Device-independent witness for the nonobjectivity of quantum dynamics

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Quantum Darwinism offers an explanation for the emergence of classical objective features (those we are used to at macroscopic scales) from quantum properties at the microscopic level. The interaction of a quantum system with its surroundings redundantly proliferates information to many parts of the environment, turning it accessible and objective to different observers. However, given that one cannot probe the quantum system directly, only its environment, how to determine whether an unknown quantum property can be deemed objective? Here we propose a probabilistic framework to analyze this question and show that objectivity implies a Bell-like inequality. Among several other results, we show quantum violations of this inequality, a device-independent proof of the nonobjectivity of quantum correlations. We also implement a photonic experiment where the temporal degree of freedom of photons is the quantum system of interest, while their polarization acts as the environment. Employing a fully black-box approach, we achieve the violation of a Bell-like inequality, thus certifying the nonobjectivity of the underlying quantum dynamics in a fully device-independent framework.

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I. INTRODUCTION

Understanding how the quantum information encoded into a microscopic system leads to classical features, those observed at the macroscopic scales, remains a central question in quantum foundations. In the early days of quantum theory, the comprehension of the quantum-classical boundary relied on arguably vague notions such as wave-particle duality [1], complementarity [2,3], and even that of a human observer [4]. Nowadays, the tools and concepts of quantum information offer a more-well-grounded framework to address those questions.

The study of decoherence [5,6], for instance, shows that quantum properties, such as coherence and entanglement, are degraded due to the interaction of a quantum system with its surrounding environment, a process that becomes more noticeable the larger the quantum system is [7], beautifully explaining some crucial aspects of the quantum to classical transition [8–10]. Simply put, decoherence selects the so-called pointer states [11], which are natural candidates for the macroscopically observed classical states obtained after a measurement, while coherent superpositions of those states are suppressed. Decoherence by itself, however, does not solve the problem of how information contained in the pointer states becomes available to different measurement apparatuses, nor how this is turned into objective information, that is, independent of observers.

That spreading of objective information is the central topic that gave rise to the idea of quantum Darwinism [11–26]. In quantum Darwinism, the environment, the same entity responsible for decoherence, is also seen as a special carrier of information about the quantum system insofar as it redundantly propagates the information of the naturally selected pointer states to many external observers. Crucially, the emergence of a classical notion of objectivity is a generic feature of quantum dynamics [22]. Irrespective of the specific modeling for the interaction with the environment, whenever the information about the pointer states is accessible to sufficiently many observers, the evolution will gradually resemble one where a specific observable is measured by all of them.

However, what if other measurements, not necessarily those related to a pointer observable, are performed? In particular, if the system-environment dynamics is not known, how can one test for objectivity or rather the absence of it? Those are precisely the questions we address in this work.

Building on the results of [22], we propose a probabilistic framework to address the question of an emergent notion of objectivity. In this probabilistic setting, we associate an observer with each part of the environment (see Fig. 1) and we show that the ability of each observer to encode and retrieve classical information about a quantum system translates into the emergence of an objective value for a measurement outcome. Objectivity here ought to be understood in the sense that it reflects a sort of common knowledge among the observers: A property of a quantum system is objective when

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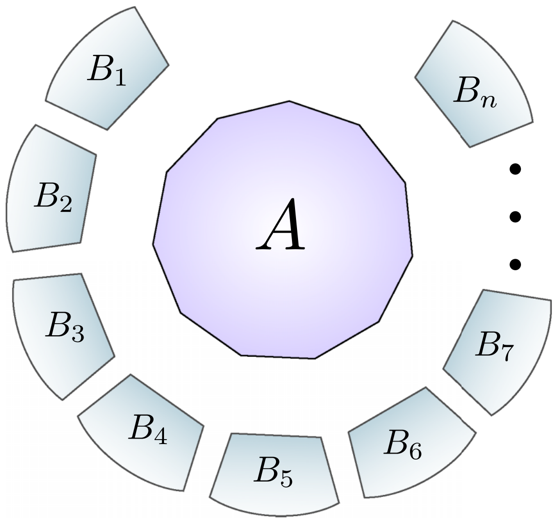

FIG. 1. General quantum Darwinism scenario. One central system A interacts with the environment described by n systems $B_1, \ldots, B_n$. As a result of this interaction, part of the information contained in A is transferred to the environment and replicated in each system B_i.

it is simultaneously agreed upon by all agents. From that, considering a particular case of two observers, we show that the Clauser-Horne-Shimony-Holt (CHSH) inequality [27], a paradigmatic Bell inequality in the study of quantum nonlocality [28], also can be turned into a witness of nonobjectivity.

More precisely, in our probabilistic setup, the violation of a CHSH inequality implies that several observers can mutually agree upon their outcome for the measurement of a given observable; still, that outcome can be completely uncorrelated from the property of the quantum system it should be related to. We also prove that if objectivity is demanded for all measurements performed by the observers in the CHSH scenario, then it implies true objectivity, reflecting agreement not only between observers but also to properties of the quantum system under scrutiny. Finally, we provide a proof-of-principle experimental realization of our framework. Employing birefringent plates placed inside a Sagnac interferometer, the temporal degree of freedom of photons gets entangled with their polarization, the first being the quantum system of interest while the latter acts as its environment within the quantum Darwinism scenario.

II. EMERGENCE OF OBJECTIVITY IN QUANTUM DARWINISM

In the following, we review the basic notions of quantum Darwinism. We give special emphasis to the standpoint of [22], where the authors proved that a well-defined notion of objectivity is a generic property of any quantum dynamics. We then prove our first result, a generalization of the findings of [22] in a general probabilistic setting, that is, not necessarily relying on (but certainly including) quantum theory.

We are interested in a general scenario where $n + 1$ quantum systems interact arbitrarily, being described at a certain instant of time by a density operator $\rho_{AB_1,\ldots,B_n}$; at this level of generality, it is irrelevant whether we refer to a closed system or a part of a larger system. The subsystem A describes the quantum system of interest and B_i stands for the different fractions of the environment. Each fraction B_i interacts with A and also possibly among themselves in such a way that the quantum information originally contained in A can be redundantly spread over the joint system. In a quantum description of the process, this information spreading is represented by a completely positive and trace-preserving map $\Lambda : \mathcal{D}(A) \to \mathcal{D}(B_1 \otimes \cdots \otimes B_n)$, where $\mathcal{D}(A)$ is the set of density operators on the Hilbert space associated with system A (similarly to the B_i's). The scenario is illustrated in Fig. 1.

Within this context, Ref. [22] makes a distinction between two notions of objectivity, that of observables and that of outcomes. The former states that the observers should extract information about the same observable of the system by probing parts of the environment, which would be associated with the pointer basis selected by the system-environment interactions. The latter considers that not only should the observable be the same, but also the value of the measurement outcome should be agreed upon by the observers.

