Letter

Observing single particles beyond the Rindler horizon

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We show that Minkowski single-particle states localized beyond the horizon modify the Unruh thermal distribution in an accelerated frame. This means that, contrary to classical predictions, accelerated observers can reveal particles emitted beyond the horizon. The method we adopt is based on deriving the explicit Wigner characteristic function for the complete description of the quantum field in the noninertial frame and can be generalized to general states.

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In classical physics, an observer in an accelerated frame cannot detect signals emitted beyond an event horizon. One can argue if this is also true in quantum physics. Starting from pioneering investigations [1,2], many authors have studied how accelerated observers can register inertial vacuum states through thermal particle detection. Here, we ask if inertial single-particle states localized beyond the horizon can be revealed by monitoring variations of the particle distribution from the thermal background. To answer this question, we adopt quantum field theory in curved space-time [3], developed in the last decades with groundbreaking results as the Hawking [4] and the Unruh effect [1,5,6].

In the Unruh effect, a thermal state replaces the vacuum when the observer is accelerated, as an outcome of the fact that quantum states are reference-frame dependent [5]. Beyond the vacuum, various authors considered more general states focusing on entangled systems (see Refs. [7–9] and references therein). Indeed, entanglement is significant because of the quantum correlation between two regions of space-time, denoted as the Rindler left and right wedge, and the need to trace over one of the two wedges to predict observable quantities in accelerate frames.

A way to describe the transition from Minkowski to Rindler frames can be made using Wigner distributions [10]. Recently, Ben-Benjamin, Scully, and Unruh have reported [10] the Wigner distribution for the Minkowski vacuum state in the right wedge and the Minkowski number states in both the right and the left wedges. However, to the best of our knowledge, the explicit expression for Minkowski number states in the right wedge, tracing out the left wedge, is still missing.

In this Letter, we compute the characteristic function [11] of single-particle states in accelerated frames. From the characteristic function, we derive the probability of finding a Rindler particle when a Minkowski particle is emitted. We show that there is a finite probability of detecting a Rindler particle as a perturbation to the Unruh thermal background, even when the Minkowski particle is localized beyond the horizon.

By following the original works of Fulling, Daviss and Unruh [1,5,6], we consider a $(1-1)$-dimensional flat space-time and the coordinate transformation (t_R, x_R) from an accelerated frame (T, X) to an inertial frame $(t, x) = [t_R(T, X), x_R(T, X)]$, $act_R(T, X) = e^{aX} \sinh acT$ and $ax_R(T, X) = e^{aX} \cosh acT$, where ac^2 is the acceleration, which is conventionally taken positive, and c the speed of light. Such a transformation covers only the right Rindler wedge. On the other hand, it is possible to cover the left Rindler wedge through the transformation $t = t_L(T, X)$ and $x = x_L(T, X)$ with $t_L(T, X)$ and $x_L(T, X)$ being identical to $t_R(T, X)$ and $x_R(T, X)$ but with opposite acceleration. In the notation we have adopted, the subscript L (R) refers to the left (right) wedge. It is possible to see a visual representation of the Minkowski and Rindler coordinates in Fig. 1, where (T_L, X_L) and (T_R, X_R) are taken as the inverse transformations of (t_L, x_L) and (t_R, x_R). However, since we are not interested in temporal evolutions of states, for the rest of this Letter we will just refer to $x_{L,R}(X)$ and $X_{L,R}(x)$ as, respectively $x_{L,R}(0, X)$ and $X_{L,R}(0, x)$.

By following again the original works of Fulling, Davis, and Unruh [1,5,6], we consider a massless free scalar field $\hat{\phi}(t, x)$. We name $\hat{a}(k)$ the annihilation operator for the Minkowski mode with momentum k, while $\hat{b}_L(K)$ $[\hat{b}_R(K)]$ the annihilation operators for the left (right) Rindler mode with momentum K.

