Limits to the observation of Unruh radiation via first-quantized hydrogenlike atoms

Riccardo Falcone [1] and Claudio Conti [1,2]

[1] *Department of Physics, University of Sapienza, Piazzale Aldo Moro 5, 00185 Rome, Italy*

[2] *Research Center Enrico Fermi, Via Panisperna 89a, 00184 Rome, Italy*

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We consider ionized hydrogenlike atoms accelerated by an external electric field to detect Unruh radiation. By applying quantum field theory in the Rindler space-time, we show that the first-quantized description for hydrogenlike atoms cannot always be adopted. This is due to the frame-dependent definition of particles as positive and negative frequency field modes. We show how to suppress such a frame-dependent effect by constraining the atomic ionization and the electric field. We identify the physical regimes with nonvanishing atomic excitation probability due to the Unruh electromagnetic background. We recognize the observational limits for the Unruh effect via first-quantized atomic detectors, which appear to be compatible with current technology. Notably, the nonrelativistic energy spectrum of the atom cannot induce coupling with the thermal radiation, even when special relativistic and general relativistic corrections are considered. On the contrary, the coupling with the Unruh radiation arises because of relativistic hyperfine splitting and the Zeeman effect.

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I. INTRODUCTION

Accelerated detectors probe noninertial quantum effects. Unruh and DeWitt [1–3] originally considered a single particle with acceleration α interacting with a scalar field via monopole coupling. In the noninertial frame, the detector reveals a thermal background [1,4,5] with temperature $T = \hbar\alpha/2\pi c k_B$, where c is the speed of light and k_B the Boltzmann constant. Similar descriptions have been used in more recent works to describe accelerated atoms as Unruh-DeWitt detectors [6–8].

More refined models, including a uniform external force as a dynamical source for the acceleration, superseded the original idealized description [9,10]. Furthermore, fully relativistic detectors have been considered [1,11].

Here we study accelerated atomic detectors by first principles within the relativistic Dirac theory in curved space-time for hydrogenlike atoms. In the nonrelativistic limit [12,13], we outline overlooked relativistic corrections and fundamental subtleties such as the frame dependence of the particle number.

Despite that the atom is initially prepared in the inertial laboratory frame as a nonrelativistic bound state with a fixed number of electrons and nuclear particles, in the comoving accelerated frame, the energy and the number of the quantum particles are different [13]. A single electron appears as a superposition of states with varying energy and particle numbers, and the electronic and nuclear structure is radically modified. The frame-dependent nature of particles—at the origin of the Unruh effect [4]—not only alters the background electromagnetic vacuum but also the electron and nuclear fields.

Such frame dependence poses limits to adopting the familiar first-quantization description of the hydrogenlike atom in its proper frame and brings up difficulties in understanding light-matter interaction with noninertial observers. Lowering the acceleration suppresses the effect on the electrons and the other nuclear particles; however, this may also suppress the Unruh background electromagnetic vacuum, with the consequent decrease in the temperature. For a nonvanishing measurement of the Unruh effect, one needs an energy gap ΔE such that $\Delta E \lesssim k_B T$. Hence the atomic spectrum must have a sufficiently fine structure to absorb the low-energy Unruh thermal photons.

Is it possible to suppress the frame-dependent effect on the electron while still detecting the electromagnetic thermal background? We give a positive answer to this question by a rigorous analysis based on the quantum field theory in curved space-time. Notwithstanding the suppressed frame-dependent effect for electrons, hyperfine splitting provides the energy gap to reveal the Unruh radiation. We identify a specific parameter region in terms of the nuclear charge number $\mathcal{Z}$ and the electric field E for the detection via first-quantized atomic detectors.

The paper is organized as follows. In Sec. II we use quantum field theory in curved space-time to describe electrons in inertial and accelerated frames and to show the frame-dependent particle content of the field. The conditions for suppressing such a frame-dependent effect are detailed in Sec. III. In Sec. IV we discuss the atomic stability by studying the interaction with the accelerating field and the nuclear electric field. In Sec. V we investigate the physical regimes to detect Unruh radiation, and we show that the relativistic hyperfine splitting is responsible for the coupling between the atom and the Unruh radiation. Conclusions are drawn in Sec. VI. Detailed calculations and proofs are provided in Appendixes A, B, and C.

II. ELECTRON FIELD IN INERTIAL AND ACCELERATED FRAME

Throughout the paper we consider the following scenario. We assume that the atom is ionized with one electron and

$\mathcal{Z} > 1$ protons and that the electron is prepared in the laboratory frame as a nonrelativistic particle. We consider a uniform electric field $\vec{E} = E\vec{e}_z$, with $E > 0$, that produces an acceleration α along $\vec{e}_z$ such that $\alpha = (\mathcal{Z} - 1)eE/M$, with e the elementary charge and M the atomic mass. Both the nucleus and electric field $\vec{E}$ are treated classically, while the electron is treated via quantum field theory of Dirac fields. In this section we study the electron field in the inertial and the accelerated frames, and we discuss the frame-dependent particle content of the field.

The inertial laboratory frame $(t, \vec{x})$ is defined by the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-c^2, 1, 1, 1)$. By using the interaction picture [12], we separate the free field theory from the interaction Lagrangian. The free electron field in the Minkowski frame $\hat{\psi}(t, \vec{x})$ reads

$$\hat{\psi}(t,\vec{x}) = \sum_{s=1}^{2}\int_{\mathbb{R}^3} d^3k[u_s(\vec{k},t,\vec{x})\hat{c}_s(\vec{k}) + v_s(\vec{k},t,\vec{x})\hat{d}_s^\dagger(\vec{k})], \tag{1}$$

where $\hat{c}_s(\vec{k})$ and $\hat{d}_s(\vec{k})$ are annihilation operators for the particle and antiparticle with momentum $\vec{k}$ and spin number s. $u_s(\vec{k}, t, \vec{x})$ and $v_s(\vec{k}, t, \vec{x})$ are the positive and negative frequency modes such that

$$u_s(\vec{k},t,\vec{x}) = (2\pi)^{-3/2}e^{-i\omega(\vec{k})t+i\vec{k}\cdot\vec{x}}\tilde{u}_s(\vec{k}), \tag{2a}$$

$$v_s(\vec{k},t,\vec{x}) = (2\pi)^{-3/2}e^{i\omega(\vec{k})t-i\vec{k}\cdot\vec{x}}\tilde{v}_s(\vec{k}), \tag{2b}$$

