Letter

Fate of multiparticle entanglement when one particle becomes classical

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(Received 18 July 2022; revised 5 February 2023; accepted 24 March 2023; published 6 April 2023)

We study the change in multiparticle entanglement if one particle becomes classical, in the sense that this particle is destructed by a measurement but the gained information is encoded into a new register. We present an estimation of this change for different entanglement measures and ways of encoding. We first simplify the numerical calculation to analyze the change in entanglement under classicalization in special cases. Second, we provide general upper and lower bounds on the entanglement change. Third, we show that the entanglement change caused only by classicalization of one qubit can still be arbitrarily large. Finally, we discuss cases where no entanglement is left under classicalization for any possible measurement. Our results shed light on the storage of quantum resources and help to develop a different direction in the field of quantum resource theories.

DOI: 10.1103/PhysRevA.107.L040401

I. INTRODUCTION

Different types of quantum resources [1] are essential for quantum information tasks, like quantum computation [2], quantum key distribution [3], and quantum metrology [4], for which they can provide a decisive advantage over the classical regime. One main problem of many quantum resources is their sensitivity to a disturbance from the environment. Their protection with tools like quantum error correction [5] is usually expensive, especially if larger systems are considered. In practice, some fraction of the particles of a larger quantum system can inevitably become classical, e.g., as the result of a measurement or decoherence process. In fact, the particles may even be completely lost.

It is a natural question to ask how multiparticle entanglement [6,7] is affected by such processes. Many works have considered the influence of decoherence on multiparticle entanglement [8–13]. Other works considered the robustness of multiparticle entanglement under particle loss [14–17]. Moreover, the sharp change in bipartite entanglement caused by the complete loss of one particle in one party has been studied as the concept of lockable entanglement [18–21]. There can, however, still be information left in the environment after loss of particles. For example, in the case of the Stern-Gerlach experiment, the remaining information is given by the location of the spots on the screen. As another example one can consider the decay of particles due to decoherence, from which it may be reasonable to gather some information from the particles before their complete decay. The usefulness of this classical information has been extensively explored in the form of the entanglement of assistance [22], where a third party (Charlie) optimizes the measurement and the resulting information to help the two original parties (Alice and Bob) reveal as much quantum entanglement as possible. Most research on the entanglement of assistance has focused on the case where the global state is pure [23–25]. As it turns out [22], the entanglement of assistance depends only on the reduced state for Alice and Bob, and the exact three-partite initial state is not important.

In this Letter we consider a different scenario: One or more particles in a multiparticle system is destructed by a measurement. The gained classical information is then encoded in a quantum state. Our question is how much the multiparticle entanglement is affected in this process of classicalization (see Fig. 1). This scenario is practically relevant because one may not have the perfect "assistance" when the size and performance of the register system are limited. Consequently, our approach can provide guidance for the storage of quantum entanglement robust to particle loss and for finding the optimal strategy of entanglement recovery with the gained classical information and a small register system. Unlike in the concept of quantum assistance, we consider mixed quantum states

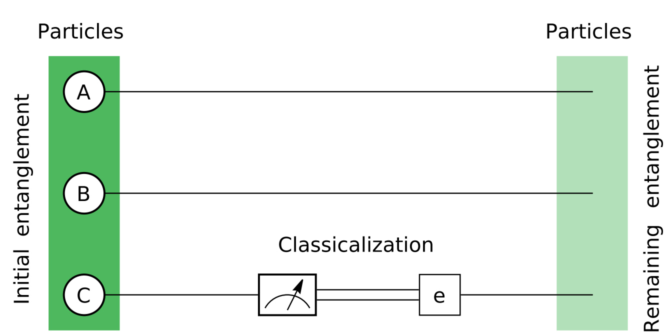

FIG. 1. The change in multiparticle entanglement if the particle C becomes classical. In this process of classicalization the particle C is first destroyed by the measurement, and then the measurement information is encoded in a new register. This Letter asks for which classicalization procedure the change in entanglement is minimal.

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where the entanglement is stored and it is not the aim of the measured party to increase the bipartite entanglement between the remaining ones. Most importantly, the initial quantum state plays a major role in the change in the entanglement due to classicalization. We stress that there are further related concepts. The so-called hidden entanglement [26] has been introduced as the difference between the entanglement without the decomposition information of a mixed state and the one with the decomposition information. In addition, the role of one particle in the change in entanglement has also been considered in distributed entanglement [27,28], where the particle is transferred from one party to another one rather than being destroyed.

