Scattering mechanism of work done on an ultracold Fermi gas by a time-dependent interaction

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We discuss the work done on repulsive ultracold Fermi gas via collisions tuned by a time-dependent magnetic ramp. If the interaction is strengthened during binary collisions, the kinetic energy of atoms increases with each such inelastic event. The net energy transfer via collisions is compared with the transfer via the mean field. It is found that for a weak interaction, with a scattering length shorter than the inverse mean momentum of atoms, the mean field dominates, while for a strong interaction, the majority of energy enters the gas via collisions.

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I. INTRODUCTION

Ultracold Fermi gases allow the study of many-body systems beyond conditions achieved in the electronic subsystem of crystals, in ^{3}He, or in nuclear matter [1–3]. Their unique advantage is the external control of the binary interaction. To this end, a complicated multichannel interaction of two alkali atoms is adjusted by the magnetic field either into the nonresonant regime or into the vicinity of the Feshbach resonance, where a corresponding effective interaction also can be tuned to large magnitudes [4,5].

The interaction strength is often modified by a magnetic ramp in the preparations of the initial state. For example, Valtolina *et al.* [6] cooled a mixture of the lowest and third-to-lowest Zeeman states of ^{6}Li atoms by evaporation in a homogeneous magnetic field of 300 G before ramping up this field to 584.5 G, where the scattering length of the third-to-lowest state equals to the second to lowest, and the excitation state was converted by the rf pulse. Next, they ramp the magnetic field down to 1 G and apply a gradient magnetic field to separate the lowest-state and excited atoms, and finally they ramp down the gradient field and ramp up the homogeneous field to the vicinity of the Feshbach resonance to achieve the desired strength of interaction.

Yet a more important role is played by the magnetic ramp applied during the measurement. Regal *et al.* [7] studied the distribution of ^{40}K atoms after simultaneously switching off the dipole traps and simultaneously reducing the binary interaction by a magnetic ramp to zero at a rate of $(2\ \mathrm{ms/G})^{-1}$ and $(6\ \mathrm{ms/G})^{-1}$. The reported distribution depends on the ramp rate.

In the present study, we focus on the collisional mechanism by which the magnetic ramp feeds energy in (or takes energy from) the repulsive Fermi gas. Let us demonstrate that such a mechanism needs to be accounted for to achieve thermodynamical consistency. The internal energy of the gas increases with increasing repulsive interaction. For the nondegenerate gas, this internal energy has two components: (1) the potential energy, which can be identified with the mean field contribution, and (2) the thermal energy closely related to the kinetic energy of individual atoms. It is intuitively clear that the increase of the binary potential enters the potential energy. In an ideal homogeneous gas, the mean field does not create forces, so conversion between the potential and kinetic energies is not possible. In the adiabatic process, however, both components of energy increase together. The mechanism which increases the kinetic energy acts during binary collisions. Due to the time-dependent interaction, the collisions are not perfectly elastic, and kinetic energy increases by a small amount in each collision.

The inelasticity of binary processes due to the time-dependent magnetic field was studied mainly with respect to the formation and dissociation of molecules [4,8–12]. The magnetic field acts as a third body in an inelastic collision and takes away the binding energy. The inelasticity we assume here is much less dramatic. Collisions maintain all properties of the elastic collisions except for the additional energy proportional to the rate of the magnetic ramp. This additional energy is very small but not negligible because of the large number of collisions.

The collision mechanism of energy transfer was outlined in [13], but its importance was not evaluated quantitatively so far. Here we show that for a sufficiently strong interaction, the energy transfer by collisions is more effective than the usually assumed transfer by the mean field, i.e., that the inelastic correction to the energy conservation in binary collisions should be included in the quasiclassical simulations of the ultracold gases with interaction controlled by the time-dependent magnetic field.

For the sake of simplicity, we assume that the momentum distribution of atoms is close to the equilibrium Fermi-Dirac distribution. For such adiabatically slow ramps, the rates of the energy transfer by collisions and by the mean field are linear functions of the ramp rate with coefficients given by the interaction strength. Systems out of equilibrium have to be treated from case to case either by analytical approaches like [14,15] or by simulations like [16–22]. The inelastic correction to the energy conservation derived in [13,23] is justified for any magnetic ramp except for the fastest magnetic ramps with the rate $(0.1\ \mu\mathrm{s/G})^{-1}$ [24] near the unitarity limit. Of course, it also does not apply to the fast optical switching of interaction [25–28].

The paper is organized as follows. In Sec. II we introduce a kinetic equation for a homogenous system with a time-

dependent binary interaction and define rates of the energy transferred by collisions. In Appendix C we derive the mean-field energy transfer; the evaluated collision mechanism is compared with it. Dilute gases are treated in Sec. III. The nondegenerate distribution makes it possible to evaluate the rates of energy transfer in the analytic form. These analytic formulas apply to a nonsymmetric system with atoms of different mass and density [29–32]. Appendix A offers a simple derivation of the energy transferred via collision in the free space. The dense gas, discussed in Sec. IV, provides a complementary picture. First, it is shown that the free-space T-matrix is not sufficient and the in-medium T-matrix is derived. The two-particle in-medium propagator entering the T-matrix is derived in Appendix B. Because of the in-medium effects, a numerical treatment is necessary. In order to reduce the number of parameters, we assume only symmetric systems of identical mass and density of both components. We also restrict our attention to the zero temperature at which all dissipative processes are frozen out. A mechanism similar to collisions is the exchange of pairs of particles; it is also not perfectly elastic. In Appendix D we simplify the energy transfer via the pair exchange to a two-dimensional integral with a single parameter—the scattering length scaled with the Fermi momentum. Section V concludes our discussion.

II. ENERGY BALANCE IN THE KINETIC EQUATION

We start our analysis from the kinetic equation of Boltzmann type for the quasiclassical dynamics of a homogeneous system of Fermi gas particles [23,33]:

$$\begin{aligned}\frac{\partial f_1}{\partial t} = &\int \frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3} f_{\bar{3}} f_{\bar{4}}[(1-f_1)(1-f_2) - f_1 f_2] \\ &\times \left|\mathcal{T}\left(\mathbf{k}+\mathbf{p}, \varepsilon_1+\varepsilon_2-\Delta_E, t-\frac{1}{2}\Delta_t\right)\right|^2 \\ &\times 2\pi\hbar^{-3}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_{\bar{3}}-\varepsilon_{\bar{4}}-2\Delta_E) \\ &- \int \frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3} f_1 f_2[(1-f_3)(1-f_4) - f_3 f_4] \\ &\times \left|\mathcal{T}\left(\mathbf{k}+\mathbf{p}, \varepsilon_1+\varepsilon_2+\Delta_E, t+\frac{1}{2}\Delta_t\right)\right|^2 \\ &\times 2\pi\hbar^{-3}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4+2\Delta_E). \end{aligned} \tag{1}$$

Subscripts indicate cumulative variables of initial and final states of collisions:

$$\begin{aligned} 1 &\equiv \uparrow, \mathbf{k}, t, \\ 2 &\equiv \downarrow, \mathbf{p}, t, \\ \bar{3} &\equiv \uparrow, \mathbf{k}-\mathbf{q}, t-\Delta_t, \\ \bar{4} &\equiv \downarrow, \mathbf{p}+\mathbf{q}, t-\Delta_t, \\ 3 &\equiv \uparrow, \mathbf{k}-\mathbf{q}, t+\Delta_t, \\ 4 &\equiv \downarrow, \mathbf{p}+\mathbf{q}, t+\Delta_t. \end{aligned} \tag{2}$$

The first scattering-in integral in the right-hand side of the kinetic equation (1) describes collisions in which atoms in states $\uparrow, \mathbf{k}-\mathbf{q}$ and $\downarrow, \mathbf{p}+\mathbf{q}$ encounter at time $t-\Delta_t$. Their correlated motion lasts for Δ_t so that atoms enter states $\uparrow, \mathbf{k}$ and $\downarrow, \mathbf{p}$ at time t. The second integral is the scattering out in which atom $\downarrow, \mathbf{p}$ kicks another atom out of state $\uparrow, \mathbf{k}$ at time t. Their correlated motion ends at $t+\Delta_t$ at states $\uparrow, \mathbf{k}-\mathbf{q}$ and $\downarrow, \mathbf{p}+\mathbf{q}$. The spin notation denotes the upper $\uparrow$ and lower $\downarrow$ Zeeman state of the atom.

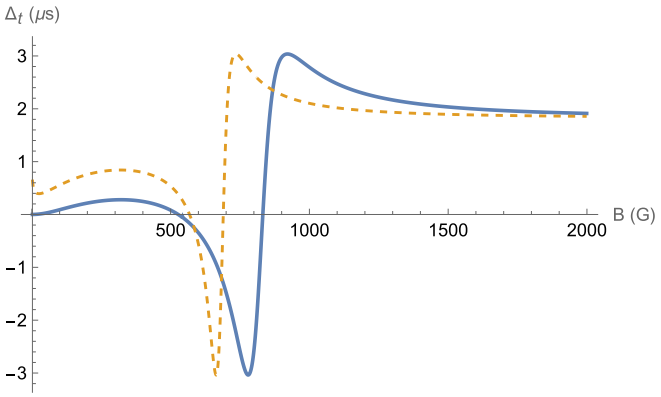

FIG. 1. Free-space collision delay as a function of the magnetic field. The relative momentum of atoms is 3×10^6 m^{-1}. A typical delay of a collision of atoms in the lowest with the second-to-lowest excited states (full; blue online) as well as of the lowest with the third-to-lowest state (dashed; orange online) is 1 μs. We have used experimentally established scattering length from [5] and the free-space T-matrix (7).

