Multiple measurements on an uncollapsed entangled two-photon state

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(Received 12 October 2022; revised 5 June 2023; accepted 24 July 2023; published 10 August 2023)

The relativity of simultaneity together with the definition of a quantum state collapse results in experimental situations where multiple measurements can be taken on an uncollapsed quantum state. The quantum state's collapse is defined to be instantaneous in a rest inertial frame of a detector performing measurements on the quantum system. The definition is consistent with the Copenhagen interpretation and in agreement with all measurements performed with detectors at rest in an arbitrary Lorentz (laboratory) frame. From the introduced collapse model follows that under certain conditions multiple measurements are allowed on the same uncollapsed quantum state. An application of the developed approach is shown on measurement of photon-pair state entangled in polarization and energy. Conditions under which two measurements can be taken on the uncollapsed photon-pair state are derived. Serious consequences follow from the allowance of multiple measurements on the same uncollapsed state. For example, the measurements taken by both detectors in this situation are uncorrelated. Moreover, all the conservation laws could be violated in individual measurements, but not in mean values. This statement is proved on the two-photon state entangled in energy. This is in contradiction with experimental results observed by the detectors in rest relative to each other. It is shown that the property of measuring uncorrelated results with detectors in relative movement is related solely to the proposed collapse model. The remaining collapse models—the preferred Lorentz frame, Aharonov-Albert, and Hellwig-Kraus—are examined and discussed with respect to the designed experiment, which involves spacelike separated measurements.

DOI: 10.1103/PhysRevA.108.023709

I. INTRODUCTION

A state of a quantum system undergoes two types of space-time evolution [1–4]. When the quantum system is undisturbed by a measurement apparatus (detector), time evolution of its quantum state (QS) is given by the Schrödinger equation. Since the time evolution of the quantum system can be uniquely predicted by solution of the equation, the time evolution is considered to be deterministic. When the quantum system interacts with the measurement apparatus, its QS undergoes a collapse. The collapse is explained as projection of the quantum system's QS into an eigenvector related to a value measured by the detector. This process is considered to be purely statistical, since only the probabilities with which the values can be measured (on the quantum system) can be predicted. The QS's collapse, according to the Copenhagen interpretation (CI), occurs at the time of the measurement on all space [1,4].

First discrepancies between the relativity of simultaneity and time evolution of a QS have been described by Bloch [5]. He studied interaction of a QS with multiple detectors positioned at various points in space-time when viewed (and evolved) in different Lorentz frames. He concluded that ambiguities in a QS may arise when it is evolved in moving Lorentz frames. But these ambiguities do not affect probabilities of the measured results. He also proposed the Lorentz-invariant QS's collapse along the backward light cone. This idea was later developed by Hellwig and Kraus [6]. Aharonov and Albert (A&A) argued the preferred reference frame models and the Hellwig-Kraus (H&K) model of wave function collapse are unsuitable [7,8]. Instead, they proposed QS collapse to occur instantaneously for any observer. They based their argumentation on measurement of the nonlocal observables. All models of the state's reduction have been summarized and discussed with respect to the three most used interpretations of quantum mechanics (QM) in a review [8] by Cohen and Hiley (C&H). The authors claim to defend the preferred Lorentz frame (PLF) model and extended it to include unitary interactions. The PLF model defines the collapse of the QS to occur instantaneously in a preferred (particular) Lorentz frame. In their work C&H [8] were exploring space-time properties of a general PLF model utilizing a single collapse hypersurface.

C&H showed [8] the A&A arguments for discarding of the PLF model to be invalid. They managed to prove that both models provide the same results of a measurement. C&H studied measurement of nonlocal observables, as proposed by A&A, on a singlet spin state $|\Psi\rangle = |+_z\rangle_1|-_z\rangle_2 - |-_z\rangle_1|+_z\rangle_2$ ($-_z$ denotes spin down projected on axis z and $+_z$ spin up in the same direction). A&A showed that utilization of immediate collapse in the measurement (on QS $|\Psi\rangle$) results in nondemolishing verification of the state. On the other hand, they argued that collapse occurring in a preferred Lorentz frame leads to different measurement results [8]. C&H managed to disproved this by an explicit calculation of the final state, which emerges after the measurement.

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C&H further argue that the A&A collapse model does not propose a unique space-time hypersurface for collapse of a QS. Because of this property, C&H seem to prefer the PLF model. It allows them to introduce unique unitary evolution of a QS including interactions. A&A are not concerned about ambiguities of the QS's collapse or physical meaning of the QS as far as it provides correct results of the measurements. We note that this treatment may appear to be incorrect if Susskind's and Maldacena's recent hypothesis in its strong form is revealed to be true [9,10]. It states that entanglement between quantum particles is physically realized by Einstein-Rosen bridges (i.e., wormholes in space-time). Then a QS of a quantum system should be treated as an object of reality with unique properties as well. We remark that analysis of closing of the exits of the wormholes by the external nearby moving observers may bring a significant shift in both topics: Susskind's and Madacena's hypothesis as well as relativity of the QS's collapse.

The H&K model requires collapse of a QS to occur before a measurement takes place [4,8]. It is the only model which preserves the collapse hypersurface under the Lorentz transformation. The property of collapsing of the QS prior to the measurement is as well called a precollapse [4]. This property is in contradiction with causality. However, there has been no argument or experiment revealing that the violation of causality can be demonstrated by a measurement. Similarly to the A&A model, the H&K model possesses QS's ambiguities (see Sec. 9 in [8]). The ambiguities have their roots in elements of the H&K model. Because of these properties, the H&K model is considered as inappropriate [8]. In Appendix B we show that the H&K model predicts results conflicting with already performed experiments. This is observed when two spacelike separated measurements are performed on an entangled state.

The PLF collapse model defines a collapse of a QS to occur instantaneously in only one chosen Lorentz frame (called preferred frame). This unique frame may not necessarily be attached to any detector or observer. In consequence, all collapse hypersurfaces related to measurements by the detectors will be parallel to each other. The preferred frame could be attached to a center-of-mass frame of a two-particle state or to a microwave background radiation frame [8]. The PLF model has already been used for studying collapse properties of an entangled electron pair [7,8].

In this paper we introduce a multiple Lorentz frame collapse model (MLF), which sets the collapse of a QS to occur immediately in a rest frame of a detector taking a measurement on the QS. Since there can be more detectors moving relative to each other and taking measurements on the QS, the collapse hypersurfaces related to individual measurements may intersect. Due to this property, the area where the QS is uncollapsed and collapsed after a series of measurements has to be properly identified.

A motivation for development of the MLF collapse model is to generalize the CI of collapse of the QS to measurement schemes, where the detectors are in relative movement to each other (relativistic schemes). The features of the MLF collapse model are the following: (1) it attaches a unique collapse hypersurface to a measurement and (2) it preserves compatibility with the CI in situations when all detectors are at rest in one laboratory Lorentz frame. These properties makes the MLF model unique among the others. In all experiments performed so far, the detectors have been at rest in one (laboratory) reference frame—mostly connected with earth's surface. In these situations, the CI has provided a unique collapse hypersurface for a particular measurement. A relativistic generalization should possess this feature and at the same time should be compatible with the CI by definition and by the predicted results in all experiments performed so far. The proposed MLF collapse model, as the only one, satisfies these requirements.

The MLF model allows for construction of a measurement scenario where two measurements on an uncollapsed polarization-entangled two-photon state is allowed. For this particular measurement, we consistently derive equations of the collapse lines and space-time characteristics of the experiment and point out the dependencies of all involved parameters. The adopted MLF model is shown to be compatible with the CI of a QS's collapse as far as all detectors are at rest in one reference frame, is independent of the time sequence of measurements and, to our best knowledge, is in agreement with all experimental observations so far. Similarly to H&K model, the PLF and MLF models include a precollapse. In the MLF model the precollapse occurs only in reference frames moving with respect to a detector taking a measurement and only on a part of the space-time. In consequence, the MLF and H&K models allows for taking multiple measurements on an uncollapsed QS, while the PLF (strictly speaking only in the preferred frame; see Appendix A) and A&A models do not. At the same time, the MLF and PLF models allow the initial QS to collapse in finite time before it gets into contact with any detector. This situation is briefly explored in Appendix A (for the PLF model) and Appendix E (for the MLF model).

In this paper the properties of the MLF collapse model are studied in detail with respect to a spacelike separated measurement on an polarization-entangled photon pair. Properties of the other collapse models in this measurement scheme are briefly examined in the Appendixes A (PLF), C (H&K), and D(A&A). In summary, there are four space-time models of QS's collapse examined in this paper:

(1) Hellwig-Kraus (H&K)—along the backward light cone [6].

(2) Preferred Lorentz frame (PLF)—along spacelike hypersurface t = const in a unique preferred Lorentz frame [8].

(3) Multiple Lorentz frame (MLF)—along spacelike hypersurface t = const in a rest frame of a detector [8].

(4) Aharonov-Albert (A&A)—along spacelike hypersurface t = const in an observer's Lorentz frame [7,11].

The idea that in a system of two entangled particles each particle can be measured before another has been proposed by Suarez [12]. He pointed out that in this measurement scheme, measurements taken on both photons are uncorrelated. He used this statement as an axiom of his new measurement theory without proving its correctness or relation to the collapse models. In connection with this idea, he developed a new measurement model called "alternative description" (AD). It was developed in order to deal with experiments involving impacts of photons on moving and stationary beam splitters. Suarez used this description for prediction of measurement outcomes in experiment involving one moving and one stationary beam splitter. Using AD, he predicted measurement outcomes which differ from prediction of quantum mechanics. The experiment has been modified and tested by Zbinden *et al.* [13,14] with results agreeing with predictions of quantum mechanics.

Two spacelike separated measurements on a two-particle quantum system in relation to time ordering of the events have already been explored [12,15], but this has not been studied in relation to the PLF and MLF collapse models. We prove that in the proposed experiment, the MLF model predicts results distinct from the A&A and PLF models. It follows that the proposed experiment either validates or disproves the MLF model in favor of the A&A and PLF models. The possibility of multiple measurements on an uncollapsed QS in connection with the MLF model raises a few questions regarding the correlations of measured results and final states, particularly, question of determination of the final state and questions about violation of conservation laws in individual experiments. The questions are addressed in this paper.

There are two ways how to judge a compatibility of a collapse model with the CI of a QS's collapse, either (1) by means of predicted results of a measurement or (2) by their definitions in a chosen Lorentz frame. Per point 1, until now, to our best knowledge, all collapse models have been revealed to predict the same results as CI, except the H&K model [8]. This is considered in scenarios where the detectors are stationary in the same reference frame. When a measuring detector (or an observer making a prediction about a measurement) is in a movement, the CI lacks explicit rules how to define the collapse hypersurface related to the measurement. In an experiment proposed in this paper, detectors in relative movement are considered to perform the measurements. It is demonstrated that the MLF model differs in predicted results from the A&A and PLF models. Moreover, we show the H&K model to predict results in contradiction with experiments [16–18] involving two spacelike separated measurements on an entangled two-particle state. The measurements are taken by the stationary detectors. Thus, the experiments reveal the H&K model to be invalid.