II. NOTATIONS AND DEFINITIONS

We focus on tripartite systems in this Letter; other multipartite systems can be analyzed similarly. We denote the initial state as ρ_{ABC}. First, suppose that one party of this state is measured in a process that completely destroys the measured party, such as the detection of the photon polarization.

Without loss of generality, we here assume that the destructive measurement $M=\{m_i\}$ acts on party C. After the measurement, the particles belonging to party C vanish, but the postmeasurement information from the associated outcome is available. That is, each classical outcome i can be encoded into a new register system E as associated postmeasurement states τ_i. We say that this encoding is perfect if $\tau_i=|i\rangle\langle i|$ for an orthogonal basis $\{|i\rangle\}$. In practice, of course, the encoding may not be perfect due to the interaction with the environment.

We can write the above process as the operation

$$\Phi_C(\rho_{ABC})=\sum_i p_i\sigma_i\otimes\tau_i, \tag{1}$$

where $p_i=\mathrm{tr}(\rho_{ABC}m_i)$, $\sigma_i=\mathrm{tr}_C(\rho_{ABC}m_i)/p_i$, and τ_i is the register state related to the outcome i. We say that this encoding is perfect if $\tau_i=|i\rangle\langle i|$ for an orthogonal basis $\{|i\rangle\}$. In practice, of course, the encoding may not be perfect due to the interaction with the environment or the limited memory of the register.

We denote by $\mathcal{N}_C$ the set of all possible operations in the form in Eq. (1) on party C. We stress that the set $\mathcal{N}_C$ is equivalent to the set of entanglement-breaking channels [29] acting on party C. So far, we have not imposed any assumption on the destructive measurements and the encoding, but in practice, there can be extra limitations on them.

Our central question is how much the global entanglement in ρ_{ABC} is changed by the operation Φ_C. The maximal change happens usually when there is no classical information left or it has not been employed, that is, τ_i are the same for all outcomes i; a similar question was explored already under the concept of lockable entanglement [18] (see Sec. V for more details). Here we are particularly interested in the minimal amount of entanglement change with remaining classical information which corresponds to the optimal operation Φ_C to keep as much entanglement as possible.

For this purpose, we define the quantity $\Delta_{\mathcal{E}}(\rho_{ABC})$ as

$$\Delta_{\mathcal{E}}(\rho_{ABC})=\min_{\Phi_C\in\mathcal{N}_C}\{\mathcal{E}[\rho_{ABC}]-\mathcal{E}[\Phi_C(\rho_{ABC})]\}, \tag{2}$$

where $\mathcal{E}$ is a tripartite entanglement measure. The practical choice of $\mathcal{E}$ may depend on the quantum information task under consideration. For the choice of entanglement measures, it is necessary to require that $\mathcal{E}$ does not increase under local operations and classical communication (LOCC) [30], called monotonicity under LOCC. In this case, $\Delta_{\mathcal{E}}(\rho_{ABC})$ is always non-negative.

Two further remarks are in order. First, if $\mathcal{E}$ is a measure of genuine multipartite entanglement, then $\Delta_{\mathcal{E}}(\rho_{ABC})=\mathcal{E}[\rho_{ABC}]$ since $\Phi_C(\rho_{ABC})$ is always separable with respect to the bipartition $AB|C$ for any Φ_C and ρ_{ABC}. Second, if we restrict the set $\mathcal{N}_C$ with limitations on measurements and register states, the amount of $\Delta_{\mathcal{E}}(\rho_{ABC})$ can be affected. One example is to consider the operations which keep the dimension of the system.

III. SIMPLIFICATION

In general it is difficult to calculate $\Delta_{\mathcal{E}}(\rho_{ABC})$ due to the complexity of characterizing the set $\mathcal{N}_C$. Here we provide a method to simplify the calculation. By default, we assume the entanglement measure $\mathcal{E}$ is monotonic under LOCC. Then we have the following observation.

Observation 1. If the entanglement measure $\mathcal{E}$ is convex, we only need to consider $M=\{m_i\}$ as an extremal point in the considered measurement set $\mathcal{M}$. More precisely,

$$\Delta_{\mathcal{E}}(\rho_{ABC})=\min_{M\in\partial\mathcal{M}}\left\{\mathcal{E}[\rho_{ABC}]-\sum_i p_i\mathcal{E}[\sigma_i\otimes|0\rangle\langle 0|]\right\}, \tag{3}$$

where $\partial\mathcal{M}$ is the set of extremal points in $\mathcal{M}$, $p_i=\mathrm{tr}(\rho_{ABC}m_i)$ and $\sigma_i=\mathrm{tr}_C(\rho_{ABC}m_i)/p_i$.

