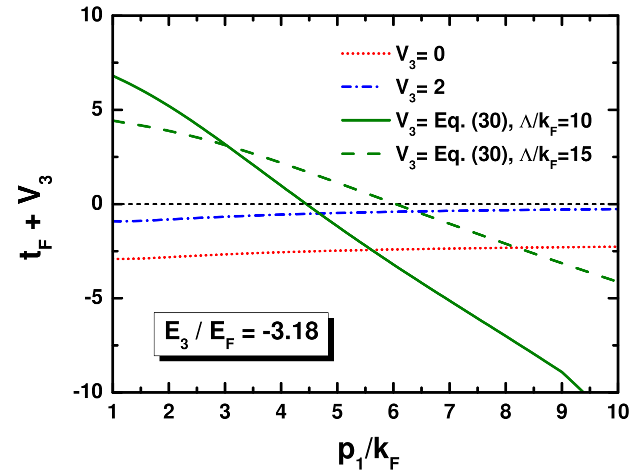

FIG. 3. Dimensionless three-body interaction kernel $t_{\rm F}(p_1/k_{\rm F}, p_2/k_{\rm F}, E_3/E_{\rm F}) + \mathcal{V}_3(p_1/k_{\rm F}, p_2/k_{\rm F}, E_3/E_{\rm F})$ as functions of p_1 with $p_2/k_{\rm F} = 1$ and $E_3/E_{\rm F} = -3.18$ for different $\mathcal{V}_3$. The three-body coupling constant U_3 is taken as in-medium critical coupling $U_c^{\rm med}$ at $1/(k_F a) = -1.0$.

Eq. (22) is also shown in Fig. 4. U_3 is normalized in the reference scheme of the critical coupling strength U_c. From Fig. 4, it is seen that the three-body clusters gradually become more tightly bound with the increase of three-body coupling constant. While pure three-body force case always exhibits squeezed Cooper triples, it is not always true in the presence of two-body attraction. It is seen that E_3 becomes finite here even when $U_3/U_c < 1$. On the other hand, indeed, the definition of the boundary of the crossover between Cooper triple phase and squeezed one also involves an ambiguity.

For comparison, we also show p-wave Cooper pairing energies $E_{2,p}$ in Figs. 2 and 4, which can be obtained from the in-medium two-body equations for the p-wave pairing [18]

$$1 + U_2 \sum_k{}' \frac{k^2}{\xi_k + \xi_{-k} - E_{2,p}} = 0. \tag{32}$$

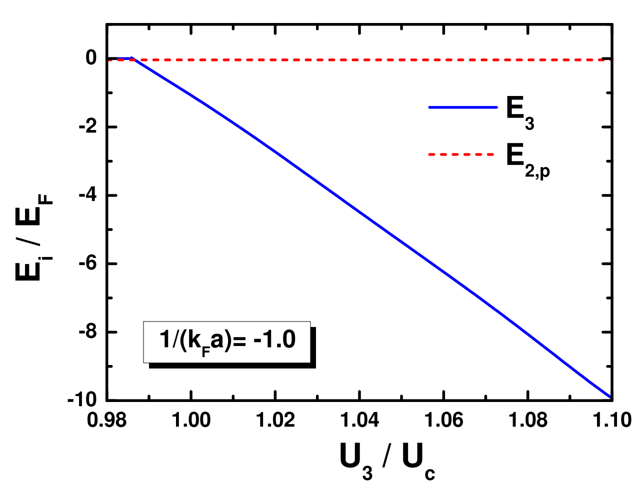

FIG. 4. In-medium three-body energy E_3 as a function of dimensionless coupling constant of three-body interaction U_3 at $1/(k_{\rm F}a) = -1.0$ solved from Eq. (22). U_3 is normalized in the reference scheme of the critical coupling strength U_c.