Moving to point 2, the A&A and MLF models are compatible with the CI by definition, while the PLF and H&K models are not. In practice, the CI is used in a laboratory Lorentz frame, where all detectors taking measurements are at rest. Therefore, a collapse model is considered compatible if a collapse of a QS is instantaneous on all space in arbitrary laboratory Lorentz frame in which the detectors are at rest. The A&A model is compatible with the CI because the collapse hypersurfaces are instantaneous in any reference frame regardless of motion of the detectors. The MLF model is compatible with the CI because the measurements taken by the detectors, which are all stationary in an arbitrary Lorentz frame, are related to instantaneous collapse hypersurfaces in the Lorentz frame. When one of the detectors in MLF model is in relative moment to the remaining detectors, compatibility of the model with CI breaks. We summarize that both the A&A and MLF models have the same definition of collapse hypersurfaces when all detectors are stationary in one reference frame. The PLF model is compatible with the CI only in the preferred Lorentz frame. Since the laboratory frame can in principle be associated with an arbitrary Lorentz frame, the collapse hypersurfaces defined by the PLF model are not generally instantaneous in the laboratory frame. Thus, the collapse hypersurfaces in the PLF model are instantaneous only if the laboratory Lorentz frame is identical to the preferred Lorentz frame. The H&K model does not agree with the CI by definition in any reference frame, since the collapse of the QS precedes the measurement in any reference frame. Thus, the PLF and H&K collapse models are not compatible with the CI, while MLF and A&A are compatible with the CI by definition.

In this article we propose an experiment in which one moving and one stationary detector perform measurements on a quantum polarization-entangled photon-pair state. The measurements are assumed to be spacelike separated. For analysis of this situation, we use the MLF model, where the frame of instantaneous collapse is attached to a rest frame of a detector. We compare the result of the experiment when the MLF collapse model is used with results if the H&K, PLF, and A&A models are utilized. In Sec. I an introduction into the topic of collapse models was provided. In Sec. II we introduce the MLF collapse model along with collapse lines in detail and show its properties when a measurement is performed by one or two detectors. We focus on cases when one of the detectors is in relative movement to other. The remaining collapse models are introduced and analyzed in Appendixes A (PLF), C (H&K), and D (A&A). The experiment covering simultaneous measurement of two detectors on an uncollapsed state is analyzed in Sec. III in relation to the MLF model. The experiment is briefly discussed in connection to the PLF, A&A, and H&K models in Appendixes A, C, and D, respectively. By careful inspection of the experiment through space-time diagrams and usage of the collapse lines, we identify the origin of two temporary QSs in the MLF model. In addition to this, we realize that a rule uniquely determining the final (permanent) state is missing in the quantum theory in combination with the MLF model. From inspection of the space-time diagrams follows that measurements on both photons are uncorrelated and can be interpreted as multiple measurements on the initial state. In Sec. IV the conservation laws are shown to be violated in the experiment, but not in mean value. In Appendix B we show how multiple measurements on an uncollapsed state can be taken on a single-photon state, which is in superposition of two distinct paths [19]. In Appendix C the H&K model is proven to provide result in contradiction with the experiments when two stationary detectors perform the measurements at two spacelike separated events on an entangled state. The A&A model and PLF model (in the preferred frame) are shown to hold the correlations between the photons. Thus, results of the proposed experiment either prove the validity of the MLF model or disprove it in favor of the A&A and PLF models. In Appendix E the situation when a precollapse occurs in the MLF model is explored.

II. RELATIVITY OF A QUANTUM STATE'S COLLAPSE IN THE MLF MODEL

We inspect the space-time evolution of a QS $|\psi\rangle$ in one spatial x and time dimension t. We work inside the Schrödinger picture and assume no interactions take place inside or outside the quantum system. Therefore, the QS changes only due to a collapse or an interaction-free evolution.

The state is assumed to undergo a measurement at time $t = T_1$. Before the measurement, we assume that the quantum system is in the state $|\psi\rangle \equiv |\psi(t)\rangle$ on all available space x. After the measurement, the quantum system is in state $|\psi_{1,i}\rangle$, which is related to the original state $|\psi\rangle$ by projection postulate

$$|\psi_{1,i}\rangle = \frac{\langle\phi_i|\psi\rangle}{|\langle\phi_i|\psi\rangle|}|\phi_i\rangle. \quad (1)$$

The state $|\phi_i\rangle$ is an eigenstate of the measured observable's operator. If not explicitly stated otherwise, the MLF collapse model is used throughout the paper.

The space-time evolution of the QS $|\psi\rangle$, which in time $t = T_1$ undergoes a measurement by a detector D at position $x = 0$, is shown in Fig. 1(a). Evolution of the state is shown in reference frame $S \equiv (ct, x)$. We assume that the detector D is in rest in the S reference frame. In the Fig. 1(a) there are also depicted the axes t' and x' of a reference frame $S' \equiv (ct', x')$ moving relative to the reference frame S with constant velocity $v > 0$. The transformation of vector coordinates between reference frames S and S' are given by the Lorentz transformation

$$\begin{aligned} ct' &= \gamma(ct - \beta x) + ct'_0, \\ x' &= \gamma(x - \beta ct) + x'_0 \end{aligned} \quad (2)$$

and its inverse

$$\begin{aligned} ct &= \gamma(ct' + \beta x') + ct_0, \\ x &= \gamma(x' + \beta ct') + x_0. \end{aligned} \quad (3)$$

From Fig. 1(a) follows that the quantum system is on all space x before the time of measurement T_1 in the uncollapsed state $|\psi\rangle$ [20]. The space-time area where the QS is uncollapsed is in yellow. After a successful measurement by the detector D at the space-time point $(T_1, 0)$, the QS $|\psi\rangle$ collapses into the state $|\psi_{1,i}\rangle$ [1] in the time of measurement $t = T_1$ on all space x in accordance with Eq. (1). The space-time area where the QS is collapsed is marked in green. It is important to note that in practice there is not a fixed time in which the quantum system collapses. Rather, there is a time interval ΔT_1 during which the quantum system interacts with the measurement apparatus and undergoes the collapse [21,22]. For simplicity, we assume the collapse to occur at a fixed time T_1. As shown in Fig. 1(a), in reference frame S the space-time line of the collapse (collapse line) is given by equation $t_c = T_1$.

The same situation of the QS's collapse can be viewed from a reference frame $S' \equiv (ct', x')$, which moves relative to the detector D (and reference frame S) with velocity $v > 0$. The space-time chart of this situation is shown in Fig. 1(b). It is apparent that the collapse line t'_c is not perpendicular to the time axis t'. It is tilted with a nonzero angle with respect to the x' axis according to

$$\begin{aligned} ct'_c(x') &= -\beta x' + \frac{cT_1}{\gamma} + \beta x'_0 + ct'_0 \\ &= -\beta x' + \frac{cT'_1}{\gamma^2} + \beta x'_0 + \beta^2 ct'_0 \\ &= -\beta x' + cT'_1 + \beta X'_1. \end{aligned} \quad (4)$$

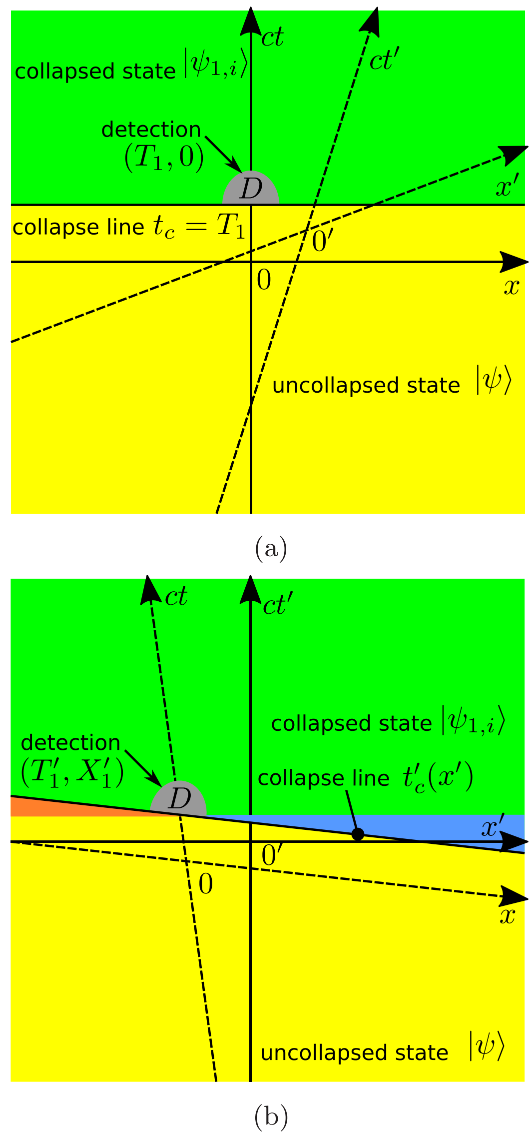

FIG. 1. (a) Space-time evolution of a QS $|\psi\rangle$ utilizing the MLF collapse model. The QS $|\psi\rangle$ undergoes a measurement at space-time point $(t, x) = (T_1, 0)$. The detector D, which performs the measurement, is in rest in reference frame $S \equiv (ct, x)$. The detector is placed at position $x = 0$. The yellow field denotes the space-time area, where the QS $|\psi\rangle$ is uncollapsed. The green area shows the space-time interval, where the QS is collapsed to state $|\psi_{1,i}\rangle$. The $S' \equiv (ct', x')$ reference frame moves relative to the reference frame S with positive velocity v. (b) Situation from (a) viewed from the reference frame S'. In reference frame S' the QS $|\psi\rangle$ has undergone a measurement at space-time point (T'_1, X'_1). The orange area denotes space-time points in reference frame S', where a subsequent measurement by a detector, at rest in reference frame S', is allowed on an uncollapsed state $|\psi\rangle$. The blue region marks space-time points, where the first measurement, by a detector in reference frame S', is performed on already collapsed state.

