Experimental discrimination of two-qubit orthogonal states via local operations and classical communication

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Quantum nonlocality, a form of nonclassicality, serves as a crucial resource in quantum information processing. Nonlocality in quantum systems is exemplified by the perfect discrimination of orthogonal quantum states through single joint measurements, which is unattainable using local operations and classical communication (LOCC). Bandyopadhyay [S. Bandyopadhyay, Phys. Rev. Lett. **106**, 210402 (2011)] established that, with LOCC alone, at most $N-1$ copies of N orthogonal quantum states are required for perfect discrimination. Here, we present the experimental verification of orthogonal quantum state discrimination employing LOCC. Our results demonstrate that up to three copies suffice for the perfect distinction of four orthogonal two-qubit states via LOCC. The verified method of perfectly distinguishing orthogonal quantum state sets offers potential applications in quantum information processes, including quantum secret sharing and quantum teleportation.

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I. INTRODUCTION

A central issue in quantum information theory is the exploration of the intrinsic relationship between quantum nonlocality and quantum entanglement [1,2]. Quantum nonlocality is evident when joint measurements on composite quantum systems yield more information about a state than local operations and classical communication (LOCC) [3–5] alone can provide. To further investigate this relationship, the local distinguishability of quantum states has garnered interest. Local distinguishability concerns the ability of two or more spatially separated observers to identify a quantum state within a set of orthogonal quantum states using only LOCC. If observers can perfectly distinguish these orthogonal states with just one copy under LOCC constraints, the states are separable, and no entanglement or nonlocality is present. However, if an entangled state exists among these orthogonal states, perfect distinguishability using a single copy and LOCC is unattainable. This implies a correlation between entanglement and quantum nonlocality. Hence, studying the problem of achieving perfect distinguishability of orthogonal quantum states with the minimum number of copies under LOCC constraints will illuminate the intrinsic link between quantum nonlocality and entanglement, advancing quantum information theory.

Ghosh *et al.* [6] initially explored the problem of achieving perfect distinguishability using LOCC, demonstrating that four Bell states cannot be perfectly distinguished with only one copy under LOCC. Walgate and Hardy [7] later proved that, for four orthogonal quantum states, LOCC can only achieve perfect distinguishability with one copy when all states are separable. For a more general case, Bandyopadhyay [8] and Walgate *et al.* [9] concluded that at most $N-1$ copies are necessary to achieve perfect distinguishability of N orthogonal quantum states using LOCC. Banik *et al.* [10] further examined the problem for two-qubit quantum states, noting that for certain special sets of orthogonal states, only one or two copies are required to achieve perfect distinguishability.

In this article, we experimentally confirmed the perfect distinguishability of any set of four orthogonal two-qubit quantum states using three copies. We also verified the perfect distinguishability of specific sets of orthogonal two-qubit quantum states with only one or two copies. Our experimental findings expose the correlation between quantum entanglement and quantum nonlocality. The perfect distinguishability method for orthogonal quantum state sets validated in our experiment has potential applications in quantum information processes such as quantum secret sharing and quantum teleportation.

II. THEORETICAL FRAMEWORK

Let us review the process of perfectly distinguishing orthogonal pure states in composite quantum systems via LOCC. For any set of N orthogonal pure states, it is possible to perfectly distinguish them using at most $N-1$ copies through LOCC. Considering two-qubit states, for a group of four orthogonal states $|\Psi_1\rangle$, $|\Psi_2\rangle$, $|\Psi_3\rangle$, and $|\Psi_4\rangle$, no more than three copies are required for perfect discrimination through LOCC. In the following, we explain how to realize such quantum state discrimination.

We first examine the prevalent scenario in which none of the four quantum states are expressible as product states. Under this circumstance, we randomly select two of the four quantum states, without loss of generality, let us assume that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are the selected states. We can always choose suitable bases to expand them as

$$\begin{aligned}|\Psi_1\rangle &= a_{11}|\chi^+\rangle|\mu^+\rangle + a_{12}|\chi^-\rangle|\omega^+\rangle,\\ |\Psi_2\rangle &= a_{21}|\chi^+\rangle|\mu^-\rangle + a_{22}|\chi^-\rangle|\omega^-\rangle,\end{aligned} \tag{1}$$

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where $|\chi^{\pm}\rangle$, $|\mu^{\pm}\rangle$, and $|\omega^{\pm}\rangle$ are all single-qubit states satisfying $\langle\chi^+|\chi^-\rangle = \langle\mu^+|\mu^-\rangle = \langle\omega^+|\omega^-\rangle = 0$, and a_{11}, a_{12}, a_{21}, and a_{22} are complex numbers satisfying $|a_{11}|^2 + |a_{12}|^2 = 1$ and $|a_{21}|^2 + |a_{22}|^2 = 1$. The first qubit of the first copy is then measured in the basis $(|\chi^+\rangle, |\chi^-\rangle)$. If the result is $|\chi^+\rangle$ ($|\chi^-\rangle$), the second qubit of the first copy is then measured in the basis $\{|\mu^+\rangle, |\mu^-\rangle\}$ ($\{|\omega^+\rangle, |\omega^-\rangle\}$). The measurement result of the second qubit being $|\mu^+\rangle$ or $|\omega^+\rangle$ ($|\mu^-\rangle$ or $|\omega^-\rangle$) indicates that the two-qubit quantum state being measured is not $|\Psi_2\rangle$ ($|\Psi_1\rangle$). Regardless of the outcome, this LOCC reduces the possible states of the two-qubit quantum state from four to three. Similarly, performing LOCC on the second copy can further reduce the possible states from three to two. Lastly, by applying another LOCC on the third copy, the specific state of the two-qubit quantum state can be determined.