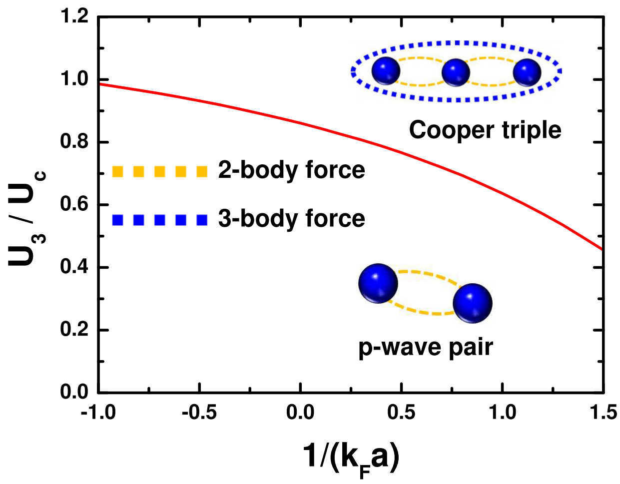

FIG. 5. Phase diagram of p-wave pair phase ($|E_{2,p}| > |E_3|$) and Cooper triple phase ($|E_3| > |E_{2,p}|$) in the plane of U_3/U_c and $1/(k_\mathrm{F}a)$. Namely, the Cooper triple (p-wave pair) phase is more stable in the regime above (below) the solid line.

In Fig. 5, we summarize the ground-state phase diagram of p-wave Cooper pairing states and the Cooper tripling state in the present model. The phase boundaries are determined in such a way that the boundary between tripling and p-wave pairing is given by $E_3 = E_{2,p}$. Such a phase diagram captures the competition between p-wave pairings and tripling. Figure 5 shows that the transition line between p-wave pair and Cooper triple phase monotonically decreases with the increasing two-body interaction. However, this critical three-body coupling cannot trivially become zero at the strong-coupling limit [$1/(k_\mathrm{F}a) \to +\infty$], as we have figured out that the solution of three-body bound state cannot be found in the absence of three-body interaction [18]. Consequently, there is a threshold for the transition line at a certain two-body coupling strength [around $1/(k_\mathrm{F}a) \simeq 2$ in our calculation] due to the competition between the fermion-dimer repulsion and U_3 in the strong-coupling regime. We note that such a threshold is not expected to be universal and would be associated with a detailed high-momentum structure of the interactions. The quantitative investigation of such a behavior point and the properties in stronger coupling regimes are out of the scope in this paper.

While the values of Λ and U_3 should be clarified to compare our results with the experiments, our phase diagram would be useful to understand the qualitative features of systems with the coexistence of two- and three-body interactions. Since the three-body ground-state energy depends on both the two- and three-body coupling constants, one can explore the competition between pairing and tripling phases by tuning the interactions in cold atomic systems.

IV. SUMMARY AND PERSPECTIVES

In this paper, we have investigated the in-medium three-body correlations (i.e., in the presence of Fermi sea) in one-dimensional spinless fermions with two- and three-body interactions. We solve the in-medium three-body equation derived from the variational approach based on the generalized Cooper problem. In contrast to the previous works [18,39], where a specific form of the constant three-body interaction was phenomenologically introduced, we have employed the antisymmetrized three-body interaction which involves a minimal momentum dependence as the leading-order contribution at low energy.

We first studied the simplified case with $U_2 = 0$, namely, the pure three-body interaction case. The three-body energies have been obtained for both the in-medium and in-vacuum cases. Unlike three-dimensional three-component Fermi gases, it has been found that the in-medium three-body bound state assisted by the Fermi-surface effect does not exist in the absence of the in-vacuum counterpart and the two-body interaction. However, there is still a nontrivial regime corresponding to the squeezed Cooper-triple phase and moreover, the one which originates from the Fermi-surface effect found in Ref. [28] in the presence of two-body attractions. Similar to the BCS-BEC crossover in the three-dimensional s-wave superfluid Fermi gas [2–4], the three-body cluster also undergoes a crossover from the Cooper tripling regime in the weak-coupling side to a regime of tightly bound trimers in the strong-coupling side when the attractive interactions increase.

We have also investigated the general case with the coexistence of two- and three-body interactions. In our previous work [18], the one-dimensional fermions were found to be stable against the three-body clustering when only two-body attractive p-wave interaction was considered. Meanwhile, by including the dimensionless constant three-body coupling, it was found that an in-medium three-body state similar to a squeezed Cooper triple appears. Similarly, in the present work, with further consideration of the antisymmetrized attractive three-body interaction, the stable three-body clusters survive as expected. The in-medium three-body cluster is found to be more tightly bound with the increase of the three-body coupling strength. Finally, we have featured a phase diagram consisting of the p-wave Cooper-pair and Cooper-triple phase in the plane of p-wave two-body coupling and three-body coupling strengths. One can explore the competition between pairing and tripling phases by tuning the interactions in cold-atomic systems.