Duration of the collision Δ_t and the energy Δ_E which time-dependent interaction potential transfers to each of colliding atoms are given by energy and time derivatives of the scattering phase shift ϕ defined from the retarded T-matrix $\mathcal{T}(\mathbf{K}, \Omega, t) = |\mathcal{T}|e^{i\phi}$ [23]:

$$\Delta_t = \frac{\partial\phi}{\partial\Omega} \quad \text{and} \quad \Delta_E = -\frac{\hbar}{2}\frac{\partial\phi}{\partial t}. \tag{3}$$

These shifts Δ represent linear-order gradient corrections to a customary local approximation of the scattering integrals. Here $\hbar\mathbf{K}$ and $\hbar\Omega$ are the sum momentum and energy of the colliding pair, respectively.

In Fig. 1 we demonstrate the collision delay for ^{6}Li. Even extreme values $\Delta_t \approx \pm 2.5$ μs are small on the scale of binding frequencies of atomic traps. In Fig. 2 we show the energy gain. One can see that gain in individual collisions is very small for the discussed case. The highest energy gain near the Feshbach resonance, $\Delta_E \approx 10$ feV, is still small compared to a typical Fermi energy, e.g., 3×10^4 feV in [6]. The gain scales with the ramp rate; for the fastest ramps $(0.1\ \mu\text{s/G})^{-1}$ its value increases by a factor of 10^4 and might achieve values which signal that one has to go beyond the quasiclassical approximation.

For an inhomogeneous system, the kinetic equation includes the drift and acceleration of quasiparticles; moreover, there are additional vector shifts $\mathbf{\Delta}$ reflecting that a colliding pair drifts during the collision and that its center of mass is accelerated meanwhile. For a general interaction (not restricted to the s-wave channel), there are finite distances of colliding particles with different directions at the beginning and end of the collision due to rotation of the pair during Δ_t; see [23] for the complete theory.

Just to briefly review history, the kinetic equation with the space nonlocality of collisions was introduced already in 1921

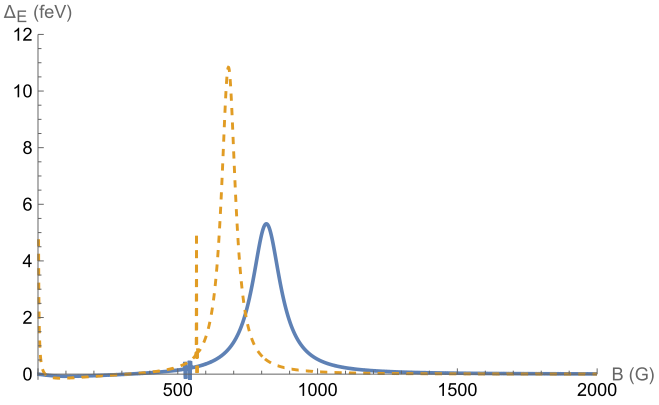

FIG. 2. Free-space energy gain for ^{6}Li-^{6}Li collision as a function of the magnetic field. Conditions are the same as in Fig. 1. The magnetic ramp rate is $(1\ \mathrm{ms/G})^{-1}$, these selected values correspond to [6]. The numerical derivative of raw experimental data from [5] leads to a noise at crossovers from the negative to the positive scattering length near 500 G.

by Enskog [34]. In 1937 Beth and Uhlenbeck [35] derived the second-order virial correction to the pressure from the momentum derivative of the scattering phase shift ϕ. Nonlocal corrections to the scattering integral based on derivatives of the scattering phase shift were developed in the 1990s for nondegenerate statistics [36–38]. All these studies work with the on-shell T-matrix, which leads to instantaneous collisions. Moreover, a natural assumption of time-independent binary interaction results in energy conservation in collisions. The finite duration of the collision was first mentioned in the theory of collisions. The energy derivative of the phase shift already was interpreted as the collision delay Δ_t in 1948 by Eisenbud [39]. This concept turned out to be useful in nuclear physics when Röpke *et al.* [40] found that correlated states lasting Δ_t are precursors of deuterons formed during the expansion of the nuclear matter after a reaction of heavy ions. Moreover, the collision duration modifies the equation of state of the nuclear matter, namely, the pressure [41]. Energy derivations of T-matrices are also the basic ingredients of the virial expansion for the ultracold Fermi gas [42,43]. The kinetic equation with collision delay was derived from the nonequilibrium Green functions [23]. The collision delay for the Feshbach resonance has been discussed by several authors [44–46].

Unlike Δ_t, the energy gain Δ_E has been mostly neglected so far. For a time-dependent interaction, the Δ_E was derived [13] using the gradient expansion originally developed for a usual time-independent interaction [23]. It was proven [13] that the total energy transferred by the scattering integrals and quasiparticle energies agrees with the energy transfer expected from the two-particle Green's function in the ladder approximation. To clarify the basic mechanism of the energy gain, Appendix A provides its simple derivation from the two-particle collision in the free space.

In the scattering integrals of Eq. (1), the essential role is played by occupation factors of single-electron states, e.g., $f_{\bar{3}}f_{\bar{4}}[(1-f_1)(1-f_2)-f_1f_2]$ of the scattering in. The channel $f_{\bar{3}}f_{\bar{4}}(1-f_1)(1-f_2)$ describes dissipative collisions where $f_{\bar{3}}f_{\bar{4}}$ provides a probability to find two atoms before the collision and $(1-f_1)(1-f_2)$ is a probability that final states are empty. Starting from the pioneering extension of the Boltzmann equation to Fermi systems [47] over the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy to derivations from the quantum statistics [48] the scattering integrals contain exclusively this dissipative channel.

The other channel $f_{\bar{3}}f_{\bar{4}}f_1f_2$ cannot be interpreted as a collision. It acts if the initial and the final states are both occupied, therefore it represents the exchange of two pairs of particles $1,2 \longleftrightarrow 3,4$. In the local approximation of the scattering integrals, $\Delta_{t,E}=0$, the exchange channel of the scattering in exactly cancels with the complementary one from the scattering out. Therefore, the pair exchange contributes only via gradient corrections to the scattering.

Multiplying the kinetic equation (1) with the quasiparticle energy ε_1 and integrating over momentum $\mathbf{k}$ one finds that the rate of the energy transfer to atomic gas via scattering integrals is a sum of the collision contributions [13]

$$\dot{\mathcal{E}}_{\mathrm{c}} = \int \frac{d\mathbf{k}\,d\mathbf{p}\,d\mathbf{q}}{(2\pi)^9} 2\Delta_E |\mathcal{T}|^2 f_{\uparrow\mathbf{k}} f_{\downarrow\mathbf{p}} (1-f_{\uparrow\mathbf{k}-\mathbf{q}})(1-f_{\downarrow\mathbf{p}+\mathbf{q}}) \times 2\pi\hbar^{-3}\delta(\varepsilon_{\uparrow\mathbf{k}}+\varepsilon_{\downarrow\mathbf{p}}-\varepsilon_{\uparrow\mathbf{k}-\mathbf{q}}-\varepsilon_{\downarrow\mathbf{p}+\mathbf{q}}) \tag{4}$$

and the pair-exchange contributions

$$\dot{\mathcal{E}}_{\mathrm{p}} = -\int \frac{d\mathbf{k}\,d\mathbf{p}\,d\mathbf{q}}{(2\pi)^9} 2\Delta_E |\mathcal{T}|^2 f_{\uparrow\mathbf{k}} f_{\downarrow\mathbf{p}} f_{\uparrow\mathbf{k}-\mathbf{q}} f_{\downarrow\mathbf{p}+\mathbf{q}} \times 2\pi\hbar^{-3}\delta(\varepsilon_{\uparrow\mathbf{k}}+\varepsilon_{\downarrow\mathbf{p}}-\varepsilon_{\uparrow\mathbf{k}-\mathbf{q}}-\varepsilon_{\downarrow\mathbf{p}+\mathbf{q}}). \tag{5}$$

With the explicit Δ_E, we can neglect all implicit shifts Δ and omit space and time arguments so that $f_1=f_{\uparrow\mathbf{k}}$, $f_2=f_{\downarrow\mathbf{p}}$, $f_1=f_{\uparrow\mathbf{k}-\mathbf{q}}$, and $f_4=f_{\downarrow\mathbf{p}+\mathbf{q}}$. For an adiabatically slow ramp, all distributions and the amplitude of the T-matrix have equilibrium values. The time dependency of the ramp enters exclusively via the energy gain Δ_E.

To simplify integrals (4) and (5), we neglect the effect of the gas on the single-atom dispersion, $\varepsilon_{\uparrow\mathbf{k}} \approx \hbar^2k^2/2m_\uparrow$. In this approximation we can separate the center-of-mass and relative motion. To this end we substitute momenta $\mathbf{k},\mathbf{p},\mathbf{q}$ with the sum momentum $\mathbf{K}=\mathbf{k}+\mathbf{p}$, the initial relative momentum $\kappa=(m_\downarrow\mathbf{k}-m_\uparrow\mathbf{p})/M$, where $M=m_\uparrow+m_\downarrow$ and the final relative momentum $\kappa'=\kappa-\mathbf{q}$. Therefore

$$\dot{\mathcal{E}}_{\mathrm{c}} = \int \frac{d\mathbf{K}\,d\kappa\,d\kappa'}{(2\pi)^9} 4\pi\hbar^{-3}\Delta_E |\mathcal{T}|^2 f_{\uparrow\frac{1}{2}\mathbf{K}+\kappa} f_{\downarrow\frac{1}{2}\mathbf{K}-\kappa} \times (1-f_{\uparrow\frac{1}{2}\mathbf{K}+\kappa'})(1-f_{\downarrow\frac{1}{2}\mathbf{K}-\kappa'})\delta\left(\frac{\hbar^2\kappa^2}{2\mu}-\frac{\hbar^2\kappa'^2}{2\mu}\right), \tag{6}$$

where $\mu=m_\uparrow m_\downarrow/M$ is the relative mass. Approximations in the pair-exchange energy transfer (5) are analogous.