Regarding the objectivity of observables, it follows that, quite generally, the map Λ can be well approximated by a measure-and-prepare map such that the reduced map for most subsets of observers is given by

$$\mathcal{E}_{\mathcal{B}}(\rho_A) = \sum_k \mathrm{tr}(\rho_A F_k)\sigma^k_{\mathcal{B}}, \tag{1}$$

where $\mathcal{B}$ is the subset of observers (or degrees of freedom of the environment being observed), $\{F_k\}_k$ is a positive-operator-valued measure (POVM) which should be the same for all subsets $\mathcal{B}$ of the same size, $\sigma^k_{\mathcal{B}}$ is the (joint) quantum state for the observers in $\mathcal{B}$, prepared according to the outcome k of F_k, and $\rho_A = \mathrm{Tr}_{B_1,\ldots,B_n}(\rho_{A,B_1,\ldots,B_n})$. More precisely, the results in [22] (i) provide an upper bound for how close a family of measure-and-prepare maps sharing the same POVM are to the true reduced evolution $\mathcal{E}_{\mathcal{B}}$ of smaller portions $\mathcal{B}$ and (ii) show that for a suitable fraction of the observers and for a large enough number of total observers, the bound gets closer to zero, meaning that all observers would agree they are obtaining information about the same property of ρ_A, determined by the observable described via the POVM $\{F_k\}_k$.

Regarding the objectivity of outcomes, Ref. [22] introduced the guessing probability of the outcome k obtained with F_k for all observers in the subset $\mathcal{B}$, tacitly assuming that the dynamics for each environment's fraction of interest has exactly the form of Eq. (1). Consider a distribution $\{p_i\}$ and a set of states $\{\sigma_i\}$ for $i \in \{1, \ldots, m\}$ and let $p_{\mathrm{guess}}(p_i, \sigma_i)$ be the guessing probability defined as

$$p_{\mathrm{guess}}(p_i, \sigma_i) = \max_{\{E_i\}} \sum_i p_i \mathrm{tr}(E_i\sigma_i), \tag{2}$$

representing the capability of the ensemble of states $\{\sigma_i\}_{i\in[m]}$ to properly encode m classical states distributed according to $\{p_i\}_{i\in[m]}$. It follows that if there exists a positive $0 < \delta < 1$ such that for every observer B_k, with $k \in \{1, \ldots, n\}$,

$$\min_{\rho\in\mathcal{D}(A)} \left[p_{\mathrm{guess}}\left(\mathrm{tr}(F_i\rho), \sigma^i_{B_k}\right)\right] \geqslant 1 - \delta, \tag{3}$$

then there exists some POVM $\{E_i^k\}$ for each B_i such that

$$\min_{\rho\in\mathcal{D}(A)}\sum_i \mathrm{tr}(F_i\rho)\mathrm{tr}\left[\left(\bigotimes_{k=1}^n E_i^k\right)\sigma^i_{B_1,\dots,B_n}\right]\geqslant 1-6n\delta^{1/4}, \quad (4)$$

where $\{F_i\}$ is an appropriate POVM and $\sigma^i_{B_k}$ the density matrix relative to the party B_k only. Qualitatively, Eq. (3) combined with Eq. (4) shows that if each B_i is capable of properly encoding the outcomes of a measurement on the A system, then one can assign an objective value to it, shared by all the B_i, in the sense that experimenters probing each single B_i will all get the same value, with high probability.

Our first goal is to extend this notion of objectivity beyond quantum-mechanical algebra machinery and instead rely on a purely probabilistic approach.[1] There are two main reasons for that. The first is that properties often seen as inherently quantum mechanical are in fact also features of generalized probability theories, including, for example, monogamy of correlations [30] and the impossibility of broadcasting information [31]. Understanding informational principles in such generalized settings often leads to deeper insights about quantum theory itself [32,33]. The other main reason for our approach is of practical relevance and relates to what is often called the device-independent approach to quantum information [34], the paradigmatic examples of which are violations of a Bell inequality and noncontextuality inequality violations and their use in cryptographic protocols [35–37]. In the device-independent setting, one can reach nontrivial conclusions about the quantum states being prepared or the measurements being performed by simply relying on the classical information obtained by measurement outcomes, without resorting to a detailed description of the experimental apparatus. In the particular case of quantum Darwinism, as we will see, it will allow us to not only define the concept of objectivity irrespective of any underlying dynamics or measurement setups, but also derive testable constraints on whether or not the statistics observed in the experiment can be deemed objective.

In our proposed setting, each agent i has access to a portion B_i of the environment surrounding A. Additionally, each agent i is free to independently choose to measure one out of many possible observables $x_i \in \{x_i^1, x_i^2, \dots, x_i^{m_i}\}$, obtaining the corresponding outcome $b_i \in \{b_i^1, b_i^2, \dots, b_i^{o_i}\}$. If we focus only on the aggregated statistics involved in this process, the scenario is thus described by a joint probability distribution

$$p(b_1,\dots,b_n|x_1,\dots,x_n)=\sum_a p(a,b_1,\dots,b_n|x_1,\dots,x_n), \quad (5)$$

where a is the outcome one would observe if a direct measurement of the system A [that measurement corresponding to the pointer-state observable (assuming it exists) defined by a given dynamics] had been performed. Each x_i represents the random variable parametrizing the choice of which observable the ith agent having access to the portion B_i of the environment measures in a given run of the experiment.

According to the Born rule, a quantum description of the same scenario is given by

$$\begin{aligned} &p(a,b_1,\dots,b_n|x_1,\dots,x_n)\\ &\quad=\mathrm{Tr}\big[\big(F_a\otimes E^{1,x_1}_{b_1}\cdots\otimes E^{n,x_n}_{b_n}\big)\rho_{A,B_1,\dots,B_n}\big], \end{aligned} \quad (6)$$

where $\rho_{A,B_1,\dots,B_n}$ is the density operator representing the quantum state shared by all the environments B_i plus the central system A and $\{E^{i,x_i}_{b_i}\}_{b_i}$ is the POVM representing a possible choice of measurement that the ith agent can realize on their fraction of the environment. It is exactly Eq. (6) that motivates a general probabilistic description where the joint distribution $p(a,b_1,\dots,b_n|x_1,\dots,x_n)$ has to fulfill three natural assumptions.

The first, called no superdeterminism, states that

$$p(a|x_1,\dots,x_n)=p(a) \quad (7)$$

for every $i\in[n]$ and for every $x_i\in\{x_i^1,x_i^2,\dots,x_i^{m_i}\}$. In other words, the choice of which observable to measure can be made by each agent independently of how the system A has been prepared or which are the pointer observables defined by a given dynamics. This is reminiscent of the measurement independence (also called free-will assumption) in Bell's theorem [38,39].

The second assumption, named no signaling, states that

$$p(b_i|a,x_1,\dots,x_n)=p(b_i|a,x_i) \quad (8)$$

for all $i\in[n]$ and for all $b_i\in\{b_i^1,b_i^2,\dots,b_i^{o_i}\}$. This condition imposes that even if we would have access to variable a defining the quantum system being probed, the statistics of what is observed by observer i should not depend on the choice of which measurement is performed by any other observer. In particular, note that this is different from the condition of locality in Bell's theorem that would state (considering the case of two observers for simplicity) that

$$p(b_1|a,b_2,x_1,x_2)=p(b_1|a,x_1) \quad (9)$$

and similarly for b_2. Specifically, locality in Bell's theorem makes the stronger assumption that the correlations between b_1 and b_2 are screened when we condition on the value of a. Together with the no-superdeterminism condition (7), only after eliminating the variable a from our description do we obtain the observational no-signaling typically considered in a Bell-like scenario, defined as $p(b_1|x_1)=\sum_{b_2}p(b_1,b_2|x_1,x_2)=\sum_{b_2}p(b_1,b_2|x_1,x_2')$ and similarly for b_2.