with $\omega(\vec{k}) = \sqrt{(mc^2/\hbar)^2 + c^2|\vec{k}|^2}$ as the mode frequency. The spinors $\tilde{u}_s(\vec{k})$ and $\tilde{v}_s(\vec{k})$ are the orthonormal solutions of the Dirac equations in momentum space:

$$\left[\omega(\vec{k})\gamma^0 - k_i\gamma^i - \frac{mc}{\hbar}\right]\tilde{u}_s(\vec{k}) = 0, \tag{3a}$$

$$\left[\omega(\vec{k})\gamma^0 - k_i\gamma^i + \frac{mc}{\hbar}\right]\tilde{v}_s(\vec{k}) = 0, \tag{3b}$$

$$\tilde{u}_s^\dagger(\vec{k})\tilde{u}_{s'}(\vec{k}) = \tilde{v}_s^\dagger(\vec{k})\tilde{v}_{s'}(\vec{k}) = \delta_{ss'}, \tag{3c}$$

$$\tilde{u}_s^\dagger(\vec{k})\tilde{v}_{s'}(-\vec{k}) = 0, \tag{3d}$$

where γ^μ are the Dirac matrices [12–14].

The comoving accelerated frame $(T, \vec{X})$ is described by the Rindler metric $g_{\mu\nu}(T, \vec{X}) = \mathrm{diag}(-c^2e^{2aZ}, 1, 1, e^{2aZ})$, with $a = \alpha/c^2$. We study the electron field by quantum field theory in Rindler space-time (see Ref. [14] for the details). The free electron field $\hat{\Psi}_\nu(T, \vec{X})$ is

$$\hat{\Psi}_\nu(T,\vec{X}) = \sum_{s=1}^{2}\int_0^\infty d\Omega\int_{\mathbb{R}^2} d^2K_\perp[U_{\nu s}(\Omega,\vec{K}_\perp,T,\vec{X})\hat{C}_{\nu s}(\Omega,\vec{K}_\perp) + V_{\nu s}(\Omega,\vec{K}_\perp,T,\vec{X})\hat{D}_{\nu s}^\dagger(\Omega,\vec{K}_\perp)], \tag{4}$$

where $\hat{C}_{\nu s}(\Omega, \vec{K}_\perp)$ and $\hat{D}_{\nu s}(\Omega, \vec{K}_\perp)$ annihilate the electron and the positron of the ν wedge with spin number s, frequency Ω, and transverse momentum $\vec{K}_\perp$ represented by the positive and negative frequency modes:

$$U_{\nu s}(\Omega,\vec{K}_\perp,T,\vec{X}) = e^{i\vec{K}_\perp\cdot\vec{X}_\perp - i\Omega T}\tilde{W}_{\nu s}(\Omega,\vec{K}_\perp,Z), \tag{5a}$$

$$V_{\nu s}(\Omega,\vec{K}_\perp,T,\vec{X}) = e^{-i\vec{K}_\perp\cdot\vec{X}_\perp + i\Omega T}\tilde{W}_{\nu s}(-\Omega,-\vec{K}_\perp,Z), \tag{5b}$$

with

$$\tilde{W}_{\nu s'}(\Omega,\vec{K}_\perp,Z_\nu(z)) = \frac{1}{2\pi^2}\sqrt{\frac{\kappa(\vec{K}_\perp)}{ca}\cosh\left(\frac{\beta}{2}\Omega\right)}\sum_{\sigma=\pm}K_{\sigma s_\nu i\Omega/ca-1/2}\left(\kappa(\vec{K}_\perp)\frac{e^{s_\nu aZ}}{a}\right)$$
$$\times\left[\frac{-s_\nu ic}{\kappa(\vec{K}_\perp)}\gamma^0\left(K_1\gamma^1 + K_2\gamma^2 + \frac{mc}{\hbar}\right)\right]^{(1-\sigma)/2}\tilde{\mathfrak{W}}_{\nu s}(\Omega,\vec{K}_\perp), \tag{6}$$

and where s_ν is the sign of the wedge (i.e., $s_{\mathrm{L}} = -1$ and $s_{\mathrm{R}} = 1$), $\kappa(\vec{K}_\perp) = \sqrt{(mc/\hbar)^2 + |\vec{K}_\perp|^2}$ is the reduced momentum, $K_\zeta(\xi)$ is the modified Bessel function of the second kind, and $\tilde{\mathfrak{W}}_{\nu s}(\Omega, \vec{K}_\perp)$ are orthonormal bases for the eigenspace of $c\gamma^0\gamma^3$ with eigenvalue 1, i.e.,

$$c\gamma^0\gamma^3\tilde{\mathfrak{W}}_{\nu s}(\Omega,\vec{K}_\perp) = \tilde{\mathfrak{W}}_{\nu s}(\Omega,\vec{K}_\perp), \tag{7a}$$

$$\tilde{\mathfrak{W}}_{\nu s}^\dagger(\Omega,\vec{K}_\perp)\tilde{\mathfrak{W}}_{\nu s'}(\Omega,\vec{K}_\perp) = \delta_{ss'}. \tag{7b}$$

In Ref. [14] we reported the following Bogoliubov transformation relating the Minkowski particle creator $\hat{c}_s(\vec{k})$ to Rindler operators:

$$\hat{c}_s(\vec{k}) = \sum_{\nu=\{\mathrm{L},\mathrm{R}\}}\sum_{s'=1}^{2}\int_{\mathbb{R}} d\Omega\int_{\mathbb{R}^2} d^2K_\perp\alpha_\nu(\vec{k},\Omega,\vec{K}_\perp)\tilde{u}_s^\dagger(\vec{k})\tilde{\mathfrak{W}}_{\nu s'}(\Omega,\vec{K}_\perp)[\theta(\Omega)\hat{C}_{\nu s'}(\Omega,\vec{K}_\perp) + \theta(-\Omega)\hat{D}_{\nu s'}^\dagger(-\Omega,-\vec{K}_\perp)], \tag{8}$$

with

$$\alpha_\nu(\vec{k},\Omega,\vec{K}_\perp)=\frac{1}{\pi}\delta^2(\vec{k}_\perp-\vec{K}_\perp)\sqrt{\frac{\kappa(\vec{k}_\perp)}{2\pi ca}\cosh\left(\frac{\pi\Omega}{ca}\right)}\sum_{\sigma=\pm}\left[s_\nu i\frac{\omega(\vec{k})+ck_3}{c\kappa(\vec{k}_\perp)}\right]^{(\sigma-1)/2}\int_{\mathbb{R}}dz\theta(s_\nu z)e^{-ik_3z}K_{\sigma s_\nu i\Omega/ca-1/2}(s_\nu\kappa(\vec{K}_\perp)z), \tag{9}$$