In this section, we examine the case where only one of the four quantum states is a product state. Without loss of generality, let $|\Psi_1\rangle$ represent this product state, which can be expanded as

$$|\Psi_1\rangle = |\eta^+\rangle|\nu^+\rangle. \tag{2}$$

The first qubit of the first copy is then measured in the basis $\{|\eta^+\rangle, |\eta^-\rangle\}$. After that, the second qubit of the first copy is then measured in the basis $\{|\nu^+\rangle, |\nu^-\rangle\}$. If the measurement outcome is $|\eta^+\rangle|\nu^+\rangle$, it indicates that the measured two-qubit state is $|\Psi_1\rangle$, and the state discrimination task is accomplished. However, if the measurement outcome differs from $|\eta^+\rangle|\nu^+\rangle$, it implies that the measured two-qubit state is not $|\Psi_1\rangle$, thus narrowing down the possible states from four to three. In such a scenario, the specific state of this two-qubit quantum system can be determined by applying the previously described method to perform corresponding LOCC on the second and third copies.

In the cases discussed above, a maximum of three copies is required to achieve state discrimination. We now consider three scenarios where state discrimination can be accomplished with only one or two copies.

(i) If suitable bases are chosen to represent $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as a sum of two terms, as demonstrated in Eq. (1), $|\Psi_3\rangle$ and $|\Psi_4\rangle$ can also be expressed as a sum of two terms in the same bases, for example:

$$\begin{aligned}|\Psi_3\rangle &= a_{31}|\chi^+\rangle|\mu^+\rangle + a_{32}|\chi^-\rangle|\omega^+\rangle,\\ |\Psi_4\rangle &= a_{41}|\chi^+\rangle|\mu^-\rangle + a_{42}|\chi^-\rangle|\omega^-\rangle,\end{aligned} \tag{3}$$

or

$$\begin{aligned}|\Psi_3\rangle &= a_{31}|\chi^+\rangle|\mu^-\rangle + a_{32}|\chi^-\rangle|\omega^-\rangle,\\ |\Psi_4\rangle &= a_{41}|\chi^+\rangle|\mu^+\rangle + a_{42}|\chi^-\rangle|\omega^+\rangle.\end{aligned} \tag{4}$$

In this case, although none of the four quantum states are product states, perfect state discrimination can be achieved with at most two copies. Assuming that $|\Psi_3\rangle$ and $|\Psi_4\rangle$ can be expanded using Eq. (3), the first qubit of the first copy is then measured in the basis $\{|\chi^+\rangle, |\chi^-\rangle\}$. If the result is $|\chi^+\rangle$ ($|\chi^-\rangle$), the second qubit of the first copy is measured in the basis $\{|\mu^+\rangle, |\mu^-\rangle\}$ ($|\omega^+\rangle, |\omega^-\rangle$) subsequently. If the measurement result of the second qubit is $|\mu^+\rangle$ or $|\omega^+\rangle$ ($|\mu^-\rangle$ or $|\omega^-\rangle$), it indicates that the measured two-qubit quantum state is not $|\Psi_2\rangle$ or $|\Psi_4\rangle$ ($|\Psi_1\rangle$ or $|\Psi_3\rangle$). Regardless of the outcome, this LOCC narrows down the possible states from four to two. Subsequently, another LOCC is applied to the second copy to determine the specific state of the two-qubit quantum state.

(ii) The second case is when only two of the four quantum states are product states. Without loss of generality, let us assume that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are product states, which can always be expanded in the following form:

$$|\Psi_1\rangle = |\eta^+\rangle|\nu^+\rangle,\ |\Psi_2\rangle = |\eta^-\rangle|\nu^-\rangle. \tag{5}$$

The first qubit of the first copy is then measured in the basis $\{|\eta^+\rangle, |\eta^-\rangle\}$. After that, the second qubit of the first copy is then measured in the basis $\{|\nu^+\rangle, |\nu^-\rangle\}$. If the measurement outcome is $|\eta^+\rangle|\nu^+\rangle$ ($|\eta^-\rangle|\nu^-\rangle$), then the two-qubit state being measured is $|\Psi_1\rangle$ ($|\Psi_2\rangle$) and the state discrimination task is completed. If the measurement outcome is $|\eta^+\rangle|\nu^-\rangle$ or $|\eta^-\rangle|\nu^+\rangle$, then the two-qubit state being measured is not $|\Psi_1\rangle$ or $|\Psi_2\rangle$, and the number of possible states is reduced from four to two. Subsequently, by performing another LOCC on the second copy, the specific state of the two-qubit state can be determined.

(iii) When all four quantum states are product states, perfect discrimination can be achieved using only one copy.

III. EXPERIMENTAL SETUP

First, we present an experiment designed to perfectly distinguish a group of orthogonal two-qubit states using three copies. To validate this state discrimination scheme, we need to first prepare an arbitrary two-qubit pure state. According to the Schmidt decomposition theorem [11], such a state can be expressed as $\alpha|\phi\rangle|\varphi\rangle + \sqrt{1-\alpha^2}|\phi^\perp\rangle|\varphi^\perp\rangle$, with $0 < \alpha < 1$, and $|\phi\rangle$, $|\varphi\rangle$, $|\phi^\perp\rangle$, and $|\varphi^\perp\rangle$ denoting single quantum pure states obeying $\langle\phi|\phi^\perp\rangle = \langle\varphi|\varphi^\perp\rangle = 0$. The apparatus utilized to generate this class of two-qubit pure states is depicted in Fig. 1. A continuous-wave 405-nm pump laser passes through a 2-mm-thick β-barium borate (β-BBO) crystal, producing an entangled photon pair, *a* and *b*, via spontaneous parametric down-conversion. The photons are then guided through a walk-off compensation device comprising a 45° half-wave plate and a 1-mm-thick β-BBO crystal, resulting in a maximally entangled two-photon state $\frac{1}{\sqrt{2}}(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b)$, where H and V represent horizontal and vertical polarization, respectively. Subsequently, photon b encounters an optical device consisting of two beam displacers and two half-wave plates (HWP1 and HWP2). This apparatus is responsible for converting H (V) to V (H) and outputting with a specific transmittance adjustable via HWP2 (HWP1). By fine-tuning the angles of HWP1 and HWP2, the state $\alpha|H\rangle_a|H\rangle_b + \sqrt{1-\alpha^2}|V\rangle_a|V\rangle_b$ is achieved. Following this, photons a and b pass through an optical assembly composed of two quarter-wave plates and a half-wave plate, which enables the implementation of any polarization-based single-qubit unitary transformation. Consequently, the two-photon state can be prepared in the desired state $\alpha|\phi\rangle_a|\varphi\rangle_b + \sqrt{1-\alpha^2}|\phi^\perp\rangle_a|\varphi^\perp\rangle_b$ [12]. Photons a and b can then be measured in any polarization basis using a detection system comprising a quarter-wave plate, a half-wave plate, a PBS, and a single-photon detector.