The proof of Observation 1 is given in Sec. A in the Supplemental Material [31]. The Observation shows that the actual calculation of $\Delta_{\mathcal{E}}(\rho_{ABC})$ can be reduced to the set of extremal points in $\mathcal{M}$, which was well characterized in Ref. [32]. In the following, we will address this problem for two special cases. In the first case the party C is a qubit, and the measurement information from the outcomes is also registered in a qubit system E [33]. For convenience, we denote by $\mathcal{N}_1$ the set of those operations, which is equivalent to the set of all entanglement-breaking channels mapping qubit to qubit. In the second case the measurement M is a dichotomic positive operator-valued measure [32], where C is not necessarily a qubit. We denote this set as $\mathcal{N}_2$.

Now we can present the following observation.

Observation 2. For a convex entanglement measure $\mathcal{E}$, if we replace $\mathcal{N}_C$ by $\mathcal{N}_1$ or $\mathcal{N}_2$ in the definition of $\Delta_{\mathcal{E}}$, then the value of $\Delta_{\mathcal{E}}(\rho_{ABC})$ can be achieved with projective measurements.

The proof of Observation 2 is given in Sec. B in the Supplemental Material [31]. Observations 1 and 2 make the numerical calculation possible with only a few parameters as in the following examples.

A. Example: Three-qubit systems

Here we look at three-qubit systems and analyze $\Delta_{\mathcal{E}}(\rho_{ABC})$ with $\mathcal{N}_1$ and $\mathcal{N}_2$. Important examples of multipartite entanglement measures that satisfy convexity and monotonicity under

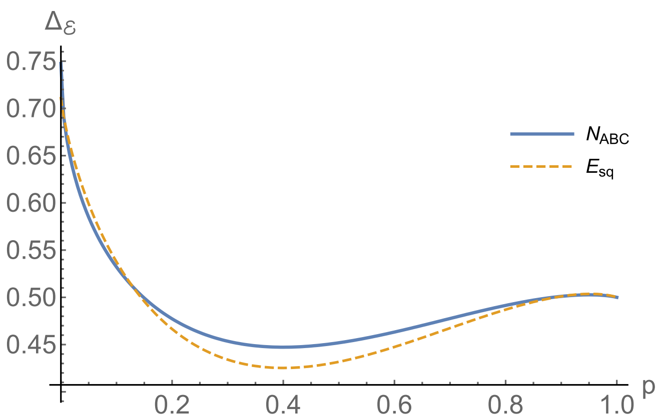

FIG. 2. $\Delta_\mathcal{E}$ with N_{ABC} and E_{sq} for $|\psi(p)\rangle = \sqrt{p}\,|\text{GHZ}\rangle + \sqrt{1-p}\,|\text{W}\rangle$.

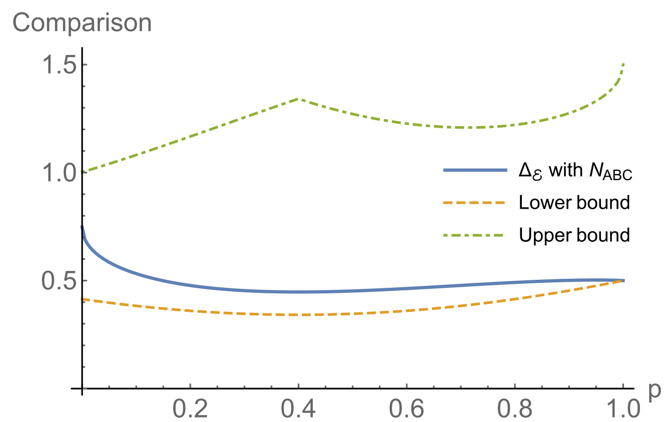

FIG. 3. Comparison between $\Delta_\mathcal{E}$ with N_{ABC} and its lower and upper bounds for the state $|\psi(p)\rangle = \sqrt{p}\,|\text{GHZ}\rangle + \sqrt{1-p}\,|\text{W}\rangle$.