and we showed the following representation of the Minkowski vacuum in the Rindler space-time:

$$|0_\mathrm{M}\rangle\propto\exp\left(-i\sum_{\nu=\{\mathrm{L},\mathrm{R}\}}s_\nu\sum_{s=1}^{2}\sum_{s'=1}^{2}\int_0^{+\infty}d\Omega\int_{\mathbb{R}^2}d^2K_\perp e^{-\pi\Omega/ca}\tilde{\mathfrak{W}}^\dagger_{\nu s}(\Omega,\vec{K}_\perp)\tilde{\mathfrak{W}}_{\bar{\nu}s'}(-\Omega,\vec{K}_\perp)\hat{C}^\dagger_{\nu s}(\Omega,\vec{K}_\perp)\hat{D}^\dagger_{\bar{\nu}s'}(\Omega,-\vec{K}_\perp)\right)|0_\mathrm{L},0_\mathrm{R}\rangle, \tag{10}$$

where $|0_\mathrm{M}\rangle$ is the Minkowski vacuum, $\bar{\nu}$ is the opposite of ν (i.e., $\bar{\nu}=\mathrm{L}$ if $\nu=\mathrm{R}$ and $\bar{\nu}=\mathrm{R}$ if $\nu=\mathrm{L}$), and $|0_\mathrm{L},0_\mathrm{R}\rangle$ is the Rindler vacuum state. By using Eqs. (8) and (10), one can see that any nonrelativistic single electron prepared in the inertial frame appears as a superposition of states with varying energy and particle number in the accelerated frame.

III. SUPPRESSING THE FRAME-DEPENDENT EFFECT ON THE ELECTRON

In the previous section we showed the frame-dependent particle content of the electron field, which is responsible for the appearance of electron states with varying energy and particle number in the accelerated frame. In this section we investigate the conditions under which such a frame-dependent effect is suppressed.

It is already known that in the case of scalar fields, the frame-dependent effect is suppressed when the acceleration α is sufficiently low and the particle state is localized in the approximately Minkowskian region of the Rindler space-time, i.e., where $g_{\mu\nu}(T,\vec{X})\approx\eta_{\mu\nu}$ [13]. Any nonrelativistic Minkowski single particle appears as a nonrelativistic Rindler particle in the accelerated frame if α is such that

$$\frac{\hbar a}{mc}\lesssim\epsilon^{3/2} \tag{11}$$

and the localization in $\vec{x}$ in such that

$$|az-1|\lesssim\epsilon, \tag{12}$$

where $\epsilon=\hbar\Omega/mc^2$ is the nonrelativistic parameter, defined as the ratio between the nonrelativistic energy $\hbar\Omega$ and the mass energy mc^2. The resulting Rindler single particle is created over the Unruh background $|0_\mathrm{M}\rangle$, which is in a superposition of Rindler particles. These background particles are mostly localized far from the region (12) and close to the Rindler horizon; hence they can be ignored for the local detection of the Unruh effect.

Here we find that the results obtained for scalar fields are also applicable to the case of Dirac fields. When $\hbar a/mc\lesssim\epsilon^{3/2}$, $|az-1|\lesssim\epsilon$, and $|\Omega|/ca\lesssim 1$, the Bessel functions appearing in Eq. (9) are approximated as [15]

$$K_{\pm i\Omega/ca-1/2}(\kappa(\vec{K}_\perp)z)\approx 0, \tag{13}$$

and when $\hbar a/mc\lesssim\epsilon^{3/2}$, $\hbar|\vec{K}_\perp|/mc\lesssim\epsilon^{1/2}$, $|az-1|\lesssim\epsilon$, and $|\Omega|/ca\gg 1$, as

$$\sqrt{\cosh\left(\frac{\pi\Omega}{ca}\right)}K_{\pm i\Omega/ca-1/2}(\kappa(\vec{K}_\perp)z)$$
$$\approx\pi\left(\frac{\hbar a}{\sqrt{2}mc}\right)^{1/3}\mathrm{Ai}\left(\left(\frac{\sqrt{2}mc}{\hbar a}\right)^{2/3}\left[\left(\frac{\hbar|\vec{K}_\perp|}{\sqrt{2}mc}\right)^2+az-\frac{\hbar\Omega}{mc^2}\right]\right), \tag{14}$$

where $\mathrm{Ai}(\xi)$ is the Airy function. By using Eqs. (13) and (14) one can prove that the electron is seen as a nonrelativistic single particle in both frames if the acceleration is constrained by Eq. (11) and the electron is localized in the region given by Eq. (12). See Appendix A for the complete proof.

IV. AVOIDING COMPLETE IONIZATION

In this section we discuss the interaction between the electron and the classic electromagnetic field. We obtain the conditions under which the atom is not completely ionized by the electric field $\vec{E}$. We show that such conditions not only guarantee the atomic stability but also suppress the frame-dependent effect described in the previous sections.

The classic electromagnetic field affecting the electron is made by the potential energy V_ext due to the external electric field $\vec{E}$ and the potential energy V_nuc due to the nuclear Coulomb interaction. In the comoving frame, V_ext is

$$V_\mathrm{ext}(Z)=\frac{1-e^{-2aZ}}{2}\frac{eE}{a} \tag{15}$$

(see Appendix B). The nuclear potential energy V_nuc is

$$V_\mathrm{nuc}(R)=-\epsilon_\mathrm{QED}^{1/2}\frac{\hbar c}{R}, \tag{16}$$

where $R=|\vec{X}|$ is the radial coordinate, $\epsilon_\mathrm{QED}=(\mathcal{Z}\alpha_0)^2$ is the quantum electrodynamics (QED) coupling, and α_0 the fine-structure constant.

The electron is pulled away from its orbit by V_ext while it is dragged by the accelerating nucleus via V_nuc. If E is sufficiently large, the electron escapes from the nuclear Coulomb barrier via quantum tunneling, compromising the atomic stability. To avoid complete ionization, we require a small E such that

$$|V_\mathrm{ext}(R_0)|\ll\left|E_0^{(0)}\right|, \tag{17}$$

with $E_0^{(0)} = -\epsilon_{\rm QED}\mu c^2/2$ as the ground state of $V_{\rm nuc}$, $\mu = (m + M_{\rm N})/mM_{\rm N} \approx m$ as the reduced mass, $M_{\rm N} \approx M$ as the nuclear mass, $R_0 = a_0/\mathcal{Z}$ as the atomic radius, and $a_0 = \hbar/mc\alpha_0$ as the Bohr radius. Hence we assume that the external force $V_{\rm ext}$ simply perturbs the spectrum of $V_{\rm nuc}$ via the Stark effect.