The Unruh effect can be obtained by representing the Minkowski vacuum state $|0_M\rangle$, defined by $\hat{a}(k)|0_M\rangle = 0$ for any $k \in \mathbb{R}$, in the representation space of the $\hat{b}_{L,R}(K)$ algebra. This leads to the following state [1],

$$|0_M\rangle \propto \exp\left[\int_{-\infty}^{+\infty} dK \exp\left(-\frac{\beta}{2}|K|\right) \times \hat{b}_L^\dagger(K)\hat{b}_R^\dagger(K)\right]|0_L, 0_R\rangle, \quad (1)$$

with $\beta = 2\pi/a$ and $|0_{L,R}\rangle$ defined by $\hat{b}_{L,R}(K)|0_{L,R}\rangle = 0$. The final expression for the Minkowski vacuum state in the right Rindler frame can be obtained by performing a partial

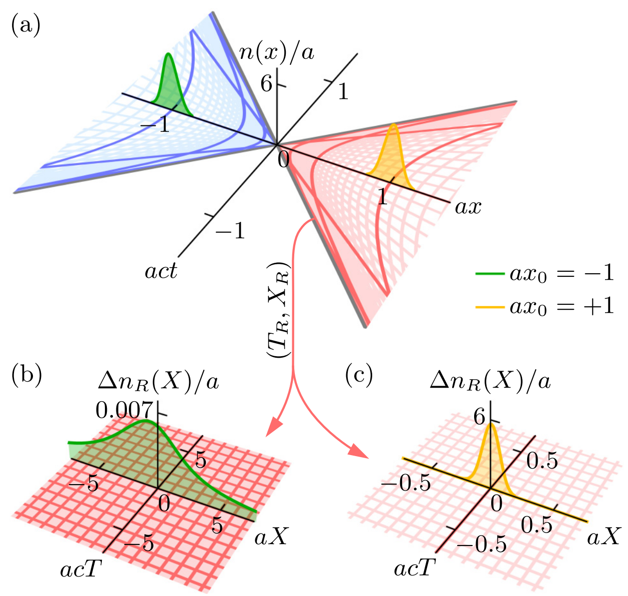

FIG. 1. Representation of the coordinate transformation $(t, x) \mapsto (T, X) = [T_R(t, x), X_R(t, x)]$ and the probability density function transformation $n(x) \mapsto \Delta n_R(X)$ for localized wave packets from the inertial to the accelerated frame. The yellow and the green lines are associated to single-particle states with Gaussian wave function $\psi(x)$ defined by Eq. (16) with $a\sigma = 1$ and $ax_0 = \pm 1$. The Rindler left and right wedges are shown in (a) through constant T and X lines. The two wedges are delimited by the Rindler horizons (gray). The profile of the probability density $n(x)$, defined by Eq. (13), for the two wave packets has been drawn. In (b) and (c), we show the constant T and X lines in the accelerated frame and the profile of $\Delta n_R(X)$, defined by Eq. (12a). The value of $\Delta n_R(X)$ gives the variation of probability density to find a Rindler particle in X with respect to the Minkowski vacuum. In (b), $\Delta n_R(X)$ is non-negligible, even if $\psi(x)$ is localized in the left wedge. On the other hand, in (c), $\Delta n_R(X)$ is larger and narrower for $ax_0 = +1$, as we expect from wave functions localized within the right wedge.

trace over the left wedge, which leads to a thermal state $\hat{\rho}_0$ with temperature $T_0 = (k_B\beta)^{-1}$, where k_B is the Boltzmann constant.

Analogously to $|0_M\rangle$, any Minkowski single-particle state $|\psi\rangle$ can be represented in the right wedge through a representative in the $\hat{b}_{L,R}(K)$ algebra and by performing a partial trace over the left wedge $\hat{\rho} = \mathrm{Tr}_L|\psi\rangle\langle\psi|$. Here, $|\psi\rangle$ is defined through a normalized wave function $\psi(x)$ such that

$$|\psi\rangle = \int_{-\infty}^{+\infty} dx\psi(x)\hat{a}^\dagger(x)|0_M\rangle, \tag{2}$$

where $\hat{a}^\dagger(x) = \int_{-\infty}^{+\infty} dk e^{-ikx}\hat{a}^\dagger(k)/\sqrt{2\pi}$ is the creation operator for a particle in position x.