Our final assumption, which we name δ objectivity, is structured as follows. Let $\delta>0$ represent an error parameter. For each agent i, denote by x_i^* their choice of measurement corresponding to the case where their outcome should be correlated with the outcome a. In a quantum description, that would precisely correspond to the pointer state observable on system A, that is, corresponding to a POVM $\{E_k\}_k$ reproducing as reliably as possible the observable $\{F_k\}_k$ emerging in the effective measure-and-prepare dynamics in Eq. (1). The

[1]A possible route to generalization of quantum Darwinism to generalized probabilistic theories (GPTs) was proposed in [29]. Unfortunately, the authors were concerned with defining what an idealized quantum Darwinism process would look like in GPTs, more precisely, a general version of a fan-out gate, and did not consider the noisy version of such a process.

outcome b_i is δ objective if for each observer we have that

$$\sum_a p(a)p(b_i = a|a, x_i^*) \geqslant 1 - \delta. \tag{10}$$

The fact that this assumption introduces a clearer notion of objectivity will become justified after our first result below. For now, note that as there is always a POVM attaining the optimal value for the guessing probability, we can create a parallel involving Eq. (6), the equation defining $p_{\rm guess}$, and the quantity in Eq. (4) as shown by

$$p_{\rm guess}(\mathrm{tr}(F_i\rho_A), \sigma_i) = \max_{\{E_i\}} \sum_i \mathrm{tr}(F_i\rho_A)\mathrm{tr}(E_i\sigma_i) \leftrightarrow \sum_a p(a)p(b_i = a|a, x^*), \tag{11}$$

$$\min_{\rho_A} \sum_i \mathrm{tr}(F_i\rho_A)\mathrm{tr}\left(\bigotimes_k E_i^k \sigma_i^{1,\dots,n}\right) \leftrightarrow \sum_a p(a)p(b_1 = b_2 = \dots = b_n = a|a, x_1^*, \dots, x_n^*). \tag{12}$$

With that, we can state our first result, proven in Sec. I of the Supplemental Material [40], justifying our δ-objectivity assumption.

Result 1. If there exists a positive $\delta \leqslant 1$ such that for every $k \in \{1, \dots, n\}$,

$$\sum_a p(a)p(b_k = a|a, x_k^*) \geqslant 1 - \delta, \tag{13}$$

then we have

$$\sum_a p(a)p(b_1 = \dots = b_n = a|a, x_1^*, \dots, x_n^*) \geqslant 1 - n\delta. \tag{14}$$

Remark. Result 1 says that a result analogous to Eq. (4) continues to hold, even in a fully probabilistic setting, and gives a stricter bound despite not using any assumption on the dynamic of the systems. Put another way, the inequality $\sum_a p(a)p(b_1 = b_2 = \dots = b_n = a|x_1^*, \dots, x_n^*) \geqslant 1 - n\delta$ expresses the possibility of assigning an objective nature to the outcome obtained by each observer. Recall that objectivity here means that regardless of the outcome obtained by each agent, that outcome is agreed upon among all the B_i's, that is, $p(b_1 = b_2 = \dots = b_n|x_1^*, \dots, x_n^*) = 1$. Furthermore, it also reflects a property related to an observable described by a POVM $\{F_k\}_k$ acting on the subsystem A. In particular, when there is perfect local agreement, i.e., when $\delta = 0$, implying $\sum_a p(b_k = a|x_k^*) = 1$ for every agent, Result 1 guarantees that $\sum_a p(b_1 = b_2 = \dots = b_n = a|x_1^*, \dots, x_n^*) = 1$. One can read this implication as saying that perfect local agreement implies perfect global agreement.

III. BELL-LIKE INEQUALITIES WITNESSING NONOBJECTIVITY

The conditions of no superdeterminism, no signaling, and δ objectivity [Eqs. (7), (8), and (10), respectively] clearly define a notion for objectivity of outcomes in the probabilistic setting. Notwithstanding, note that those conditions involve the outcome a that by assumption is not directly observable, as any information about it can only be obtained indirectly, by correlations of it with the outcomes b_i. Thus, similarly to Bell's theorem, a plays the role of a latent or hidden variable. However, the conjunction of assumptions (7), (8), and (10) does imply testable constraints, exactly Bell-like inequalities, for the observed correlations among the outcomes b_i.

The fact that Bell inequalities emerge as testable constraints is a natural consequence of using a probabilistic framework. More generally, Bell inequalities emerge whenever two ingredients are put together: (i) a probabilistic description of physical processes based on measurement outcomes and (ii) the fact that some of the events or variables relevant to the underlying dynamics are not empirically accessible and thus have to be eliminated from the description of the problem. The first point is precisely what defines the device independence of the approach. The second is at the core of local hidden variable models (and hence in Bell's theorem) but is much more general in what is called marginal problems. In the case of quantum Darwinism, the marginal problem emerges due to the fact that we cannot access the quantum system directly but rather indirectly, measuring the environments that have interacted with it.

Within this context, we consider the particular case of only two observers ($n = 2$). Each observer has two possible measurements available to them and each measurement is dichotomic, that is, $x_1, x_2, b_1, b_2 \in \{0, 1\}$. Moreover, we specify x_1^* as $x_1 = 0$ and x_2^* as $x_2 = 0$ (recall that each x_i^* corresponds to the special case where the outcome should be correlated with the outcome a). We can then state our second result.

Result 2. Any observed correlation $p(b_1, b_2|x_1, x_2)$ compatible with the conditions (7), (8), and (10) fulfills the inequality

$$\mathrm{CHSH}_{\delta,\epsilon} = \langle B_1^0 B_2^0 \rangle + \langle B_1^0 B_2^1 \rangle - \langle B_1^1 B_2^0 \rangle + \langle B_1^1 B_2^1 \rangle \leqslant 2 + 4\delta - 2\epsilon, \tag{15}$$

with $\langle B_1^0 B_2^0 \rangle = 1 - 2\epsilon$, where $\langle B_1^{x_1} B_2^{x_2} \rangle = \sum (-1)^{b_1+b_2} p(b_1, b_2|x_1, x_2)$ is the expectation value of the observables corresponding to inputs x_1 and x_2.

Note that Eq. (15) is a relaxed version of the CHSH inequality [27] with one additional constraint. In Eq. (15) we impose that $\langle B_1^0 B_2^0 \rangle = 1 - 2\epsilon$ to mean that both observers are in agreement (up to a discordance factor of 2ϵ) whenever they decide to measure the special inputs $x_1^* = 0$ and $x_2^* = 0$, respectively. It might appear curious that the CHSH inequality also bounds the set of objective correlations, which as discussed previously is defined by a different set of assumptions compared to Bell's theorem. We remark, however, that the same inequality also appears in different contexts, for instance, the study of the local friendliness [41], a device-independent approach to the measurement problem in quantum theory.