Equation (17) reads

$$aR_0 \ll \epsilon_{\rm QED}\frac{(\mathcal{Z}-1)m}{2M}, \tag{18}$$

or, equivalently,

$$E \ll \frac{(\mathcal{Z}\alpha_0)^3}{2}\frac{m^2c^3}{\hbar e}. \tag{19}$$

Notice that the electron is localized inside the region $R \lesssim R_0$ since $V_{\rm nuc}$ dominates over $V_{\rm ext}$. Notice also that $\epsilon_{\rm QED} \ll 1$ and $(\mathcal{Z}-1)m \ll M$. Hence, from Eqs. (15) and (18) one concludes that the electron is localized where the electric field is approximately uniform.

Equation (17) guarantees a lifetime τ for the atom that is exponentially increasing for decreasing electric field. Indeed, by using the WKB approximation, one can find the following ionization rate [16]:

$$\frac{1}{\tau} \approx \frac{16}{\hbar R_0 eE}\left(E_0^{(0)}\right)^2 \exp\left(\frac{4E_0^{(0)}}{3R_0eE}\right). \tag{20}$$

Notice that $\epsilon_{\rm QED}mc^2$ is the order of the nonrelativistic atomic energies. Indeed, the spectrum of $V_{\rm nuc}$ is

$$E_n^{(0)} = -\frac{\epsilon_{\rm QED}}{2(n+1)^2}\mu c^2. \tag{21}$$

By comparing Eq. (18) with Eq. (12), one finds out that the localization condition (12) is already met by configurations that satisfy Eq. (18). Furthermore, Eq. (18) leads to

$$\frac{\hbar a}{mc} \ll \epsilon_{\rm QED}^{3/2}\frac{(\mathcal{Z}-1)m}{2M}, \tag{22}$$

which is a sufficient condition for Eq. (11). By constraining E and $\mathcal{Z}$ accordingly to Eq. (19), one guarantees the atom stability and the first-quantization electron description in the accelerated frame. The atom does not ionize and the electron appears as a nonrelativistic single-particle bound state in both frames.

V. DETECTING UNRUH RADIATION VIA HYPERFINE SPLITTING

In this section we consider the interaction between the accelerated atom and the electromagnetic Unruh background [1,2,7]. We show the conditions under which the coupling between electron and Unruh radiation produce measurable effects. We study the spectrum of the relativistic hydrogenlike atom in Rindler space-time with uniform external electric field and we show that the coupling is induced by the hyperfine splitting. Finally, we plot the regime of parameters for the observability of the Unruh effect.

In the accelerated frame, as a consequence of the Unruh radiation the electron can be excited by absorbing a photon with the energy $\Delta E_n = E_n - E_0$ of the nth electronic transition. The event is detectable if

$$\Delta E_n \lesssim k_B T \tag{23}$$

and if the atom has a sufficiently large lifetime such that

$$\tau \gtrsim \frac{\hbar}{\Delta E_n}. \tag{24}$$

Equation (23) guarantees a nonvanishing probability for the electron to interact with photons described by the following Boltzmann distribution:

$$P_{\rm B} = \frac{1}{e^{\Delta E_n/k_{\rm B}T} - 1}. \tag{25}$$

A more refined constraint than Eq. (23) can be imposed by assuming a lower bound for $P_{\rm B}$, i.e.,

$$P_{\rm B} < P_{\rm min}, \tag{26}$$

with $P_{\rm min} < 1$. Equation (24), instead, ensures that the absorption spectrum of the atom is narrow around ΔE_n. Given the exponential growth of the atom lifetime for smaller E [see Eq. (20)], it is safe to assume that Eq. (24) gives an almost exact lower limit for τ, i.e.,

$$\tau > \frac{\hbar}{\Delta E_n}. \tag{27}$$

The states and energies of the spectrum E_n are the solutions of the Dirac equation in Rindler space-time for hydrogenlike atoms with the interaction potential $V_{\rm ext}$. They can be computed perturbatively by considering the nonrelativistic hydrogenlike spectrum $E_n^{(0)}$ [see Eq. (21)] perturbed by $V_{\rm ext}$ and by relativistic corrections coming from the Rindler-Dirac equation.

The energy gaps of the unperturbed Hamiltonian $\Delta E_n^{(0)} = E_n^{(0)} - E_0^{(0)}$ in Eq. (21) are of the order

$$\Delta E_n^{(0)} \sim \epsilon_{\rm QED}mc^2. \tag{28}$$

By plugging Eq. (28) in Eq. (23), one finds that the lower bound for the electric field is

$$E \gtrsim \frac{2\pi(\mathcal{Z}\alpha_0)^2}{\mathcal{Z}-1}\frac{mMc^3}{\hbar e}, \tag{29}$$

which is way larger than the upper bound (19). Hence, $\Delta E_n^{(0)}$ does not induce coupling with the electromagnetic background for any stable configuration.

Perturbations of $E_n^{(0)}$ do not significantly change the energies gaps, unless they break the spin degeneracy of the atomic ground state. In that case the first level $E_0^{(0)}$ splits into the actual ground state E_0 and the first excited state E_1, with $\Delta E = E_1 - E_0 \ll \epsilon_{\rm QED}mc^2$.

In Appendix C we show that the Rindler-Dirac equation for the hydrogenlike atom with potentials $V_{\rm nuc}$ and $V_{\rm ext}$ have a degenerate minimum energy level. Hence the external electron field $V_{\rm ext}$ and the special and general relativity corrections do not break the spin degeneracy of $E_0^{(0)}$.

One has to look at the hyperfine structure to see a split of $E_0^{(0)}$ due to quantum electrodynamics corrections. The electron-nucleus interaction via spin-spin coupling generates the following energy gap [17]:

$$\Delta E_{\mathrm{hf}} = \begin{cases} \frac{1}{3\pi}(2I+1)\mathcal{Z}^3\alpha_0^4 g \frac{m^2c^2}{M_{\mathrm{P}}} & \text{if } I \neq 0 \\ 0 & \text{if } I = 0 \end{cases}, \tag{30}$$

with M_{P} as the proton mass. I is the quantum number such that $|\vec{I}|^2 = I(I+1)$, where $\vec{I}$ is the nucleus spin. g is the effective g factor, defined as follows: $\vec{\mu} = (g\hbar e/2M_{\mathrm{P}})\vec{I}$, where $\vec{\mu}$ is the magnetic moment of the nucleus resulting from its spin.

Notice that the selection rule that forbids transitions between levels with vanishing azimuthal quantum number $\ell = 0$ breaks down due to the Stark effect. Hence the absorption of photons coupled to the hyperfine structure is allowed.