Equation (2) can be put into the following form,

$$|\psi\rangle = \int_{-\infty}^{+\infty} dK[\tilde{\psi}_-(K)\hat{b}_R(K) + \tilde{\psi}_+(K)\hat{b}_R^\dagger(K)]|0_M\rangle, \tag{3}$$

with

$$\tilde{\psi}_\pm(K) = \frac{e^{-\theta(\pm 1)\beta|K|}}{n_0(K)} \int_{-\infty}^{+\infty} dX \frac{e^{\mp iKX}}{\sqrt{2\pi}} \left\{\psi_R(X)\left[\theta(\pm 1) + \tilde{f}_{R\pm}\left(\mp\frac{K}{a}\right)\right] + \psi_L(-X)\tilde{f}_{L\pm}\left(\mp\frac{K}{a}\right)\right\}, \tag{4a}$$

$$\tilde{f}_{L,R\pm}(\kappa) = -\theta(s_{L,R})\theta(\pm 1) + \frac{1}{2\pi}\sqrt{|\kappa|}\Gamma(i\kappa)\Gamma\left(\frac{1}{2} - i\kappa\right) \times \exp\left[\pm\theta(s_{L,R})\pi|\kappa| \pm is_{L,R}\,\mathrm{sgn}(\kappa)\frac{\pi}{4}\right], \tag{4b}$$

$$\psi_{L,R}(X) = \sqrt{a|x_{L,R}(X)|}\psi(x_{L,R}(X)), \tag{4c}$$

$$s_L = -1, \quad s_R = 1, \quad n_0(K) = (e^{\beta|K|} - 1)^{-1}. \tag{4d}$$

A proof for Eq. (3) is given in the Supplemental Material (SM) [12]. The key element for such a proof is provided by the following identity,

$$\hat{b}_{L,R}^\dagger(K)|0_M\rangle = \exp\left(\frac{\beta}{2}|K|\right)\hat{b}_{R,L}(K)|0_M\rangle, \tag{5}$$

which holds for any $K \in \mathbb{R}$. Equation (5) states that the creation of a Rindler particle in the left (right) wedge over the Minkowski vacuum background is equivalent to the destruction of a Rindler particle in the right (left) wedge, up to an $\exp(\beta|K|/2)$ factor. Owing to Eq. (5), we can give the following interpretation to the functions $\tilde{\psi}_\pm(K)$ that appear in Eq. (3). $\tilde{\psi}_+(K)$ [$\tilde{\psi}_-(K)$] can be seen as the wave function of a Rindler particle created (destroyed) over the Minkowski vacuum background in the right wedge, or, up to an $\exp(\beta|K|/2)$ factor, as a Rindler particle destroyed (created) in the left wedge.

$\psi_R(X)$, on the other hand, can be interpreted as a transformed version of the wave function $\psi(x)$ in terms of the infinitesimal probability function $n(x)dx = |\psi(x)|^2dx$. Indeed, from Eq. (4c), it is possible to notice that for $x > 0$, $|\psi(x)|^2dx$ is equivalent to $|\psi_R(X)|^2dX$, up to the coordinate transformation $x \mapsto X = X_R(x)$.

By taking the partial trace of $|\psi\rangle\langle\psi|$ over the left wedge, Eq. (3) results in the following expression for the transformed single-particle state,

$$\hat{\rho} = \int_{-\infty}^{+\infty} dK[\tilde{\psi}_-(K)\hat{b}_R(K) + \tilde{\psi}_+(K)\hat{b}_R^\dagger(K)]\hat{\rho}_0 \times \int_{-\infty}^{+\infty} dK'[\tilde{\psi}_-^*(K')\hat{b}_R^\dagger(K') + \tilde{\psi}_+^*(K')\hat{b}_R(K')]. \tag{6}$$