Result 1 implies that $\delta \geqslant \epsilon/2$, while $\delta \leqslant \epsilon/2$ would correspond to the Darwinistic case,[2] where the disagreement δ between the observers and the latent variable A follows directly from the disagreement ϵ between the observers themselves. Thus, any value $\mathrm{CHSH}_{\delta,\epsilon} > 2$ implies that $\delta > \epsilon/2$,

[2]Note that, in principle, one could always reduce to $\delta = \epsilon/2$ in this case, by a suitable choice of the variable a.

witnessing nonobjectivity even in the case of nonperfect agreement between the observers ($\epsilon > 0$).

Considering the case where $\delta = 0$ and $\epsilon = 0$, our next result shows that quantum theory can violate the $\mathrm{CHSH}_{0,0}$ inequality while respecting $\langle B_1^0 B_2^0\rangle = 1$. In other words, we may have apparent agreement between the observers, where their outcomes do not reflect a property of the system A which they assume to be fully correlated with.

Result 3. Quantum theory allows a violation of $\mathrm{CHSH}_{0,0}$ up to the value $\frac{5}{2}$ while respecting $\langle B_1^0 B_2^0\rangle = 1$. In particular, the maximal violation allows us to self-test a maximally two-qubit entangled state, which at the same time certifies one bit of randomness and also implies a monogamy relation. In other words, even though the observers agree among themselves, the outcome of each one of them is completely uncorrelated from system A.

It is worth noting that, in this case, the inequality (15) can also be put in the form $\langle B_1^0 B_2^1\rangle - \langle B_1^1 B_2^0\rangle + \langle B_1^1 B_2^1\rangle \leqslant 1 + 4\delta$, which is similar to the original inequality that appeared in Bell's paper [42]. Here, contrary to the original derivation, obtained assuming perfect anticorrelation for spin measurements in the same direction, the bounds are recovered by means of the objectivity assumption only.

In the following, we will discuss in more depth the consequences of these results, while a detailed proof is presented in Sec. III of [40]. Note that the violation $\mathrm{CHSH}_{0,0} = \frac{5}{2}$ is achieved considering the state

$$|\psi\rangle_{B_1B_2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{16}$$

and choosing $B_j^0 = \sigma_z$ for $j = 1, 2$ and

$$B_1^1 = -\frac{\sigma_z}{2} - \frac{\sqrt{3}}{2}\sigma_x, \tag{17}$$

$$B_2^1 = \frac{\sigma_z}{2} - \frac{\sqrt{3}}{2}\sigma_x. \tag{18}$$

As detailed in Sec. III of [40], the proof that $\mathrm{CHSH}_{0,0} = \frac{5}{2}$ is the maximum quantum violation relies on the idea that together with the agreement condition $\langle B_1^0 B_2^0\rangle = 1$, it allows us to self-test the maximally entangled state. Recall that the possibility of performing a self-test is a sufficient condition to ensure that the quantum probability distribution achieving $\mathrm{CHSH}_{0,0} = \frac{5}{2}$ is unique, as discussed in Ref. [43]. Combining that uniqueness with the convex nature of the set of quantum correlations, it thus follows that $\mathrm{CHSH}_{0,0} = \frac{5}{2}$ is the maximal possible violation. Otherwise, if there was a distribution leading to a higher violation, there would be different manners to mix it with other probability distributions (say, the ones leading to maximal violation of other symmetries of this inequality) in order to obtain two different correlations reaching $\mathrm{CHSH}_{0,0} = \frac{5}{2}$, a situation that would forbid the possibility of self-testing.

Furthermore, following the arguments of Ref. [44], we can state that being an extreme point of the set of quantum behaviors ensures that any third party event is uncorrelated with the outcomes of the observers, i.e., it holds that any realization a of some third variable A is such that $p(a, b_1, b_2|x_1, x_2) = p(b_1, b_2|x_1, x_2)p(a)$. Finally, because the $\mathrm{CHSH}_{0,0}$ inequality is invariant under the transformation $b_1' = (b_1 + 1)\mathrm{mod}2$ (the same holds for a similar transformation of b_2) and the behavior leading to its maximal violation is unique, we can certify a bit of randomness [45], either b_1 or b_2. In particular, the certification of a random bit and the fact that any third party is uncorrelated implies that the probability of guessing the outcome of one of the participants is always $\frac{1}{2}$.

It is worth noting that seen from the perspective of Bell's theorem, Result 3 also has interesting consequences for randomness certification. Differently from the usual setup that requires a violation of $\mathrm{CHSH}_{0,0} = 2\sqrt{2}$ to certify one bit of randomness, the agreement condition $\langle B_1^0 B_2^0\rangle = 1$ permits the same to reach with a smaller CHSH inequality violation. Furthermore, the standard scenario with $\mathrm{CHSH}_{0,0} = 2\sqrt{2}$ requires that a third input is measured by either of the parties if one wishes to directly establish a common secret bit between them. This follows from the fact that in this case, $\langle B_1^0 B_2^0\rangle = \langle B_1^0 B_2^1\rangle = -\langle B_1^1 B_2^0\rangle = \langle B_1^1 B_2^1\rangle = 1/\sqrt{2}$, that is, the measurement outcomes are not completely correlated. In turn, in our case, the condition $\langle B_1^0 B_2^0\rangle = 1$ already guarantees the perfectly correlated secret bit.

Moving beyond the case of the maximal violation $\mathrm{CHSH}_{0,0} = \frac{5}{2}$, we can also probe, via a semidefinite program detailed in Sec. IV of [40], the minimum value of δ required to explain a given value of $\mathrm{CHSH}_{\delta,\epsilon}$, with the results shown in Fig. 2. There we also consider the effect of imperfections on the agreement between the observers, that is, we allow $\langle B_1^0 B_2^0\rangle = 1 - 2\epsilon$, a condition of relevance for experimental tests of our witness. Interestingly, we observe that for any value of ϵ there is always a quantum violation of the $\mathrm{CHSH}_{\delta,\epsilon}$ inequality leading to $\delta = \frac{1}{2}$, that is, the outcomes of the observers are completely uncorrelated from the outcome a of the central system they are supposedly probing. Within the range $0 \leqslant \epsilon \leqslant \frac{1}{2}(1 - 1/\sqrt{2})$, as we increase the ϵ we also increase the maximum quantum violation of $\mathrm{CHSH}_{0,0}$. From this point on, that corresponds to $\mathrm{CHSH}_{\delta,\epsilon} = 2\sqrt{2}$, and for $\langle B_1^0 B_2^0\rangle = 1/\sqrt{2}$ we see the opposite behavior, since the maximum possible violation of $\mathrm{CHSH}_{\delta,\epsilon}$ decreases as we increase ϵ in the range $\frac{1}{2}(1 - 1/\sqrt{2}) \leqslant \epsilon \leqslant \frac{1}{2}$.