The measurement space-time point $(T_1, 0)$ has been transformed into the S' reference frame, to point (T'_1, X'_1). Equation (4) is a linear equation with respect to independent variable x' with negative slope. The line related to the equation obviously passes through the point of measurement (T'_1, X'_1); see Fig. 1(b). From Eq. (4) follows that the "speed"

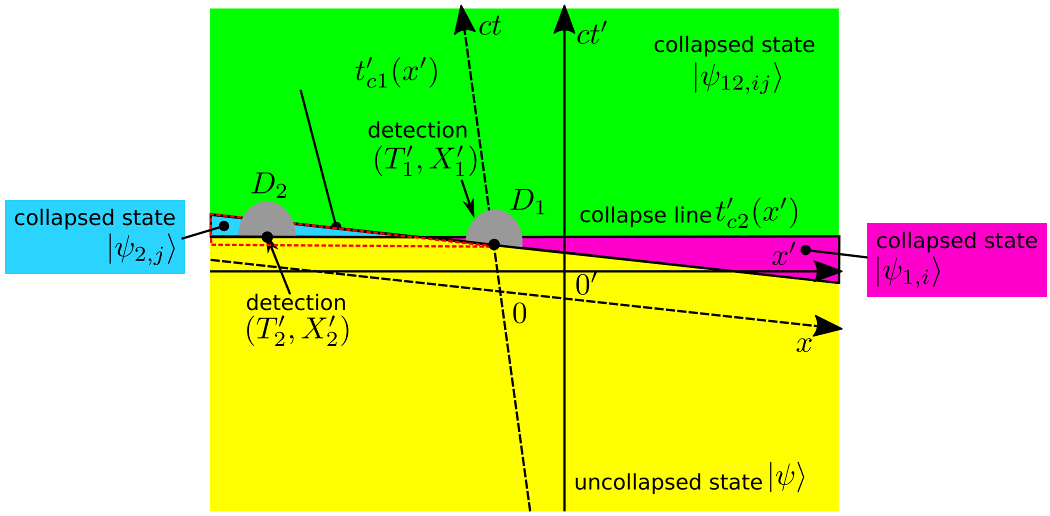

FIG. 2. Space-time evolution of a QS $|\psi\rangle$, which collapses upon measurements by detectors D_1 and D_2, shown in reference frame $S' \equiv (ct', x')$. The reference frame $S \equiv (ct, x)$ moves in the negative direction of axis x'. The collapse of QS $|\psi\rangle$ is triggered by two measurements: by detector D_1 at space-time point (T'_1, X'_1) and detector D_2 at space-time point (T'_2, X'_2). The measurement performed by detector D_1, at rest in reference frame S, divides the space-time diagram by collapse line t'_{c1} into areas where the QS is uncollapsed (yellow area) and collapsed to state $|\psi_{1,i}\rangle$ (magenta area). The measurement taken by detector D_2 divides the space-time diagram by collapse line t_{c2} into areas where the state is collapsed to state $|\psi_{2,j}\rangle$ (blue area) after the measurement and where the state is uncollapsed before the measurement. The triangle with a red dashed border denotes the area where a second measurement can be performed on an uncollapsed state $|\psi\rangle$ in reference frame S' after the first measurement has been taken by detector D_1 at space-time point (X'_1, T'_1) in reference frame S. The large green area in the upper part of the figure is given by the intersection of collapsed areas of detectors D_1 (magenta) and D_2 (blue). The QS which occupies this area is $|\psi_{12,ij}\rangle$.

of the QS's collapse triggered in reference frame S is finite in reference frame S' and equal to $-c/\beta$, which is in absolute value always grater than c.

From an inspection of Fig. 1(b) follows that after the measurement is taken in time T'_1, at point X'_1 by the detector D at rest in reference frame S, there are places x' at which a second measurement on the uncollapsed state $|\psi\rangle$ can be taken at a time t' subsequent to the measurement time T'_1. The triangle-shaped area of the space-time points satisfying these requirements is depicted in Fig. 1(b) in orange. The lower line of the triangle is given by

$$t' = T'_1, \tag{5}$$

while the upper line of the triangle $t'_c(x')$ is given by Eq. (4). Both lines have an ending point in the measurement point (T'_1, X'_1). In order to keep the measurement by detector D unaffected, the second measurement has to be performed by a detector at rest in reference frame S' at the space-time point located in the orange triangle in Fig. 1(b). In this scenario, both detectors take measurements on an uncollapsed QS $|\psi\rangle$. We develop this idea in detail at the end of this section. An evolution of the QS $|\psi\rangle$ subjected to a measurement by the detector D with an arbitrary constant velocity, as described in this paragraph, can be inspected as well by means of the PLF, H&K, and A&A collapse models. They are introduced in detail in Appendixes A, C, and D, respectively.

If a second measurement in reference frame S' was taken at space-time point in the blue area [see Fig. 1(b)], the measurement is considered to be taken on the already collapsed state $|\psi\rangle$. The measurement, as observed in reference frame S', is taken prior to the first measurement by detector D. For a detailed discussion of this case, see Appendix E.

The time interval $\Delta t'_c$ at position x', in which a measurement on an uncollapsed state $|\psi\rangle$ can be taken in the moving reference frame S' after the first measurement has been taken by the detector D in the reference frame S at space-time point (T'_1, X'_1), is expressed by

$$\begin{aligned}\Delta t'_c(x') &= t'_c(x') - T'_1\\ &= -\frac{\beta}{c}x' + \beta^2(t'_0 - T'_1) + \frac{\beta x'_0}{c}\\ &= \frac{\beta}{c}(-x' + X'_1).\end{aligned} \tag{6}$$

From a geometrical point of view, the time difference $\Delta t'_c$ expresses a height of the orange triangle in Fig. 1(b) at the arbitrary point x'. This expression is positive only at positions $x' < X'_1$, which is in agreement with placement of the orange triangle in Fig. 1(b). The spatial position of the detection point X'_1 (in reference frame S') is equal to

$$X'_1 = -v(T'_1 - t'_0) + x'_0. \tag{7}$$

In Fig. 2 it is shown how the second measurement performed by detector D_2 in rest in reference frame S' at space-time point (T'_2, X'_2) on the uncollapsed state $|\psi\rangle$ will affect its space-time evolution. Further, the detector D will be denoted as D_1 and the collapse line related to its measurement t_c as t_{c1}. We assume that detector D_2 remains at rest at fixed point X'_2 in reference frame S'. We further assume that this detector performs a measurement which causes the uncollapsed state $|\psi\rangle$ to be projected to state $|\psi_{2,j}\rangle$ upon detection of a value related to an eigenvector $|\Psi_j\rangle$ according to the projection postulate in Eq. (1). The detection performed by the detector D_2 is related to the collapse line t_{c2}. The line divides the space-time to the area where the measured state $|\psi\rangle$ is uncollapsed (the yellow area in Fig. 2) and where it is collapsed after measurement by detector D_2 (the union of blue and green areas).

There are four different collapse-related space-time regions which boundaries are given by collapse lines t_{c1} and t_{c2}; see Fig. 2. In the yellow space-time region the QS is in the original uncollapsed state $|\psi\rangle$. The $|\psi_{1,i}\rangle$ state originates after detection by detector D_1. It occupies the magenta space-time region in the central-right part of Fig. 2. The state $|\psi_{1,i}\rangle$ is uniquely related to the uncollapsed state $|\psi\rangle$ through the projection postulate in Eq. (1). The same is valid for state $|\psi_{2,j}\rangle$ and the blue region in the central-left part of Fig. 2, but with relation to detector D_2. The QS $|\psi_{12,ij}\rangle$ emerges as the final collapsed state when both measurements (projections) by detectors D_1 and D_2 are taken into account. *While the states $|\psi_{1,i}\rangle$ and $|\psi_{2,j}\rangle$ are uniquely given by the projection postulate, the final QS $|\psi_{12,ij}\rangle$ cannot be uniquely determined in some cases.* We will address this issue on an example of measurement of a polarization-entangled photon-pair state in Sec. IV.

The detection points (T_1', X_1') and (T_2', X_2') in Fig. 2 are spacelike separated. Therefore, the time sequence of detections by the detectors D_1 and D_2 is observer-dependent [23]. It follows that it is not possible to generally determine which measurement triggered the collapse of the QS $|\psi\rangle$ first. But according to the space-time evolution diagram in Fig. 2, it is possible to determine *regardless of the reference frame* which detector has been measuring on a collapsed or an uncollapsed QS.

The scenario when the measurements are taken by the two detectors D_1 and D_2 (as shown in Fig. 2 and Fig. 3) has been investigated as well with utilization of the PLF, H&K, and A&A collapse models in Appendixes A, C, and D, respectively. From the inspection with the H&K model follows if both detectors D_1 and D_2 take spacelike separated measurements, the measurements do not preserve correlations present in the initial state $|\psi\rangle$. This occurs regardless of the state of motion of both detectors. Since the early 2000s, there have been experiments measuring correlations of entangled photon pairs at two spacelike separated events with stationary detectors [13,14,16,18]. None of those experiments have reported violation of the correlations. Therefore, the H&K model provides predictions conflicting with the experimental observations. On the other hand the PLF and A&A models predict the correlations to be preserved regardless of the state of motion of the detectors; see Appendixes A and D. These predictions are the opposite to MLF model in the proposed experiment. Therefore, the experiment involving multiple measurements on an uncollapsed state may either prove or disprove the validity of the MLF collapse model.

III. PROPOSAL OF EXPERIMENT INVOLVING TWO MEASUREMENTS ON AN UNCOLLAPSED ENTANGLED PHOTON-PAIR STATE

We investigate the detection of a photon-pair state entangled in polarizations and frequency according to the experimental layout in Fig. 2. Conditions by which measurements by detectors D_1 and D_2 are taken on an uncollapsed state are derived. We assume the detector D_1 to be located in a spatial origin $x = 0$ of a coordinate system $S \equiv (t, x)$ and the other D_2 in a spatial origin $x' = 0$ of the coordinate system $S' \equiv (t', x')$. Detector D_2 moves relative to detector D_1 with positive velocity $v > 0$. The photon-pair source is assumed to be in rest in reference frame S. It will be resolved at which experimental conditions two measurements on the uncollapsed QS will be allowed.