As we have mentioned before, an alternative representation for the state $\hat{\rho}$ can be provided through the following characteristic function [11],

$$\chi[\xi, \xi^*] = \mathrm{Tr}\left[\hat{\rho}\exp\left(\int_{-\infty}^{+\infty} dK\xi(K)\hat{b}_R^\dagger(K)\right) \times \exp\left(-\int_{-\infty}^{+\infty} dK\xi^*(K)\hat{b}_R(K)\right)\right]. \tag{7}$$

Owing to Eq. (6), we can write $\chi[\xi, \xi^*]$ in terms of functional derivatives of the characteristic function for the thermal state

$\chi_0[\xi,\xi^*]$ (SM [12]):

$$\chi[\xi,\xi^*] = \left\{1 - \left|\int_{-\infty}^{+\infty} dK n_0(K)[\tilde{\psi}_-(K)\xi(K) - e^{\beta|K|}\tilde{\psi}_+(K)\xi^*(K)]\right|^2\right\}\chi_0[\xi,\xi^*]. \quad (8)$$

Finally, the explicit expression for $\chi[\xi,\xi^*]$ can be obtained from Eq. (8) supplemented with Eqs. (4) and the already known expression for $\chi_0[\xi,\xi^*]$ [11]:

$$\chi_0[\xi,\xi^*] = \exp\left(-\int_{-\infty}^{+\infty} dK n_0(K)|\xi(K)|^2\right). \quad (9)$$

Functional derivatives of the characteristic function $\chi[\xi,\xi^*]$ allow us to extract different mean values of $\hat{\rho}$ [11]. In this way, we compute the probability density function of $\hat{\rho}$, defined as

$$\langle \hat{n}_R(X)\rangle_{\hat{\rho}} = \int_{-\infty}^{+\infty} dK \frac{e^{-iKX}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dK' \frac{e^{iK'X}}{\sqrt{2\pi}} \times \frac{\delta}{\delta\xi(K)}\left(-\frac{\delta}{\delta\xi^*(K')}\right)\chi[\xi,\xi^*]|_{\xi=0}, \quad (10)$$

where $\hat{n}_R(X) = \hat{\tilde{b}}_R^\dagger(X)\hat{\tilde{b}}_R(X)$ is the particle density operator and $\hat{\tilde{b}}_R(X) = \int_{-\infty}^{+\infty} dK e^{iKX}\hat{b}_R(K)/\sqrt{2\pi}$ is the annihilation operator in X. Equation (10) results into the following equation (SM [12]),

$$\Delta n_R(X) = n_+(X) + n_-(X), \quad (11)$$

with

$$\Delta n_R(X) = \langle \hat{n}_R(X)\rangle_{\hat{\rho}} - \langle \hat{n}_R(X)\rangle_{\hat{\rho}_0}, \quad (12a)$$

$$n_\pm(X) = \left|\int_{-\infty}^{+\infty} dK \frac{e^{\pm iKX}}{\sqrt{2\pi}} n_0(K) e^{\theta(\pm 1)\beta|K|}\tilde{\psi}_\pm(K)\right|^2. \quad (12b)$$

$\Delta n_R(X)$, defined by Eq. (12a), represents the difference in the probability density function between the Minkowski single-particle and the Minkowski vacuum state in terms of Rindler particles. An accelerated observer measuring a nonvanishing $\Delta n_R(X)$ can infer the presence of a Minkowski particle. Figure 1 shows $\Delta n_R(X)$ for Gaussian wave functions in comparison with the probability density function in the Minkowski space-time, defined as

$$n(x) = \langle \hat{a}^\dagger(x)\hat{a}(x)\rangle_{|\psi\rangle\langle\psi|}. \quad (13)$$