LOCC are the multipartite negativity [34] and multipartite squashed entanglement [21,35]:

$$N_{ABC}(\rho_{ABC}) = N_{AB|C} + N_{BC|A} + N_{AC|B}, \tag{4}$$

$$E_{sq}(\rho_{ABC}) = \min_{\gamma_{ABCX}} \tfrac{1}{2} I(A:B:C|X). \tag{5}$$

Here $N_{X|Y} = |\sum_{\lambda_i<0} \lambda_i|$ is the negativity for a bipartition $X|Y$ with eigenvalues λ_i of the partially transposed state ρ^{T_Y} with respect to subsystem Y, where $Y = A, B, C$. Also, $I(A:B:C|X) = S(AX) + S(BX) + S(CX) - S(ABCX) - 2S(X)$ is the quantum conditional mutual information, where γ_{ABCX} is any extension of ρ_{ABC}, i.e., $\rho_{ABC} = \text{tr}_X[\gamma_{ABCX}]$, and $S(M)$ is the von Neumann entropy of system M. For a pure state ρ_{ABC}, the quantum conditional mutual information can be simplified as $I(A:B:C|X) = S(A) + S(B) + S(C)$, which is independent of system X.

As the first example, we consider the superposition of Greenberger-Horne-Zeilinger (GHZ) states and W states:

$$|\psi(p)\rangle = \sqrt{p}\,|\text{GHZ}\rangle + \sqrt{1-p}\,|W\rangle\,, \tag{6}$$

where $0 \leqslant p \leqslant 1$, $|\text{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$, and $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$. The numerical relation between $\Delta_\mathcal{E}$ and p is presented in Fig. 2 for $\mathcal{E} = N_{ABC}, E_{sq}$; details about the optimization method are given in in Sec. C in the Supplemental Material [31]. Interestingly, we find that the maximal value of $\Delta_\mathcal{E}(|\psi\rangle)$ is given by the W state, while the minimal value is not achieved by the GHZ state but the state at $p = 0.4$. We remark that both $N_{ABC}(\,|\psi(p)\rangle\,)$ and $E_{sq}(\,|\psi(p)\rangle\,)$ are minimized when $p = 0.4$. However, it is an open problem to understand why this state also has minimal entanglement change.

Moreover, let us consider a three-qutrit case and compute the tuple of $\Delta_\mathcal{E}$ for $\mathcal{E} = (N_{ABC}, E_{sq})$. The GHZ state $\sum_{i=0}^{2} |iii\rangle\,/\sqrt{3}$ has $(1.667,\ 0.792489)$, while the state $(|012\rangle + |120\rangle + |201\rangle + |021\rangle + |210\rangle + |102\rangle)/\sqrt{6}$ has $(1.86747,\ 0.971332)$. More details are given in Sec. C in the Supplemental Material [31].

IV. GENERAL BOUNDS

In general, it may be hard to obtain the exact value of $\Delta_\mathcal{E}(\rho_{ABC})$ for some entanglement measure $\mathcal{E}$. To address this situation, we now derive upper and lower bounds that can be used for the estimation. First, we present a general lower bound.

Observation 3. For a convex entanglement measure $\mathcal{E}$ and for the set $\mathcal{N}_C$, we have

$$\Delta_\mathcal{E}(\rho_{ABC}) \geqslant \min_{|x\rangle} \{\mathcal{E}[\rho_{ABC}] - \mathcal{E}[\sigma_{|x\rangle} \otimes |0\rangle\langle 0|]\}, \tag{7}$$

where $|x\rangle$ is a measurement direction on party C and $\sigma_{|x\rangle} = \langle x|\rho_{ABC}|x\rangle/\text{tr}[\langle x|\rho_{ABC}|x\rangle]$ is a normalized state.

The proof of Observation 3 is given in Sec. D in the Supplemental Material [31]. This lower bound can be used to characterize the complete entanglement loss, as we will see in Sec. VI.

Furthermore, suppose that we remove all the classical information of the measurement outcomes, that is, we encode all the measurement outcomes into the same state $|0\rangle$. Then we find an upper bound:

$$\Delta_\mathcal{E}(\rho_{ABC}) \leqslant \tilde{\Delta}_\mathcal{E}(\rho_{ABC}) \tag{8}$$

for any convex entanglement measure $\mathcal{E}$, where

$$\tilde{\Delta}_\mathcal{E}(\rho_{ABC}) = \mathcal{E}[\rho_{ABC}] - \mathcal{E}[\rho_{AB} \otimes |0\rangle\langle 0|], \tag{9}$$

with $\rho_{AB} = \text{tr}_C(\rho_{ABC})$. We remark that $\tilde{\Delta}_\mathcal{E}(\rho_{ABC})$ is the maximal entanglement change since we can always map any encoding into the state $|0\rangle\langle 0|$ with a local operation on the system C.