III. DILUTE GAS

In a dilute gas, all contributions to the energy transfer that require three or more particles are negligible. In particular, the pair-exchange contribution $\dot{\mathcal{E}}_{\mathrm{p}}$ vanishes because it involves four atoms; see Eq. (5). Due to this, the energy transfer is analytically solvable.

Since the three-particle encounters vanish, the binary collisions are approximated by the free-space T-matrix

$$\mathcal{T} \approx \mathcal{T}_0 = \frac{2\pi\hbar^3}{\mu}\frac{a}{1+ia\kappa}. \tag{7}$$

Similarly, the energy gain (3) has the free-space value $\Delta_E \approx \Delta_E^0$. The scattering length a depends on the time via the time-dependent magnetic field. From the free-space phase shift $\phi^0 = \pi - \arctan(\kappa a)$, the energy gain results as

$$\Delta_E^0 = \frac{\hbar}{2}\frac{\kappa}{1+\kappa^2a^2}\frac{\partial a}{\partial t}. \tag{8}$$

In the dilute gas the Pauli blocking is negligible, $1 - f_{\frac{1}{2}\mathbf{K}\pm\kappa} \approx 1$. For the s-wave scattering, the scattering rate $|\mathcal{T}|^2$ and the energy gain Δ_E are independent of the final relative momentum allowing for the analytical integration over κ' in Eq. (6):

$$\dot{\mathcal{E}}_c = \frac{2\mu}{\pi\hbar^5}\int\frac{d\mathbf{K}d\kappa}{(2\pi)^6}\kappa\Delta_E^0|\mathcal{T}_0|^2 f_{\uparrow\frac{1}{2}\mathbf{K}+\kappa}f_{\downarrow\frac{1}{2}\mathbf{K}-\kappa}. \tag{9}$$

Finally, approximating the Fermi-Dirac distribution by the Boltzmann distribution for dilute gas, the r.h.s. of Eq. (9) is separated into the product of functions of the sum and relative momentum using

$$f_{\uparrow\frac{1}{2}\mathbf{K}+\kappa}f_{\downarrow\frac{1}{2}\mathbf{K}-\kappa} = n_\uparrow n_\downarrow\sqrt{\frac{(2\pi\hbar^2)^3}{(\mu k_B T)^3}}e^{-\frac{\hbar^2\kappa^2}{2\mu k_B T}} \times\sqrt{\frac{(2\pi\hbar^2)^3}{(Mk_B T)^3}}e^{-\frac{\hbar^2K^2}{2Mk_B T}}. \tag{10}$$

The free-space T-matrix $\mathcal{T}_0$ and the energy gain Δ_E^0 are independent of the sum momentum $\mathbf{K}$, therefore from Eqs. (7)–(10) follows

$$\dot{\mathcal{E}}_c = \frac{\partial a}{\partial t}\frac{2\hbar^2 n_\uparrow n_\downarrow}{\pi\mu}\sqrt{\frac{(2\pi\hbar^2)^3}{(\mu k_B T)^3}}\int_0^\infty d\kappa\frac{\kappa^4a^2}{(1+\kappa^2a^2)^2}e^{-\frac{\hbar^2\kappa^2}{2\mu k_B T}}. \tag{11}$$

By introducing the dimensionless scattering length

$$\alpha = a\frac{1}{\hbar}\sqrt{2\mu k_B T} \tag{12}$$

and momentum $\kappa = \tilde{\kappa}\sqrt{2\mu k_B T}/\hbar$, Eq. (11) is recast,

$$\dot{\mathcal{E}}_c = \frac{\partial a}{\partial t}\frac{\hbar^2 n_\uparrow n_\downarrow}{\mu}\mathcal{C}, \tag{13}$$

where the dimensionless energy transfer via collisions

$$\mathcal{C} = 2^4\sqrt{\pi}\int_0^\infty d\tilde{\kappa}\frac{\tilde{\kappa}^4\alpha^2}{(1+\tilde{\kappa}^2\alpha^2)^2}e^{-\tilde{\kappa}^2} = \frac{4\pi}{\alpha^5}\left[2\alpha(1+\alpha^2) - (2+3\alpha^2)e^{\frac{1}{\alpha^2}}\sqrt{\pi}\,\mathrm{Erfc}\left(\frac{1}{\alpha}\right)\right] \tag{14}$$

is shown in Fig. 3. The energy transfer by collisions $\mathcal{C}$ is quadratic in α for a weak interaction. It directly follows from the finite value of the energy gain (8) in the weak interaction limit ($a \to 0$ with the rate of ramp $\partial a/\partial t$ fixed) and linearly vanishing T-matrix (7).

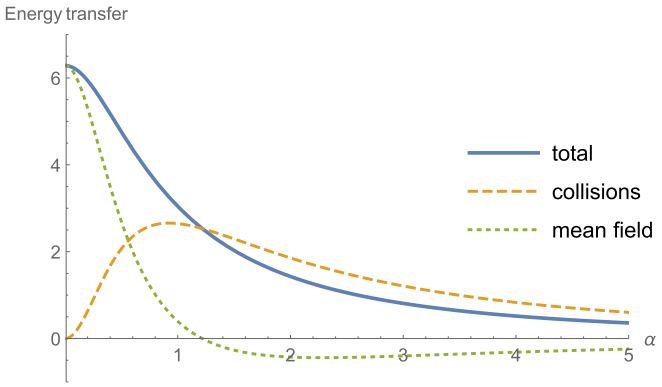

FIG. 3. Dimensionless energy transfer as a function of the dimensionless scattering length $\alpha = a\sqrt{2\mu k_B T}/\hbar$ at high temperatures. For a weak interaction, $\alpha \ll 1$, the collision energy transfer $\mathcal{C}$ is a small correction to the mean-field transfer $\mathcal{M}$. For a strong interaction, $\alpha > 1$, the collision transfer is the dominant contribution to the total transfer $\mathcal{C} + \mathcal{M}$.

To see the magnitude of $\mathcal{C}$ on the scale of the total energy transfer, in Fig. 3 we also plot the dimensionless energy transfer by mean field (C10) evaluated in Appendix C 1. The energy transfer by mean field $\mathcal{M}$ has features expected from the two-particle wave function [49,50]. For a weak interaction, the wave function fully penetrates the repulsive potential, therefore the mean-field energy (C7) linearly increases with the interaction strength, which gives $\mathcal{M}(0) = 2\pi$. An increasing potential expels the wave function from the interaction region; therefore $\mathcal{M}$ goes down. For $\alpha > 1.2$, the loss of interaction energy due to expelled wave function exceeds the gain due to stronger interaction and $\mathcal{M}$ is negative.

For the weak interaction the mean field dominates, but around $\alpha \approx 0.5$ functions $\mathcal{C}$ and $\mathcal{M}$ cross, and the collision energy transfer becomes dominant. In the unitarity limit, $\alpha \to \infty$, both contributions fall as $1/\alpha^2$, the energy transfer by collisions $\mathcal{C} \to 8\mu k_B T/\hbar^2 a^2$ and the energy transfer by mean field $\mathcal{M} \to -4\mu k_B T/\hbar^2 a^2$. Due to the collision transfer, the total energy transfer is positive for all values of the interaction strength in agreement with the intuitive expectation that the energy of the gas increases with the repulsive interaction. Briefly, the energy transfer via collisions is negligible for the weak interaction, but for the strong interaction it is indispensable.

IV. DENSE GAS AT ZERO TEMPERATURE

In the dilute gas discussed above, the only scale of the atomic momentum is the thermal de Broglie wavelength. This allowed us to compare individual mechanisms of energy transfer using the dimensionless parameter α. In dense gas, the most important scale is the Fermi momentum. To have a single momentum scale, we focus on the zero temperature limit and assume a symmetric system with $n_\uparrow = n_\downarrow$ and $m_\uparrow = m_\downarrow = m$ in which the Fermi momenta of both components equal $k_F^\uparrow = k_F^\downarrow \equiv k_F$, the sum mass and the relative mass read $M = 2m$ and $\mu = m/2$, and the relative momentum is $\kappa = \frac{1}{2}(\mathbf{k} - \mathbf{p})$.

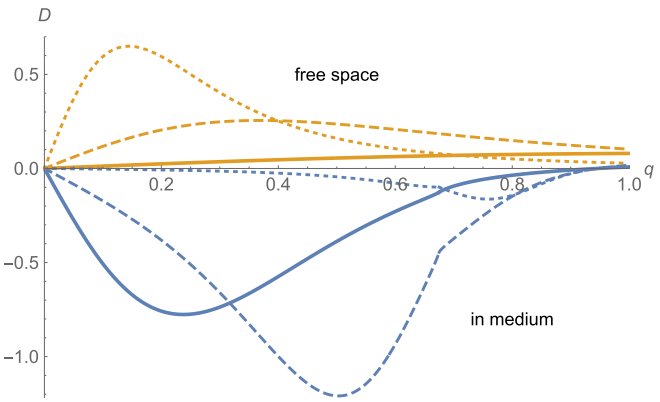

FIG. 4. Free-space vs in-medium energy transfer in a single collision. The in-medium (negative lines; blue online) collision gain weighted with the scattering rate, $|\mathcal{T}|^2\Delta_E = (\partial a/\partial t)D$, is compared with its free-space approximation (positive lines; orange online) as a function of the relative momentum $\kappa = qk_\mathrm{F}$ for the sum momentum $K = 0.65k_\mathrm{F}$. For all interaction strengths, weak $\vartheta = ak_\mathrm{F} = 0.5$ (full lines), moderate $\vartheta = \pi/2$ (dashed lines), and strong $\vartheta = 4$ (dotted lines), the in-medium values are negative while their free-space approximation is positive.