Our results would be useful for further investigation of unconventional superconductors and superfluids. Moreover, an in-medium three-body bound state with the existence of a non-negligible three-body interaction also paves a promising way for the study of higher-order clusters associated with the Fermi-surface effect. The medium effect on bound trimers in higher dimensions such as the super Efimov state [56] would also be an interesting topic. More detailed studies on quantum correlations would also be worth investigating [57].

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APPENDIX A: EXPECTATION VALUE OF HAMILTONIAN

In this Appendix, we show the detailed expressions for the expectation values of each term in the Hamiltonian. By applying the trial wave function (9) to the Hamiltonian (1), the expectation values for the kinetic and two-body interaction parts are then obtained as [18]

$$\begin{aligned}\langle\Psi_3|K|\Psi_3\rangle &= \sum_{p_1,p_2,p_3,p_1',p_2',p_3'}{}^{\prime} \Omega^*_{p_1,p_2}\Omega_{p_1',p_2'}(\xi_{p_1}+\xi_{p_2}+\xi_{p_3})\epsilon_{p_1,p_2,p_3}\epsilon_{p_1',p_2',p_3'}\delta_{p_3',-p_1'-p_2'}\delta_{p_3,-p_1-p_2}\\ &= 2\sum_{p_1,p_2}{}^{\prime}(\xi_{p_1}+\xi_{p_2}+\xi_{-p_1-p_2})\Omega^*_{p_1,p_2}[\Omega_{p_1,p_2}+\Omega_{p_2,-p_1-p_2}+\Omega_{-p_1-p_2,p_1}],\end{aligned} \tag{A1}$$

with the Levi-Civita symbol ϵ_{p_1,p_2,p_3} and

$$\begin{aligned}\langle\Psi_3|V_2|\Psi_3\rangle &= \frac{U_2}{2}\sum_{k_1,k_2}{}^{\prime}\sum_{k_1',k_2'}{}^{\prime}\sum_{p_1,p_2,p_3}{}^{\prime}\sum_{p_1',p_2',p_3'}{}^{\prime}\left(\frac{k_1-k_2}{2}\right)\left(\frac{k_1'-k_2'}{2}\right)\Omega^*_{p_1,p_2}\Omega_{p_1',p_2'}\delta_{p_1+p_2,-p_3}\delta_{p_1'+p_2',-p_3'}\\ &\quad\times\langle \mathrm{FS}|F_{p_1,p_2,p_3}B^\dagger_{k_1,k_2}B_{k_1',k_2'}F^\dagger_{p_1',p_2',p_3'}|\mathrm{FS}\rangle\\ &\equiv 2v_{21}+v_{22},\end{aligned} \tag{A2}$$

respectively, where we defined

$$\begin{aligned}v_{21} &= \frac{U_2}{2}\sum_{p_1,p_2,q}{}^{\prime}\Omega^*_{p_1,p_2}[(p_1-p_2)(2q-p_1-p_2)\Omega_{-p_1-p_2,q}+(2p_2+p_1)(2q+p_1)\Omega_{p_1,q}\\ &\quad+(-2p_1-p_2)(2q+p_2)\Omega_{p_2,q}],\end{aligned} \tag{A3a}$$

and

$$\begin{aligned}v_{22} &= \frac{U_2}{2}\sum_{p_1,p_2,q}{}^{\prime}\Omega^*_{p_1,p_2}[(p_1-p_2)(2q-p_1-p_2)\Omega_{q,-q+p_1+p_2}+(2p_2+p_1)(2q+p_1)\Omega_{q,-q-p_1}\\ &\quad+(-2p_1-p_3)(2p_1+p_2)\Omega_{q,-q-p_2}].\end{aligned} \tag{A3b}$$