$n_\pm(X)$ derive from $\hat{\rho}$ of Eq. (6) through the contribution of, respectively, $\tilde{\psi}_\pm(K)$. Therefore, they are associated with the Rindler particles respectively created and destroyed over the Minkowski vacuum background in the right wedge. Their explicit form with respect to $\psi_{L,R}(X)$ reads (SM [12])

$$n_\pm(X) = |\theta(\pm 1)\psi_R(X) + \psi_{R\pm}(X) + \psi_{L\pm}(X)|^2, \quad (14)$$

with

$$\psi_{L,R\pm}(X) = \int_{-\infty}^{+\infty} d\xi\, \psi_{L,R}\left(s_{L,R}\frac{\xi}{a}\right) f_{L,R\pm}(\xi - aX), \quad (15a)$$

$$f_{L,R\pm}(\xi) = \int_{-\infty}^{+\infty} d\kappa \frac{e^{i\kappa\xi}}{2\pi}\tilde{f}_{L,R\pm}(\kappa). \quad (15b)$$

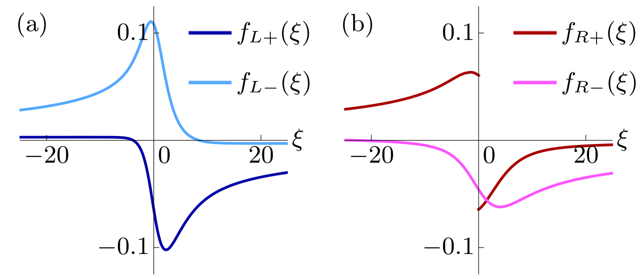

FIG. 2. Profiles of $f_{L,R\pm}(\xi)$ defined by Eq. (15b) which have been numerically derived through a Fourier transform of $\tilde{f}_{L,R\pm}(\xi)$ defined by Eq. (4b).

$\psi_R(X)$ and $\psi_{L,R+}(X)$ of Eq. (14) play the role of superposed wave functions which result in a probability density function $n_+(X)$ describing Rindler particles created over $|0_M\rangle$ in the right wedge. $\psi_{L,R-}(X)$, on the other hand, refer to Rindler particles destroyed in the right wedge. Within the decomposition of the wave functions $\psi_R(X) + \psi_{L+}(X) + \psi_{R+}(X)$ and $\psi_{L-}(X) + \psi_{R-}(X)$, $\psi_{L\pm}(X)$ derive from the left-wedge part of $\psi(x)$, i.e., $\psi(x)$ for $x < 0$, while $\psi_R(X)$ and $\psi_{R\pm}(X)$ from the right-wedge part of $\psi(x)$.

The presence of $\psi_{L\pm}(X)$ in Eq. (14) implies that values of the wave function beyond the horizon give nonvanishing contributions to $\langle \hat{n}_R(X)\rangle_{\hat{\rho}}$. Even a state with small values of $|\psi(x)|$ for $x < 0$ can still be detected in the right Rindler wedge. We remark that this effect is not due to the right tail of the wave function, since the corresponding contribution is exponentially smaller than the leading one, as detailed below with a specific example.

We can argue that the result may change if we use a Lorentz-invariant normalization for $\psi(x)$ [13], since left-wedge values of the wave function are normalization dependent. Nevertheless, we have verified that left-wedge values of $\psi(x)$ appear in $\Delta n_R(X)$ even when we use a Lorentz-invariant normalization (SM [12]).

$f_{L,R\pm}(\xi)$, shown in Fig. 2, are localized around $\xi = 0$. This means that $\Delta n_R(X)$ receives most contributions from $\psi_L(X')$ and $\psi_R(X')$ from $X' \approx -X$ and $X' \approx X$, respectively, and within a region $\Delta X' \sim a^{-1}$. In the case of Fig. 2(a), this implies that most of the contributions for $\psi_{L\pm}(X)$ come from $\psi_L(X')$ when $x_L(X') = -x_R(X)$, or, equivalently, from $\psi(-x_R(X))$. Moreover, wave functions localized in the left wedge, i.e., with small values of $|\psi(x)|$ for $x > 0$, are characterized by a $\Delta n_R(X)$ whose main contributions come from $\psi_{L\pm}(X)$, since $\psi_R(X)$ is defined by right-wedge values of $\psi(x)$. This means that $\Delta n_R(X) \approx |\psi_{L+}(X)|^2 + |\psi_{L-}(X)|^2$ and that most of the contributions for $\Delta n_R(X)$ come from $\psi(x)$, with x as the specular point of X in the Minkowski space-time with respect to the horizon, i.e., $x = -x_R(X)$.