IV. RESULTS

We now present a detailed account of our experimental procedure. Initially, we randomly selected a set of orthogonal

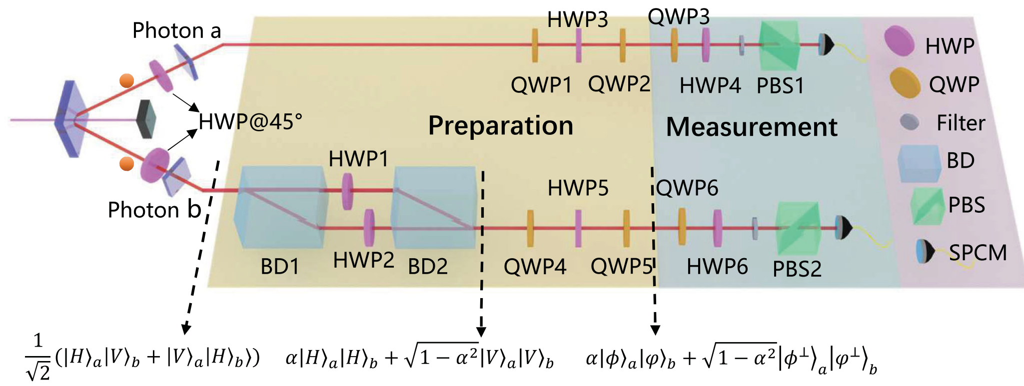

FIG. 1. Experimental setup for distinguishing two-qubit quantum states via LOCC. A continuous-wave laser with a wavelength of 405 nm is incident on a 2-mm-thick β-BBO crystal, generating a pair of photons (a and b) through type-II spontaneous parametric down-conversion. Following passage through the optical components in the yellow region, an arbitrary two-qubit quantum state is prepared. Subsequently, photon a (b) is directed into the measurement apparatus consisting of QWP3, HWP4, and PBS1 (QWP6, HWP6, and PBS2) for projection measurements. HWP, half-wave plate; QWP, quarter-wave plate; BD, beam displacer; PBS, polarization beam splitter; SPCM, single-photon counting module.

quantum states, denoted as S_1, with the following forms:

$$|\Psi_1\rangle = 0.6114|HH\rangle + (0.2147 + 0.3119i)|HV\rangle + (0.0076 - 0.2762i)|VH\rangle + (-0.5862 + 0.2508i)|VV\rangle,$$
$$|\Psi_2\rangle = 0.3656|HH\rangle + (0.0955 - 0.3878i)|HV\rangle + (-0.3359 - 0.5503i)|VH\rangle + (0.2633 - 0.4709i)|VV\rangle,$$
$$|\Psi_3\rangle = 0.5710|HH\rangle + (-0.0712 + 0.3908i)|HV\rangle + (-0.1185 + 0.4830i)|VH\rangle + (0.5111 - 0.0871i)|VV\rangle,$$
$$|\Psi_4\rangle = 0.4081|HH\rangle + (-0.3076 - 0.6668i)|HV\rangle + (0.4554 + 0.2309i)|VH\rangle + (-0.0728 + 0.1682i)|VV\rangle.$$

Calculations indicate that three copies of the quantum state are required for perfect discrimination via LOCC in this case. In the experiment, we randomly prepared one of these quantum states, using $|\Psi_1\rangle$ as an example to demonstrate our specific experimental procedure. As depicted in Fig. 2(a), we first performed LOCC operation L_1 on the initial copy of $|\Psi_1\rangle$ by measuring photon a in the basis $\{|\chi_1^+\rangle, |\chi_1^-\rangle\}$. Depending on the measurement outcome, we measured photon b in either the basis $\{|\mu_1^+\rangle, |\mu_1^-\rangle\}$ or $\{|\omega_1^+\rangle, |\omega_1^-\rangle\}$ [13]. The vectors of these bases are as follows:

$$|\chi_1^+\rangle = \begin{pmatrix} 0.3553 \\ 0.6538 - 0.6681i \end{pmatrix}, \quad |\chi_1^-\rangle = \begin{pmatrix} 0.6538 + 0.6681i \\ -0.3553 \end{pmatrix},$$
$$|\mu_1^+\rangle = \begin{pmatrix} -0.6716 \\ 0.5904 + 0.4476i \end{pmatrix}, \quad |\mu_1^-\rangle = \begin{pmatrix} 0.5904 - 0.4476i \\ 0.6716 \end{pmatrix},$$
$$|\omega_1^+\rangle = \begin{pmatrix} 0.6703 \\ 0.6073 + 0.4264i \end{pmatrix}, \quad |\omega_1^-\rangle = \begin{pmatrix} 0.6073 - 0.4264i \\ -0.6703 \end{pmatrix}.$$

The LOCC operation L_1 can eliminate one possible state between candidate options $|\Psi_1\rangle$ and $|\Psi_2\rangle$.

Subsequently, we apply another LOCC operation, L_2, to the second copy of $|\Psi_1\rangle$ to exclude one possible state between the candidate options $|\Psi_3\rangle$ and $|\Psi_4\rangle$. In the experiment, the probability of obtaining the result $+1$ (-1) after L_2 is $q_2^+ = 0.5964 \pm 0.0128$ ($q_2^- = 0.4036 \pm 0.0128$), corresponding to the exclusion of $|\Psi_4\rangle$ ($|\Psi_3\rangle$). When obtaining the result $+1$ (-1), we apply LOCC operation L_3 (L_4) to the third copy of $|\Psi_1\rangle$ to exclude one possible state between candidate options $|\Psi_1\rangle$ and $|\Psi_3\rangle$ ($|\Psi_4\rangle$). In the experiment, the probability of obtaining the result $+1$ (-1) after L_3 is $q_3^+ = 0.9779 \pm 0.0038$ ($q_3^- = 0.0221 \pm 0.0038$), corresponding to the exclusion of $|\Psi_3\rangle$ ($|\Psi_1\rangle$), and the probability of obtaining the result $+1$ (-1) after L_4 is $q_4^+ = 0.9819 \pm 0.0042$ ($q_4^- = 0.0181 \pm 0.0042$), corresponding to the exclusion of $|\Psi_4\rangle$ ($|\Psi_1\rangle$). The specific measurement bases for the three LOCC operations L_2, L_3, and L_4 are provided in the Appendix. Here we note that the experiment was conducted 50 times to ascertain the statistical error associated with the values q_1^+, q_1^-, q_2^+, q_2^-, q_3^+, q_3^-, q_4^+, and q_4^-.