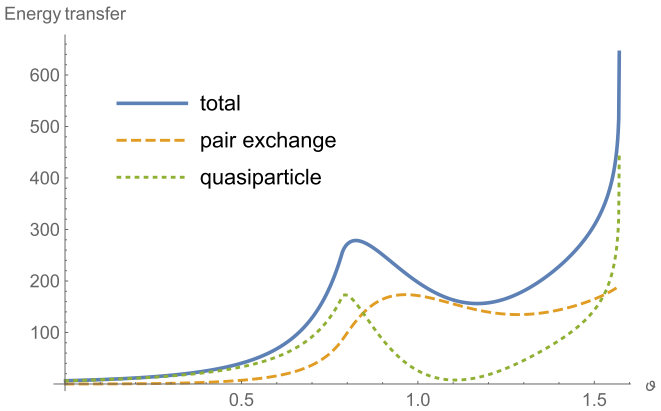

FIG. 5. Dimensionless energy transfer at $T = 0$. The (total) energy transfer $\mathcal{P} + \mathcal{Q}$ is dominated by the quasiparticle contribution $\mathcal{Q}$ for a weak interaction and by the pair exchange contribution $\mathcal{P}$ for the strong interaction $\vartheta = ak_\mathrm{F} > 1$.

At zero temperature, collisions freeze out, $\dot{\mathcal{E}}_\mathrm{c} = 0$. To evaluate the pair-exchange energy transfer $\dot{\mathcal{E}}_\mathrm{p}$, we need to calculate the in-medium T-matrix.

The inverse form of the ladder approximation

$$\frac{1}{\mathcal{T}} = \frac{1}{\mathcal{V}} - \mathcal{G} \tag{15}$$

links the in-medium T-matrix with its free-space value $1/\mathcal{T}_0 = 1/\mathcal{V} - \mathcal{G}_0$ as

$$\frac{1}{\mathcal{T}} = \frac{1}{\mathcal{T}_0} + 2\mathcal{G}'. \tag{16}$$

The Green function is decomposed into its free-space part and in-medium corrections, $\mathcal{G} = \mathcal{G}_0 - 2\mathcal{G}'$. For the free-space T-matrix we use approximation (7), and the T-matrix is thus parameterized by the scattering length a rather than the interaction potential $\mathcal{V}$. The two-particle Green function is evaluated in Appendix B [Eq. (B21)], where the reader can find more details about the T-matrix.

The collision gain

$$\Delta_E = -\frac{\hbar}{2}\mathrm{Im}\left[\frac{1}{\mathcal{T}}\frac{\partial\mathcal{T}}{\partial t}\right] \tag{17}$$

weighted with the scattering rate $|\mathcal{T}|^2$ is shown in Fig. 4. The free-space value is positive for any magnitude of the scattering length. In contrast, the in-medium value is always negative. These signs follow from signs of two-particle spectral functions: $-2\mathrm{Im}\mathcal{G} < 0$ and $-2\mathrm{Im}\mathcal{G}_0 > 0$ below the Fermi energy; see Fig. 6 in Appendix B. Apparently, the in-medium propagation of internal states of the T-matrix cannot be approximated by the free-space value.

At zero temperature, the Fermi-Dirac distribution of quasiparticles is the step function, $f_{\uparrow\mathbf{k}} = \theta(E_\mathrm{F} - \hbar^2k^2/(2m))$, where $E_\mathrm{F} = \hbar^2k_\mathrm{F}^2/(2m)$ is the Fermi energy. The pair-exchange energy transfer (5) thus reads

$$\dot{\mathcal{E}}_\mathrm{p} = -\int \frac{d\mathbf{K}d\kappa d\kappa'}{(2\pi)^9} 2\Delta_E|\mathcal{T}|^2 2\pi\hbar^{-3}\delta\left(\frac{\hbar^2\kappa^2}{2\mu} - \frac{\hbar^2\kappa'^2}{2\mu}\right)$$
$$\times\theta\left(E_\mathrm{F} - \frac{\hbar^2\left|\frac{1}{2}\mathbf{K}+\kappa\right|^2}{2m}\right)\theta\left(E_\mathrm{F} - \frac{\hbar^2\left|\frac{1}{2}\mathbf{K}-\kappa\right|^2}{2m}\right)$$
$$\times\theta\left(E_\mathrm{F} - \frac{\hbar^2\left|\frac{1}{2}\mathbf{K}+\kappa'\right|^2}{2m}\right)\theta\left(E_\mathrm{F} - \frac{\hbar^2\left|\frac{1}{2}\mathbf{K}-\kappa'\right|^2}{2m}\right). \tag{18}$$

This nine-dimensional integration can be reduced to the two dimensions using symmetries of the system.

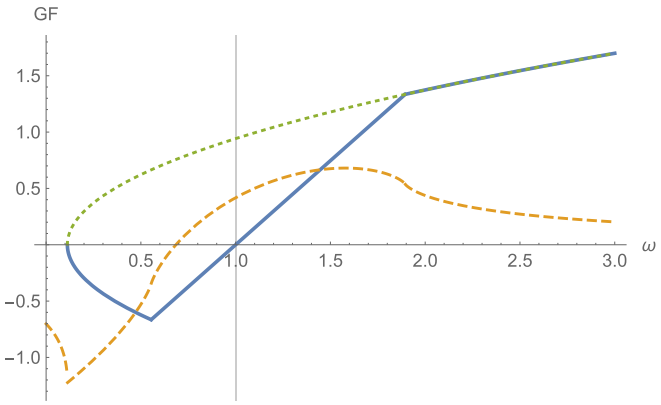

FIG. 6. Dimensionless in-medium two-particle Green function as a function of dimensionless energy $\omega = \hbar\Omega/(2E_\mathrm{F})$ for $|\mathbf{K}| = 0.67k_\mathrm{F}$. The free-space spectral function $\tilde{\mathcal{A}}_0(\omega, \tilde{\mathbf{K}})$ (dotted; green online) is a simple parabola everywhere positive. The in-medium spectral function $\tilde{\mathcal{A}}(\omega, \tilde{\mathbf{K}})$ (full; blue online) starts from negative values, crosses zero at twice Fermi energy, $\omega = 1$, and later merges to the free-space spectrum. The in-medium correction to the two-particle Green function $-2\mathrm{Re}\tilde{\mathcal{G}}'(\omega, \tilde{\mathbf{K}})$ (dashed; orange online) has nonanalytic points at the breakpoints of the spectral function. The vertical at line $\omega = 1$ indicates the maximum energy of two particles $2E_\mathrm{F}$.

In the numerical treatment we use scaled variables $\tilde{K} = K/k_{\mathrm{F}}$, $\tilde{\kappa} = \kappa/k_{\mathrm{F}}$ and the dimensionless scattering length

$$\vartheta = ak_{\mathrm{F}}. \tag{19}$$

The T-matrix $\mathcal{T} = \frac{4\pi\hbar^3}{mk_{\mathrm{F}}}\tilde{\mathcal{T}}$

$$\tilde{\mathcal{T}}(\tilde{\kappa}, \tilde{K}) = \frac{1}{\frac{1}{\vartheta} + i\tilde{\kappa} + 2\tilde{\mathcal{G}}'(\tilde{\kappa}, \tilde{K})} \tag{20}$$

depends on the dimensionless Green function (B26). Using the single-component density $n_\uparrow = n_\downarrow = k_{\mathrm{F}}^3/6\pi^2$ and the relative mass $\mu = m/2$ we find

$$\dot{\mathcal{E}}_{\mathrm{p}} = \frac{\partial a}{\partial t}\frac{\hbar^2 n_\uparrow n_\downarrow}{\mu}\mathcal{P}, \tag{21}$$

where the dimensionless function

$$\begin{aligned}\mathcal{P} = \frac{144\pi}{\vartheta^2}\int_0^2 d\tilde{K}\int_0^1 d\tilde{\kappa}\tilde{\kappa}\,\mathrm{Im}[\tilde{\mathcal{T}}]|\tilde{\mathcal{T}}|^2 \\ \times\theta\left(1-\frac{1}{4}\tilde{K}^2-\tilde{\kappa}^2\right) \\ \times\left\{\tilde{K}^2\tilde{\kappa}^2\,\theta\left[1-\left(\frac{1}{2}\tilde{K}+\tilde{\kappa}\right)^2\right]\right. \\ \left.+\left(1-\frac{1}{4}\tilde{K}^2-\tilde{\kappa}^2\right)^2\theta\left[+\left(\frac{1}{2}\tilde{K}+\tilde{\kappa}\right)^2-1\right]\right\}\end{aligned} \tag{22}$$

depends only on the dimensionless scattering length ϑ. Details of the derivation can be found in Appendix D.

Function $\mathcal{P}$ is compared in Fig. 5 with the energy transfer $\mathcal{Q}$ via quasiparticle energy $\sigma_{s\mathbf{k}}$:

$$\begin{aligned}\dot{\mathcal{E}}_{\mathrm{q}} &= \frac{\partial a}{\partial t}\frac{\hbar^2 n_\uparrow n_\downarrow}{\mu}\mathcal{Q} \\ &= \frac{\partial a}{\partial t}\frac{\partial}{\partial a}\frac{1}{2}\sum_{s=\uparrow\downarrow}\int\frac{d\mathbf{k}}{(2\pi)^3}\sigma_{s\mathbf{k}}f_{s\mathbf{k}}.\end{aligned} \tag{23}$$

See Appendix C 2 for details. For a very weak interaction, the quasiparticle mechanism dominates with a dimensionless mean-field value $\mathcal{Q}(0) = 2\pi$ while the collision contribution vanishes, $\mathcal{P}(0) = 0$. The in-medium effect on the T-matrix leads to enhancement of the energy transfer by pair exchanges to values $\mathcal{P} \sim 200$, and similar values are reached by the quasiparticle contribution $\mathcal{Q}$. The quasiparticle mechanism has a peak of effectiveness at the scattering length $ak_{\mathrm{F}} = 0.8$, and the effectiveness of pair exchanges has a broad maximum near $ak_{\mathrm{F}} \approx 1$. For the strong interaction $ak_{\mathrm{F}} > 0.9$, the pair-exchange energy transfer dominates.