It is also possible to notice from Fig. 2(b) that $|f_{R\pm}(\xi)| \ll 1$. Therefore, if $\psi(x)$ is localized in a region $\mathcal{R}$, i.e., $|\psi(x)|$ is small outside a finite region $\mathcal{R}$, and if $\mathcal{R}$ is in the right wedge and with a width $\Delta x \ll a^{-1}$, then $\psi_{R\pm}(X)$ are expected to be negligible with respect to $\psi_R(X)$. The same happens for states with $\Delta x \sim a^{-1}$ but with $\mathcal{R}$ far away from the origin with respect to a, i.e., $x \gg a^{-1}$ for any $x \in \mathcal{R}$. This last result can be motivated by the fact that the transformed region $x_R(\mathcal{R})$

becomes way smaller than a^{-1} when $x \gg a^{-1}$ for any $x \in \mathcal{R}$ and, therefore $\psi_R(X)$ is non-negligible within a region way smaller than a^{-1}. In summary, $|\psi_{R\pm}(X)| \ll |\psi_R(X)|$ for any X and for wave functions well localized in the right wedge, i.e., when $\Delta x \ll a^{-1}$ and $x \gtrsim a^{-1}$ for any $x \in \mathcal{R}$ or when $x \gg a^{-1}$ for any $x \in \mathcal{R}$. Moreover, for such states, $\psi_{L\pm}(X)$ are negligible and therefore $\Delta n_R(X) \approx |\psi_R(X)|^2$. In other words, $\psi_R(X)$ acts as a probability amplitude for wave functions well localized in the right wedge and it appears as the dominant term in Eq. (11), with $\psi_{L,R\pm}(X)$ as small corrective terms.

We remark that the wave function can still have infinite tails. To give a quantitative example, we consider a normalized Gaussian wave functions, whose localization degree is given by the variance σ:

$$\psi(x) = \frac{1}{\sqrt[4]{\pi}\sqrt{\sigma}} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right). \tag{16}$$

We are interested in the limit $a\sigma \to 0$ for fixed $x_0 \neq 0$ and the limit $ax_0 \to \pm\infty$ for fixed σ, which correspond to the case of well-localized states in the left or right wedge. It is possible to prove that for Gaussian wave functions the limit $a\sigma \to 0$ is equivalent to $a|x_0| \to \infty$ up to a translation of $\Delta n_R(X)$ with respect to X (SM [12]). More specifically, it is possible to prove that when $x_0 \neq 0$, any transformation $x_0 \mapsto \alpha x_0$ with $\alpha > 0$ acting on $\psi_R(X)$ is equivalent to $\sigma \mapsto \sigma/\alpha$, $aX \mapsto aX - \ln\alpha$. This also applies to $\psi_{L,R\pm}(X)$ and $\Delta n_R(X)$. Given the invariance under the transformation $x_0 \mapsto x_0/\alpha$, $\sigma \mapsto \sigma/\alpha$, $aX \mapsto aX - \ln\alpha$ for any $\alpha > 0$, the functions $\psi_R(X)$, $\psi_{L,R\pm}(X)$, and $\Delta n_R(X)$ can be put in a form depending on $\mathrm{sgn}(x_0)$, $\sigma/|x_0|$, and $X - X_R(|x_0|)$ instead of σ, x_0, and X. This feature is adopted in Fig. 3, where we show $\Delta n_R(X)$ for different σ/x_0.