By performing the corresponding LOCC operations on three copies of the quantum state, we can ultimately determine the state of the quantum system. Based on the four sets of experimental results shown in Fig. 2(a), we can calculate the probability of predicting the quantum state as $|\Psi_1\rangle$ to be $q_1^+(q_2^+q_3^+ + q_2^-q_4^+) = 0.9669 \pm 0.0042$. Similarly, we conducted state discrimination experiments on cases where the input quantum states were $|\Psi_2\rangle$, $|\Psi_3\rangle$, and $|\Psi_4\rangle$, with probabilities of correctly predicting the quantum state being 0.9595 ± 0.0047, 0.9199 ± 0.0037, and 0.9439 ± 0.0034, respectively, as shown in Fig. 2(b). Here we note that the quantum state measurement in our experiment, which exclusively involves single-qubit projection measurements, yields high accuracy with a minimal contribution to overall errors. Conversely, due to the prevalent usage of entangled states with inherently imperfect preparation fidelity, these form the primary source of experimental error.

In addition to completing the experiment for discriminating the quantum state set S_1, we conducted experimental verification of orthogonal quantum state discrimination for several other cases. First, we randomly generated a set of orthogonal quantum states, S_2, containing one product state and three entangled states. Unlike the discrimination of

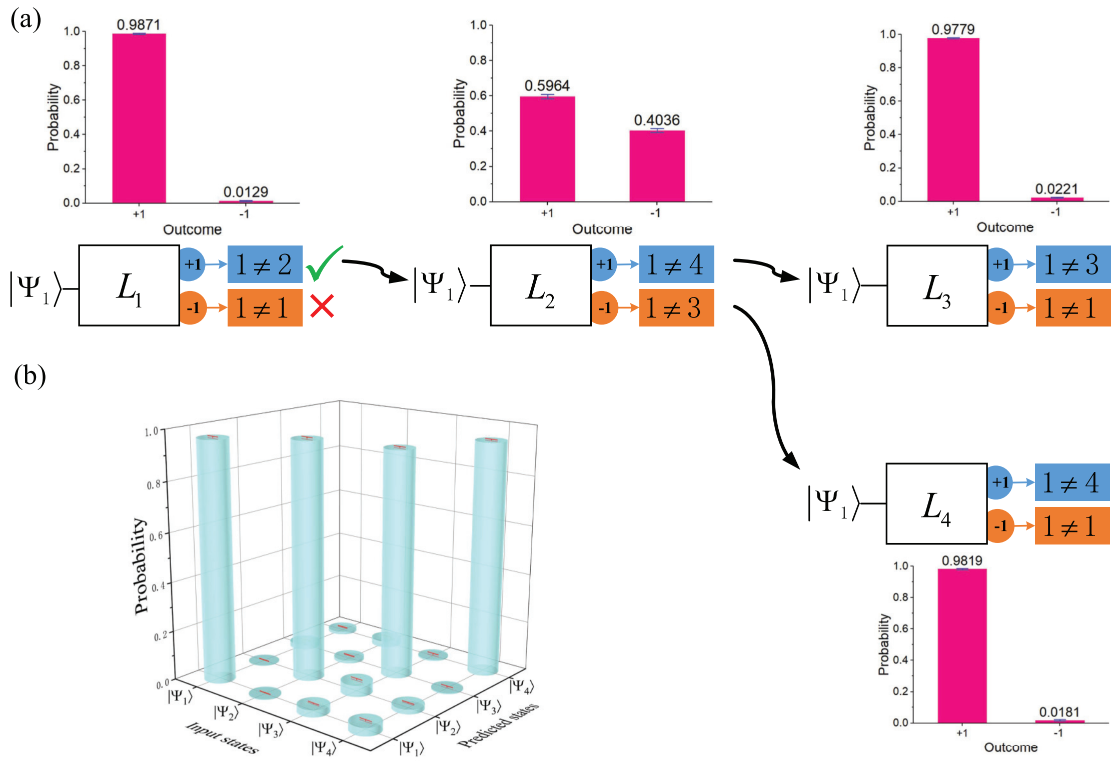

FIG. 2. Experimental results for two-qubit state discrimination using local operations and classical communication (LOCC). (a) Flowchart illustrating the process to identify the input quantum state, $|\Psi_1\rangle$, with three copies prepared. The first copy undergoes LOCC operation L_1; a $+1$ result probability of 0.9871 ± 0.0032 significantly diminishes the likelihood of the input state being $|\Psi_2\rangle$. The second copy is subjected to LOCC operation L_2, with $+1$ and -1 result probabilities of 0.5964 ± 0.0128 and 0.4036 ± 0.0128, respectively. A $+1$ (-1) result from L_2 eliminates the possibility of the input state being $|\Psi_4\rangle(|\Psi_3\rangle)$. LOCC operations L_3 (L_4) on the third copy further discount $|\Psi_3\rangle(|\Psi_4\rangle)$ with high probability, confirming the input state as $|\Psi_1\rangle$. The pink bar chart depicts probability distributions for measurement outcomes after the LOCC operation. (b) Predicted results for different input quantum states. For an input state of $|\Psi_1\rangle$ $(|\Psi_2\rangle, |\Psi_3\rangle, |\Psi_4\rangle)$, the experimentally predicted probability of identification as $|\Psi_1\rangle$ $(|\Psi_2\rangle, |\Psi_3\rangle, |\Psi_4\rangle)$ is 0.9669 ± 0.0042 $(0.9595 \pm 0.0047, 0.9199 \pm 0.0037, 0.9439 \pm 0.0034)$.