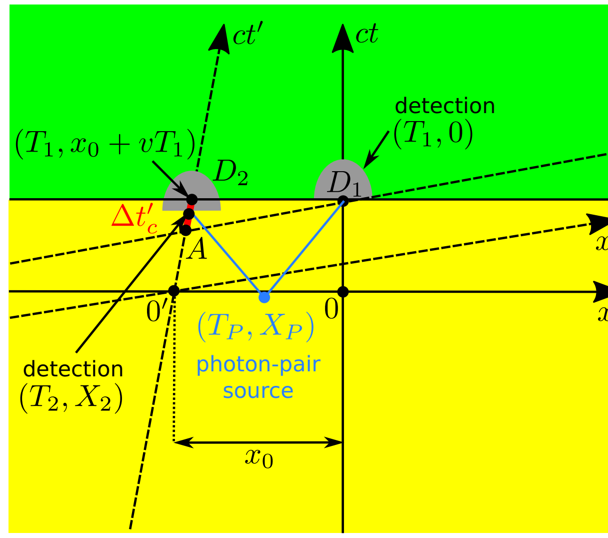

FIG. 3. Space-time layout of the two-photon detection. A polarization-entangled photon pair is emitted by a source, marked as blue dot. One photon propagates in the positive direction of axis x and the other in the negative direction of axis x, marked as blue lines. We assume the photon-pair source to remain in rest in reference frame $S \equiv (ct, x)$. Meanwhile, the detector D_2 moves in the positive direction of axis x with velocity v. The detector D_2 is located in spatial origin $x' = 0$ of reference frame $S' \equiv (ct', x')$. Detector D_1 remains at rest in spatial origin $x = 0$ of reference frame S. Detector D_1 detects a photon at space-time point $(T_1, 0)$, while detector D_2 detects a photon in time interval $(T_1', T_1' + \Delta t_c')$, where T_1' is time of detection T_1 expressed in reference frame S' and $\Delta t_c'$ time difference given by Eq. (6). By these conditions, both detectors measure an uncollapsed two-photon state.

First, we define space-time scheme of the two-photon detection experiment; see Fig. 3. Let us assume that the detection of a photon by the detector D_1 occurs at time T_1. Further, we fix the constant t_0 in Eq. (3) by requirement that the origin $0' \equiv (t' = 0, x' = 0)$ of reference frame S' has a time component in reference frame $S \equiv (t, x)$ equal to zero, $t = 0$. In other words, we require the clocks at origins (detectors) in both frames S and S' to be synchronized at time $t' = t = 0$. By this assumption, using Eq. (21) we have $t_0 = 0$. When the synchronization occurs, the origin $0'$ (of the reference frame S') is located at position $x = x_0$ in reference frame S. The equation of motion of detector D_2 in reference frame S is given by the expression

$$x_2(t) = vt + x_0. \tag{8}$$

In reference frame S, the detector D_2 performs the measurement first at time T_2 in the time interval

$$T_2 \in (T_1 - \Delta t_c, T_1) \tag{9}$$

at position X_2; see Fig. 3. In Fig. 3 the point of detection (T_2, X_2) of detector D_2 on the uncollapsed state lies on the red line, which is a world line of the detector D_2. The point of the earliest detection by detector D_2 is marked as A. The relation between the time differences $\Delta t_c'$, in Eq. (6) and its counterpart Δt_c in reference frame S is given by equation for time dilatation

$$\Delta t_c = \gamma \Delta t_c'. \tag{10}$$

The time interval for time T_2' when the detector D_2 performs a measurement on the uncollapsed state $|\psi\rangle$ can be expressed as

$$T_2' \in (T_1', T_1' + \Delta t_c') \equiv \left(T_1', T_1' + \frac{\beta}{c} X_1'\right), \tag{11}$$

which follows from Fig. 3, Eq. (6), and the position of detector D_2 in reference frame S' at coordinate $x' = 0$. Since the right corner point of the interval has to be larger than the left one it follows that $X_1' > 0$. X_1' denotes the position of detector D_1 in reference frame S'. This means that in order to perform detection on the uncollapsed state, the detector D_2 has to be moving towards the detector D_1 from the negative side of axis x. Or in other words, the detector D_1 has to be ahead of detector D_2, positioned at positive coordinate X_1', which is in agreement with the layout in Fig. 3. The variables in the corner points of the interval in Eq. (11) can be expressed by means of variables in reference frame S as

$$T_1' = \gamma T_1, \tag{12}$$

$$X_1' = \gamma(-x_0 - vT_1) = \gamma \Delta X_{12}, \tag{13}$$

where ΔX_{12} is spatial distance between detectors D_1 and D_2 in time of detection $t = T_1$ of detector D_1 in reference frame S. From the combination of Eqs. (6), (10), and (13) follows that the time difference Δt_c is equal to

$$\Delta t_c = \gamma^2 \frac{\beta}{c} \Delta X_{12} \approx \frac{\beta}{c} \Delta X_{12} = \frac{v}{c^2} \Delta X_{12} \approx \Delta t_c'. \tag{14}$$

The approximation in Eq. (14) follows from the assumption that we neglect terms with powers in β higher than one and keep only the first power of β as the most significant contribution. Equation (14) puts into relation the time differences $\Delta t_c'$ and Δt_c during which a detection can be taken by detector D_2 on the uncollapsed state, velocity v of detector D_2, and distance ΔX_{12} of detectors D_1 and D_2 in time T_1. Time T_1 is time of detection of a photon by detector D_1.

The QS $|\psi\rangle$ collapses after result of the measurement is stored in a classical memory of detector D_2. With regard to the current state of electronics, we estimate the time between detection and storage at the order of 10^{-10} s = 0.1 ns. Based on this assumption, the magnitude of the product of the distance between detectors ΔX_{12} and velocity v of detector D_2 can be derived from Eq. (14):

$$v \Delta X_{12} = c^2 10^{-10}\ \text{s} \approx 10^7\ \text{m}^2\ \text{s}^{-1}. \tag{15}$$

If the moving detector was located in a vehicle moving on the earth's surface, the maximum allowed velocity v could be at the order of 10^2 m s^{-1} = 360 km h^{-1}. When substituted into Eq. (15), the distance at which the photon pair should propagate ΔX_{12} is at the order 10^5 m = 100 km. If the detector D_2 was placed on an orbital station [15], a two-dimensional model in spatial coordinates considering noncollinear motion of the station and the photons would have to be developed [24].

The position X_P and time of emission T_P of the photon pair (see Fig. 3) have to be properly tailored in order to hit the detectors D_1 and D_2 at required times T_1 and T_2 [25]. We propose the time of detection T_2 by the moving detector D_2 to take place just after space-time point A at coordinates

$$T_2 = T_A + \varepsilon, \tag{16}$$

$$X_2 = vT_2 + x_0 = X_A + v\varepsilon;\ \varepsilon > 0; \tag{17}$$

see Fig. 3. The parameter ε determines time difference between time of detection T_2 and time T_A. It transforms as time difference $\varepsilon = \gamma\varepsilon'$. This would provide detector D_2 the longest available time $\Delta t_c - \varepsilon$ for resolving of polarization of the photon and storage of the result. From this requirement and equations of motion for photons $x(t) = \pm c(t - T_P) + X_P$ follows that the position X_P and time T_P have to obey the equations

$$X_P - cT_P = -cT_1, \tag{18}$$

$$X_P + cT_P = X_2 + cT_2. \tag{19}$$

Equations (18) and (19) represent the set of two equations for two unknowns X_P and T_P. Their solution is

$$X_P = \frac{X_2 + cT_2 - cT_1}{2}, \tag{20}$$

$$cT_P = \frac{X_2 + cT_2 + cT_1}{2}. \tag{21}$$

The set of Eqs. (20) and (21) determine the position X_P of the photon-pair source and time of their emission T_P such that one photon from the pair hits detector D_1 at time T_1 at place X_1 and the other reaches moving detector D_2 at time T_2 at place X_2.

The time T_A and place X_A, which are related to time of detection T_2 and place of detection X_2 of detector D_2 through Eqs. (16) and (17), are not independent parameters. Therefore we need to determine how they are related to the velocity v of detector D_2, time of detection T_1 and position x_0 of detector D_2 at time $t = 0$. We have assumed that the detector D_2 gets into contact with a photon in time $T_2 = T_A + \varepsilon$, where T_A is time related to space-time point A, shown in Fig. 3. From Fig. 3 follows that $T_A' = T_1'$ at point $x = 0$. From this, we can derive space-time coordinates of point (T_A, X_A) in reference frame S:

$$cT_A = \gamma^2(cT_1 + \beta x_0), \tag{22}$$

$$X_A = \beta\gamma^2(cT_1 + \beta x_0) + x_0 = \gamma^2(vT_1 + x_0). \tag{23}$$

Equation (23) for point X_A can be verified by utilization of Eq. (6) and Eq. (8) of motion for spatial origin $x' = 0$ of reference frame S' in reference frame S. By setting $t = T_1 - \Delta t_c$

in the equation of motion

$$X_A = x_2(T_1 - \Delta t_c), \tag{24}$$

Eq. (23) emerges as well.

From Eqs. (20), (21), (22), and (23) follows that the only independent parameters of the experiment are the velocity v of the detector D_2, its position x_0 at time $t = 0$, and time T_1 of detection of the photon by detector D_1. The space-time point (T_2, X_2) of detection of detector D_2 is set by means of these parameters through Eqs. (16), (17), (22), and (23). The space-time point of emission of the photon pair (T_P, X_P) is determined by the independent parameters as well in Eqs. (20) and (21).

An experiment similar to the proposed one has been performed by Zbinden *et al.* [13,14], with regard to theoretical works of Suarez [12,26]. In particular, the goal of the experiment was to verify Suarez's theoretical model of measurement called AD. It predicted results distinct from quantum mechanics. The experiment disproved the AD theory in favor of well-established quantum mechanics. The authors did not perform measurements where both signal and idler photons' polarizations (or frequencies) were resolved. Instead, they left properties of the measured idler photon as unresolved. In Appendix B we have sketched an experiment proposed by Suarez [19]. In this experiment two measurements on an uncollapsed *single-photon state* $|\psi\rangle$ are performed. The analysis of this experiment can be made in the same way as the analysis of the experiment utilizing the photon pair in Sec. III.

IV. CONSEQUENCES OF MULTIPLE MEASUREMENTS ON AN UNCOLLAPSED TWO-PHOTON STATE

The QS $|\psi\rangle$ subjected to detection is a maximally entangled state in polarizations V and H [27,28]:

$$|\psi\rangle = \iint_0^\infty \frac{d\omega_s\, d\omega_i}{\sqrt{2}} e^{i(\omega_s+\omega_i)t}\phi(\omega_s,\omega_i) \quad [|V(\omega_s)\rangle_1|H(\omega_i)\rangle_2 + |H(\omega_s)\rangle_1|V(\omega_i)\rangle_2]. \tag{25}$$

ϕ denotes two-photon amplitude. The H and V polarization states are assumed to be orthogonal to each other and related to transversally polarized photons. Let us assume that photon state labeled by subscript 1 is related to photon mode 1 propagating in the positive direction of axis x. In the same way, the photon state labeled with subscript 2 is related to photon mode 2 propagating in the negative direction of axis x. The polarization of the photon mode 1 is resolved on detector D_1 and polarization of photon in mode 2 by detector D_2. The maximally entangled QS in Eq. (25) has been utilized in order to show the violation of quantum correlations, contained in the entanglement, in the measurement scheme proposed in Sec. III.