The limit of well-localized wave functions is identified with $\sigma/|x_0| \to 0$. In Fig. 3, we show how Gaussian wave functions give the same results expected for the general case. Specifically, Fig. 3(a) shows that for wave packets well localized in the left wedge, $\Delta n_R(X) \approx |\psi_{L+}(X)|^2 + |\psi_{L-}(X)|^2$. Figure 3(b) shows that $\Delta n_R(X) \approx |\psi_R(X)|^2$ when $\sigma/|x_0| \to 0$ and $x_0 > 0$. This result can be proven analytically (SM [12]),

$$|\psi_R(X)|^2 = \begin{cases} \delta(X - X_R(x_0)) + O\left[\frac{|x_0|}{\sigma}\exp\left(-\frac{|x_0|^2}{2\sigma^2}\right)\right] & \text{if } x_0 > 0, \\ O\left[\frac{|x_0|}{\sigma}\exp\left(-\frac{x_0^2}{2\sigma^2}\right)\right] & \text{if } x_0 < 0, \end{cases} \tag{17a}$$

$$|\psi_{L,R\pm}(X)|^2 = \begin{cases} F_{L,R\pm}(X) + o\left(\frac{\sigma}{|x_0|}\right) & \text{if } s_{L,R}x_0 > 0, \\ o\left[\frac{\sigma}{|x_0|}\exp\left(-\frac{x_0^2}{2\sigma^2}\right)\right] & \text{if } s_{L,R}x_0 < 0, \end{cases} \tag{17b}$$

with

$$F_{L,R\pm}(X) = \frac{\sigma}{|x_0|} 2\sqrt{\pi} a f_{L,R+}^2[aX_R(|x_0|) - aX]. \tag{18}$$

From Eqs. (17) we obtain the explicit limit $\sigma/|x_0| \to 0$ of $\Delta n_R(X)$ for Gaussian wave functions. When $x_0 < 0$, $\Delta n_R(X) \to 0$ with leading term $F_{L+}(X) + F_{L-}(X)$ which is proportional to $\sigma/|x_0|$. When the degree of localization of the particle increases, the probability of detection in the right wedge decreases. Nevertheless, if $\sigma \ll |x_0|$ but $\sigma \neq 0$, the profile of $\Delta n_R(X)$ is approximately $F_{L+}(X) + F_{L-}(X)$, as in Fig. 3(a). Accelerated observers can still see a difference with respect to the vacuum state, even when the particle is localized beyond the horizon. The result does not depend on the presence of a tail in the right Rindler wedge, since most of the contributions for $\Delta n_R(X)$ come from values of $\psi(x)$ beyond the horizon. Indeed, $\psi_R(X)$ and $\psi_{R\pm}(X)$ are vanishing with exponential orders, while $\psi_{L\pm}(X)$ are linear in $\sigma/|x_0|$.

The peak of $F_{L+}(X) + F_{L-}(X)$ in $X = X_R(|x_0|)$ results in a maximum probability to find the particle in $X = X_R(-x_0)$.