S_1, the number of copies required for S_2 is not fixed. In some instances, only one copy is needed, while in others, three copies remain necessary for state discrimination. The experimental results for S_2 discrimination are shown in Fig. 3(a), with the correct prediction probabilities for the four measured quantum states being 0.96 ± 0.0062, 0.9521 ± 0.0067, 0.9514 ± 0.0066, and 0.9612 ± 0.0052, respectively. We also randomly generated an orthogonal quantum state set,

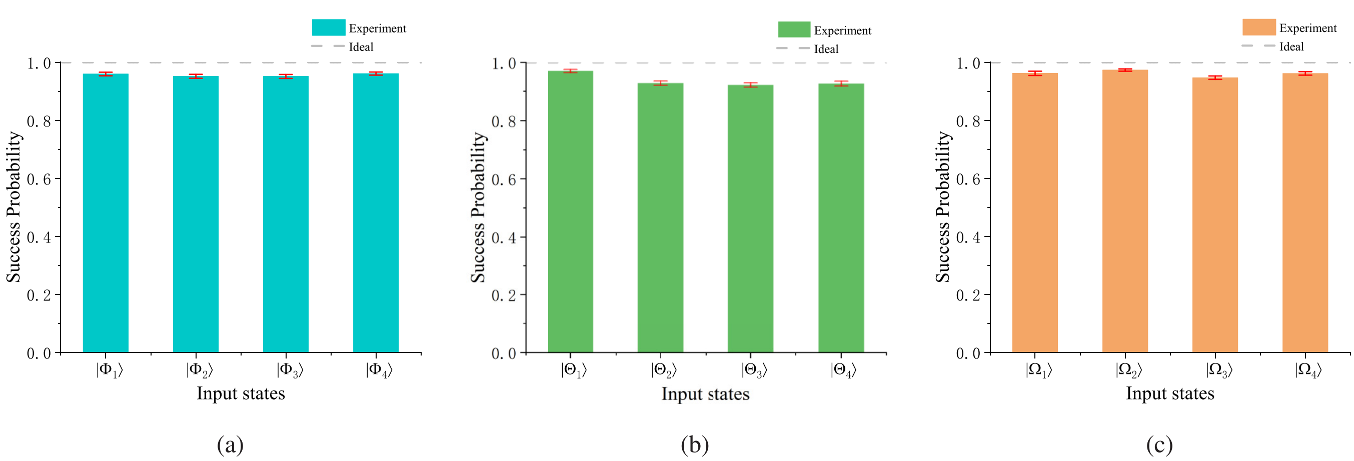

FIG. 3. Experimental results for distinguishing two-qubit states under three specific conditions. (a) Predictions for input quantum states with the quantum state set S_2. S_2 comprises one product state and three entangled states, distinguishable using up to three copies of the input state. (b) Predictions for input quantum states with the quantum state set S_3. S_3 consists of four entangled states, distinguishable using two copies of the input state. (c) Predictions for input quantum states with the quantum state set S_4. S_4 includes two product states and two entangled states, distinguishable using up to two copies of the input state.

S_3, containing four entangled states which are all expandable in the form of Eqs. (1) and (3) [or Eq. (4)]. Unlike the discrimination of S_1, only two copies are needed for S_3. The experimental results for S_3 discrimination are shown in Fig. 3(b), with the correct prediction probabilities for the four measured quantum states being 0.9709 ± 0.0053, 0.9276 ± 0.0076, 0.9214 ± 0.0073, and 0.9261 ± 0.0081, respectively. Furthermore, we randomly generated an orthogonal quantum state set, S_4, containing two product states and two entangled states. The number of copies required for S_4 discrimination is not fixed. In some cases, only one copy is needed, while in others, two copies are required. The experimental results for S_4 discrimination are shown in Fig. 3(c), with the correct prediction probabilities for the four measured quantum states being 0.9622 ± 0.0075, 0.9732 ± 0.004, 0.9469 ± 0.006, and 0.9617 ± 0.0061, respectively. For the specific forms of the quantum states in S_2, S_3, and S_4, as well as the required LOCC operations for their state discrimination, please refer to the Appendix.

V. SUMMARY

In summary, we have experimentally verified the state discrimination of two-qubit orthogonal quantum state sets using LOCC under various conditions, including the general case requiring three copies and cases requiring only one or two copies. As an easily implemented quantum operation, LOCC holds widespread application within the domain of quantum information. In its varied uses, Lim *et al.* [14] deployed LOCC to perform approximate partial transpose for entanglement determination in quantum states, while He *et al.* [15] utilized LOCC to achieve conversion between entanglement and coherence. Similarly, Laing *et al.* [16] effectively employed LOCC for the flawless discrimination of two distinct quantum processes. LOCC, serving as a restrictive form of measurement, tends to offer less information relative to global measurements. This disparity often underscores the nonclassicality of the quantum system under inspection. Perfect discrimination of orthogonal quantum states can be achieved using a single copy via global measurements. Conversely, LOCC typically necessitates multiple copies, with the requisite number correlating with the entanglement attributes of the quantum states set. In our experiment, we confirm that when utilizing only LOCC operations, the count of copies required for perfect discrimination of a set of two-qubit orthogonal states is indeed intimately associated with their entanglement properties. Our research on employing LOCC to discriminate orthogonal quantum states contributes to understanding the relationship between quantum nonlocality and quantum entanglement and has potential applications in quantum information processing, including quantum communication and quantum secret sharing.

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APPENDIX: CONCRETE REPRESENTATION OF QUANTUM STATE SETS AND LOCC OPERATIONS FOR STATE DISCRIMINATION

In the main text, we have presented the specific form of the quantum state set S_1. A total of six LOCC operations, L_1 through L_6, can be utilized to distinguish states in S_1. The operations L_1 and L_2 can eliminate one possible state between candidate pairs $|\Psi_1\rangle$ and $|\Psi_2\rangle$, and $|\Psi_3\rangle$ and $|\Psi_4\rangle$, respectively. Similarly, L_3, L_4, L_5, and L_6 can eliminate one possible state between candidate pairs $|\Psi_1\rangle$ and $|\Psi_3\rangle$, $|\Psi_1\rangle$ and $|\Psi_4\rangle$, $|\Psi_2\rangle$ and $|\Psi_3\rangle$, and $|\Psi_2\rangle$ and $|\Psi_4\rangle$, respectively. The specific forms of the basis vectors corresponding to L_2, L_3, L_4, L_5, and L_6 are provided below.