As the scattering length approaches $ak_{\mathrm{F}} = \pi/2$, the quasiparticle contribution diverges. This is likely connected with the phase transition to the ferromagnetism [51], which (in the simplest approximation) has the quantum critical point at this interaction strength [52]. In this region, both components of the presented energy transfer should be taken with reservations because the theory of the ferromagnetic phase is based on the reconstruction of the single-particle energy spectrum while the present theory benefits from the non-self-consistent approximation. Moreover, near the ferromagnetic state, the T-matrix does not cover the most important collision processes given by ferromagnetic fluctuations [53].

V. CONCLUSIONS

Binary collisions with time-dependent interaction are not perfectly elastic. The nonelasticity is rather small. For a single ^{6}Li-^{6}Li collision at nK temperature and magnetic ramp rate $(1\ \mathrm{ms/G})^{-1}$, the energy gained or lost is at least by four orders of magnitude smaller than the kinetic energy of atoms. Nevertheless, the net energy transferred in this way represents an important part of the work done on the system after a larger number of collisions happening during the magnetic ramp. We have compared this collision mechanism with the energy transfer via the mean field.

For dilute gases at significantly higher temperature than the Fermi energy, $k_{\mathrm{B}}T \gg E_{\mathrm{F}}$, we have obtained the rate of the energy transfer in the analytic form. As shown in Fig. 3, the mean field dominates for a weak interaction when the scattering length is smaller than the thermal de Broglie's wavelength $a \ll \hbar/\sqrt{8\mu k_{\mathrm{B}}T}$. Both mechanisms supply energy with equal power for $a = \hbar/\sqrt{8\mu k_{\mathrm{B}}T}$, and for even stronger interactions the scattering mechanism dominates. We note that this result applies also to dilute Bose gases because atoms satisfy the Boltzmann distribution.

For dense Fermi gases at zero temperature, the binary collisions are frozen out, but their role in the energy transfer is played by pair-exchange processes. The energy transfer calculation in this regime requires numerical treatment. As shown in Fig. 5, the rate of the energy transfer is enhanced compared to the dilute system. Basic features, however, are common. For a weak interaction, $a \ll 1/k_{\mathrm{F}}$, the rate of the energy transfer is given by the Hartee-type mean field as in the dilute gas. Due to the three-particle processes, the quasiparticle type of energy transfer dominates over the pair-exchange mechanism up to the scattering length $ak_{\mathrm{F}} \sim 0.8$ when both become comparable. For the strong interaction, $ak_{\mathrm{F}} > 1$, the pair-exchange mechanism is dominant. Our discussion does not apply to the very strong interaction, $ak_{\mathrm{F}} > \pi/2$, when the gas is ferromagnetic.

We have focused our attention on homogeneous Fermi systems with time-dependent interaction. In systems with strongly inhomogeneous interaction strength [28], two additional nonlocal corrections to the scattering integral emerge. First, colliding pairs fly together during the collision time Δ_t, and, second, binary collisions do not strictly conserve the sum momentum. All such nonlocal corrections can be implemented in quasiclassical simulation codes in a manner similar to simulations of the nuclear reactions [23,54]. Since the nonlocal corrections modify the equation of state [55], one can expect that they affect density profile and collective motion. Moreover, the energy transferred by collisions enters the kinetic energy in a random way increasing the local temperature while the energy transferred by the mean-field enters binary forces which drive collective flows.

For dilute gases, all nonlocal corrections to the energy balance in collisions are simple functions of the relative momentum. Such corrections do not increase the runtime of simulations; see [23,54]. On the other hand, in dense gases, the nonlocal corrections depend on demanding calculations

of the in-medium T-matrix. Still, simulations of realistic experimental conditions with in-medium scattering rates are feasible [20].

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APPENDIX A: ENERGY GAIN IN FREE SPACE

In this Appendix, we outline how the energy gain Δ_E emerges from the scattering of two particles in the free space. For a more detailed derivation applicable also to dense systems, we refer the reader to [23], where the kinetic equation (1) with gradient corrections to the scattering integral (3) is derived from the gradient expansion of the collision integral for the nonequilibrium Green functions.

Let us assume two colliding particles, $\uparrow \mathbf{k}$ and $\downarrow \mathbf{p}$. The initial state of their collision is a product of two plane waves

$$\psi^{(2)}(\mathbf{r}', \mathbf{r}'') = e^{i\mathbf{k}\cdot\mathbf{r}'} e^{i\mathbf{p}\cdot\mathbf{r}''}. \tag{A1}$$

In the barycentric coordinates, $\mathbf{R} = (m_\uparrow \mathbf{r}' + m_\downarrow \mathbf{r}'')/M$ and $\mathbf{r} = \mathbf{r}' - \mathbf{r}''$ with $M = m_\uparrow + m_\downarrow$, this two-particle wave function

$$\psi^{(2)}(\mathbf{R}, \mathbf{r}) = e^{i\mathbf{K}\cdot\mathbf{R}} e^{i\kappa\cdot\mathbf{r}} \tag{A2}$$

splits into a function of the sum momentum $\mathbf{K} = \mathbf{k} + \mathbf{p}$ and the relative momentum $\kappa = (m_\downarrow \mathbf{k} - m_\uparrow \mathbf{p})/M$. The interaction does not act on the center-of-mass coordinate $\mathbf{R}$. We thus focus on the relative motion.

In the stationary case, the wave scattered by the contact potential,

$$\psi'_\kappa(r) = -\frac{\mu}{2\pi\hbar^2 r} e^{i\kappa r} \mathcal{T}_0\left(\frac{\hbar\kappa^2}{2\mu}\right), \tag{A3}$$

is isotropic and depends on the strength of potential via the T-matrix. Its argument $\frac{\hbar^2\kappa^2}{2\mu}\frac{1}{\hbar}$ is proportional to the kinetic energy, where $\mu = m_\downarrow m_\uparrow/M$ is the relative mass.

The corresponding time-dependent wave function

$$\psi'_\kappa(r,t) \approx -\frac{\mu}{2\pi\hbar^2 r} e^{i\kappa r - i\frac{\hbar\kappa^2}{2\mu}t} \mathcal{T}_0\left(\frac{\hbar\kappa^2}{2\mu}, t - \frac{r\mu}{\hbar\kappa}\right) \tag{A4}$$

adiabatically depends on a slowly changing T-matrix [56]. The time of collision, $t - \frac{r\mu}{\hbar\kappa}$, is obtained from the time of observation t after subtraction of the traveling time $\frac{r\mu}{\hbar\kappa}$.

The relative energy and momentum of scattered particles are given by the frequency of oscillations in time and space, respectively. They are influenced by the phase ϕ of the T-matrix, $\mathcal{T}_0 = |\mathcal{T}_0| e^{i\phi}$. For slow changes, we can expand the phase

$$\phi\left(\frac{\hbar\kappa^2}{2\mu}, t - \frac{r\mu}{\hbar\kappa}\right) \approx \phi\left(\frac{\hbar\kappa^2}{2\mu}, 0\right) + \frac{\partial\phi}{\partial t}\left(t - \frac{r\mu}{\hbar\kappa}\right). \tag{A5}$$

Substituting into (A4) the energy after the collision is deduced to be

$$E' = \frac{\hbar^2\kappa^2}{2\mu} - \hbar\frac{\partial\phi}{\partial t} \tag{A6}$$

and the momentum

$$\kappa' = \kappa - \frac{\partial\phi}{\partial t}\frac{\mu}{\hbar\kappa}. \tag{A7}$$

Note that these final values are consistent: Keeping the linear terms in gradient $E' = \hbar^2\kappa'^2/2\mu$.

The energy gain $\Delta_E = -\frac{\hbar}{2}\frac{\partial\phi}{\partial t}$ per particle enters the energy conservation, i.e., δ function in Eq. (1) with the factor of 2.

APPENDIX B: IN-MEDIUM T-MATRIX

In this Appendix, we derive the T-matrix with the Pauli blocking and the pair exchange in the internal propagation. We restrict our attention to the zero temperature. In-medium T-matrix at finite temperature has been evaluated, for instance, by Chiacchiera *et al.* [57].

The retarded T-matrix in the ladder approximation

$$\mathcal{T}(\Omega, \mathbf{K}) = \mathcal{V} + \mathcal{V}\mathcal{G}(\Omega, \mathbf{K})\mathcal{T}(\Omega, \mathbf{K}) \tag{B1}$$

sums all orders of the interaction potential $\mathcal{V}$. The contact potential $\mathcal{V}$ is independent of momentum and the relevant element of the two-particle Green's function $\mathcal{G}$ depends only on the sum energy $\hbar\Omega$ and the sum momentum $\hbar\mathbf{K}$.

The Green's function, $\mathcal{G} = \mathrm{Re}\mathcal{G} - \frac{i}{2}\mathcal{A}$ is obtained by the Kramers-Kronig relation

$$\mathrm{Re}\mathcal{G}(\Omega', \mathbf{K}) = -\int \frac{d\Omega}{2\pi} \frac{\wp}{\Omega - \Omega'} \mathcal{A}(\Omega, \mathbf{K}) \tag{B2}$$

from the two-particle spectral function $\mathcal{A}$. We will introduce the occupation factors into the spectral function.