At last, the expectation value for the three-body interaction part reads

$$\begin{aligned}\langle\Psi_3|V_3|\Psi_3\rangle &= U_3\sum_{k_1,k_2,k_3}{}^{\prime}\sum_{k_1',k_2',k_3'}{}^{\prime}\sum_{p_1,p_2,p_3}{}^{\prime}\sum_{p_1',p_2',p_3'}{}^{\prime}\left(\frac{k_1-k_2}{2}\right)\left(\frac{k_1'-k_2'}{2}\right)\left(\frac{k_2-k_3}{2}\right)\left(\frac{k_2'-k_3'}{2}\right)\left(\frac{k_3-k_1}{2}\right)\left(\frac{k_3'-k_1'}{2}\right)\\ &\quad\times\Omega^*_{p_1,p_2}\Omega_{p_1',p_2'}\delta_{k_1+k_2+k_3,k_1'+k_2'+k_3'}\delta_{p_1+p_2,-p_3}\delta_{p_1'+p_2',-p_3'}\langle\mathrm{FS}|c_{p_3}c_{p_2}c_{p_1}c^\dagger_{k_1}c^\dagger_{k_2}c^\dagger_{k_3}c_{k_3'}c_{k_2'}c_{k_1'}c^\dagger_{p_1'}c^\dagger_{p_2'}c^\dagger_{p_3'}|\mathrm{FS}\rangle\\ &= \frac{9U_3}{16}\sum_{p_1,p_2}{}^{\prime}\sum_{p_1',p_2'}{}^{\prime}(p_1-p_2)(p_1'-p_2')(p_1+2p_2)(p_1'+2p_2')(2p_1+p_2)(2p_1'+p_2')\Omega^*_{p_1,p_2}\Omega_{p_1',p_2'}.\end{aligned} \tag{A4}$$

APPENDIX B: DERIVATION OF EQ. (21)

In this Appendix, we show the detailed derivations of Eq. (21). The amplitude Ω_{p_1,p_2} can be expressed in terms of $\mathcal{A}(p_1,p_2)$, $\mathcal{B}(p_2)$, and $\mathcal{C}$ as,

$$\begin{aligned}\Omega_{p_1,p_2}+\Omega_{p_2,p_3}+\Omega_{p_3,p_1} &= -\frac{\frac{U_2}{2}[(p_1-p_2)\mathcal{B}(p_3)+(p_2-p_3)\mathcal{B}(p_1)+(p_3-p_1)\mathcal{B}(p_2)]}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}\\ &\quad+\frac{9U_3}{16}\mathcal{C}\frac{\mathcal{X}_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}.\end{aligned} \tag{B1}$$

The above equation can be also recast into

$$\mathcal{B}(p_2)\left[1+\frac{U_2}{2}\sum_{p_1}{}^{\prime}\frac{(p_3-p_1)^2}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}\right] = -U_2\sum_{p_1}{}^{\prime}\frac{(p_2-p_3)(p_3-p_1)\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}+\frac{9U_3}{16}\mathcal{C}\sum_{p_1}{}^{\prime}\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{B2}$$

By multiplying $\mathcal{X}_{p_1,p_2,p_3}\delta_{p_1+p_2+p_3,0}$ on both sides in Eq. (B1) and taking the momentum summation with respect to p_1, p_2, p_3, one has

$$-3\mathcal{C} = -\frac{U_2}{2}\sum_{p_1,p_2}{}' \frac{\mathcal{X}_{p_1,p_2,p_3}[(p_1-p_2)\mathcal{B}(p_3)+(p_2-p_3)\mathcal{B}(p_1)+(p_3-p_1)\mathcal{B}(p_2)]}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)} + \frac{9U_3}{16}\mathcal{C}\sum_{p_1,p_2}{}' \frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{B3}$$

Since the antisymmetric tensor $\mathcal{X}_{p_1,p_2,p_3}$ satisfies

$$\mathcal{X}_{p_2,p_1,p_3} = (p_2-p_1)(p_1-p_3)(p_3-p_2) = -(p_1-p_2)(p_2-p_3)(p_3-p_1) \equiv -\mathcal{X}_{p_1,p_2,p_3}, \tag{B4}$$

the first term in the right-hand side of the above equation can be recast into,

$$\sum_{p_1,p_2}{}' \frac{\mathcal{X}_{p_1,p_2,p_3}(p_2-p_3)\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)} + \sum_{p_1,p_2}{}' \frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)\mathcal{B}(p_2)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)} + \sum_{p_1,p_2}{}' \frac{\mathcal{X}_{p_1,p_2,p_3}(p_1-p_2)\mathcal{B}(p_3)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)} = 3\sum_{p_1,p_2}{}' \frac{\mathcal{X}_{p_1,p_2,p_3}(p_2-p_3)\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{B5}$$