The basis vector corresponding to L_2 is

$$|\chi_2^+\rangle = \begin{pmatrix} 0.91 \\ 0.4029 - 0.0979i \end{pmatrix}, \quad |\chi_2^-\rangle = \begin{pmatrix} 0.4029 + 0.0979i \\ -0.91 \end{pmatrix},$$
$$|\mu_2^+\rangle = \begin{pmatrix} 0.7565 \\ 0.4650 + 0.4599i \end{pmatrix}, \quad |\mu_2^-\rangle = \begin{pmatrix} 0.465 - 0.4599i \\ -0.7565 \end{pmatrix},$$
$$|\omega_2^+\rangle = \begin{pmatrix} 0.7576 \\ -0.5787 - 0.3019i \end{pmatrix}, |\omega_3^-\rangle = \begin{pmatrix} -0.5787 + 0.3019i \\ -0.7576 \end{pmatrix}.$$

The basis vector corresponding to L_3 is

$$|\chi_3^+\rangle = \begin{pmatrix} 0.6675 \\ 0.7144 - 0.2099i \end{pmatrix}, \quad |\chi_3^-\rangle = \begin{pmatrix} 0.7144 + 0.2099i \\ -0.6675 \end{pmatrix},$$
$$|\mu_3^+\rangle = \begin{pmatrix} 0.7713 \\ -0.6109 + 0.1788i \end{pmatrix}, \quad |\mu_3^-\rangle = \begin{pmatrix} -0.6109 - 0.1788i \\ -0.7713 \end{pmatrix},$$
$$|\omega_3^+\rangle = \begin{pmatrix} 0.5807 \\ 0.8090 - 0.091i \end{pmatrix}, \quad |\omega_3^-\rangle = \begin{pmatrix} 0.8090 + 0.0910i \\ -0.5807 \end{pmatrix}.$$

The basis vector corresponding to L_4 is

$$|\chi_4^+\rangle = \begin{pmatrix} 0.0562 \\ -0.9474 + 0.3152i \end{pmatrix}, \ |\chi_4^-\rangle = \begin{pmatrix} -0.9474 - 0.3152i \\ -0.0562 \end{pmatrix},$$
$$|\mu_4^+\rangle = \begin{pmatrix} 0.3801 \\ -0.257 - 0.8885i \end{pmatrix}, \ |\mu_4^-\rangle = \begin{pmatrix} -0.257 + 0.8885i \\ -0.3801 \end{pmatrix},$$
$$|\omega_4^+\rangle = \begin{pmatrix} 0.8623 \\ 0.2396 + 0.446i \end{pmatrix}, \ |\omega_4^-\rangle = \begin{pmatrix} 0.2396 - 0.446i \\ -0.8623 \end{pmatrix}.$$

The basis vector corresponding to L_5 is

$$|\chi_5^+\rangle = \begin{pmatrix} 0.9678 \\ 0.1172 + 0.2228i \end{pmatrix}, \ |\chi_5^-\rangle = \begin{pmatrix} 0.1172 - 0.2228i \\ -0.9678 \end{pmatrix},$$
$$|\mu_5^+\rangle = \begin{pmatrix} -0.3654 \\ 0.0152 + 0.9307i \end{pmatrix}, \ |\mu_5^-\rangle = \begin{pmatrix} 0.0152 - 0.9307i \\ 0.3654 \end{pmatrix},$$
$$|\omega_5^+\rangle = \begin{pmatrix} 0.8416 \\ 0.3403 + 0.4194i \end{pmatrix}, \ |\omega_5^-\rangle = \begin{pmatrix} 0.3403 - 0.4194i \\ -0.8416 \end{pmatrix}.$$

The basis vector corresponding to L_6 is

$$|\chi_6^+\rangle = \begin{pmatrix} 0.6882 \\ 0.5599 - 0.4614i \end{pmatrix}, \ |\chi_6^-\rangle = \begin{pmatrix} 0.5599 + 0.4614i \\ -0.6882 \end{pmatrix},$$
$$|\mu_6^+\rangle = \begin{pmatrix} 0.687 \\ 0.7107 + 0.1514i \end{pmatrix}, \ |\mu_6^-\rangle = \begin{pmatrix} 0.7107 - 0.1514i \\ -0.687 \end{pmatrix},$$
$$|\omega_6^+\rangle = \begin{pmatrix} 0.8396 \\ -0.432 + 0.3294i \end{pmatrix}, \ |\omega_6^-\rangle = \begin{pmatrix} -0.432 - 0.3294i \\ -0.8396 \end{pmatrix}.$$

We also present the specific form of quantum state set S_2:

$$|\Phi_1\rangle = (-0.3482 - 0.8107i)|HH\rangle + (0.2477 - 0.1119i)|HV\rangle + (-0.2530 - 0.2662i)|VH\rangle + (0.0805 - 0.0794i)|VV\rangle,$$
$$|\Phi_2\rangle = (-0.0360 + 0.0128i)|HH\rangle + (0.0588 + 0.6239i)|HV\rangle + (-0.4351 - 0.0453i)|VH\rangle + (-0.5783 + 0.2830i)|VV\rangle,$$
$$|\Phi_3\rangle = (-0.3149 - 0.2942i)|HH\rangle + (-0.2409 + 0.5446i)|HV\rangle + (0.5160 + 0.4122i)|VH\rangle + (0.1535 - 0.0043i)|VV\rangle,$$
$$|\Phi_4\rangle = (-0.1094 + 0.1496i)|HH\rangle + (-0.3480 + 0.2402i)|HV\rangle + (-0.2503 - 0.4184i)|VH\rangle + (0.6388 + 0.3756i)|VV\rangle.$$

There are four LOCC operations, L_7 through L_{10}, that can be employed to distinguish states in S_2. The operation L_7 determines if the quantum state is $|\Phi_1\rangle$ or not, while L_8, L_9, and L_{10} can eliminate one possible state between candidate pairs $|\Phi_2\rangle$ and $|\Phi_3\rangle$, $|\Phi_2\rangle$ and $|\Phi_4\rangle$, and $|\Phi_3\rangle$ and $|\Phi_4\rangle$, respectively. The specific forms of the basis vectors corresponding to L_7, L_8, L_9, and L_{10} are as follows.