Generalizing our scenario, we can now consider the case where system A has not only one but actually two properties, corresponding to the outcomes a_0 and a_1, that we assume to be correlated with the outcomes of measurements performed by the two observers. In this case, the conditions (7), (8), and (10) now assume a joint probability distribution $p(a_0, a_1, b_1, b_2|x_1, x_2)$ and in particular, assuming $\delta = 0$ for simplicity, the objectivity condition implies that

$$\begin{aligned} p(b_1 = a_i|x_1 = i, x_2) = 1, \\ p(b_2 = a_i|x_1, x_2 = i) = 1. \end{aligned} \tag{19}$$

Put differently, if $x_i = 0$ then the outcome b_i should be correlated with a_0; if $x_i = 1$ then the outcome b_i should be correlated with a_1. Similarly to the previous case, it follows that $\mathrm{CHSH}_{\delta,0}$ constrains the correlations compatible with this scenario. However, as a corollary of our next result, proven in Sec. V of [40], differently from the previous case, if we impose a stronger notion of agreement among the observers for all possible measurements, that is, $p(b_1 = \cdots = b_n|x_1 = \cdots = x_n = x) = 1$ for all settings x, then there are no quantum violations of the objectivity conditions.

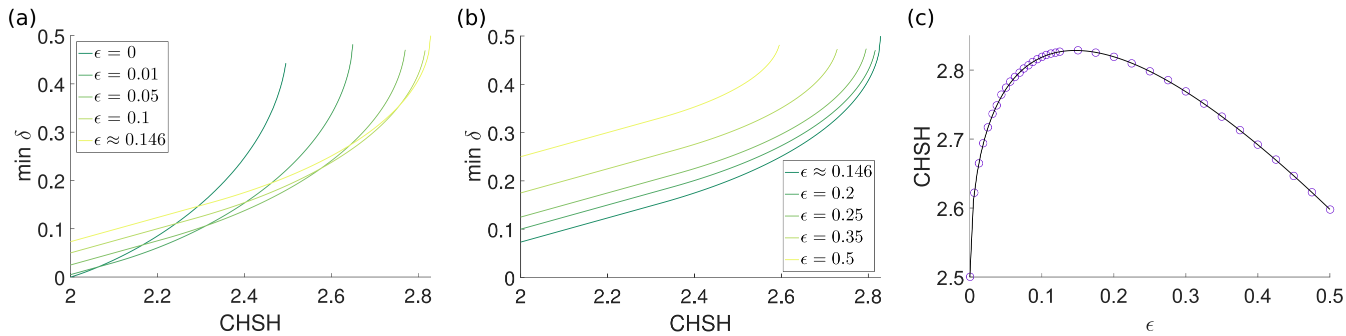

FIG. 2. (a) and (b) Minimal values possible for δ as a function of the value of $\text{CHSH}_{\delta,\epsilon}$ corresponding to the observable distribution $p(b_1, b_2|x_1, x_2)$. Results were obtained using the third level of the Navascues-Pironio-Acin hierarchy [46]. Different curves correspond to different values for outcome agreement between observers, given by the constraint $\sum_{b_1} p(b_1 = b_2|x_1 = x^*, x_2 = x^*) = 1 - \epsilon$ or equivalently $\langle B_1^0 B_2^0\rangle = 1 - 2\epsilon$. A change in the behavior for the maximal violation of the CHSH inequality can be seen between values (a) $\langle B_1^0 B_2^0\rangle \leqslant 1/\sqrt{2}$ and (b) $\langle B_1^0 B_2^0\rangle \geqslant 1/\sqrt{2}$, where the former increases with increasing ϵ while the latter decreases with increasing ϵ. At the same time, the restriction $\delta \geqslant \epsilon/2$ from Eq. (14) can be observed throughout the graphics, with saturation occurring in all cases for $\text{CHSH}_{\delta,\epsilon} = 2$. It also can be seen that a sharp rise in δ occurs near maximal violation for each ϵ, which leads to numerical instabilities in these regions. For this reason, no curve reaches $\delta = 0.5$ and terminal points are different for each curve. (c) Optimal values for the violation of the CHSH inequality with the constraint $\langle B_1^0 B_2^0\rangle = 1 - 2\epsilon$, with explicit quantum realizations found numerically. The points achieve the self-testing criterion of [47], satisfying $\arcsin(\langle B_1^0 B_2^0\rangle) + \arcsin(\langle B_1^0 B_2^1\rangle) + \arcsin(\langle B_1^1 B_2^0\rangle) - \arcsin(\langle B_1^1 B_2^1\rangle) = \pi$, together with the constraints $\langle B_1^0 B_2^1\rangle = \langle B_1^1 B_2^0\rangle = -\langle B_1^1 B_2^1\rangle$. The analytical curve is obtained by combining the equations, resulting in $\text{CHSH}_{\delta,\epsilon} = 1 - 2\epsilon + 3\sin[\frac{\pi}{3} - \frac{1}{3}\arcsin(1 - 2\epsilon)]$.

Result 4. For any number $n \geqslant 2$ of observers, if we impose that $p(b_1 = \cdots = b_n|x_1 = \cdots = x_n) = 1$ for all possible values of x, then all quantum correlations are compatible with the assumptions (7), (8), and (19).

This shows that if the observers agree on the outcomes of all measurements being performed, then the observed correlations are necessarily compatible with the underlying statistics where the observed outcomes are correlated with the property of the system they are probing, represented by the probability distribution $p(a)$.

It is worth remarking here that such nonobjectivity bounds can also serve as witnesses of postquantum correlations. Differently from Result 4 for quantum correlations, no-signaling correlations, those respecting Eqs. (7) and (8), a set of correlations that includes the quantum set, do allow for violations even in the case where the observers agree on the outcomes of all measurements being performed. To illustrate this, it is enough to consider the paradigmatic Popescu-Rohrlich (PR) box [32], given by

$$p(b_1, b_2|x_1, x_2) = \tfrac{1}{2}\delta_{b_1\oplus b_2, x_1\bar{x_2}}. \tag{20}$$

The PR box is such that the observers agree on the outcomes of both possible measurement inputs but at the same time can violate the CHSH inequality up to its algebraic maximum of $\text{CHSH}_{0,0} = 4$. In general, via Result 4, the violation of the CHSH inequality under the constraint of concordance between the observers implies directly the postquantum nature of the correlations.

IV. PROOF-OF-PRINCIPLE EXPERIMENTAL SETUP

In the following, we describe a proof-of-principle photonic experiment realizing a physical interaction dynamics whose output state can be naturally mapped into the quantum Darwinism scenario. In the spirit of Bell's theorem, the constraints we want to test do not depend on any specific dynamics and do not need to assume any precise physical description.

In our scheme, we identify the temporal degree of freedom of photons as the observed system A, while the polarization of two photons represents a pair of observer systems B_1 and B_2, identified with the environment [see Fig. 3(a)]. The two photons interact with a birefringent crystal, thus coupling the polarization and the temporal delay. Both the generation and the interaction occur within a Sagnac interferometer, after which the photons are spatially separated and their (possibly) entangled polarization carries information on the temporal delay.