In this way, we obtain

$$\mathcal{C} = \left[1+\frac{3U_3}{16}\sum_{p_1,p_2}{}' \frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E)}\right]^{-1} \frac{U_2}{2}\sum_{p_1,p_2}{}' \frac{(p_2-p_3)\mathcal{X}_{p_1,p_2,p_3}\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{B6}$$

By combining Eqs. (B2) and (B6), we obtain the closed equation for $\mathcal{B}(p)$ and E_3 as

$$\mathcal{B}(p_2)\left[\frac{1}{U_2}+\sum_{p_1}{}' \frac{(p_1+p_2/2)^2}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3}\right] = \sum_{p_1}{}' \frac{(p_1+2p_2)(p_1+p_2/2)\mathcal{B}(p_1)}{\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3} + \frac{\frac{9U_3}{16}\sum'_{p_1}\frac{\mathcal{X}_{p_1,p_2,p_3}(p_3-p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}{1+\frac{3U_3}{16}\sum'_{p_1,p_2}\frac{\mathcal{X}^2_{p_1,p_2,p_3}}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}}\sum_{p_1,p_2}{}' \frac{(p_2+p_1/2)\mathcal{X}_{p_1,p_2,p_3}\mathcal{B}(p_1)}{2(\xi_{p_1}+\xi_{p_2}+\xi_{p_3}-E_3)}. \tag{B7}$$

[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Feshbach resonances in ultracold gases, Rev. Mod. Phys. **82**, 1225 (2010).
[2] M. Randeria and E. Taylor, Crossover from Bardeen-Cooper-Schrieffer to Bose-Einstein condensation and the unitary Fermi gas, Annu. Rev. Condens. Matter Phys. **5**, 209 (2014).
[3] G. C. Strinati, P. Pieri, G. Röpke, P. Schuck, and M. Urban, The BCS–BEC crossover: From ultra-cold Fermi gases to nuclear systems, Phys. Rep. **738**, 1 (2018).
[4] Y. Ohashi, H. Tajima, and P. van Wyk, BCS–BEC crossover in cold atomic and in nuclear systems, Prog. Part. Nucl. Phys. **111**, 103739 (2020).
[5] C. Ticknor, C. A. Regal, D. S. Jin, and J. L. Bohn, Multiplet structure of Feshbach resonances in nonzero partial waves, Phys. Rev. A **69**, 042712 (2004).
[6] C. H. Schunck, M. W. Zwierlein, C. A. Stan, S. M. F. Raupach, W. Ketterle, A. Simoni, E. Tiesinga, C. J. Williams, and P. S. Julienne, Feshbach resonances in fermionic ^{6}Li, Phys. Rev. A **71**, 045601 (2005).
[7] Y. Inada, M. Horikoshi, S. Nakajima, M. Kuwata-Gonokami, M. Ueda, and T. Mukaiyama, Collisional properties of p-wave Feshbach molecules, Phys. Rev. Lett. **101**, 100401 (2008).
[8] T. Nakasuji, J. Yoshida, and T. Mukaiyama, Experimental determination of p-wave scattering parameters in ultracold ^{6}Li atoms, Phys. Rev. A **88**, 012710 (2013).
[9] V. Gurarie and L. Radzihovsky, Resonantly paired fermionic superfluids, Ann. Phys. (NY) **322**, 2 (2007).
[10] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Tuning p-wave interactions in an ultracold Fermi gas of atoms, Phys. Rev. Lett. **90**, 053201 (2003).
[11] J. Zhang, E. G. M. van Kempen, T. Bourdel, L. Khaykovich, J. Cubizolles, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, p-wave Feshbach resonances of ultracold ^{6}Li, Phys. Rev. A **70**, 030702(R) (2004).
[12] M. Waseem, T. Saito, J. Yoshida, and T. Mukaiyama, Two-body relaxation in a Fermi gas at a p-wave Feshbach resonance, Phys. Rev. A **96**, 062704 (2017).
[13] J. Yoshida, T. Saito, M. Waseem, K. Hattori, and T. Mukaiyama, Scaling law for three-body collisions of identical fermions with p-wave interactions, Phys. Rev. Lett. **120**, 133401 (2018).
[14] F. Ç. Top, Y. Margalit, and W. Ketterle, Spin-polarized fermions with p-wave interactions, Phys. Rev. A **104**, 043311 (2021).