By plugging Eq. (30) in Eq. (26), one finds that the atomic hyperfine structure produces a measurable Boltzmann distribution when $I \neq 0$ and when

$$\left[\exp\left(\frac{2(2I+1)\mathcal{Z}^3\alpha_0^4 g}{3(\mathcal{Z}-1)}\frac{Mm^2c^3}{M_{\mathrm{P}}\hbar eE}\right) - 1\right]^{-1} < P_{\mathrm{min}}. \tag{31}$$

Furthermore, the atom has a sufficiently long lifetime when [see Eqs. (27) and (30)]

$$\frac{\hbar eE}{m^2c^3}\exp\left(\frac{2(\mathcal{Z}\alpha_0)^3}{3}\frac{m^2c^3}{\hbar eE}\right) > \frac{12\pi\,\mathcal{Z}^2\alpha_0}{(2I+1)g}\frac{M_{\mathrm{P}}}{m}. \tag{32}$$

Equations (31) and (32) define the regime of parameters E, $\mathcal{Z}$, M, I, g for the detection of the Unruh effect via first-quantized atomic detectors. The results are shown in Fig. 1, where for some nuclear configurations we plot the range of validity for the electric field E. In Fig. 1 we also plot the Boltzmann distribution

$$P_{\mathrm{B}} = \left[\exp\left(\frac{(2I+1)\mathcal{Z}^3\alpha_0^4 g}{3(\mathcal{Z}-1)}\frac{Mm^2c^3}{M_{\mathrm{P}}\hbar eE}\right) - 1\right]^{-1} \tag{33}$$

and the Unruh temperature

$$T = \frac{\mathcal{Z}-1}{2\pi}\frac{\hbar eE}{k_{\mathrm{B}}Mc}. \tag{34}$$

To overcome background noises, a sufficiently large T is needed and requires high ionization $\mathcal{Z}$.

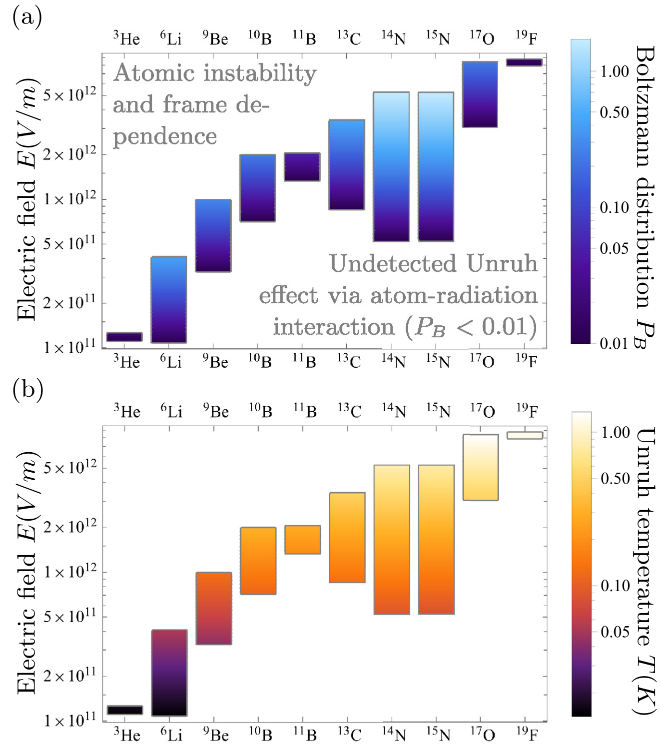

FIG. 1. Observation window for the Unruh effect via first-quantized atomic detectors. The constrained variable is the accelerating electric field E for each nuclear configuration $\mathcal{Z}$, M, I, g [18]. The upper limit for E [see Eq. (32)] guarantees the stability of the atomic bound state and the first-quantized description of the electron in the accelerated frame. Above this limit, the electron escapes from the Coulomb potential via tunneling and it appears as a superposition of states with different energies and number of particles in the accelerated frame. The lower limit [see Eq. (31)], instead, ensures the detection of the Unruh effect from the electromagnetic thermal background via light-matter interaction. Below such limits, the hyperfine structure of the atom produces an energy gap that is too large for the Boltzmann distribution to be detected. In (a) we show the Boltzmann distribution for the atom-radiation interaction [see Eq. (33)]. In (b) we show the Unruh temperature T of the electromagnetic background in the accelerated frame for different configurations [see Eq. (34)].

VI. CONCLUSIONS

Various experimental proposals have been reported to test the Unruh effect. The proposals include the depolarization of electrons in storage rings [19,20], Penning traps [21], and ultraintense lasers [22,23]. The growing interest is motivated by the ever-improving experimental equipment that allows to reach high accelerations [24]. Besides electrons, also uniformly accelerated protons have been considered as Unruh-DeWitt detectors via acceleration-induced weak-interaction decay [25–27] and photon emission [28,29].

In this manuscript we analyzed the electron in the accelerated atom by the nonrelativistic limit of a Dirac field in Rindler space-time [12–14]. While considering hyperfine splitting, we addressed three problems: (i) the instability of the atom due to a strong accelerating field; (ii) the frame-dependent nature of the electron; and (iii) the detectability of the Unruh effect due to the electromagnetic radiation. We have shown that (i) and (ii) impose an upper boundary condition for the electric field accelerating the ionized hydrogenlike atom. Taking into account (iii), we determined an observation window in the E and $\mathcal{Z}$ plane [see Fig. 1]. Surprisingly, quantitative estimates unveil that the effect can be detected with electric fields within the reach of the modern technologies of high-power lasers and nuclear magnetic resonance.

As a concluding remark, we discuss the possibility to extend our results by including the Zeeman effect. The presence of a uniform magnetic field $\vec{B}$ parallel to the accelerating electric field $\vec{E}$ has the following effects: (i) it does not affect the accelerating trajectory of the atom, and (ii) it induces an energy splitting at the ground states similarly to the hyperfine splitting. Hence, for atoms that have vanishing hyperfine splitting (e.g., nuclei with zero spin), only the Zeeman effect produces a measurable energy gap that couples to the Unruh radiation. We remark that, at variance with the hyperfine splitting, the strength of the magnetic field $\vec{B}$ can be controlled and induces an arbitrarily small energy gap. Consequently, no lower bound for the electric field $\vec{E}$ occurs.