Consider the scheme in Fig. 3(b). A nonlinear crystal is placed within the interferometer and a laser pump beam enters the interferometer along an input of a dual-wavelength polarizing beam splitter (DPBS) and its interaction with the crystal generates pairs of photons with orthogonal polarizations. The pump passes through the interferometer in the clockwise and counterclockwise directions and the relative amplitude of these contributions depends on the polarization state of the pump beam at the input of the DPBS. Note that this represents a common scheme for the generation of polarization-entangled photon pairs [48]. A birefringent plate is inserted in a Sagnac interferometer after (seen from the counterclockwise path) the crystal. In the clockwise direction, the pump is unaffected by the plate, inducing only an inconsequential phase shift on the pump beam, which has a coherence time much larger than the introduced delay. In particular, the maximum delay introduced inserting the plates is around 4 ps, which is five order of magnitude smaller than the pump coherence time of approximately 100 ns. On the other hand, the counterclockwise generated photon pair has a coherence time of the same order of magnitude as of the introduced time shift and thus it is affected by the polarization-dependent delay, whereas the clockwise term is not.

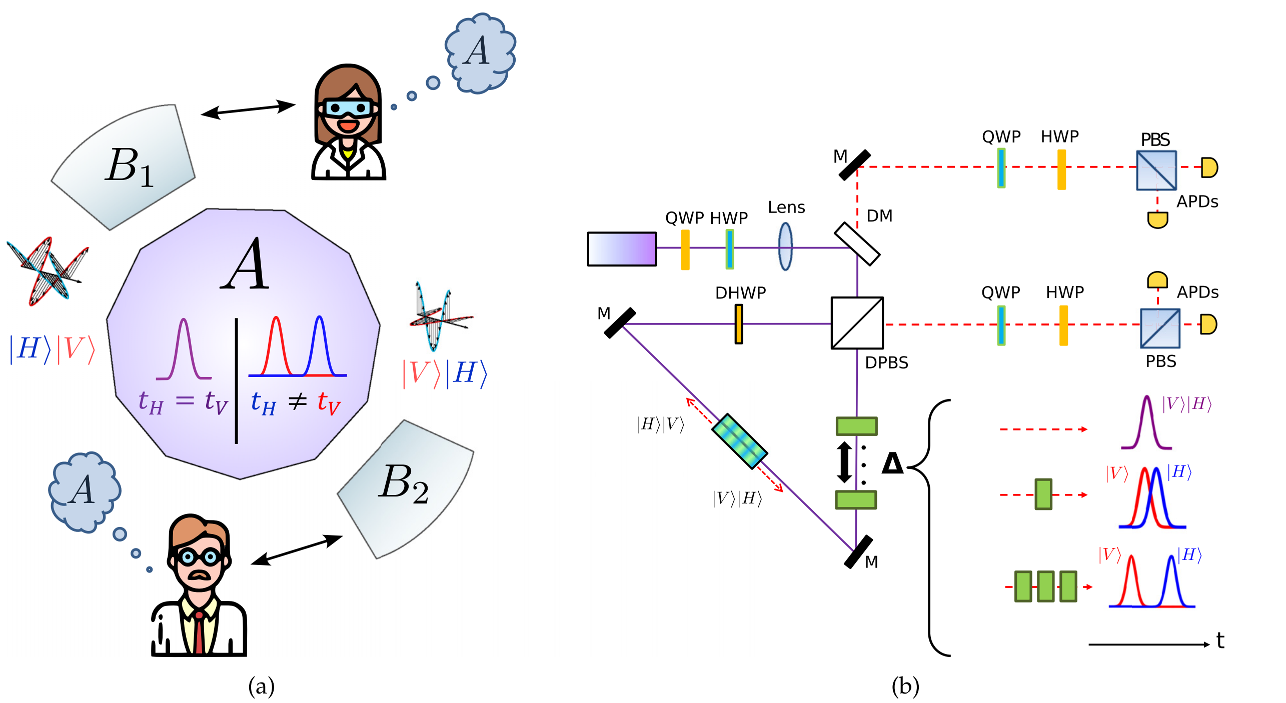

FIG. 3. Proof-of-principle experimental setup. (a) Conceptual scheme of the experiment, where the polarization of photons represents the environments while the temporal delay represents the quantum system of interest. (b) Experimental setup. A Sagnac-based polarization entangled photon source generates pairs of degenerate photons at 808 nm. Inside the Sagnac interferometer, the interaction of the photons generated in the counterclockwise direction interacts with the birefringent plates which couple the polarizations with the temporal degree of freedom. The strength of the interaction is parametrized by Δ, indicating the overlap between the temporal wave functions of the horizontal and vertical polarizations, and is varied by changing the thickness of the birefringent plates. Finally, the photons are collected and detected by single-photon detectors, where M denotes mirror; PBS, polarizing beam splitter; HWP, half waveplate; QWP, quarter waveplate; DM, dichroic mirror; DHWP, dual-wavelength half waveplate; and APDs, avalanche photodiode detectors.

In order to map the experimental scheme into a quantum Darwinism scenario, we can interpret the interferometer with the birefringent plate as a device mediating the interaction between the temporal degree of freedom of the photons, i.e., our system, and the polarization of the outgoing modes, i.e., the environment where information proliferates. More specifically, consider the joint temporal state of the two photons generated inside the interferometer. This can be described by a certain amplitude function $f(t_H, t_V) = g(t_H - t_V) = g(\Delta t)$ which depends only on the time difference between the photons in the two modes if the pump is monochromatic. Let us call $|0\rangle_A$ the state associated with the temporal mode $g_0(\Delta t)$ of the photons in the standard case with no birefringent plate. When the birefringent plate is present it inserts a delay τ between the horizontal and vertical polarization modes, so that we will simply have $g_\tau(\Delta t) = g_0(\Delta t + \tau)$, that we can associate with the state $|g_\tau\rangle_A$. This state will in general have some overlap with $|0\rangle_A$, which we can quantify as

$$\Delta = {}_A\langle 0|g_\tau\rangle_A \propto \int d\Delta t\, g_0^*(\Delta t) g_0(\Delta t + \tau). \tag{21}$$

We will call $|1\rangle_A = (\mathbb{I} - |0\rangle_A\langle 0|)|g_\tau\rangle_A$ the orthogonal component of $|g_\tau\rangle_A$ to $|0\rangle_A$. After the interferometer the two photons are split in two directions B_1 and B_2. The state of the photons can now be described by the polarization in each mode B_1 and B_2 and the temporal mode A. For the counterclockwise generation, the effective temporal state will be in a superposition $\Delta|0\rangle_A + \sqrt{1-\Delta^2}|1\rangle_A$ and, consequently, the state at the output of the interferometer will be

$$\frac{1}{\sqrt{2}}(\Delta|H\rangle_{B_1}|V\rangle_{B_2} - |V\rangle_{B_1}|H\rangle_{B_2})|0\rangle_A + \sqrt{\frac{1-\Delta^2}{2}}|H\rangle_{B_1}|V\rangle_{B_2}|1\rangle_A. \tag{22}$$

Tracing out the temporal degree of freedom A, the final state of the two observer systems is given by

$$\rho_f = |\Delta|^2|\Psi^-\rangle\langle\Psi^-| + (1-|\Delta|^2)\rho_{mix}, \tag{23}$$

where $|\Psi^-\rangle$ is the singlet polarization state and ρ_{mix} is the mixed state $\rho_{mix} = (|HV\rangle\langle HV| + |VH\rangle\langle VH|)/2$.