The polarizations measured by detectors D_1 and D_2 on state $|\psi\rangle$ in Eq. (25) can be uncorrelated if the two measurements are performed on the uncollapsed state. If the detectors D_1 and D_2 were placed in rest with respect to each other, the first polarization measurement by detector D_1, would project the state $|\psi\rangle$ into state, where only one predictable result can be obtained by polarization measurement with detector D_2. The polarizations obtained by the detectors would be orthogonal to each other. On the other hand, if we assume the detector D_2 measuring mode 2 to be moving and the measurement scheme is obeyed as proposed, both detectors D_1 and D_2 are measuring on the uncollapsed state $|\psi\rangle$. This means that both detectors have an equal chance of measurement of the V polarized photon as well as horizontally polarized photon H. Therefore, there is 50% probability that both photons would be measured with the same polarization. In this case the measured photon-pair state would be $|V\rangle_1|V\rangle_2$ or $|H\rangle_1|H\rangle_2$. These measurement results are equivalent to the situation when each detector performs a measurement on its own copy of an uncollapsed QS.

If the polarization measurement (by either detector D_1 or D_2) is taken on one of the photons in state $|\psi\rangle$ in Eq. (25), polarization of the other photon immediately after the measurement is uniquely given but not at an arbitrary subsequent time. Let us assume a situation when both detectors D_1 and D_2 measured V polarization without frequency discrimination. For now, we assume that both photons have been detected nondestructively [29,30]. From inspection of the Fig. 2 follows that the states $|\psi_{1,i}\rangle$ and $|\psi_{2,j}\rangle$ are determined with the projection postulate in Eq. (1) from state $|\psi\rangle$ in Eq. (25) utilizing projectors $\hat{P} = \int d\omega\, |V(\omega)\rangle_{11}\langle V(\omega)|$ and $\hat{P} = \int d\omega\, |V(\omega)\rangle_{22}\langle V(\omega)|$, respectively. Since the frequency ω of the measured photon is not resolved, integration over variable ω is carried out. In this case the projected states $|\psi_{1,V}\rangle$ and $|\psi_{2,V}\rangle$ (the index of a state is related to the mode and polarization of the measured photon) are equal to

$$|\psi_{1,V}\rangle = \iint_0^\infty d\omega_s\, d\omega_i\, \phi(\omega_s,\omega_i)e^{i(\omega_s+\omega_i)t} \times |V(\omega_s)\rangle_1|H(\omega_i)\rangle_2, \tag{26}$$

$$|\psi_{2,V}\rangle = \iint_0^\infty d\omega_s\, d\omega_i\, \phi(\omega_s,\omega_i)e^{i(\omega_s+\omega_i)t} \times |H(\omega_s)\rangle_1|V(\omega_i)\rangle_2. \tag{27}$$

According to Fig. 2, the states $|\psi_{1,V}\rangle$ and $|\psi_{2,V}\rangle$ are temporary. On the other hand, the final state $|\psi_{12,VV}\rangle$ (see Fig. 2) is permanent from time point of view. *But there is no rule by which this state can be determined.* The state $|\psi_{12,VV}\rangle$ cannot simply be defined as either $|\psi_{1,V}\rangle$ or $|\psi_{2,V}\rangle$, since they are different and there is no reason to prefer one against another. It can be suggested that the final state $|\psi_{12,VV}\rangle$ should be given by results of the measurements—the vertical polarizations of both photons by assumption—but this had to be verified by the experiment.

The possibility of obtaining the uncorrelated measurement results from state $|\psi\rangle$ in Eq. (25) leads to violation of conservation laws, but not in mean value. We demonstrate this on the energy conservation law. If the photon pair originated in the process of spontaneous parametric down-conversion, the photons in modes 1 and 2 have the sum frequency equal to frequency of the pump beam

$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i. \tag{28}$$

At conditions when both detectors D_1 and D_2 are at rest, measured frequencies of both photons ω_s and ω_i have to obey this equation. But in the case when both detectors D_1 and D_2 measure frequency on an uncollapsed state $|\psi\rangle$, the energy conservation law in Eq. (28) has to be obeyed independently by frequency ω_s and ω_i. This follows from the statement that both measurements on the QS $|\psi\rangle$ are uncorrelated. In consequence, *the sum of measured frequencies ω_s and ω_i does not have to be equal to ω_p in individual measurements.*

The probability density $p(\omega_s, \omega_i)$ of detecting a signal photon with frequency ω_s and idler photon with frequency ω_i when both detectors D_1 and D_2 are at rest, is equal to

$$p(\omega_s, \omega_i) = |\phi(\omega_s, \omega_i)|^2. \tag{29}$$

On the other hand, if frequencies of both photons ω_s and ω_i are measured on the uncollapsed state, the probability density $p_N(\omega_s, \omega_i)$ of detection of a signal photon with frequency ω_s and idler photon with frequency ω_i on an uncollapsed state is given by

$$p_N(\omega_s, \omega_i) = \int_0^\infty d\bar{\omega}_i\, p(\omega_s, \bar{\omega}_i) \int_0^\infty d\bar{\omega}_s\, p(\bar{\omega}_s, \omega_i). \tag{30}$$

The different expression for probability density p_N emerges from an assumption that when the frequency of one of the photons is measured on an uncollapsed state, the frequency of the other is still not determined. The probability density $p_N(\omega_s, \omega_i)$ is function separable in variables ω_s and ω_i, while function $p(\omega_s, \omega_i)$ is nonseparable.

The mean value of energy $\langle E\rangle$ in QS $|\psi\rangle$ measured by static detectors is equal to

$$\langle E\rangle = \hbar\langle\omega_s\rangle_p + \hbar\langle\omega_i\rangle_p. \tag{31}$$

The subscript p in the mean values $\langle\omega_s\rangle_p$ and $\langle\omega_i\rangle_p$ denotes averaging with respect to probability density p defined in Eq. (29). It follows that the mean value of energy $\langle E\rangle$ in the $|\psi\rangle$ state divided by $\hbar$ is given by the sum of the mean values $\langle\omega_s\rangle_p$ and $\langle\omega_i\rangle_p$ of measured frequencies ω_s and ω_i computed with probability density $p = p(\omega_s, \omega_i)$ defined in Eq. (29). If the frequencies ω_s and ω_i are both measured on the uncollapsed state $|\psi\rangle$ the mean value of energy $\langle E\rangle$ in QS $|\psi\rangle$ has to be computed with probability density $p_N(\omega_s, \omega_i)$ defined in Eq. (30). It is straightforward to show that the mean value of measured signal frequency ω_s is the same for both probability densities p_N and p [31]. This holds for the mean value of the idler frequency ω_i as well. Therefore the mean value of energy $\langle E\rangle$ in state $|\psi\rangle$ is the same regardless of utilization of multiple measurements on the uncollapsed state.

V. CONCLUSION

We have shown the diagrams of space-time evolution of a general QS subjected to a measurement by a detector in the reference frame of the detector and in the reference frame moving with constant velocity relative to the detector. The evolution has been studied with utilization of the multiple Lorentz frame (MLF), preferred Lorentz frame (PLF), Hellwig-Kraus (H&K), and Aharonov-Albert (A&A) collapse models. The evolution has been analyzed as well in cases when the measurement is taken by two detectors, which detections are spacelike separated. Compatibility of all models with the Copenhagen interpretation (CI) has been examined. From the MLF space-time model of a QS collapse follows that multiple measurements can be taken on the same uncollapsed QS. On the other hand, the PLF and A&A models are revealed to lack this type of measurement in the considered measurement scheme. The H&K model has been proved to provide an invalid result in measurement scenarios of this type. In particular, it predicts results conflicting with experiments involving spacelike separated measurements

With the MLF model we show that under certain conditions one stationary and one moving detector can perform measurements on uncollapsed photon-pair state entangled in polarizations and energy. Allowance of multiple measurements on the uncollapsed QS results in uncorrelated measurement results even if the uncollapsed state is maximally correlated. This is opposite to the PLF and A&A model, where the measurements are correlated. With the PLF collapse model, all predictions have to be made in the preferred frame in order to avoid back-in-time collapses affecting results of the former measurements. The uncorrelated measurements in MLF model lead to violation of conservation laws and unavailability to determine the collapsed QS, which originates after all detections take place. For one example of frequency measurement on the photon-pair state, it is discussed that the energy conservation is violated in individual measurements but not in mean value. The result of the proposed two-photon experiment determines wether the MLF model is valid while leaving the A&A and PLF models generally invalid. An alternative less demanding experiment which allows for multiple measurement on a single photon state has been sketched. It involves a single-photon state superposed in two distinct paths.

The proposed photon-pair experiment can be generalized on an entangled multiphoton state with multiple detectors, each moving with its own velocity. As a consequence, multiple measurements—more than two—on an uncollapsed QS are allowed in the presented MLF collapse model.

ACKNOWLEDGMENTS

The author gratefully acknowledges J. Peřina Jr. and L. Richterek from Palacký University, Faculty of Sciences for useful discussions, corrections, and editing of the manuscript.

APPENDIX A: PREFERRED LORENTZ FRAME COLLAPSE MODEL

In preferred Lorentz frame (PLF) collapse model, collapse of a QS is instantaneous in one unique Lorentz frame. It will be denoted as $S^{\mathrm{P}} \equiv (ct^{\mathrm{P}}, x^{\mathrm{P}})$. For an example, see Fig. 4(a), where two collapses, triggered by the measurements of detectors D_1 and D_2, occur. This means that in all other Lorentz frames, a collapse hypersurface will be a spacelike hypersurface spanning into both the past and future [8]. For an example, see Fig. 4(b), where the collapse lines are viewed from reference frame $S \equiv (ct, x)$ moving with positive constant velocity along axis x^{P}. The preferred Lorentz frame can be associated with the center of mass of an entangled two-particle system or microwave background radiation [8]. We remark that in the preferred Lorentz frame S^{P}, orientation of the collapse hypersurface is not dependent on speed of an observer either taking measurement with a detector or making a prediction of a measurement. As a result, for the observer at rest in preferred Lorentz frame S^{P}, the PLF and A&A collapse