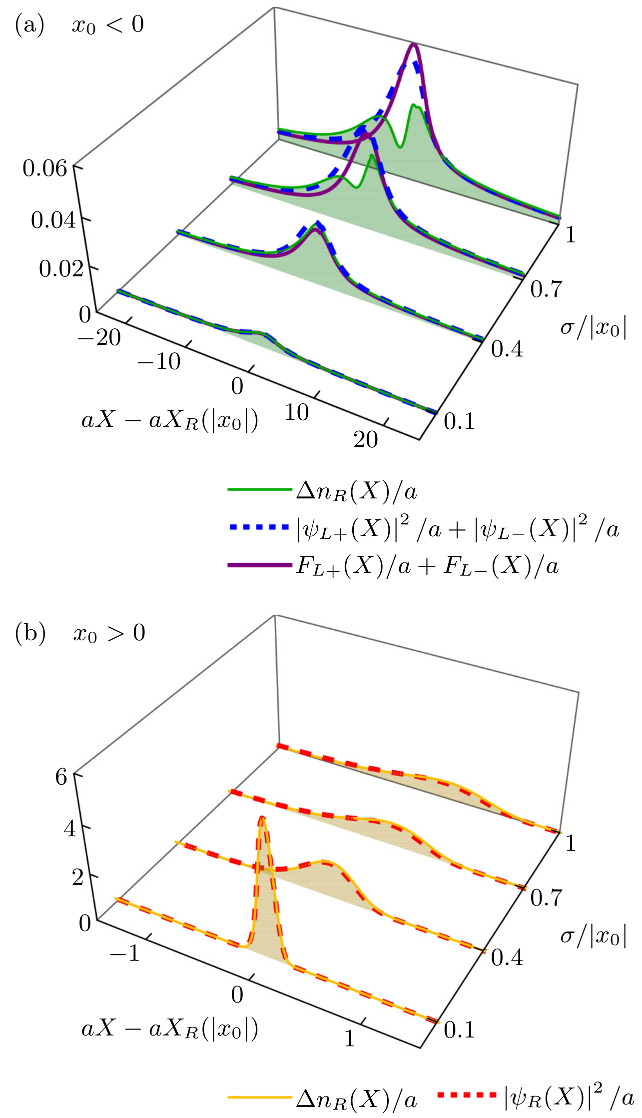

FIG. 3. Profile of $\Delta n_R(X)$ defined by Eq. (12a) for different Gaussian single-particle states (16). In (a), the configurations are defined by $x_0 < 0$ and different values of $\sigma/|x_0|$. $|\psi_{L+}(X)|^2 + |\psi_{L-}(X)|^2$ is the dominant contribution of Eq. (11) when $\sigma/|x_0| \to 0$ and $x_0 < 0$. We also show the function $F_{L+}(X) + F_{L-}(X)$ defined in Eq. (18). In (b), the profile of $\Delta n_R(X)$ and $|\psi_R(X)|^2$ for configurations with $x_0 > 0$ is shown. In this case, the dominant contribution to $\Delta n_R(X)$ is $|\psi_R(X)|^2$, which, in the particular case of Gaussian wave functions, has Eq. (17a) as the distributional limit.

In the Minkowski space-time, such a point corresponds to the specular counterpart of x_0 with respect to the horizon: $x_R(X) = -x_0$.

When $x_0 > 0$, the distributional limit of $\Delta n_R(X)$ is $\delta(X - X_R(x_0))$, as in Fig. 3(b). The single-particle appears perfectly localized in both inertial and accelerated frame at the same position (up to the coordinate transformation).

In conclusion, we have provided a complete description for single-particle states in accelerated frames $\hat{\rho}$ through their characteristic functions $\chi[\xi, \xi^*]$. By the derivatives of $\chi[\xi, \xi^*]$, we obtain original expressions for the right-wedge density function $\langle \hat{n}_R(X) \rangle_{\hat{\rho}}$ for a general state. A significant outcome of this theoretical analysis is that $\langle \hat{n}_R(X) \rangle_{\hat{\rho}}$ receives non-negligible contributions from left-wedge values of $\psi(x)$. This points toward the possibility for single-particle quantum states to tunnel from the left to the right wedge, across the Rindler horizon. We want to point out that such a result does not depend on the particular form of $\psi(x)$. Nevertheless, we have tested the extreme case in which almost all the wave function is localized beyond the horizon. Specifically, we have considered in detail the case of Gaussian wave function $\psi(x)$ and verified that in the limit of high locality degree, i.e. $\sigma/|x_0| \to 0$, the dominant term of $\Delta n_R(X)$ is related to left-wedge values of $\psi(x)$ while the contributions coming from the right tail go to zero exponentially faster.

The use of the characteristic function has played a crucial role for deriving the results for single-particle states. Possible generalizations for $\chi[\xi, \xi^*]$ in the case of general Minkowski-Fock states can be obtained through the use of the same identities that have led to Eq. (8), such as Eq. (5). The development of an explicit form for such characteristic functions has been reported in Ref. [14].

Analyses of the results in the context of quantum information are out of the scope of the present Letter. However, it is worth mentioning that in spite of the nonlocal effect described here, no superluminal communication between the inertial and the accelerated observer occurs when they are separated by the Rindler horizon. The accelerated observer detects the presence of single particles emitted by the inertial observer in the other wedge; however, the two observers cannot use this effect to communicate. One can see this by assuming that the two observers are supplied with Unruh-DeWitt detectors [1,2] as a particle emitter and detector and by using the same arguments of Refs. [15,16] to prove that no violation of causality holds. We believe that a complete discussion about this topic is worthwhile for dedicated future works.

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