We proceed in two steps. First, we discuss $\mathcal{G}$ in the free space to link the ladder approximation (B1) with the parametric form (7). Second, we derive the in-medium two-particle Green's function from which we evaluate the in-medium T-matrix.

1. Free space

In the space-time representation, the free-space spectral function is a product of two single-particle spectral functions

$$\mathcal{A}_0(\mathbf{x}, t; \mathbf{y}, t') = A_\uparrow(\mathbf{x}, t; \mathbf{y}, t') A_\downarrow(\mathbf{x}, t; \mathbf{y}, t'). \tag{B3}$$

It is sufficient to evaluate only elements of identical initial and final times and coordinates because we study a system with instantaneous and contact interaction.

In the Fourier representation

$$A_\uparrow(\omega, \mathbf{k}) = 2\pi\delta\left(\hbar\omega - \frac{\hbar^2 k^2}{2m_\uparrow}\right), \tag{B4}$$

therefore

$$\begin{aligned}\mathcal{A}_0(\Omega, \mathbf{K}) &= \int d\mathbf{x} e^{-i\mathbf{K}\mathbf{x}} \int dt e^{i\Omega t} \\ &\quad \times A_\uparrow(\mathbf{x}, t; \mathbf{0}, 0) A_\downarrow(\mathbf{x}, t; \mathbf{0}, 0) \\ &= \frac{\mu}{\pi\hbar^4} \sqrt{2\mu\left(\hbar\Omega - \frac{\hbar^2 K^2}{2M}\right)}.\end{aligned} \tag{B5}$$

Since $\mathcal{A} = -2\mathrm{Im}\mathcal{G}$, from Eq. (B1) we get

$$\mathcal{T}_0(\Omega, \mathbf{K}) = \frac{1}{\frac{1}{\mathcal{V}} - \mathrm{Re}\mathcal{G}_0(\Omega, \mathbf{K}) + i\frac{\mu}{2\pi\hbar^4}\sqrt{2\mu\left(\hbar\Omega - \frac{\hbar^2K^2}{2M}\right)}}. \tag{B6}$$

From the T-matrix (B6) we can define a function

$$a(\Omega, \mathbf{K}) = \frac{\mu}{2\pi\hbar^3}\frac{1}{\frac{1}{\mathcal{V}} - \mathrm{Re}\mathcal{G}_0(\Omega, \mathbf{K})} \tag{B7}$$

in terms of which the off-shell T-matrix reads

$$\mathcal{T}_0(\Omega, \mathbf{K}) = \frac{2\pi\hbar^3}{\mu}\frac{a}{1 + ia\frac{1}{\hbar}\sqrt{2\mu\left(\hbar\Omega - \frac{\hbar^2K^2}{2M}\right)}}. \tag{B8}$$

On the energy shell, the sum energy is a sum of the center-of-mass kinetic energy and the relative energy, $\hbar\Omega = \hbar^2K^2/2M + \hbar^2\kappa^2/2\mu$. In this spirit, we can switch between the off-shell and on-shell T-matrix using substitution

$$\kappa = \frac{1}{\hbar}\sqrt{2\mu\left(\hbar\Omega - \frac{\hbar^2K^2}{2M}\right)}. \tag{B9}$$

The on-shell T-matrix results from Eq. (B8) as

$$\mathcal{T}_0(\kappa, \mathbf{K}) = \mathcal{T}_0(\Omega, \mathbf{K})|_{\Omega = \frac{\hbar K^2}{2M} + \frac{\hbar\kappa^2}{2\mu}} = \frac{2\pi\hbar^3}{\mu}\frac{a}{1 + ia\kappa}. \tag{B10}$$

The parametric form of the T-matrix (7) is the free-space T-matrix $\mathcal{T} \approx \mathcal{T}_0$ with function a approximated by a constant. We use the free-space T-matrix (7) as our starting point.

2. In-medium two-particle Green's function

In a degenerate system, the two-particle spectral function depends on the occupation of single-particle states

$$\begin{aligned}\mathcal{A}(\Omega, \mathbf{K}) &= \int \frac{dz}{2\pi}\frac{dk}{(2\pi)^3}[G^>_\uparrow(\mathbf{k}, z)G^>_\downarrow(\mathbf{K} - \mathbf{k}, \Omega - z) \\ &\quad - G^<_\uparrow(\mathbf{k}, z)G^<_\downarrow(\mathbf{K} - \mathbf{k}, \Omega - z)] \\ &= \int \frac{dk}{(2\pi)^3}\frac{2\pi}{\hbar}\delta(\hbar\Omega - \varepsilon_{\uparrow\mathbf{k}} - \varepsilon_{\downarrow\mathbf{K}-\mathbf{k}}) \\ &\quad \times (1 - f_{\uparrow\mathbf{k}} - f_{\downarrow\mathbf{K}-\mathbf{k}}),\end{aligned} \tag{B11}$$

where

$$\begin{aligned}G^<_\uparrow(\mathbf{k}, z) &= 2\pi\delta(\hbar z - \varepsilon_{\uparrow\mathbf{k}})f_{\uparrow\mathbf{k}} \\ G^>_\uparrow(\mathbf{k}, z) &= 2\pi\delta(\hbar z - \varepsilon_{\uparrow\mathbf{k}})(1 - f_{\uparrow\mathbf{k}})\end{aligned} \tag{B12}$$

are the particle and hole correlation functions. The spectral function is thus a difference

$$\mathcal{A} = \mathcal{A}_0 - 2\mathcal{A}' \tag{B13}$$

of the free-space function $\mathcal{A}_0$ given by Eq. (B5) and the in-medium contribution

$$\begin{aligned}\mathcal{A}'(\Omega, \mathbf{K}) &= \int \frac{d\kappa}{(2\pi)^3}2\pi\hbar^{-1}\delta\left(\hbar\Omega - \frac{\hbar^2K^2}{2M} - \frac{\hbar^2\kappa^2}{2\mu}\right) \\ &\quad \times \theta\left(E_\mathrm{F} - \frac{\hbar^2}{2m}\left|\frac{1}{2}\mathbf{K} + \kappa\right|^2\right).\end{aligned} \tag{B14}$$

We have approximated the quasiparticle energy by the bare one and used that atoms of both Zeeman states have equal mass and density.

The high-density spectral function can be solved analytically. The integration in spherical coordinates gives a linear section

$$\begin{aligned}\mathcal{A}_1(\Omega, \mathbf{K}) &= \frac{m^2}{4\pi\hbar^5K}\theta\left(\hbar\Omega - 2E_\mathrm{F} - \frac{\hbar^2K^2}{2m} + \frac{\hbar^2Kk_\mathrm{F}}{m}\right) \\ &\quad \times \theta\left(2E_\mathrm{F} + \frac{\hbar^2K^2}{2m} + \frac{\hbar^2Kk_\mathrm{F}}{m} - 2\hbar\Omega\right) \\ &\quad \times (2E_\mathrm{F} - \hbar\Omega)\end{aligned} \tag{B15}$$

and a square root section

$$\begin{aligned}\mathcal{A}_2(\Omega, \mathbf{K}) &= \frac{m}{4\pi\hbar^4}\sqrt{m\left(\hbar\Omega - \frac{\hbar^2K^2}{4m}\right)}\,\theta\left(\hbar\Omega - \frac{\hbar^2K^2}{4m}\right) \\ &\quad \times \left[\theta\left(2E_\mathrm{F} + \frac{\hbar^2K^2}{2m} + \frac{\hbar^2Kk_\mathrm{F}}{m} - \hbar\Omega\right)\right. \\ &\quad \left. + \theta\left(2E_\mathrm{F} + \frac{\hbar^2K^2}{2m} - \frac{\hbar^2Kk_\mathrm{F}}{m} - \hbar\Omega\right)\right].\end{aligned} \tag{B16}$$

The in-medium spectral function $\mathcal{A} = \mathcal{A}_0 - 2\mathcal{A}_1 - 2\mathcal{A}_2$ for zero temperature is shown in Fig. 6. As one can see, in the energy region corresponding to occupied states, the in-medium spectral function $\mathcal{A}$ is negative, while the free-space function $\mathcal{A}_0$ is positive.

The spectral function yields the imaginary part of the retarded Green function, $\mathcal{A}'(\Omega, \mathbf{K}) = -2\mathrm{Im}\mathcal{G}'(\Omega, \mathbf{K})$. With the help of the Kramers-Kronig relation (B2) we find real parts of in-medium corrections:

$$\mathrm{Re}\mathcal{G}_1(\Omega, \mathbf{K}) = \frac{mk_\mathrm{F}}{4\pi^2\hbar^3} + \frac{m^2(\hbar\Omega - 2E_\mathrm{F})}{8\pi^2\hbar^5K}\ln\left|\frac{E_+ - E_r}{E_- - E_r}\right| \tag{B17}$$

and

$$\begin{aligned}\mathrm{Re}\mathcal{G}_2(\Omega, \mathbf{K}) &= \frac{m^{\frac{3}{2}}}{4\pi^2\hbar^4}\sum_\pm\left(-\sqrt{E_\pm}\right. \\ &\quad \left. + \mathrm{sgn}E_r\sqrt{E_r}\,\mathrm{ArcTanh}\sqrt{\frac{E_\pm}{E_r}}\right)\end{aligned} \tag{B18}$$

with the relative energy

$$E_r = \hbar\Omega - \frac{\hbar^2K^2}{4m} \tag{B19}$$

and positions of singularities

$$E_\pm = E_\mathrm{F} + \frac{\hbar^2(k_\mathrm{F} \pm K)^2}{2m} - \frac{\hbar^2K^2}{4m}. \tag{B20}$$

The real part of the in-medium correction to the two-particle Green function $-2\mathrm{Re}\mathcal{G}' = -2\mathrm{Re}\mathcal{G}_1 - 2\mathrm{Re}\mathcal{G}_2$ for zero temperature is shown in Fig. 6. The in-medium correction to

the two-particle Green function is

$$\mathcal{G}' = \mathrm{Re}\mathcal{G}_1 + \mathrm{Re}\mathcal{G}_2 - i\tfrac{1}{2}\mathcal{A}_1 - i\tfrac{1}{2}\mathcal{A}_2. \tag{B21}$$

The in-medium T-matrix is given by Eq. (16). All functions are needed only on the energy shell, where

$$\hbar\Omega = \frac{\hbar^2\kappa^2}{m} + \frac{\hbar^2 K^2}{4m}. \tag{B22}$$

The T-matrix is thus a function of two amplitudes of momentum $\mathcal{T}(\kappa, K)$.