Let us compare $\Delta_\mathcal{E}$ with its lower and upper bounds using the tripartite negativity N_{ABC}. Figures 3 and 4 illustrate the cases of the pure three-qubit state $|\psi(p)\rangle$ in Eq. (6) and the mixed three-qubit state $\rho(q) = q\rho_{\text{GHZ}} + (1-q)\rho_{\text{W}}$, where $\rho_{\text{GHZ}} = |\text{GHZ}\rangle\langle\text{GHZ}|$ and $\rho_{\text{W}} = |W\rangle\langle W|$. We find that the lower bound is relatively close to $\Delta_\mathcal{E}$, especially if the state approximates the GHZ state. The gap between $\Delta_\mathcal{E}$ and $\tilde{\Delta}_\mathcal{E}$ shows that the postmeasurement information is more relevant for the GHZ state than for the W state.

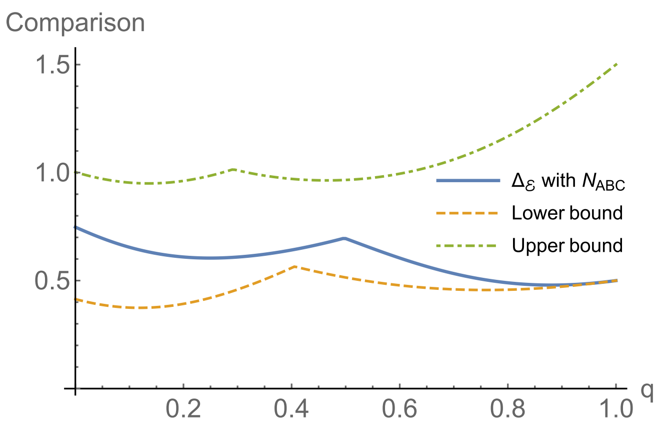

FIG. 4. Comparison between $\Delta_{\mathcal{E}}$ with N_{ABC} and its lower and upper bounds for the state $\rho(q) = q\rho_{\rm GHZ} + (1-q)\rho_{\rm W}$.

Next, let us connect entanglement change to quantum discord. For that, we consider the multipartite relative entropy of entanglement, which is the sum of the relative entropies of entanglement [36] for all bipartitions, i.e.,

$$R_{ABC}(\rho_{ABC}) = R_{AB|C} + R_{BC|A} + R_{AC|B}, \tag{10}$$

where $R_{X|Y} = \min_{\sigma\in\rm SEP} S(\rho_{XY}||\sigma)$ is the relative entropy of entanglement for a bipartition $X|Y$, $S(\rho||\sigma) = \mathrm{tr}[\rho\,(\log_2\rho - \log_2\sigma)]$ is the von Neumann relative entropy, and SEP is the set of bipartite separable states.

Similarly, the amount of quantum discord [37] can be also measured by the relative entropy: $D_{XY}(\rho_{XY}) = \min_{\rho'\in\Lambda} S(\rho_{XY}||\rho')$, where Λ is the set of quantum-classical states $\rho' = \sum_i p_i\sigma_i \otimes |i\rangle\langle i|$ with orthonormal basis $\{|i\rangle\}$. Now we can formulate the following two observations.

Observation 4. When the entanglement measure $\mathcal{E}$ is the tripartite relative entropy of entanglement R_{ABC}, we have

$$R_{AB|C}(\rho_{ABC}) \leqslant \Delta_{\mathcal{E}}(\rho_{ABC}) \leqslant 3D_{AB|C}(\rho_{ABC}). \tag{11}$$

Observation 5. More generally, if $D_{AB|C}(\rho_{ABC}) = 0$, then we have $\Delta_{\mathcal{E}}(\rho_{ABC}) = 0$ for any entanglement measure $\mathcal{E}$.