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APPENDIX A: SUPPRESSING THE FRAME-DEPENDENT EFFECT FOR DIRAC SINGLE PARTICLES

In this section we adapt the discussion of Ref. [13] to the case of Dirac fields. We consider a single fermionic particle $|\psi\rangle$ prepared by an inertial observer, and we derive its representation in the Rindler-Fock space by using the Bogoliubov transformation (8). We assume that the acceleration is constrained by Eq. (11) and that the particle is nonrelativistic and localized in (12). We show that the Rindler-Fock representative for $|\psi\rangle$ is approximated by a nonrelativistic Rindler single particle created over the Minkowski vacuum $|0_{\rm M}\rangle$, which, in turns, is mainly populated by Rindler particles and antiparticles localized far from (12).

We start from the general definition of Minkowski single particles $|\psi\rangle$, with wave function $\psi(x)$. The state is defined as follows:

$$|\psi\rangle = \hat{\mathfrak{C}}_\psi^\dagger |0_{\rm M}\rangle, \tag{A1}$$

with

$$\hat{\mathfrak{C}}_\psi = \sum_{s=1}^{2} \int_{\mathbb{R}^3} d^3k \tilde{\psi}_s^*(\vec{k}) \hat{c}_s(\vec{k}) \tag{A2}$$

as the particle annihilator and with $\tilde{\psi}_s(\vec{k})$ as the wave function in the spin-momentum representation, i.e.,

$$\psi(\vec{x}) = \sum_{s=1}^{2} \int_{\mathbb{R}^3} d^3k \tilde{\psi}_s(\vec{k}) u_s(\vec{k}, 0, \vec{x}). \tag{A3}$$

The nonrelativistic condition for the particle is

$$\tilde{\psi}_s(\vec{k}) \approx 0 \quad \text{if } \frac{\hbar|\vec{k}|}{mc} \gg \epsilon^{1/2}. \tag{A4}$$

The localization condition, instead, is

$$\psi(\vec{x}) \approx 0 \quad \text{if } |az - 1| \gg \epsilon. \tag{A5}$$

By using the Bogoliubov transformation (8), we write $\hat{\mathfrak{C}}_\psi^\dagger$ in terms of Rindler operators:

$$\begin{aligned}\hat{\mathfrak{C}}_\psi = &\sum_{\nu=\{\rm L,R\}} \sum_{s=1}^{2} \sum_{s'=1}^{2} \int_{\mathbb{R}^3} d^3k \int_{\mathbb{R}} d\Omega \int_{\mathbb{R}^2} d^2K_\perp \tilde{\psi}_s^*(\vec{k}) \alpha_\nu(\vec{k}, \Omega, \vec{K}_\perp) \tilde{u}_s^\dagger(\vec{k}) \tilde{\mathfrak{W}}_{\nu s'}(\Omega, \vec{K}_\perp)\\ &\times [\theta(\Omega) \hat{C}_{\nu s'}(\Omega, \vec{K}_\perp) + \theta(-\Omega) \hat{D}_{\nu s'}^\dagger(-\Omega, -\vec{K}_\perp)].\end{aligned} \tag{A6}$$

As a consequence of the Dirac delta function appearing in the explicit form of $\alpha_\nu(\vec{k}, \Omega, \vec{K}_\perp)$ [see Eq. (9)] and the nonrelativistic condition (A4), $\hat{\mathfrak{C}}_\psi^\dagger$ creates Rindler particles and destroys Rindler antiparticles with nonrelativistic transverse momentum, i.e., $\hbar|\vec{K}_\perp|/mc \lesssim \epsilon^{1/2}$. Furthermore, Eqs. (9) and (A4) lead to the following approximation:

$$\begin{aligned}\hat{\mathfrak{C}}_\psi \approx &\sum_{\nu=\{\rm L,R\}} \sum_{s=1}^{2} \sum_{s'=1}^{2} \int_{\mathbb{R}^3} d^3k \int_{\mathbb{R}} d\Omega \int_{\mathbb{R}^2} d^2K_\perp \tilde{\psi}_s^*(\vec{k}) \beta_\nu(\vec{k}, \Omega, \vec{K}_\perp) \tilde{u}_s^\dagger(\vec{k}) \tilde{\mathfrak{W}}_{\nu s'}(\Omega, \vec{K}_\perp)\\ &\times [\theta(\Omega) \hat{C}_{\nu s'}(\Omega, \vec{K}_\perp) + \theta(-\Omega) \hat{D}_{\nu s'}^\dagger(-\Omega, -\vec{K}_\perp)],\end{aligned} \tag{A7}$$

with

$$\beta_\nu(\vec{k}, \Omega, \vec{K}_\perp) = \frac{1}{\pi} \delta^2(\vec{k}_\perp - \vec{K}_\perp) \sqrt{\frac{m}{2\pi\hbar a} \cosh\left(\frac{\pi\Omega}{ca}\right)} \sum_{\sigma=\pm} (s_\nu i)^{(\sigma-1)/2} \int_{\mathbb{R}} dz \theta(s_\nu z) e^{-ik_3 z} K_{\sigma s_\nu i\Omega/ca - 1/2}(s_\nu \kappa(\vec{K}_\perp) z). \tag{A8}$$

By recalling Eq. (2a), we write Eq. (A7) in the following way:

$$\hat{\mathfrak{C}}_\psi \approx \sum_{\nu=\{\rm L,R\}} \sum_{s=1}^{2} \int_{\mathbb{R}} d\Omega \int_{\mathbb{R}^2} d^2K_\perp \chi_{\nu s}(\Omega, \vec{K}_\perp) [\theta(\Omega) \hat{C}_{\nu s}(\Omega, \vec{K}_\perp) + \theta(-\Omega) \hat{D}_{\nu s}^\dagger(-\Omega, -\vec{K}_\perp)], \tag{A9}$$

with

$$\begin{aligned}\chi_{\nu s}(\Omega, \vec{K}_\perp) = &\frac{1}{2\pi^2} \sqrt{\frac{m}{\hbar a} \cosh\left(\frac{\pi\Omega}{ca}\right)} \int_{\mathbb{R}^3} d^3x \theta(s_\nu z) \sum_{\sigma=\pm} (s_\nu i)^{(\sigma-1)/2} K_{\sigma s_\nu i\Omega/ca - 1/2}(s_\nu \kappa(\vec{K}_\perp) z)\\ &\times \sum_{s'=1}^{2} \int_{\mathbb{R}^3} d^3k \tilde{\psi}_{s'}^*(\vec{k}) u_{s'}^\dagger(\vec{k}, 0, \vec{x}) \tilde{\mathfrak{W}}_{\nu s}(\Omega, \vec{K}_\perp).\end{aligned} \tag{A10}$$