In this way, an interaction occurs between the polarization of the two photons and their time degree of freedom by means of the same birefringent plate. After such an interaction, the polarization of the photons, i.e., the environment systems B_1 and B_2, carry information about the time degree of freedom defined as the observed system A. The strength of this interaction can be tuned by changing the thickness of the birefringent plates. To illustrate this, we describe two extremal conditions. When no birefringent plate is present no interaction occurs; thus the temporal state of the photons is uncorrelated with respect to the polarization. In this case, $\Delta = 1$ and, from Eq. (23), the final polarization state of the photons is a maximally entangled state $\frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$.

Conversely, when the thickness of the birefringent plate introduces a temporal delay much greater than the coherence time of the photons, $\Delta \to 0$ and then the global state, from Eq. (22), will be $(|HV\rangle \otimes |1\rangle_A - |VH\rangle \otimes |0\rangle_A)/\sqrt{2}$. From Eq. (23), tracing out the time degree of freedom, the polarization state will be the mixed state $(|HV\rangle\langle HV| + |VH\rangle\langle VH|)/2$.

To resume, considering the state of the photonic polarizations in Eq. (23), we have that when there is maximum coupling ($\Delta = 0$) the polarization values of the two photons effectively correspond to the presence ($|H\rangle$ for the first photon and $|V\rangle$ for the second one) or absence ($|V\rangle$ for the first photon and $|H\rangle$ for the second one) of a temporal delay. Conversely, when no coupling is present, no information on the presence of temporal delay is stored in the polarization of the photons. From our perspective, polarization plays the role of an environment, mediating the interactions between the (indirectly) observed system, here the temporal delay, and the measurement apparatus.

V. EXPERIMENTAL RESULTS

We performed the measurements using four different delays. For each delay, there is a corresponding strength of the interaction, parametrized by the overlap Δ_i, with $i = 1, 2, 3, 4$, from Eq. (22). The trivial case with $\Delta = 1$ is obtained using the standard quantum source without the necessity of inserting delay plates. On the other side, by inserting birefringent plates with two different lengths, we inserted polarization dependent delays of $t \approx 0.9$ and 2.17 ps, reaching overlaps of $\Delta = 0.91$ and 0.59, respectively. The extreme case with $\Delta \to 0$ is reached using a potassium titanyl phosphate crystal of 6 mm length, which implies a time delay of approximately 4 ps.

Once the overlap Δ_i is fixed, the measurements are performed by varying the agreement $\langle B_1^0 B_2^0\rangle = 1 - 2\epsilon$ between the measurements of the two observers. For each agreement, the violation of the CHSH inequality is optimized, using information from quantum state tomography [49]. More specifically, from the tomography, we extract the values of the rotation angles of the measurement waveplates, able to reach, within experimental errors, the desired agreement and the corresponding maximum CHSH parameter achievable by the generated state.

The results are shown in Fig. 4. For the case with maximum interaction $\Delta = 0$, the polarization of the two photons becomes maximally entangled with the time degree of freedom and consequently, from the monogamy of entanglement [50], no entanglement is possible between the polarizations. Thus, no violation of the CHSH inequality is observed (see green points in Fig. 4). In this case, one can argue the emergence of objectivity since not only do the observers measuring the polarization of the two photons agree among themselves, but their measurement outcomes can indeed correspond to an objective property, in our proof-of-principle experiment represented by the time degree of freedom of the photons.

Conversely, when the interaction is absent ($\Delta = 1$), the experimental entangled state is able to violate the CHSH inequality up to a value $S^{\text{expt}} = 2.475 \pm 0.008$ with an observed agreement $\langle B_1^0 B_2^0\rangle = 1 - 2\epsilon = 0.956 \pm 0.002$, that is, $\epsilon = 0.022 \pm 0.001$. Using the $\text{CHSH}_{\delta,\epsilon}$ inequality (15), valid for general no-signaling correlations, we see that this corresponds to $\delta \geqslant 0.124 \pm 0.002$. The effect becomes more pronounced when assuming the validity of quantum theory for all systems involved. A numerical computation approximating the quantum set by a superset of allowed distributions [46] returns $\delta = 0.49 \pm 0.01$, revealing that the observed variables should be almost completely uncorrelated with any candidate for the system A property. We thus obtain solid experimental evidence of the (noisy) apparent agreement, where even if the two observers have agreement on the supposedly measured value, the latter cannot correspond to an objective property of the quantum system of interest.

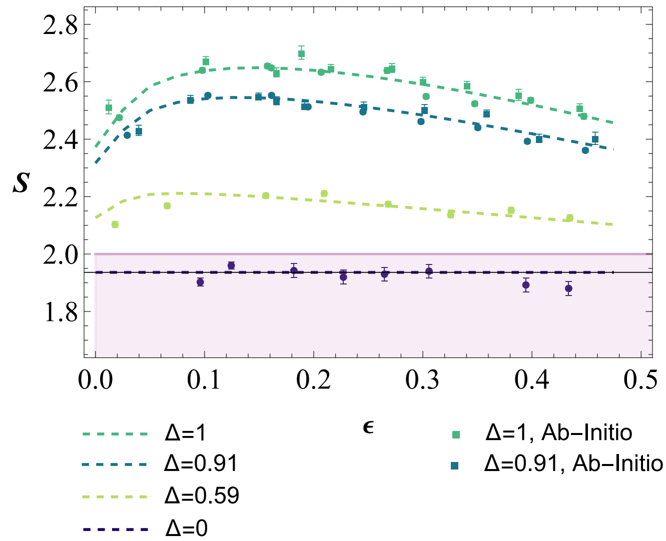

FIG. 4. Experimental data. Optimal values of the experimental CHSH parameter as a function of the constraint $\langle B_1^0|B_2^0\rangle = 1 - 2\epsilon$ for different values of temporal overlap Δ between the wave functions of the two polarizations. The dashed lines represent the optimal values calculated from the theoretical model of the experimental state. Moreover, for $\Delta = 1$ and 0.91, there are also reported results obtained using the *ab initio* approach, which are indicated by squares. When no temporal overlap is present ($\Delta = 0$), we observe $S \approx 1.94 < 2$, due to an additional uncorrelated noise contribution, i.e., $\rho_{\text{dep}} \propto \mathbb{1}/4$, on the generated state. The error bars are calculated assuming Poissonian statistics.