The proofs of Observations 4 and 5 are given in Secs. E and F in the Supplemental Material [31]. From Observation 5, the condition $D_{AB|C}(\rho_{ABC}) = 0$ is a sufficient condition for $\Delta_{\mathcal{E}}(\rho_{ABC}) = 0$ for any measure $\mathcal{E}$. On the other hand, this is not a necessary condition. For instance, if the initial state ρ_{ABC} is fully separable, clearly, $\Delta_{\mathcal{E}}(\rho_{ABC}) = 0$, but this does not mean $D_{AB|C}(\rho_{ABC}) = 0$. From the conceptional perspective, quantum discord is the difference in quantum correlation before and after a projective measurement, whereas $\Delta_{\mathcal{E}}(\rho_{ABC})$ quantifies the difference in entanglement, which is only one sort of quantum correlation.

V. LOCKABILITY

Previous works [18–20] have studied a similar issue under the name of lockability of entanglement measures. In that case, one asks for the quantitative change in entanglement by the loss of one particle (e.g., one qubit) *within* one party. For example, in the bipartite scenario, one considers the situation where Alice and Bob both have five qubits, and then one asks how the entanglement changes if Alice loses one of her qubits. If the entanglement change can be arbitrarily large, the entanglement measure is called lockable. For instance, all convex entanglement measures are known to be lockable, while the relative entropy of entanglement is not [18].

The lockable entanglement is related to our consideration in the following sense. For a given tripartite state ρ_{ABC}, if we choose the convex entanglement measure $\mathcal{E}$ to measure only the entanglement in the bipartition $A|BC$ (or $AC|B$), then $\tilde{\Delta}_{\mathcal{E}}$ defined in Eq. (9) is the quantity considered in lockable entanglement. More precisely, for any convex entanglement measure $\mathcal{E}$ for the bipartition $A|BC$, we have

$$\tilde{\Delta}_{\mathcal{E}}(\rho_{ABC}) = \mathcal{E}[\rho_{ABC}] - \mathcal{E}[\rho_{AB}], \tag{12}$$

where we used $\mathcal{E}[\rho_{AB}\otimes|0\rangle\langle 0|] = \mathcal{E}[\rho_{AB}]$; see Theorem 2 in Ref. [38].

In order to understand the difference between the behavior of entanglement under classicalization and the lockability problem, one has to analyze the role of the information coming from the measurement results. We know already from Figs. 3 and 4 that this information makes some difference for the entanglement change. In the following, we will show that this difference can be arbitrarily large.

A. Example: Flower state

First, let us consider the so-called flower state in $(d\otimes d\otimes 2)$-dimensional systems [19]:

$$\omega_{ABC} = \frac{2}{d(d+1)}P_{AB}^{(+)} \otimes \frac{d+1}{2d}|0\rangle\langle 0|_C + \frac{2}{d(d-1)}P_{AB}^{(-)} \otimes \frac{d-1}{2d}|1\rangle\langle 1|_C, \tag{13}$$

where $P_{AB}^{(\pm)}$ are the projections onto the symmetric and antisymmetric subspaces, that is, $P_{AB}^{(\pm)} = (\mathbb{1}_{AB} \pm V_{AB})/2$ with the SWAP operator V_{AB} acting as $V_{AB}\,|v_A\rangle\otimes|v_B\rangle = |v_B\rangle\otimes|v_A\rangle$.

Notice that the quantum discord of ω_{ABC} for the bipartition $AB|C$ is zero, i.e., $D_{AB|C}(\omega_{ABC}) = 0$. From Observation 5, we conclude that $\Delta_{\mathcal{E}}(\omega_{ABC}) = 0$ for any entanglement measure $\mathcal{E}$. However, we have $\tilde{\Delta}_{\mathcal{E}}(\omega_{ABC}) = \mathcal{E}(\omega_{ABC}) > 0$ because $\mathrm{tr}_C(\omega_{ABC})\otimes|0\rangle\langle 0|$ is fully separable. In fact, if the entanglement measure $\mathcal{E}$ is taken to be the squashed entanglement, then $\mathcal{E}(\omega_{ABC})$ can be arbitrarily large [19]. This directly implies that the difference $\tilde{\Delta}_{\mathcal{E}} - \Delta_{\mathcal{E}}$ can be arbitrarily large if d is properly chosen. Hence, although the information from the measurement at the flower state is only one bit, a large amount of entanglement can be saved by collecting it.