[15] J. Levinsen, N. R. Cooper, and V. Gurarie, Strongly resonant *p*-wave superfluids, Phys. Rev. Lett. **99**, 210402 (2007).

[16] M. Waseem, J. Yoshida, T. Saito, and T. Mukaiyama, Unitarity-limited behavior of three-body collisions in a *p*-wave interacting Fermi gas, Phys. Rev. A **98**, 020702(R) (2018).

[17] L. Zhou and X. Cui, Stretching *p*-wave molecules by transverse confinements, Phys. Rev. A **96**, 030701(R) (2017).

[18] Y. Guo and H. Tajima, Stability against three-body clustering in one-dimensional spinless *p*-wave fermions, Phys. Rev. A **106**, 043310 (2022).

[19] Y.-T. Chang, R. Senaratne, D. Cavazos-Cavazos, and R. G. Hulet, Collisional loss of one-dimensional fermions near a *p*-wave Feshbach resonance, Phys. Rev. Lett. **125**, 263402 (2020).

[20] V. Venu, P. Xu, M. Mamaev, F. Corapi, T. Bilitewski, J. P. D'Incao, C. J. Fujiwara, A. M. Rey, and J. H. Thywissen, Unitary *p*-wave interactions between fermions in an optical lattice, Nature (London) **613**, 262 (2023).

[21] A. S. Marcum, F. R. Fonta, A. M. Ismail, and K. M. O'Hara, Suppression of three-body loss near a *p*-wave resonance due to quasi-1D confinement, arXiv:2007.15783.

[22] K. G. Jackson, C. J. Dale, J. Maki, K. G. Xie, B. A. Olsen, D. J. Ahmed-Braun, S. Zhang, and J. H. Thywissen, Emergent *s*-wave interactions between identical fermions in quasi-one-dimensional geometries, Phys. Rev. X **13**, 021013 (2023).

[23] Y. Guo and H. Tajima, Competition between pairing and tripling in one-dimensional fermions with coexistent *s*- and *p*-wave interactions, Phys. Rev. B **107**, 024511 (2023).

[24] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Theory of superconductivity, Phys. Rev. **108**, 1175 (1957).

[25] P. Niemann and H.-W. Hammer, Pauli-blocking effects and Cooper triples in three-component Fermi gases, Phys. Rev. A **86**, 013628 (2012).

[26] T. Kirk and M. M. Parish, Three-body correlations in a two-dimensional SU(3) Fermi gas, Phys. Rev. A **96**, 053614 (2017).

[27] S. Akagami, H. Tajima, and K. Iida, Condensation of Cooper triples, Phys. Rev. A **104**, L041302 (2021).

[28] H. Tajima, S. Tsutsui, T. M. Doi, and K. Iida, Three-body crossover from a Cooper triple to a bound trimer state in three-component Fermi gases near a triatomic resonance, Phys. Rev. A **104**, 053328 (2021).

[29] H. Tajima, S. Tsutsui, T. M. Doi, and K. Iida, Cooper triples in attractive three-component fermions: Implication for hadron-quark crossover, Phys. Rev. Res. **4**, L012021 (2022).

[30] G. Röpke, A. Schnell, P. Schuck, and P. Nozières, Four-particle condensate in strongly coupled fermion systems, Phys. Rev. Lett. **80**, 3177 (1998).

[31] N. Sandulescu, D. Negrea, J. Dukelsky, and C. W. Johnson, Quartet condensation and isovector pairing correlations in $n = z$ nuclei, Phys. Rev. C **85**, 061303(R) (2012).

[32] V. V. Baran and D. S. Delion, A quartet BCS-like theory, Phys. Lett. B **805**, 135462 (2020).

[33] Y. Guo, H. Tajima, and H. Liang, Cooper quartet correlations in infinite symmetric nuclear matter, Phys. Rev. C **105**, 024317 (2022).

[34] Y. Guo, H. Tajima, and H. Liang, Biexciton-like quartet condensates in an electron-hole liquid, Phys. Rev. Res. **4**, 023152 (2022).

[35] J. E. Drut, J. R. McKenney, W. S. Daza, C. L. Lin, and C. R. Ordóñez, Quantum anomaly and thermodynamics of one-dimensional fermions with three-body interactions, Phys. Rev. Lett. **120**, 243002 (2018).