By using Eq. (A3), Eq. (A10) reads as

$$\chi_{\nu s}(\Omega, \vec{K}_\perp) = \frac{1}{2\pi^2}\sqrt{\frac{m}{\hbar a}\cosh\left(\frac{\pi\Omega}{ca}\right)}\int_{\mathbb{R}^3} d^3x\theta(s_\nu z)\sum_{\sigma=\pm}(s_\nu i)^{(\sigma-1)/2}K_{\sigma s_\nu i\Omega/ca-1/2}(s_\nu\kappa(\vec{K}_\perp)z)\psi^\dagger(\vec{x})\tilde{\mathfrak{W}}_{\nu s}(\Omega, \vec{K}_\perp). \tag{A11}$$

The localization condition (A5) can be used in Eq. (A11) to obtain the following approximation:

$$\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \approx \frac{1}{2\pi^2}\sqrt{\frac{m}{\hbar a}\cosh\left(\frac{\pi\Omega}{ca}\right)}\int_{\mathbb{R2}^3} d^3x\theta(\epsilon - |az-1|)\sum_{\sigma=\pm} i^{(\sigma-1)/2} \times K_{\sigma i\Omega/ca-1/2}(\kappa(\vec{K}_\perp)z)\psi^\dagger(\vec{x})\tilde{\mathfrak{W}}_{\mathrm{R}s}(\Omega, \vec{K}_\perp), \tag{A12a}$$

$$\chi_{\mathrm{L}s}(\Omega, \vec{K}_\perp) \approx 0, \tag{A12b}$$

which means that $\hat{\mathfrak{C}}_\psi^\dagger$ creates Rindler particles and destroys Rindler antiparticles in the right wedge only. By using again Eq. (A3), Eq. (A12a) reads as

$$\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \approx \frac{1}{2\pi^2}\sqrt{\frac{m}{\hbar a}\cosh\left(\frac{\pi\Omega}{ca}\right)}\sum_{s'=1}^{2}\int_{\mathbb{R}^3} d^3k\tilde{\psi}_{s'}^*(\vec{k})\sum_{\sigma=\pm} i^{(\sigma-1)/2}\int_{\mathbb{R}^3} d^3x\theta(\epsilon - |az-1|) \times u_{s'}^\dagger(\vec{k}, 0, \vec{x})\tilde{\mathfrak{W}}_{\mathrm{R}s}(\Omega, \vec{K}_\perp)K_{\sigma i\Omega/ca-1/2}(\kappa(\vec{K}_\perp)z). \tag{A13}$$

We now consider the limit of low acceleration (11), and we use Eqs. (13) and (14) in Eq. (A13). By using Eq. (13), we find that $\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp)$ is vanishing when $|\Omega|/ca \lesssim 1$, and hence $\hat{\mathfrak{C}}_\psi^\dagger$ creates Rindler particles and destroys Rindler antiparticles with frequency $|\Omega| \gg ca$. By using Eq. (14), one can write Eq. (A13) in the limit $|\Omega| \gg ca$ as follows:

$$\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \approx \frac{1-i}{2\pi}\sqrt{\frac{m}{\hbar a}}\left(\frac{\hbar a}{\sqrt{2}mc}\right)^{1/3}\sum_{s'=1}^{2}\int_{\mathbb{R}^3} d^3k\tilde{\psi}_{s'}^*(\vec{k})\int_{\mathbb{R}^3} d^3x\theta(\epsilon - |az-1|)u_{s'}^\dagger(\vec{k}, 0, \vec{x})\tilde{\mathfrak{W}}_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \times \mathrm{Ai}\left(\left(\frac{\sqrt{2}mc}{\hbar a}\right)^{2/3}\left[\left(\frac{\hbar|\vec{K}_\perp|}{\sqrt{2}mc}\right)^2 + az - \frac{\hbar\Omega}{mc^2}\right]\right). \tag{A14}$$

Owing to Eq. (2a), Eq. (A14) reads as

$$\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \approx \frac{1-i}{(2\pi)^{5/3}}\sqrt{\frac{m}{\hbar a}}\left(\frac{\hbar a}{\sqrt{2}mc}\right)^{1/3}\sum_{s'=1}^{2}\int_{\mathbb{R}^3} d^3k\tilde{\psi}_{s'}^*(\vec{k})\tilde{u}_{s'}^\dagger(\vec{k})\tilde{\mathfrak{W}}_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \times \int_{\mathbb{R}^3} d^3x\theta(\epsilon - |az-1|)e^{-i\vec{k}\cdot\vec{x}}\mathrm{Ai}\left(\left(\frac{\sqrt{2}mc}{\hbar a}\right)^{2/3}\left[\left(\frac{\hbar|\vec{K}_\perp|}{\sqrt{2}mc}\right)^2 + az - \frac{\hbar\Omega}{mc^2}\right]\right). \tag{A15}$$

The nonrelativistic condition (A4) in Eq. (A15) leads to

$$\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \approx \frac{1-i}{(2\pi)^{5/3}}\sqrt{\frac{m}{\hbar a}}\left(\frac{\hbar a}{\sqrt{2}mc}\right)^{1/3}\sum_{s'=1}^{2}\int_{\mathbb{R}^3} d^3k\theta\left(\epsilon^{1/2} - \frac{\hbar|\vec{k}|}{mc}\right)\tilde{\psi}_{s'}^*(\vec{k})\tilde{u}_{s'}^\dagger(\vec{k})\tilde{\mathfrak{W}}_{\mathrm{R}s}(\Omega, \vec{K}_\perp) \times \int_{\mathbb{R}^3} d^3x\theta(\epsilon - |az-1|)e^{-i\vec{k}\cdot\vec{x}}\mathrm{Ai}\left(\left(\frac{\sqrt{2}mc}{\hbar a}\right)^{2/3}\left[\left(\frac{\hbar|\vec{K}_\perp|}{\sqrt{2}mc}\right)^2 + az - \frac{\hbar\Omega}{mc^2}\right]\right). \tag{A16}$$