B. Example: *n* pairs of Bell states

On the other hand, we will see that the entanglement change $\Delta_{\mathcal{E}}$ can also be arbitrarily large even if only one qubit has become classical. As an example, let us consider a pure state made of n pairs of Bell states $|\Psi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. We label the ith pair of particles by a_i, b_i. Suppose that party A owns particles $\{a_i\}_{i=1}^n$, party B owns particles $\{b_i\}_{i=1}^{n-1}$, and party C owns particle b_n. We denote this state as $\beta_{ABC} =$

$|\Psi^+\rangle\langle\Psi^+|^{\otimes n}$. Now we can present the following observation, which is proven in Sec. G in the Supplemental Material [31].

Observation 6. For the entanglement measure $\mathcal{E}$ to be the tripartite negativity N_{ABC}, we have

$$\Delta_{\mathcal{E}}(\beta_{ABC}) = 2^{n-2} + 1/2. \tag{14}$$

Thus, $\Delta_{\mathcal{E}}(\beta_{ABC})$ can be arbitrary large if n is properly chosen.

Inspired by those two examples, an interesting question arises about whether entanglement measures $\mathcal{E}$ and states ρ_{ABC} exist such that both $\Delta_{\mathcal{E}}(\rho_{ABC})$ and $\tilde{\Delta}_{\mathcal{E}}(\rho_{ABC}) - \Delta_{\mathcal{E}}(\rho_{ABC})$ can be arbitrarily large in the sense that they are not limited by the size of C, even if C is only a qubit. We leave this question for further research.

VI. COMPLETE ENTANGLEMENT LOSS UNDER CLASSICALIZATION

By definition, $\Delta_{\mathcal{E}}(\rho_{ABC}) \leqslant \mathcal{E}[\rho_{ABC}]$ always holds. We are now concerned about the case where this inequality is saturated, i.e., $\Delta_{\mathcal{E}}(\rho_{ABC}) = \mathcal{E}[\rho_{ABC}]$, or, equivalently, $\max_{\Phi_C\in\mathcal{N}_C} \mathcal{E}[\Phi_C(\rho_{ABC})] = 0$.

First of all, Observation 3 implies a sufficient condition for complete entanglement loss under classicalization, which can be formulated as follows.

Condition 1. If, after a projective measurement in any direction $|x\rangle$ on C, the postmeasurement state $\sigma_{|x\rangle} \propto \langle x|\rho_{ABC}|x\rangle$ is always separable, then the entanglement is completely lost under classicalization.

Clearly, Condition 1 is stronger than the condition that the reduced state ρ_{AB} is separable. For instance, let us consider the GHZ state. Its reduced state $\mathrm{tr}_C[\rho_{\mathrm{GHZ}}]$ is separable, but its postmeasurement state $\sigma_{|x\rangle}$ can be entangled if measurement bases are $\{|+\rangle\,, |-\rangle\}$.

The existence of genuine multipartite entangled states which satisfy Condition 1, however, was already reported in Ref. [39]. We propose observations using Condition 1 and provide more examples in Secs. H and I in the Supplemental Material [31].

VII. CONCLUSION AND DISCUSSION

Multiparticle quantum entanglement is an important quantum resource, and the preservation of entanglement is a practical issue. We have studied the change in multiparticle entanglement under classicalization of one particle. Clearly, the results usually depend on the choice of the entanglement quantifier, and the change in entanglement is difficult to compute. We provided simplifications for important special scenarios and upper and lower bounds for the general case. One crucial question is whether one small part like one qubit can change quantum resources like quantum entanglement a lot. Our results showed that the entanglement change can still be arbitrarily large even with complete measurement information remaining. In addition, the measurement information can also make an arbitrarily large difference. Finally, we provided conditions under which quantum entanglement is always completely lost under classicalization.

While we focused on the difference between the original quantum resource and the remaining resource if one party becomes classical, the behavior of quantum resources *during* the quantum to classical transition is also interesting, and it may have a richer theoretical structure. We believe that our work paves the way to designing concepts for quantum resource storage and may help in the development of a different direction in the field of quantum resource theories.

ACKNOWLEDGMENTS

We thank H. C. Nguyen, M. Plávala, B. Yadin, and X.-D. Yu for discussions. This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation, Projects No. 447948357 and No. 440958198), the Sino-German Center for Research Promotion (Project No. M-0294), the ERC (Consolidator Grant No. 683107/TempoQ), the German Ministry of Education and Research (Project QuKuK, BMBF Grant No. 16KIS1618K), the Humboldt Foundation, and the DAAD.

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