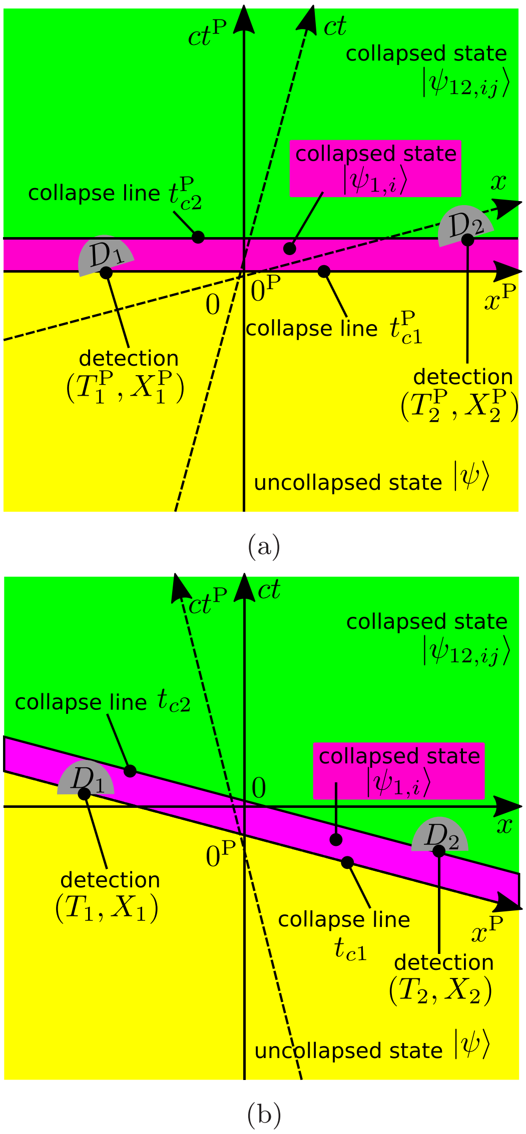

FIG. 4. (a) Space-time evolution of a QS $|\psi\rangle$ viewed from the preferred Lorentz frame $S^{\mathrm{P}} \equiv (ct^{\mathrm{P}}, x^{\mathrm{P}})$. For determination of the collapse lines t^{P}_{c1} and t^{P}_{c2} the PLF collapse model is utilized. The initial QS $|\psi\rangle$ undergoes two measurements at space-time points $(T^{\mathrm{P}}_1, X^{\mathrm{P}}_1)$ and $(T^{\mathrm{P}}_2, X^{\mathrm{P}}_2)$ by detectors D_1 and D_2, respectively. Both detectors are assumed to be at rest in reference frame $S \equiv (ct, x)$, which axes are depicted. The reference frame S propagates along the positive direction of axis x^{P} with constant velocity v. After measurement by detector D_1, the QS $|\psi\rangle$ collapses to QS $|\psi_{1,i}\rangle$, which occupies the magenta space-time region. After measurement by detector D_2, the QS $|\psi_{1,i}\rangle$ collapses to final QS $|\psi_{12,ij}\rangle$, which is located in the green space-time region. The magenta box in which the caption of the QS $|\psi_{1,i}\rangle$ is placed highlights only the affiliation of the caption to the magenta space-time region. (b) Space-time evolution of a QS $|\psi\rangle$ in Fig. 4(a), viewed from reference frame $S \equiv (ct, x)$. Collapse line t_{c1} (t_{c2}) is collapse line t^{P}_{c1} (t^{P}_{c2}) viewed from reference frame S.

models are equivalent. As far as all the detectors are at rest in the preferred Lorentz frame and observers are equipped with detectors, all the studied models except H&K (PLF, MLF, and A&A) are equivalent with respect to predicted results of measurements, orientation of collapse hypersurfaces (lines), and compatibility with the CI.

Let us examine an experimental scenario when two detectors D_1 and D_2 are taking spacelike separated measurements on the polarization-entangled photon-pair state in Eq. (25). Since PLF collapse hypersurfaces' orientation is the same regardless of motion of the detectors, for simplicity we will assume both of the detectors (D_1 and D_2) to be at rest in reference frame $S \equiv (ct, x)$. We further assume the reference frame S to propagate in the positive direction of axis x^{P} with constant velocity v. The examined experimental situation viewed from reference frame S and S^{P} is shown in Figs. 4(a) and 4(b). We note that the detection events $(T^{\mathrm{P}}_1, X^{\mathrm{P}}_1)$ and $(T^{\mathrm{P}}_2, X^{\mathrm{P}}_2)$; $T^{\mathrm{P}}_1 < T^{\mathrm{P}}_2$ are placed in such a way that they change their temporal order when viewed from reference frame S; $T_1 > T_2$ [see Figs. 4(a) and 4(b)].

The measurement scheme described from the Lorentz preferred frame $S^{\mathrm{P}} \equiv (ct^{\mathrm{P}}, x^{\mathrm{P}})$ [see Fig. 4(a)] resembles a familiar scheme obtained with the CI. But in this situation, both detectors D_1 and D_2 are in movement. In the scheme the initial QS $|\psi\rangle$ undergoes two measurements: first, at point $(T^{\mathrm{P}}_1, X^{\mathrm{P}}_1)$ by moving detector D_1, where the QS collapses from initial state $|\psi\rangle$ (defined at all points of the yellow region) to state $|\psi_{1,i}\rangle$ (defined at each point of the magenta region). Then a second measurement by detector D_2 at space-time point $(T^{\mathrm{P}}_2, X^{\mathrm{P}}_2)$ is performed with an associated collapse of state $|\psi_{1,i}\rangle$ to final state $|\psi_{12,ij}\rangle$. From this perspective, all studied phenomena are in full agreement with the CI.

When the same situation as in Fig. 4(a) is inspected from reference frame S, the temporal order of measurement at spatial points X_1 and X_2 reverses ($T_1 > T_2$). In such a case, we may experience a collapse of a subsequent measurement at space-time point (T_1, X_1) affecting the previously measured result at space-time point (T_2, X_2). What is depicted in Fig. 4(b) is the final situation when both measurements by detectors D_1 and D_2 are taken into account. Let us analyze the situation depicted in Fig. 4(b) from start step by step. First, we assume that there is an initial state $|\psi\rangle$ defined by Eq. (25) undergoing a measurement with detector D_2 at space-time point (T_2, X_2). This triggers a collapse associated with collapse line t_{c2}. This collapse line is instantaneous in the preferred Lorentz frame S^{P}. We stress that the collapse line t_{c2} leads above the second detection point (T_1, X_1) [see Fig. 4(b)] and leaves the QS at this point unaffected by the measurement. After measurement by the detector D_2, the state $|\psi\rangle$ collapses to state $|\psi_{2,j}\rangle = \langle\psi|\Psi_{2,j}\rangle|\Psi_{2,j}\rangle$, where the state $|\Psi_{2,j}\rangle$ is associated with the value measured by the detector D_2. For simplicity, the state $|\psi_{2,j}\rangle$ is kept unnormalized. State $|\psi_{2,j}\rangle$ will temporarily occupy the green region in Fig. 4(b) until a second measurement after time $T_1 - T_2$ at space-time point (T_1, X_1) is performed.

At time T_1, subsequent to time T_2 in reference frame S [see Fig. 4(b)], a measurement by detector D_1 at space-time point (T_1, X_1) is performed. This measurement takes place on the uncollapsed state $|\psi\rangle$, as explained above. Therefore, it is tempting to arrive to a conclusion that the measurements by the detectors D_1 and D_2 can be uncorrelated, since both of them are taken on the initial entangled state $|\psi\rangle$. But what has to be taken into account is the backward-in-time propagating collapse line t_{c1} associated with the letter measurement by detector D_1 at space-time point (T_1, X_1). This collapse line causes the initial QS $|\psi\rangle$ to collapse at the point of first measurement (T_2, X_2) and even in finite time before the first measurement takes place to state $|\psi_{1,i}\rangle$ [it occupies the magenta region in Fig. 4(b)]. This collapse (associated with collapse line t_{c1}) ensures that the measurements at both space-time points (T_2, X_2) and (T_1, X_1) are correlated, although the measurement at space-time point (T_2, X_2) has been initially assumed to be taken on the uncollapsed state $|\psi\rangle$. In consequence, in the green region of Fig. 4(b), the final state $|\psi_{12,ij}\rangle$ emerges. It takes both projective measurements by detectors D_1 and D_2 into account.

This backward-in-time collapse (associated with line t_{c1}) in reference frame S may affect a polarization value initially measured by detector D_2. This occurs because the measurement scenario in Fig. 4(a) is inspected in reference frame S, where the measurement points (T_1, X_1) and (T_2, X_2) exchange their temporal order [see Figs. 4(a) and 4(b)]. It is hard to accept that a measurement which occurs first (by detector D_2) may be affected by a measurement after it (by detector D_1). Particularly, it can be assumed that detector D_2 measured V polarization (with no frequency discrimination) first [see Fig. 4(b)]. Then, at time $T_1 > T_2$, a subsequent measurement is performed by detector D_2 on the initial state $|\psi\rangle$ as well. This measurement may result into polarization V as well. But the latter measurement causes collapse backward in time t, which changes the polarization measured by detector D_1 from V to H, in order to keep correlations established in the initial state $|\psi\rangle$. Thus, all predictions of measurement outcomes with PLF model have to be made in the preferred frame S^{P} in order to avoid backward-in-time collapse situations, which are hard to interpret. In the preferred Lorentz frame S^{P}, the measurements by detectors D_1 and D_2 are always correlated. Therefore, the predicted results of measurements by the PLF and A&A (see Appendix D) collapse models on the polarization entangled state $|\psi\rangle$ in Eq. (25) are the same. Backward-in-time collapse lines appear in the MLF model and H&K model as well. But they do not enforce correlations backward in time like in the case of the PLF model. The H&K and MLF models either leave both measured results uncorrelated or keep the measured results correlated since the initial measurement, depending on the particular experimental scheme.

APPENDIX B: MULTIPLE MEASUREMENTS ON AN UNCOLLAPSED SINGLE-PHOTON STATE SUPERPOSED IN TWO DISTINCT PATHS

In Sec. III an experimental scenario involving multiple measurements on an uncollapsed photon-pair state entangled in energy and polarization has been described. This type of experiment requires generation of a photon-pair state and utilization of both photons during the noncollapsing measurements. The generation of a photon-pair state is a second-order process. Thus, the generation rate of the photon-pairs is low in comparison with the first-order processes. Moreover, generation of a photon-pair simultaneously entangled in energy and polarization, as required by the proposed experiment [see Eq. (25)], can be challenging. Therefore, we briefly outline an experiment in which multiple measurements on an uncollapsed state is performed on a single-photon state superposed in two distinct paths. Since the experiment does not include entangled particles, it does not show consequences of the uncollapsing measurements on the correlations. It is shown that the uncollapsing measurement on the single-photon state superposed in two distinct paths may result in cloning of the single-photon state or its complete destruction. This experiment has been already proposed by Suarez [19].