The basis vector corresponding to L_7 is

$$|\chi_7^+\rangle = \begin{pmatrix} 0.6844 - 0.6196i \\ 0.1777 - 0.3407i \end{pmatrix}, \ |\chi_7^-\rangle = \begin{pmatrix} 0.1777 + 0.3407i \\ -0.6844 - 0.6196i \end{pmatrix},$$
$$|\mu_7^+\rangle = \begin{pmatrix} -0.6771 - 0.6745i \\ 0.2038 - 0.2124i \end{pmatrix}, \ |\mu_7^-\rangle = \begin{pmatrix} 0.2038 + 0.2124i \\ 0.6771 - 0.6745i \end{pmatrix}.$$

The basis vector corresponding to L_8 is

$$|\chi_8^+\rangle = \begin{pmatrix} 0.5439 \\ 0.55 + 0.6338i \end{pmatrix}, \ |\chi_8^-\rangle = \begin{pmatrix} 0.55 - 0.6338i \\ -0.5439 \end{pmatrix},$$
$$|\mu_8^+\rangle = \begin{pmatrix} -0.4065 \\ -0.6889 + 0.6001i \end{pmatrix}, \ |\mu_8^-\rangle = \begin{pmatrix} -0.6889 - 0.6001i \\ 0.4065 \end{pmatrix},$$
$$|\omega_8^+\rangle = \begin{pmatrix} 0.6698 \\ -0.1246 + 0.732i \end{pmatrix}, \ |\omega_8^-\rangle = \begin{pmatrix} -0.1246 - 0.732i \\ -0.6698 \end{pmatrix}.$$

The basis vector corresponding to L_9 is

$$|\chi_9^+\rangle = \begin{pmatrix} -0.6217 \\ 0.2321 + 0.7481i \end{pmatrix}, \quad |\chi_9^-\rangle = \begin{pmatrix} 0.2321 - 0.7481i \\ 0.6217 \end{pmatrix},$$
$$|\mu_9^+\rangle = \begin{pmatrix} -0.9409 \\ -0.2578 + 0.2198i \end{pmatrix}, \quad |\mu_9^-\rangle = \begin{pmatrix} -0.2578 - 0.2198i \\ 0.9409 \end{pmatrix},$$
$$|\omega_9^+\rangle = \begin{pmatrix} -0.3126 \\ -0.7837 + 0.5368i \end{pmatrix}, \quad |\omega_9^-\rangle = \begin{pmatrix} -0.7837 - 0.5368i \\ 0.3126 \end{pmatrix}.$$

The basis vector corresponding to L_{10} is

$$|\chi_{10}^+\rangle = \begin{pmatrix} 0.3085 \\ 0.489 - 0.8159i \end{pmatrix}, \quad |\chi_{10}^-\rangle = \begin{pmatrix} 0.489 + 0.8159i \\ -0.3085 \end{pmatrix},$$
$$|\mu_{10}^+\rangle = \begin{pmatrix} 0.8878 \\ 0.4334 - 0.1547i \end{pmatrix}, \quad |\mu_{10}^-\rangle = \begin{pmatrix} 0.4334 + 0.1547i \\ -0.8878 \end{pmatrix},$$
$$|\omega_{10}^+\rangle = \begin{pmatrix} -0.7146 \\ 0.3756 + 0.5901i \end{pmatrix}, \quad |\omega_{10}^-\rangle = \begin{pmatrix} 0.3756 - 0.5901i \\ 0.7146 \end{pmatrix}.$$

Additionally, we provide the specific form of quantum state set S_3:

$$|\Theta_1\rangle = (0.2531 + 0.0829i)|HH\rangle + (0.2118 - 0.3579i)|HV\rangle + (-0.6450 - 0.1866i)|VH\rangle + (-0.1600 - 0.5288i)|VV\rangle,$$
$$|\Theta_2\rangle = (0.1894 - 0.1384i)|HH\rangle + (-0.0295 + 0.8368i)|HV\rangle + (-0.3469 - 0.1790i)|VH\rangle + (-0.2951 + 0.0665i)|VV\rangle,$$
$$|\Theta_3\rangle = (-0.1492 + 0.6445i)|HH\rangle + (-0.2058 - 0.0001i)|HV\rangle + (-0.4998 + 0.1049i)|VH\rangle + (0.2465 + 0.4453i)|VV\rangle,$$
$$|\Theta_4\rangle = (0.3916 - 0.5319i)|HH\rangle + (-0.1582 - 0.2421i)|HV\rangle + (-0.2119 - 0.3017i)|VH\rangle + (0.3657 + 0.4587i)|VV\rangle.$$

Three LOCC operations, L_{11}, L_{12}, and L_{13}, can be used to distinguish states in S_3. The operation L_{11} determines if the quantum state belongs to the subset composed of $|\Theta_1\rangle$ and $|\Theta_2\rangle$ or to the subset composed of $|\Theta_3\rangle$ and $|\Theta_4\rangle$. Moreover, L_{12} and L_{13} can eliminate one possible state between candidate pairs $|\Theta_1\rangle$ and $|\Theta_2\rangle$, and $|\Theta_3\rangle$ and $|\Theta_4\rangle$, respectively. The specific forms of the basis vectors corresponding to L_{11}, L_{12}, and L_{13} are provided below.