One can now use the limits of the Airy function to show that if $\hbar\Omega/mc^2 - 1 \ll -\epsilon$, then $\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp)$ is exponentially vanishing [13]. Conversely, if $\hbar\Omega/mc^2 - 1 \gg \epsilon$, the Airy function appearing in Eq. (A16) oscillates way faster than e^{-ik_3z}[13], leading to a vanishing integration with respect to z. Consequently, $\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp)$ is nonvanishing only when $|\hbar\Omega/mc^2 - 1| \lesssim \epsilon$ (i.e., Ω is positive and nonrelativistic). As a result, $\hat{\mathfrak{C}}_\psi^\dagger$ only creates a nonrelativistic right-Rindler particle and can be approximated by the following identity:

$$\hat{\mathfrak{C}}_\psi \approx \sum_{s=1}^{2}\int_{\mathbb{R}} d\Omega\theta\left(\epsilon - \left|\frac{\hbar\Omega}{mc^2} - 1\right|\right)\int_{\mathbb{R}^2} d^2K_\perp \times \theta\left(\epsilon^{1/2} - \frac{\hbar|\vec{K}_\perp|}{mc}\right)\chi_{\mathrm{R}s}(\Omega, \vec{K}_\perp)\hat{C}_{\mathrm{R}s}(\Omega, \vec{K}_\perp). \tag{A17}$$

Equation (A17) can be plugged in Eq. (A1) to give a Rindler-Fock representation of the Minkowski single particle. The operator $\hat{\mathfrak{C}}_{\psi}^{\dagger}$ creates a nonrelativistic right-Rindler single particle over the Minkowski vacuum background $|0_{\mathrm{M}}\rangle$, which, in turn, is given by Eq. (10).

By using Eq. (10) and the fact that $e^{-\pi\Omega/ca}$ is exponentially vanishing when $\Omega/ca \gg 1$, we find that the Minkowski vacuum is mainly populated by Rindler particles with frequency $\Omega \lesssim ca$. Such particles are localized far from the region $|aZ| \lesssim \epsilon$. Indeed, by using Eqs. (5), (6), and (13) one can prove that the modes representing right-Rindler particles and antiparticles with $\Omega \lesssim ca$ are vanishing in $|aZ| \lesssim \epsilon$. The result is that Eq. (A1) is approximately a nonrelativistic right-Rindler single particle created over a sea of Rindler particles and antiparticles localized far from $|aZ| \lesssim \epsilon$.

APPENDIX B: UNIFORM ELECTRIC FIELD IN THE ACCELERATED FRAME

Here we consider an electric field $\vec{E}$ that appears uniform in the inertial frame, i.e., $\vec{E}(t,\vec{x}) = E\vec{e}_z$, and we compute the consequent electromagnetic field in the accelerated frame. The electromagnetic tensor in the inertial frame is

$$F^{\mu\nu} = \frac{E}{c}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \tag{B1}$$

The coordinate transformation $(t,\vec{x}) \mapsto (T,\vec{X})$ from the Minkowski to the right-Rindler frame is such that

$$cat = e^{aZ}\sinh(caT), \quad x = X, \quad y = Y, \tag{B2a}$$
$$az = e^{aZ}\cosh(caT), \tag{B2b}$$

which can be inverted to give

$$caT = \tanh^{-1}\left(\frac{ct}{z}\right), \quad X = x, \quad Y = y, \tag{B3a}$$
$$aZ = \frac{1}{2}\log[(az)^2 - (cat)^2]. \tag{B3b}$$

From Eq. (B3) one can compute the following Jacobian matrix:

$$\frac{\partial X^{\mu}}{\partial x^{\nu}} = \begin{pmatrix} e^{-aZ}\cosh(caT) & 0 & 0 & c^{-1}e^{-aZ}\sinh(caT) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ ce^{-aZ}\sinh(caT) & 0 & 0 & e^{-aZ}\cosh(caT) \end{pmatrix}. \tag{B4}$$

The components of the electromagnetic tensor in the noninertial frame $(T,\vec{X})$ can be obtained by using Eq. (B4) as follows:

$$\frac{\partial X^{\mu}}{\partial x^{\alpha}}\frac{\partial X^{\nu}}{\partial x^{\beta}}F^{\alpha\beta} = e^{-2aZ}\frac{E}{c}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \tag{B5}$$

Equation (B5) states that in the accelerated frame, $\vec{E}$ appears as an electric field along Z with magnitude $e^{-2aZ}E$. The consequent potential energy V_{ext} that is vanishing for $Z = 0$ is reported by Eq. (15).

APPENDIX C: SPIN DEGENERACY OF ACCELERATED HYDROGENLIKE ATOM

Here we consider the energy potential V_{ext} and the special and general relativity corrections to the accelerated hydrogenlike atom. We show that the spin degeneracy of the first energy level is not lifted.

The full Rindler-Dirac equation with potentials V_{nuc} and V_{ext} describing the electron in the accelerated frame is

$$i\hbar\partial_0\Psi = H\Psi, \tag{C1}$$

with the following Hamiltonian (see, for instance, Ref. [12]):

$$\begin{aligned} H = &- i\hbar c^2\gamma^0\gamma^3\partial_3 - \frac{i}{2}\hbar\alpha\gamma^0\gamma^3 + e^{aZ}\gamma^0(-i\hbar c^2\gamma^1\partial_1 \\ &- i\hbar c^2\gamma^2\partial_2 + mc^3) + V_{\mathrm{nuc}} + V_{\mathrm{ext}}. \end{aligned} \tag{C2}$$

Notice that H is symmetric with respect to the following unitary operators:

$$U_1 = ic\gamma^0 P_1\gamma^2\gamma^3, \quad U_2 = ic\gamma^0\gamma^1 P_2\gamma^3, \tag{C3}$$

where P_i is the parity operator for the ith coordinate, i.e., $P_1 : X \mapsto -X$ and $P_2 : Y \mapsto -Y$. Indeed, one can prove that H commutes with U_1 and U_2,

$$[U_1, H] = 0, \quad [U_2, H] = 0, \tag{C4}$$

by using the following anticommutative properties:

$$\{P_1, \partial_1\} = 0, \quad \{P_2, \partial_2\} = 0, \tag{C5a}$$
$$\{\gamma^{\mu}, \gamma^{\nu}\} = -2\eta^{\mu\nu}. \tag{C5b}$$

Equation (C5b), the hermiticity of γ^0, P_1, and P_2, and the antihermiticity of γ^i can be used to prove the unitarity of U_1 and U_2. Furthermore, Eq. (C5b) leads to the following anticommutative relation:

$$\{U_1, U_2\} = 0. \tag{C6}$$

Equation (C4) implies that H and U_1 are simultaneously diagonalizable. The same occurs for H and U_2. However, U_1 and U_2 are not compatible, since, as a consequence of Eq. (C6), they do not commute.

Consider a state Ψ which is simultaneously an eigenstate of H and U_2. The noncompatibility between U_1 and U_2 implies that Ψ is not an eigenstate of U_1. Hence $U_1\Psi$ is a different state from Ψ. However, $U_1\Psi$ is still an eigenstate of H with the same energy of Ψ. We find that the Hamiltonian H is at least two-degenerate for each energy level.

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