3. Scaled variables

The Fermi momentum k_F is a natural scale for all momenta. For integrations, we use scaled variables $\tilde{K} = K/k_\mathrm{F}$ and $\tilde{\kappa} = \kappa/k_\mathrm{F}$ in terms of which the in-medium correction to the two-particle Green's function $\mathcal{G}' = \frac{mk_\mathrm{F}}{4\pi\hbar^3}\tilde{\mathcal{G}}'$ from Eq. (B21) is built of spectral functions

$$\begin{aligned}\tilde{\mathcal{A}}_1(\tilde{\kappa}, \tilde{K}) &= \frac{1 - \tilde{\kappa}^2 - \frac{1}{4}\tilde{K}^2}{\tilde{K}}\theta(1 + \tilde{K} - 2\tilde{\kappa}^2) \\ &\quad\times\theta\left(\tilde{\kappa}^2 - \frac{1}{4}\tilde{K}^2 - 1 + \tilde{K}\right) \\ \tilde{\mathcal{A}}_2(\tilde{\kappa}, \tilde{K}) &= \tilde{\kappa}\left[\theta\left(1 + \tilde{K} - \tilde{\kappa}^2 + \frac{1}{4}\tilde{K}^2\right)\right. \\ &\quad\left.+\theta\left(1 - \tilde{K} - \tilde{\kappa}^2 + \frac{1}{4}\tilde{K}^2\right)\right]\end{aligned} \tag{B23}$$

and real parts

$$\begin{aligned}\mathrm{Re}\tilde{\mathcal{G}}_1(\tilde{\kappa}, \tilde{K}) &= \frac{1}{\pi} + \frac{\tilde{\kappa}^2 + \frac{1}{4}\tilde{K}^2 - 1}{2\pi\tilde{K}}\ln\left|\frac{\tilde{E}_+ - \tilde{E}_r}{\tilde{E}_- - \tilde{E}_r}\right| \\ \mathrm{Re}\tilde{\mathcal{G}}_2(\tilde{\kappa}, \tilde{K}) &= \frac{1}{\pi}\sum_\pm\left(\tilde{\kappa}\,\mathrm{ArcTanh}\sqrt{\frac{\tilde{E}_\pm}{\tilde{E}_r}} - \sqrt{\frac{\tilde{E}_\pm}{2}}\right)\end{aligned} \tag{B24}$$

with

$$\begin{aligned}\tilde{E}_r &= 2\tilde{\kappa}^2 \\ \tilde{E}_\pm &= 1 + (1 \pm \tilde{K})^2 - \tfrac{1}{2}\tilde{K}^2.\end{aligned} \tag{B25}$$

The scaled in-medium correction to the two-particle Green function thus is

$$\tilde{\mathcal{G}}' = \mathrm{Re}\tilde{\mathcal{G}}_1 + \mathrm{Re}\tilde{\mathcal{G}}_2 - i\tfrac{1}{2}\tilde{\mathcal{A}}_1 - i\tfrac{1}{2}\tilde{\mathcal{A}}_2. \tag{B26}$$

APPENDIX C: QUASIPARTICLE ENERGY TRANSFER

The total energy of interacting Fermi gas is given by the Kadanoff-Baym formula,

$$\mathcal{E} = \sum_{s=\uparrow\downarrow}\int\frac{d\omega}{2\pi}\frac{d\mathbf{k}}{(2\pi)^3}A_s(\omega, \mathbf{k})\frac{1}{2}\left(\omega\hbar + \frac{\hbar^2k^2}{2m_s}\right)f(\omega\hbar). \tag{C1}$$

Its time derivative covers both contributions, from scattering and quasiparticle energies. The former is hidden in the off-shell part of the spectral function A [13,23]; the quasiparticle contribution obtains from the quasiparticle approximation of the spectral function

$$A_\uparrow(\omega, \mathbf{k}) = 2\pi\delta(\omega\hbar - \varepsilon_{\uparrow\mathbf{k}}). \tag{C2}$$

The quasiparticle energy

$$\varepsilon_{\uparrow\mathbf{k}} = \frac{\hbar^2k^2}{2m_\uparrow} + \sigma_{\uparrow\mathbf{k}} \tag{C3}$$

is given by the real part of the self-energy on the energy shell, $\sigma_{\uparrow\mathbf{k}} = \mathrm{Re}\Sigma(\varepsilon_{\uparrow\mathbf{k}}/\hbar, \mathbf{k})$; therefore the total energy in the quasiparticle approximation is a sum of the bare kinetic energy and the interaction

$$\mathcal{E}_\mathrm{q} = \frac{1}{2}\sum_{s=\uparrow\downarrow}\int\frac{d\mathbf{k}}{(2\pi)^3}\sigma_{s\mathbf{k}}f_{s\mathbf{k}}. \tag{C4}$$

The time derivative $\dot{\mathcal{E}}_\mathrm{q}$, taken under the condition that collisions are absent, $\dot{f} \to 0$, represents the energy transfer via quasiparticle energies and will be compared with the energy transfer during collisions $\dot{\mathcal{E}}_\mathrm{c}$ and exchanges of pairs $\dot{\mathcal{E}}_\mathrm{p}$.

The self-energy is expressed in terms of the T-matrix, which is in the ladder approximation [23]

$$\begin{aligned}\Sigma_\uparrow(\omega, \mathbf{k}) &= \int\frac{d\Omega}{2\pi}\frac{d\mathbf{Q}}{(2\pi)^3}\mathcal{T}(\Omega, \mathbf{Q})G^<_\downarrow(\Omega - \omega, \mathbf{Q} - \mathbf{k}) \\ &\quad - \int\frac{d\Omega}{2\pi}\frac{d\mathbf{Q}}{(2\pi)^3}\mathcal{T}^<(\Omega, \mathbf{Q})\bar{G}_\downarrow(\Omega - \omega, \mathbf{Q} - \mathbf{k}).\end{aligned} \tag{C5}$$

Our notation follows the generalized Kadanoff-Baym formalism [58] except for suppressed symbols of retarded or advanced functions. Here the bar denotes the complex conjugation, i.e., G and $\bar{G}$ are the retarded and advanced Green function, respectively.

The first term of self-energy (C5) represents the mean-field interaction of particle $\uparrow\mathbf{k}$ with particle $\downarrow\mathbf{Q} - k$ via the retarded T-matrix (16). The second term of self-energy (C5) depends on (at least) three particles. Indeed, writing the correlation function of the T-matrix in terms of the two-particle Green's function, $\mathcal{T}^< = \mathcal{T}G^<_\uparrow G^<_\downarrow\bar{\mathcal{T}}$, one can see that $\mathcal{T}^<$ is nonzero only if both intermediate states are occupied. To evaluate this part we will benefit from the equilibrium relation $\mathcal{T}^<(\Omega, \mathbf{Q}) = -2\mathrm{Im}\mathcal{T}(\Omega, \mathbf{Q})f_\mathrm{BE}(\Omega)$, where f_BE is the Bose-Einstein distribution.

For the sake of simplicity, we use free-space energy spectrum for internal Green's functions of the self-energy, $G^<_\downarrow(\Omega-\omega, \mathbf{Q}-\mathbf{k}) = f_{\downarrow\mathbf{Q}-\mathbf{k}}2\pi\delta(\Omega\hbar-\omega\hbar - |\mathbf{Q}-\mathbf{k}|^2\hbar^2/2m_\downarrow)$ and $\bar{G}_\downarrow(\Omega - \omega, \mathbf{Q} - \mathbf{k}) = 1/(\Omega\hbar-\omega\hbar - |\mathbf{Q}-\mathbf{k}|^2\hbar^2/2m_\downarrow)$, and the free-space approximation of the frequency argument, $\sigma_{\uparrow\mathbf{k}} = \mathrm{Re}\Sigma(\hbar k^2/2m_\uparrow, \mathbf{k})$. The approximation of the quasiparticle energy is specified.