The basis vector corresponding to L_{11} is

$$|\chi_{11}^+\rangle = \begin{pmatrix} 0.9264 \\ 0.356 + 0.1226i \end{pmatrix}, \quad |\chi_{11}^-\rangle = \begin{pmatrix} 0.356 - 0.1226i \\ -0.9264 \end{pmatrix},$$
$$|\mu_{11}^+\rangle = \begin{pmatrix} 0.1776 \\ -0.9828 + 0.0498i \end{pmatrix}, \quad |\mu_{11}^-\rangle = \begin{pmatrix} -0.9828 - 0.0498i \\ -0.1776 \end{pmatrix},$$
$$|\omega_{11}^+\rangle = \begin{pmatrix} 0.8353 \\ 0.4424 + 0.3265i \end{pmatrix}, \quad |\omega_{11}^-\rangle = \begin{pmatrix} 0.4424 - 0.3265i \\ -0.8353 \end{pmatrix}.$$

The basis vector corresponding to L_{12} is

$$|\chi_{12}^+\rangle = \begin{pmatrix} 0.8954 \\ -0.2898 + 0.3381i \end{pmatrix}, \quad |\chi_{12}^-\rangle = \begin{pmatrix} -0.2898 - 0.3381i \\ -0.8954 \end{pmatrix},$$
$$|\mu_{12}^+\rangle = \begin{pmatrix} 0.9685 \\ -0.0762 - 0.2372i \end{pmatrix}, \quad |\mu_{12}^-\rangle = \begin{pmatrix} -0.0762 + 0.2372i \\ -0.9685 \end{pmatrix},$$
$$|\omega_{12}^+\rangle = \begin{pmatrix} 0.6135 \\ 0.5387 + 0.5774i \end{pmatrix}, \quad |\omega_{12}^-\rangle = \begin{pmatrix} 0.5387 - 0.5774i \\ -0.6135 \end{pmatrix}.$$

The basis vector corresponding to L_{13} is

$$|\chi_{13}^+\rangle = \begin{pmatrix} 0.5708 \\ 0.1661 + 0.8041i \end{pmatrix}, \quad |\chi_{13}^-\rangle = \begin{pmatrix} 0.1661 - 0.8041i \\ -0.5708 \end{pmatrix},$$
$$|\mu_{13}^+\rangle = \begin{pmatrix} -0.9321 \\ 0.1807 + 0.3141i \end{pmatrix}, \quad |\mu_{13}^-\rangle = \begin{pmatrix} 0.1807 - 0.3141i \\ 0.9321 \end{pmatrix},$$
$$|\omega_{13}^+\rangle = \begin{pmatrix} 0.5076 \\ 0.5351 + 0.6753i \end{pmatrix}, \quad |\omega_{13}^-\rangle = \begin{pmatrix} 0.5351 - 0.6753i \\ -0.5076 \end{pmatrix}.$$

Finally, we introduce the specific form of quantum state set S_4:

$$|\Omega_1\rangle = (0.4418 + 0.3212i)|HH\rangle + (0.3314 + 0.1857i)|HV\rangle + (-0.4274 - 0.4393i)|VH\rangle + (-0.3311 - 0.2685i)|VV\rangle,$$

$$|\Omega_2\rangle = (-0.2979 - 0.3049i)|HH\rangle + (0.4769 + 0.3849i)|HV\rangle + (-0.2156 - 0.3128i)|VH\rangle + (0.3607 + 0.4102i)|VV\rangle,$$

$$|\Omega_3\rangle = (-0.0545 - 0.4274i)|HH\rangle + (-0.4533 - 0.3738i)|HV\rangle + (-0.3053 - 0.5610i)|VH\rangle + (0.1483 - 0.1981i)|VV\rangle,$$

$$|\Omega_4\rangle = (0.4394 + 0.3759i)|HH\rangle + (-0.3601 + 0.0721i)|HV\rangle + (-0.2584 + 0.0731i)|VH\rangle + (0.5663 + 0.3715i)|VV\rangle.$$

Two LOCC operations, L_{14} and L_{15}, can be applied to distinguish states in S_4. The operation L_{14} determines if the quantum state is $|\Omega_1\rangle$ or not, while L_{15} can eliminate one possible state between candidate pair $|\Omega_3\rangle$ and $|\Omega_4\rangle$. The specific forms of the basis vectors corresponding to L_{14} and L_{15} are as follows.

The basis vector corresponding to L_{14} is

$$|\chi_{14}^+\rangle = \begin{pmatrix} -0.4236 + 0.5130i \\ 0.5661 - 0.4866i \end{pmatrix}, \quad |\chi_{14}^-\rangle = \begin{pmatrix} 0.5661 + 0.4866i \\ 0.4236 + 0.5130i \end{pmatrix},$$
$$|\mu_{14}^+\rangle = \begin{pmatrix} -0.5578 - 0.6023i \\ -0.4345 - 0.3704i \end{pmatrix}, \quad |\mu_{14}^-\rangle = \begin{pmatrix} -0.4345 + 0.3704i \\ 0.5578 - 0.6023i \end{pmatrix}.$$

The basis vector corresponding to L_{15} is

$$|\chi_{15}^+\rangle = \begin{pmatrix} 0.9968 \\ -0.0658 + 0.0454i \end{pmatrix}, \quad |\chi_{15}^-\rangle = \begin{pmatrix} -0.0658 - 0.0454i \\ -0.9968 \end{pmatrix},$$
$$|\mu_{15}^+\rangle = \begin{pmatrix} 0.5373 \\ 0.6162 - 0.5758i \end{pmatrix}, \quad |\mu_{15}^-\rangle = \begin{pmatrix} 0.6162 + 0.5758i \\ -0.5373 \end{pmatrix},$$
$$|\omega_{15}^+\rangle = \begin{pmatrix} 0.9478 \\ 0.1788 + 0.2639i \end{pmatrix}, \quad |\omega_{15}^-\rangle = \begin{pmatrix} 0.1788 - 0.2639i \\ -0.9478 \end{pmatrix}.$$

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[12] We have prepared a total of 16 different two-qubit quantum states in the experiment, with their fidelities ranging from 0.9529 ± 0.0024 to 0.9855 ± 0.0026.

[13] For the sake of experimental convenience, we have employed the method described in Liang *et al.* [16] and Lim *et al.* [14] to implement one-way LOCC equivalently. This approach involves simultaneous projection measurements on two photons. Photon a is measured in the basis of $|\chi_1^+\rangle$ and $|\chi_1^-\rangle$, while photon b has two sets of measurement bases: $|\mu_1^+\rangle$ and $|\mu_1^-\rangle$, as well as $|\omega_1^+\rangle$ and $|\omega_1^-\rangle$. Subsequently, based on the obtained results, we reconstruct the same measurement outcomes as those achieved by LOCC.

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