In order to shift our experimental evidence for nonobjectivity closer to the spirit of the device-independent paradigm, we also perform measurements using the *ab initio* approach introduced in [51], where experimental violations of classical constraints were found and optimized in a fully black-box scenario, without any knowledge of the generated state and the measurement apparatuses. More specifically, while usually in an experiment one tries to violate some inequality using precise knowledge of the employed experimental apparatus, in an *ab initio* approach one does not assume any prior information and, based only on the (noisy) output statistics, adaptively learns the optimal values of some controllable parameters, in order to optimize a given cost function, such as the violation of a Bell-like inequality.

In our experiment there are eight parameters to be optimized by the algorithm, corresponding to the values of the angles of pairs of waveplates (one pair for each measurement station) for each of the four measurements needed to evaluate the CHSH parameter. In particular, the optimization first reaches the target value of the agreement $\langle B_1^0 B_2^0 \rangle = 1 - 2\epsilon$ tuning the four involved waveplate parameters and then it reaches a global optimum for the CHSH value tuning the other four parameters associated with the remaining CHSH measurements $\{B_1^1, B_2^1\}$. Details on the *ab initio* optimization protocol can be found in Sec. VII of [40]. The results on the values of the CHSH experimentally achieved with the *ab initio* approach are shown in Fig. 4.

The experimental points collected with the *ab initio* approach can reach values higher than the ones achieved using quantum state tomography information. This is possible because within an *ab initio* framework errors in the characterization of the optical setup, such as the optical axes of waveplates, can be compensated automatically by the optimization process. For all the curves with $\Delta \neq 0$, for each value of the observed agreement, the CHSH is violated, consequently witnessing a degree of nonobjectivity in a device-independent way.

We remark that our work is not the first to explore quantum Darwinism experimentally; see, for instance, [52–54]. Those experiments, however, relied on the computation of the mutual information between the quantum system and its environment. Our framework allows us to reason about the emergence of Darwinism simply by observing the correlations induced by the quantum system into the environment. As detailed above and also in [40], in our experiment the quantum system of interest is encoded in the temporal degree of freedom of single photons, while the interacting environment is represented by the polarization degree of freedom of the single photons.

In such a picture, we can interpret the Sagnac interferometer, as a whole, as a black box through which we can control the correlation between the system and the environment. In this black-box picture, it is possible to modify the system state, i.e., the temporal degree of freedom of single photons, only by inserting a birefringent plate. The whole interaction between time and polarization is carried out inside the interferometer. At its output, we wish to infer if the photons have been delayed or not by measuring the polarization of the photon pair, i.e., the environment which previously interacted with the system and towards which such information proliferates. As we show, we can estimate the degree of objectivity of the dynamics without knowing anything about the inner working of the black box, that is, only from the observed correlations between the environments. This stresses the device independence of our experiment, in the sense that the lack of objectivity does not rely on an exact description of the dynamics being probed, an aspect that is emphasized by the black-box optimization algorithm [51] employed in the real-time execution of the experiment.

VI. DISCUSSION

Comprehending how microscopic quantum features give rise to the observed macroscopic properties is a central goal of decoherence theory and in particular of quantum Darwinism. Importantly, the emergence of objectivity, that is, the fact that different observers agree on the properties of a quantum system under observation, can be seen as a generic aspect as long as the information of the quantum system is successfully outspread to the environment it is interacting with. It is unclear, however, how to witness the presence or rather the absence of such objectivity in practice. Can we witness nonobjectivity by simply probing the environment, without any knowledge of the underlying dynamics?

To answer in the affirmative to this question, we establish a probabilistic framework to cast objectivity through operational lens, building on the results of [22]. Within this setting, we propose three properties defining what is to be expected from a generic objective behavior, i.e., no superdeterminism, no signaling, and δ objectivity, the latter stating that $p(b_i = a|x_i^*) \geqslant 1 - \delta$, where x_i^* denotes the measurement for which the observer should try to recover as best as possible the information about the system A as encoded in the probability $p(a)$. Those conditions play a role similar to what the concept of local realism implies for Bell's theorem [28]. In particular, the notion of δ objectivity is justified by our first result stating that the local agreement between a given observer and the quantum system of interest translates into a global notion of agreement between all observers having access to some part of the environment.

We then showed that a generalization of the seminal CHSH inequality [27] constrains the set of possible correlations compatible with the three aforementioned assumptions. We showed that Bell inequalities are a relevant tool for the understanding of quantum Darwinism and the emergence of objectivity. Nonetheless, it is important to note that Bell inequalities appear in many other areas of quantum information far beyond its original purpose, such as dimension witnesses [55], self-testing [43], communication complexity problems [56], and the measurement problem [41,57]. It also emerges in totally different contexts such as game theory [58], causal inference [59], knowledge integration of expert systems in artificial intelligence [60], and database theory and privacy aspects of databases [61].

The violation of the Bell-like inequality we derive offers a device-independent witness of the nonobjectivity of the underlying process at the same time that it naturally quantifies how much one should give up objectivity in order to explain the observed correlations. Further, we proved that quantum mechanics allows for violations of this inequality and, in particular, leads to a monogamy relation between the agreement with the internal degree of freedom and the one among the observers. This implies that even though the observers agree among themselves, their outcomes can be completely uncorrelated from the system they supposedly should be correlated with, a phenomenon that we have experimentally probed using a photonic setup where the quantum property of interest is encoded in the temporal degree of freedom of photons, the polarization of which plays the role of the environment to which the information should redundantly proliferate.

For scenarios where the probed system has more than one property of interest, we demonstrated that if the observers have to agree on measurement outcomes of all performed measurements, then quantum correlations are compatible with the assumptions of no superdeterminism, no signaling, and objectivity, that is, they cannot violate any Bell-like inequality, a result that can be violated by correlations beyond those allowed by quantum theory and that thus can be employed as a test for postquantumness.

This connection between quantum Darwinism, and the notion of objectivity it entails, with Bell inequalities and device-independent quantum information processing is a bridge that deserves further investigation. For instance, it would be interesting to generalize our results to a larger number of observers and consider measurements with more outcomes. At the same time, one should understand paradigmatic dynamics considered in the literature of quantum Darwinism [13–20] under this alternative perspective and explore the connections with other objectivity measures [62] valid in the quantum framework. It is worth remarking also that our approach could lead to substantial refinements in recent tests for the emergence of objectivity [52–54]. Nevertheless, we should also note that another bridge connecting quantum Darwinism, Spekkens contextuality, and quantum information has also been recently developed. Adapting the prepare-and-measure scenario into the usual environment as a witness framework, in Ref. [24] the authors managed to prove that Spekkens noncontextuality for each observer follows through whenever the environment proliferates the information about the central system appropriately. Their notion of classicality differs from ours, insofar as here we consider mutual agreement rather than noncontextuality as a signature of classicality (and our connection with foundations of quantum mechanics is via Bell scenarios). Additionally, our work goes a step further, as we investigate our theoretical findings with a proof-of-principle experiment.

Finally, we notice that the δ-objectivity constraint we consider here is mathematically very similar to the notion of absoluteness of events employed to analyze a generalization of the Wigner friend experiment [41,57,63] on the foundation of quantum theory. Apprehending further the connections between quantum Darwinism or objectivity and Wigner's friend experiment or absoluteness of events is another relevant research direction that we hope our results might trigger.

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