We assume that the initial single-photon state $|\psi\rangle$ is superposed both in frequency ω and paths l1 and l2 according to

$$|\psi\rangle = \int_0^\infty d\omega\, \frac{\phi(\omega)}{\sqrt{2}} a_{l1}^\dagger(\omega) e^{i\omega t}|0\rangle + \frac{\phi(\omega)}{\sqrt{2}} a_{l2}^\dagger(\omega) e^{i\omega t}|0\rangle. \tag{B1}$$

The creation operator $a_{l1}^\dagger(\omega)$ ($a_{l1}^\dagger(\omega)$) creates a photon with frequency ω in path l1 (l2). The state is superposed in two distinct paths denoted as l1 and l2, since the multiple measurements on an uncollapsed photon state requires the photon to be on two distinct places in the same time. The QS in Eq. (B1) can be obtained by sending a single-photon QS $|\Phi\rangle = \int_0^\infty d\omega\, \phi(\omega) a^\dagger(\omega) e^{i\omega t}|0\rangle$ to a 50:50 beam splitter with l1 and l2 paths on its outputs. The superposition in frequency ω in state $|\psi\rangle$ in Eq. (B1) weighted by a function $\phi(\omega)$ emerges from requirement of the photon to be localized in a time pulse narrower than 10^{-10} s.

The space-time diagram of propagating of the photon along the paths l1 and l2 including measurement is analogical to Fig. 3. Path l1 can be connected with path of the photon propagating in the negative direction of axis x and path l2 with the path of the photon propagating in the positive direction of axis x. The scheme of measurement by detectors D_1 (on path l1) and D_2 (on path l2) remains the same. In analogy to photon-pair's case, the photon is observed by both detectors D_1 and D_2 with probability 1/4. If the detections by the detectors D_1 and D_2 were nondestructive, two real photons would propagate away from them after the detection. With the same probability no photon can be detected as well. This situation would result in destruction of both photons. In the rest of the cases, a photon is detected by either detector D_1 or D_2, as usual. When two photons are observed simultaneously, they are clones with respect to all degrees of freedom except the spatial ones.

APPENDIX C: HELLWIG-KRAUS COLLAPSE MODEL

Let us study a space-time evolution of a QS $|\psi\rangle$, which is measured at space-time point $(T_1, 0)$ in reference frame S [see Fig. 5(a)]. The measurement triggers the QS's collapse according to the H&K collapse model. It is assumed that detector D is at rest in reference frame S at spatial coordinate $x = 0$. The H&K model requires the collapse lines to be aligned along the past light cone with an apex in a measurement point; see Fig. 5(a). Since the speed of light c is invariant in all reference frames, a collapse line preserves its slope in all reference frames. For completeness, the process of measurement on the QS $|\psi\rangle$ viewed from moving reference frame S' is shown in Fig. 5(b).

The H&K model requires QS $|\psi\rangle$ to start its collapse before the measurement takes place; see Fig. 5(a). We admit that the MLF and PLF model suffers from this problem as well. Utilizing the MLF model [see Fig. 1(b)], in a reference

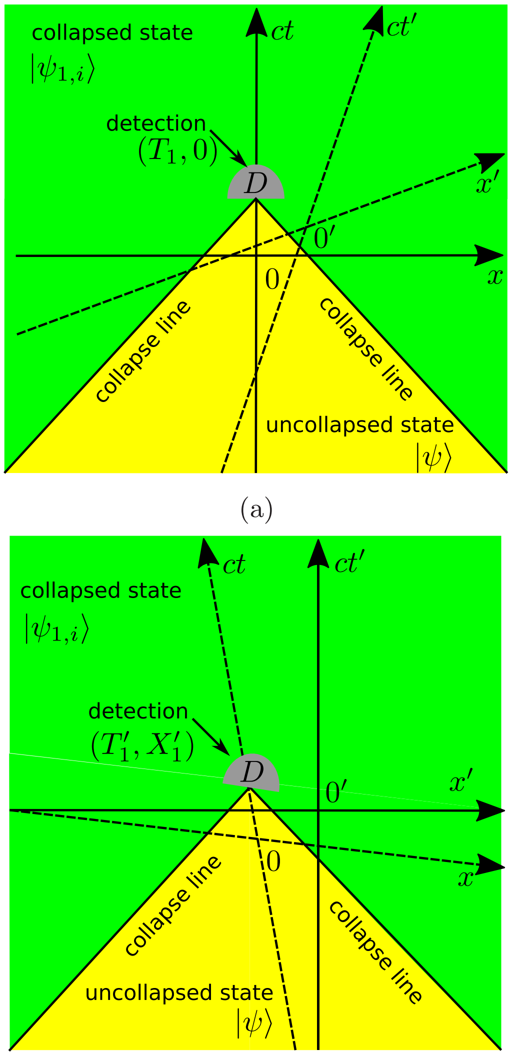

FIG. 5. (a) Space-time evolution of a QS $|\psi\rangle$ in reference frame $S \equiv (ct, x)$. The QS undergoes a measurement by a stationary detector D at space-time point $(T_1, 0)$. After the measurement the QS $|\psi\rangle$ collapses into state $|\psi_{1,i}\rangle$. The space-time distribution of the collapse is determined by the H&K model. The collapse takes place along the past light cone with an apex in the measurement point $(T_1, 0)$. The space-time regions of collapsed and uncollapsed states are divided by the collapse lines $x(t) = \pm c(t - T_1)$. Axes x' and ct' of reference frame $S' \equiv (ct', x')$ moving with positive velocity v in reference frame S are depicted. (b) The situation in (a) but viewed from the reference frame S'.

frame moving relative to the detector D a precollapse can be observed. Particularly, in the space-time area $(x' > X_1) \wedge (t' < T_1')$. For a detailed discussion of this case, see Appendix E. When considering the H&K model again, the space-time point of measurement in the future is predetermined and cannot be avoided in any reference frame. Its sudden change would demand changing of the collapsed QS $|\psi_{1,i}\rangle$ to an uncollapsed QS $|\psi\rangle$ in the past [see Fig. 1(b)].

In Fig. 6 we investigate a situation when two spacelike separated detectors D_1 and D_2 perform the measurements on an initial state $|\psi\rangle$. We prove that from the definition of the H&K

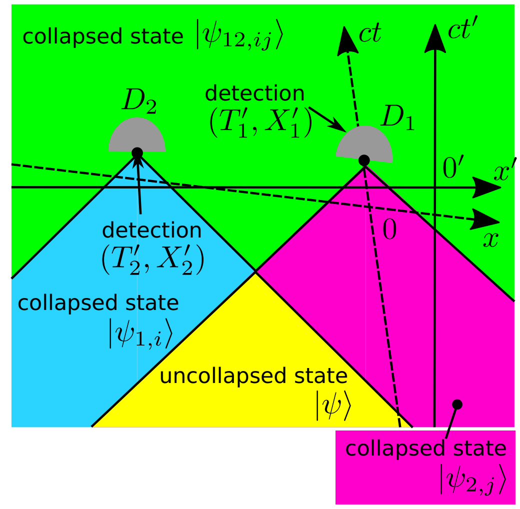

FIG. 6. Space-time evolution of a QS $|\psi\rangle$ subjected to two spacelike separated measurements by detectors D_1 and D_2. In this diagram the H&K collapse model is used. The evolution is viewed in reference frame S'. The detections are taken at space-time points (T_1', X_1'), by D_1 and (T_2', X_2'), by D_2. Detector D_1 (D_2) remains at rest in reference frame $S \equiv (ct, x)$ [$S' \equiv (ct', x')$]. Measurement by detector D_1 on state $|\psi\rangle$ causes its collapse to state $|\psi_{1,i}\rangle$. Accordingly, measurement of detector D_2 on state $|\psi\rangle$ causes its collapse to state $|\psi_{2,j}\rangle$. State $|\psi_{12,ij}\rangle$ originates by taking into account measurements of both detectors.

model immediately follows that the measurements performed on two entangled particles *are not* correlated, regardless of the state of motion of the detectors [32].

Let us keep the same assumptions as in an analysis of the MLF model; the detector D_1 (D_2) remains at rest in reference frame S (S'). The reference frame S' propagates along the positive direction of axis x with velocity $v > 0$ in reference frame S. The space-time diagram of this situation in Fig. 6 is viewed from reference frame S'.

Initially, the state is prepared in state $|\psi\rangle$, then the measurements are taken by the detectors D_1 and D_2 (see Fig. 6). Either detector causes the QS to collapse outside their backward light cones. The area inside the intersection of both backward light cones is in yellow, and all its points are occupied by the initial QS $|\psi\rangle$. The area outside the backward light cone of detector D_1 intersected with backward light cone of detector D_2 is in blue. This area is taken by QS $|\psi_{1,i}\rangle$. The QS $|\psi_{1,i}\rangle$ is created by measurement of detector D_1 on initial QS $|\psi\rangle$, due to the H&K model. Therefore, with regard to Eq. (1), the relation between state $|\psi_{1,i}\rangle$ and $|\psi\rangle$ is given by the projection postulate

$$|\psi_{1,i}\rangle = {}_1\langle\phi_i|\psi\rangle|\phi_i\rangle_1. \tag{C1}$$

In Eq. (C1) we have assumed that detector D_1 measured the value d_i related to the eigenstate $|\phi_i\rangle$. For simplicity, we kept the state $|\psi_{1,i}\rangle$ in Eq. (C1) unnormalized. In the same way, the magenta region related to state $|\psi_{2,j}\rangle$ originating by measurement of detector D_2 on initial QS $|\psi\rangle$ is defined by

$$|\psi_{2,j}\rangle = {}_2\langle\Psi_j|\psi\rangle|\Psi_j\rangle_2. \tag{C2}$$

The final state $|\psi_{12,ij}\rangle$ takes into account both measurements by detectors D_1 and D_2 and is equal to

$$|\psi_{12,ij}\rangle = {}_1\langle\phi_i|_2\langle\Psi_j|\psi\rangle|\phi_i\rangle_1|\Psi_j\rangle_2. \tag{C3}$$

From Eqs. (C1) and (C2) immediately follows that both measurements of detectors D_1 and D_2 take place on the initial uncollapsed state $|\psi\rangle$. Therefore, the measurements contradict predictions of QM. In particular, the measurements should be uncorrelated. Cohen and Hiley [8] arrived at the same conclusion. In addition, they suggested that the H&K model can be redefined such that both measurements by the detectors D_1 and D_2 are taken on an already collapsed state $|\psi_{12,ij}\rangle$.

Since a backward light cone preserves its shape regardless of the value of the constant velocity of a detector, the uncorrelated measurements must be observed in experiments where both detectors D_1 and D_2 are stationary and their detections are spacelike separated. Fortunately, experiments of this type have already been performed [16–18]. The goal of all the cited works has been to prove violation of Bell's inequalities with spacelike separated detections. Particularly, Salart *et al.* [16] and Scarini *et al.* [17] have been utilizing entanglement of photon pairs in a time domain, while Weihs *et al.* have been utilizing entanglement in polarizations, as proposed in this paper. No contradictions with predictions of QM or violation of correlations have been reported. Therefore, the H&K collapse model should be considered as generally invalid.