[36] J. R. McKenney, A. Jose, and J. E. Drut, Thermodynamics and static response of anomalous one-dimensional fermions via a quantum Monte Carlo approach in the worldline representation, Phys. Rev. A **102**, 023313 (2020).

[37] I. E. Mazets, T. Schumm, and J. Schmiedmayer, Breakdown of integrability in a quasi-1D ultracold bosonic gas, Phys. Rev. Lett. **100**, 210403 (2008).

[38] T. Tanaka and Y. Nishida, Thermal conductivity of a weakly interacting Bose gas in quasi-one-dimension, Phys. Rev. E **106**, 064104 (2022).

[39] Y. Sekino and Y. Nishida, Field-theoretical aspects of one-dimensional Bose and Fermi gases with contact interactions, Phys. Rev. A **103**, 043307 (2021).

[40] Y. Sekino and Y. Nishida, Quantum droplet of one-dimensional bosons with a three-body attraction, Phys. Rev. A **97**, 011602(R) (2018).

[41] Y. Nishida, Universal bound states of one-dimensional bosons with two- and three-body attractions, Phys. Rev. A **97**, 061603(R) (2018).

[42] L. Pricoupenko, Pure confinement-induced trimer in one-dimensional atomic waveguides, Phys. Rev. A **97**, 061604(R) (2018).

[43] G. Guijarro, A. Pricoupenko, G. E. Astrakharchik, J. Boronat, and D. S. Petrov, One-dimensional three-boson problem with two- and three-body interactions, Phys. Rev. A **97**, 061605(R) (2018).

[44] T. Cheon and T. Shigehara, Fermion-boson duality of one-dimensional quantum particles with generalized contact interactions, Phys. Rev. Lett. **82**, 2536 (1999).

[45] M. Valiente, Bose-Fermi dualities for arbitrary one-dimensional quantum systems in the universal low-energy regime, Phys. Rev. A **102**, 053304 (2020).

[46] M. Valiente, Universal duality transformations in interacting one-dimensional quantum systems, Phys. Rev. A **103**, L021302 (2021).

[47] D. S. Petrov, Three-body interacting bosons in free space, Phys. Rev. Lett. **112**, 103201 (2014).

[48] A. Hammond, L. Lavoine, and T. Bourdel, Tunable three-body interactions in driven two-component Bose-Einstein condensates, Phys. Rev. Lett. **128**, 083401 (2022).

[49] Y. Guo and H. Tajima, Medium-induced bosonic clusters in a Bose-Fermi mixture: Towards simulating cluster formations in neutron-rich matter, arXiv:2308.04738.

[50] H.-W. Hammer, A. Nogga, and A. Schwenk, *Colloquium*: Three-body forces: From cold atoms to nuclei, Rev. Mod. Phys. **85**, 197 (2013).

[51] A. Watanabe, S. Nakai, Y. Wada, K. Sekiguchi, A. Deltuva, T. Akieda, D. Etoh, M. Inoue, Y. Inoue, K. Kawahara, H. Kon, K. Miki, T. Mukai, D. Sakai, S. Shibuya, Y. Shiokawa, T. Taguchi, H. Umetsu, Y. Utsuki, M. Watanabe *et al.*, Proton-^{3}He elastic scattering at intermediate energies, Phys. Rev. C **103**, 044001 (2021).

[52] X. Cui, Universal one-dimensional atomic gases near odd-wave resonance, Phys. Rev. A **94**, 043636 (2016).

[53] H. Tajima, S. Tsutsui, T. M. Doi, and K. Iida, Unitary *p*-wave Fermi gas in one dimension, Phys. Rev. A **104**, 023319 (2021).

[54] S. Tan, Three-boson problem at low energy and implications for dilute Bose-Einstein condensates, Phys. Rev. A **78**, 013636 (2008).

[55] Z. Wang and S. Tan, The three-body scattering hypervolume of identical fermions in one dimension, arXiv:2302.13685.

[56] Y. Nishida, S. Moroz, and D. T. Son, Super Efimov effect of resonantly interacting fermions in two dimensions, Phys. Rev. Lett. **110**, 235301 (2013).

[57] P. Kościk and T. Sowiński, Universality of internal correlations of strongly interacting *p*-wave fermions in one-dimensional geometry, Phys. Rev. Lett. **130**, 253401 (2023).