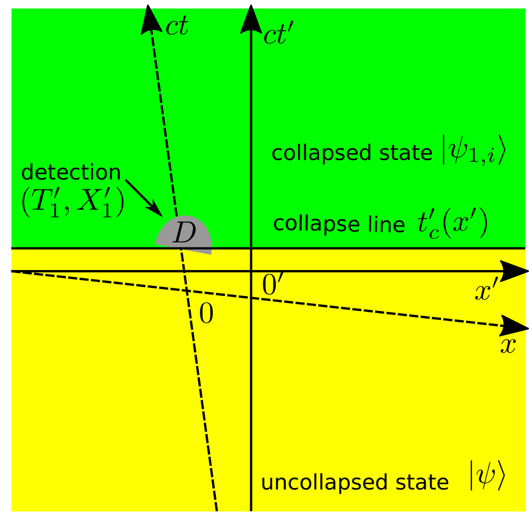

FIG. 7. Space-time diagram of evolution of a QS $|\psi\rangle$ in reference frame $S' \equiv (ct', x')$ undergoing measurement by detector D at space-time point (T_1', X_1'). The detector D is stationary in reference frame $S \equiv (ct, x)$. The evolution of the QS takes into account the A&A collapse model. After the measurement, QS $|\psi\rangle$ collapses into state $|\psi_{1,i}\rangle$ according to Eq. (1). The evolution of the QS $|\psi\rangle$ is depicted in reference frame S', which moves in the positive direction of axis x. The boundary dividing the collapsed and uncollapsed states is marked as a collapse line.

APPENDIX D: AHARONOV-ALBERT COLLAPSE MODEL

The A&A collapse model requires a collapse of a QS to be instantaneous for any observer [7,8]. In this model the observer does not have to be equipped with a detector. To introduce the model let us assume, as usual, a situation where a single detector D takes a measurement on QS $|\psi\rangle$ [see Fig. 1(a)]. The detector is assumed to be stationary in reference frame S, and the measurement occurs at space-time point $(T_1, 0)$. Let us further assume that the observer is associated with the detector D. Then the space-time scheme of the measurement is the same like in Fig. 1(a); i.e., the collapse line related to measurement of detector D is instantaneous in reference frame S.

Let the observer be stationary in reference frame S', which is moving with velocity $v > 0$ in reference frame S. Then the collapse line created by measurement of detector D on QS $|\psi\rangle$ is instantaneous in the observer's reference frame S'; see Fig. 7.

In the same way as for the MLF, H&K, and PLF models, measurement performed by two detectors D_1 and D_2 can be analyzed by means of the A&A model; see Fig. 8. The detector D_1 (D_2) is assumed to be stationary in reference frame S (S'). The reference frame S' propagates with constant positive velocity v in reference frame S. The observer is associated with detector D_2. In the A&A model, regardless of velocity of both detectors, the collapse lines will always be instantaneous in observer's reference frame (S'). Therefore, the measurements by detectors D_1 and D_2 will always be sequential and preserve correlations established in the initial state $|\psi\rangle$. With respect to collapse lines, the A&A model is equivalent to

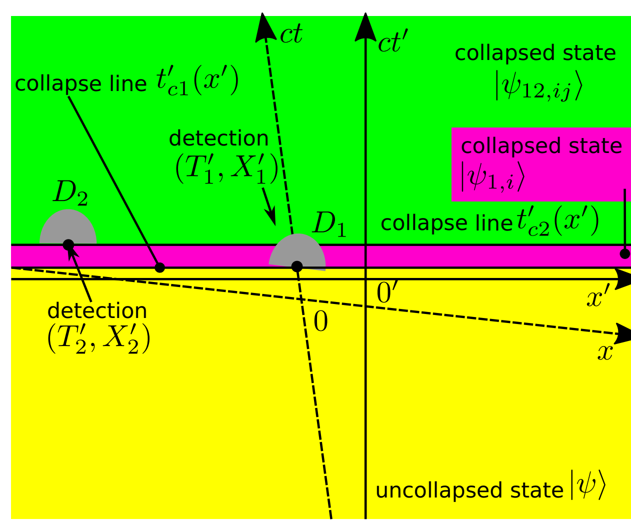

FIG. 8. Space-time diagram showing evolution of a QS $|\psi\rangle$ according to the A&A collapse model in reference frame $S' \equiv (ct', x')$. The QS $|\psi\rangle$ is measured by two detectors: D_1 at space-time point (T_1', X_1') and D_2 at space-time point (T_2', X_2'). Detectors D_1 and D_2 are at rest in reference frames $S \equiv (ct, x)$ and S', respectively. The observer is associated with detector D_2. Reference frame S' moves in the positive direction of axis x. After measurement by detector D_1, the QS $|\psi\rangle$ collapses into state $|\psi_{1,i}\rangle$, according to Eq. (1). After the second measurement by detector D_2 the state $|\psi_{1,i}\rangle$ collapses into state $|\psi_{12,ij}\rangle$.

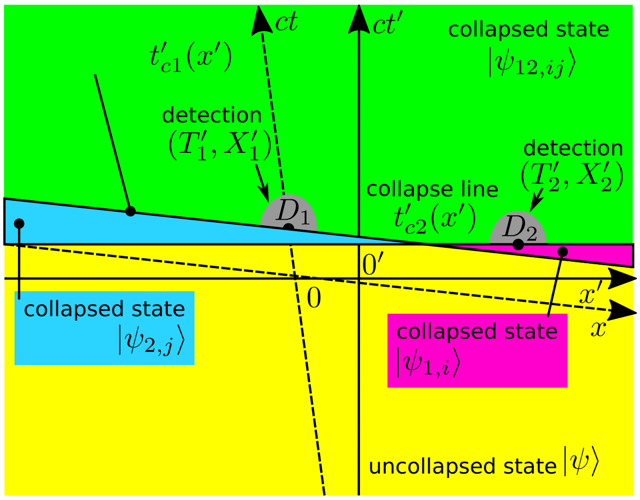

FIG. 9. Space-time scheme of evolution of a QS $|\psi\rangle$. The QS is subjected to two measurements by detectors D_1 and D_2. The detector D_1 (D_2) is at rest in reference frame S (S'). The reference frame S' moves in the positive direction of axis x with constant velocity. The detectors D_1 and D_2 are taking measurements at events (T'_1, X'_1) and (T'_2, X'_2), respectively. The detectors are positioned to take measurements on already collapsed states $|\psi_{1,i}\rangle$ and $|\psi_{2,j}\rangle$. The utilized collapse model is MLF attached to a rest frame of a detector. The space-time area occupied by the initial state $|\psi\rangle$ is in yellow. The space-time area occupied by state $|\psi_{1,i}\rangle$, which originates after projective measurement of detector D_1, is magenta. QS $|\psi_{2,j}\rangle$ occupies a blue region. The regions are divided by collapse lines t'_{c1} and t'_{c2}. Do not be confused by a small rectangular area with a distinct color inside a larger one; its intention is to highlight the relation of its caption to the area that the caption belongs to.

MLF model when all detectors are stationary in the observer's reference frame.

APPENDIX E: TRIGGERING A COLLAPSE OF A QS PRIOR TO A MEASUREMENT IN THE MLF COLLAPSE MODEL

The detector D_2 in Fig. 2 can be arranged in space x' such that both detectors D_1 and D_2 perform a measurement on already collapsed states. This occurs when detector D_2 is placed in the blue location in Fig. 2, where the state $|\psi\rangle$ is already collapsed to state $|\psi_{1,i}\rangle$. For the space-time scheme of this situation, see Fig. 9. The detector D_1 (at rest in reference frame S) performs a measurement on state $|\psi_{2,j}\rangle$, which is given by a projective measurement of detector D_2 on state $|\psi_{1,i}\rangle$. This situation is similar to one which always emerges when the H&K model is used; see Fig. 6 and its discussion in Appendix B. To our best knowledge, this situation has been briefly considered only by Zbinden *et al.* [13,14], based on work of Suarez [12,26]. Zbinden *et al.* [13,14] analyzed this situation by means of Suarez's theoretical model of relativistic measurement called the "alternative description" (AD). For more details, see Sec. I. This situation is treated in this Appendix by means of the MLF collapse model.

From Fig. 9 follows mathematical relations between collapsed and uncollapsed states:

$$\begin{aligned}|\psi_{12,ij}\rangle &= \hat{P}_{1,i}|\psi_{2,j}\rangle, \quad |\psi_{12,ij}\rangle = \hat{P}_{2,j}|\psi_{1,i}\rangle;\\ |\psi_{1,i}\rangle &= \hat{P}_{1,i}|\psi_{2,j}\rangle, \quad |\psi_{2,j}\rangle = \hat{P}_{2,j}|\psi_{1,i}\rangle;\\ \hat{P}_{1,i} &= |\phi_i\rangle_{11}\langle\phi_i|, \quad \hat{P}_{2,j} = |\Psi_j\rangle_{22}\langle\Psi_j|.\end{aligned} \tag{E1}$$

Operators $\hat{P}_{1,i}$ and $\hat{P}_{2,j}$ are related to a projective measurements by detectors D_1 and D_2, respectively. After the measurement by detector D_1 (D_2), the state $|\psi_{2,j}\rangle$ ($|\psi_{1,i}\rangle$) collapses into state $|\psi_{1,i}\rangle$ ($|\psi_{2,j}\rangle$) according to the definition of the projective measurement in Eq. (1).

From the set of Eqs. (E1) follows that the states $|\psi_{12,ij}\rangle$, $|\psi_{1,i}\rangle$ and $|\psi_{2,j}\rangle$ all have to be equal to

$$|\psi_{12,ij}\rangle = |\psi_{1,i}\rangle = |\psi_{2,j}\rangle = |\phi_i\rangle_1|\Psi_j\rangle_2. \tag{E2}$$

Therefore, the magenta and blue regions in Fig. 9 should be attached to green and be occupied by the already collapsed state $|\psi_{12,ij}\rangle$. In summary, in this situation detectors D_1 and D_2 perform a measurement on an already collapsed state $|\psi_{12,ij}\rangle$. The initial state $|\psi\rangle$ collapses to final state $|\psi_{12,ij}\rangle$ in finite time before any measurement by either detector D_1 or D_2 takes place.

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[24] In this case the time dilatation caused by general relativity effects would have to be considered as well.

[25] In practice, the photon pair can be generated by a femto-second pump-beam pulse. It generates photon pair with pulse lengths in order of hundreds of femtoseconds [33]. Therefore, the pulse lengths of photons in the pair are much smaller than the time required for detection and storage $\Delta t_s \sim 0.1$ ns. This guarantees successful detection of both photons in the pair at required times T_1 and T_2 with negligible error.

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