1. Mean-field energy transfer in the dilute gas

In this subsection we evaluate the time derivative of the quasiparticle part of the total energy (C4) for the nondegenerate dilute gas. In the dilute gas, the three-particle part of self-energy (C5) vanishes. The real part of self-energy thus

equals the mean field

$$\sigma_{\uparrow\mathbf{k}} = \int \frac{d\Omega}{2\pi}\frac{d\mathbf{Q}}{(2\pi)^3}\mathrm{Re}\mathcal{T}_0(\Omega,\mathbf{Q}) \times 2\pi\delta\left(\hbar\Omega - \frac{\hbar^2k^2}{2m_\uparrow} - \frac{\hbar^2|\mathbf{Q}-\mathbf{k}|^2}{2m_\downarrow}\right) f_{\downarrow\mathbf{Q}-\mathbf{k}} = \frac{1}{\hbar}\int \frac{d\mathbf{p}}{(2\pi)^3}\mathrm{Re}\mathcal{T}_0\left(\frac{\hbar k^2}{2m_\uparrow} + \frac{\hbar p^2}{2m_\downarrow}, \mathbf{p}+\mathbf{k}\right) f_{\downarrow\mathbf{p}}. \quad \text{(C6)}$$

Using self-energy (C6) with the T-matrix (7) in Eq. (C4), one finds

$$\mathcal{E}_\mathrm{q} = \frac{2\pi\hbar^2}{\mu}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{p}}{(2\pi)^3}\frac{a}{1+a^2\kappa^2} f_{\downarrow\mathbf{p}} f_{\uparrow\mathbf{k}}. \quad \text{(C7)}$$

By the time derivative of the energy density (C7) under the condition of absent collisions $\dot{f}=0$, one obtains the mean-field energy transfer to the gas

$$\dot{\mathcal{E}}_\mathrm{q} = \frac{2\pi\hbar^2}{\mu}\frac{\partial a}{\partial t}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{p}}{(2\pi)^3}\frac{1-a^2\kappa^2}{(1+a^2\kappa^2)^2} f_{\downarrow\mathbf{p}} f_{\uparrow\mathbf{k}}. \quad \text{(C8)}$$

This mean-field energy transfer can also be recast

$$\dot{\mathcal{E}}_\mathrm{q} = \frac{\partial a}{\partial t}\frac{\hbar^2 n_\uparrow n_\downarrow}{\mu}\mathcal{M} \quad \text{(C9)}$$

using the dimensionless function of rescaled scattering length (12)

$$\mathcal{M} = 2^3\sqrt{\pi}\int_0^\infty dx \frac{x^2-\alpha^2x^4}{(1+\alpha^2x^2)^2}e^{-x^2} = -\frac{4\pi}{\alpha^5}\left[\alpha(2+\alpha^2) - 2\sqrt{\pi}(1+\alpha^2)e^{\frac{1}{\alpha^2}}\mathrm{Erfc}\left(\frac{1}{\alpha}\right)\right]. \quad \text{(C10)}$$

2. Quasiparticle energy transfer at $T = 0$

The quasiparticle energy (C5) has mean-field and three-particle contributions. These parts have different properties, and it is more convenient to treat them separately. We thus split the self-energy into two parts corresponding to the first and second terms of the T-matrix approximation (C5),

$$\sigma^{(1)}_{\uparrow\mathbf{k}} = \frac{1}{\hbar}\int \frac{d\mathbf{Q}}{(2\pi)^3}\mathrm{Re}\mathcal{T}\left(\frac{\hbar k^2}{2m} + \frac{\hbar Q^2}{2m}, \mathbf{Q}+\mathbf{k}\right)\theta\left(k_\mathrm{F}^2 - Q^2\right) \quad \text{(C11)}$$

and

$$\sigma^{(2)}_{\uparrow\mathbf{k}} = -\int \frac{d\Omega}{2\pi}\frac{d\mathbf{Q}}{(2\pi)^3}\frac{\mathcal{T}^<(\Omega,\mathbf{Q})}{\hbar\Omega - \frac{\hbar^2k^2}{2m} - \frac{\hbar^2|\mathbf{Q}-\mathbf{k}|^2}{2m}}. \quad \text{(C12)}$$

In Eq. (C12) the denominator represents the real part of the retarded Green function, and the frequency integral over its singularity is the Cauchy principal value.

The energy transfer $\dot{\mathcal{E}}_\mathrm{q} = \dot{\mathcal{E}}^{(1)}_\mathrm{q} + \dot{\mathcal{E}}^{(2)}_\mathrm{q}$ is obtained by the time derivative of the energy density (C4). The part corresponding to self-energy (C11) is

$$\dot{\mathcal{E}}^{(1)}_\mathrm{q} = \int \frac{d\mathbf{k}}{(2\pi)^3}\dot{\sigma}^{(1)}_{\uparrow\mathbf{k}}\theta\left(k_\mathrm{F}^2-k^2\right) = \frac{1}{\hbar}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{Q}}{(2\pi)^3}\mathrm{Re}\dot{\mathcal{T}}\left(\frac{\hbar k^2}{2m} + \frac{\hbar Q^2}{2m}, \mathbf{Q}+\mathbf{k}\right) \times \theta\left(k_\mathrm{F}^2-Q^2\right)\theta\left(k_\mathrm{F}^2-k^2\right) = \frac{1}{\hbar}\int \frac{d\kappa}{(2\pi)^3}\frac{d\mathbf{K}}{(2\pi)^3}\mathrm{Re}\dot{\mathcal{T}}(\kappa,K) \times \theta\left(k_\mathrm{F}^2 - \left|\frac{1}{2}\mathbf{K}+\kappa\right|^2\right)\theta\left(k_\mathrm{F}^2 - \left|\frac{1}{2}\mathbf{K}-\kappa\right|^2\right). \quad \text{(C13)}$$

Again, we integrate over the direction of $\mathbf{K}$ and the azimuthal angle of κ,

$$\dot{\mathcal{E}}^{(1)}_\mathrm{q} = \frac{1}{4\pi^4\hbar}\int_0^{k_\mathrm{F}} d\kappa\kappa\int_0^{2k_\mathrm{F}} dKK\mathrm{Re}\dot{\mathcal{T}}(\kappa,K) \times \theta\left(k_\mathrm{F}^2 - \frac{1}{4}K^2 - \kappa^2\right) \times \left\{K\kappa\,\theta\left[k_\mathrm{F}^2 - \left(\frac{1}{2}K+\kappa\right)^2\right] + \left(k_\mathrm{F}^2 - \frac{1}{4}K^2 - \kappa^2\right)\theta\left[\left(\frac{1}{2}K+\kappa\right)^2 - k_\mathrm{F}^2\right]\right\}. \quad \text{(C14)}$$

The time derivative of the T-matrix (16) follows from the free-space value (7)

$$\dot{\mathcal{T}} = \frac{\mu}{2\pi\hbar^3}\frac{\partial a}{\partial t}\frac{1}{a^2}\mathcal{T}^2. \quad \text{(C15)}$$

The final two-dimensional integration was done numerically in the dimensionless variables,

$$\dot{\mathcal{E}}^{(1)}_\mathrm{q} = \frac{\partial a}{\partial t}\frac{\hbar^2 n_\uparrow n_\downarrow}{\mu}\mathcal{Q}^{(1)}, \quad \text{(C16)}$$

with

$$\mathcal{Q}^{(1)} = \frac{18\pi}{\vartheta^2}\int_0^1 d\tilde{\kappa}\tilde{\kappa}\int_0^2 d\tilde{K}\tilde{K}\mathrm{Re}[\tilde{\mathcal{T}}^2] \times \theta\left(1 - \frac{1}{4}\tilde{K}^2 - \tilde{\kappa}^2\right) \times \left\{\tilde{K}\tilde{\kappa}\,\theta\left[1 - \left(\frac{1}{2}\tilde{K}+\tilde{\kappa}\right)^2\right] + \left(1 - \frac{1}{4}\tilde{K}^2 - \tilde{\kappa}^2\right)\theta\left[\left(\frac{1}{2}\tilde{K}+\tilde{\kappa}\right)^2 - 1\right]\right\}. \quad \text{(C17)}$$

The energy transfer corresponding to self-energy (C12)

$$\dot{\mathcal{E}}^{(2)}_\mathrm{q} = -\int \frac{d\Omega}{2\pi}\frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{Q}}{(2\pi)^3}\frac{\dot{\mathcal{T}}^<(\Omega,\mathbf{Q})\theta\left(k_\mathrm{F}^2-k^2\right)}{\hbar\Omega - \frac{\hbar^2k^2}{2m} - \frac{\hbar^2|\mathbf{Q}-\mathbf{k}|^2}{2m}} \quad \text{(C18)}$$

depends on the correlation function of the T-matrix $\mathcal{T}^<$. Function $\mathcal{T}^<$ is nonzero only for positive frequencies. This can be seen from its equation of motion $\mathcal{T}^< = \mathcal{T}G^<_\uparrow G^<_\downarrow\bar{\mathcal{T}}$, which gives nonzero contribution only if the frequency argument equals the sum energy of two internal states, $\hbar\Omega = \varepsilon_\uparrow + \varepsilon_\downarrow$. For the same reason, function $\mathcal{T}^<$ vanishes for $\hbar\Omega > 2E_\mathrm{F}$. In equilibrium $\mathcal{T}^<(\Omega,\mathbf{Q}) = -2\mathrm{Im}\mathcal{T}(\Omega,\mathbf{Q})f_\mathrm{BE}(\Omega)$. The lower limit of frequency integration is guaranteed by the T-matrix,

$\mathrm{Im}\mathcal{T}(\Omega, \mathbf{Q}) = 0$ for $\Omega < \hbar^2 Q^2/4m$. The upper limit provides the Bose-Einstein distribution, which at zero temperature reads $f_{\mathrm{BE}}(\Omega) = -\theta(2E_{\mathrm{F}} - \hbar\Omega)$. Therefore, in the spherical coordinates

$$\dot{\mathcal{E}}_{\mathrm{q}}^{(2)} = -\frac{m}{8\pi^5\hbar^2}\int_0^\infty dQQ^2\int_{\frac{\hbar Q^2}{4m}}^{\frac{2E_{\mathrm{F}}}{\hbar}} d\Omega \mathrm{Im}\dot{\mathcal{T}}(\Omega, Q) \times \int_0^{k_{\mathrm{F}}} dkk^2\int_{-1}^1 dz \frac{1}{\frac{m\Omega}{\hbar} - k^2 - \frac{1}{2}Q^2 + zQk}. \quad \text{(C19)}$$

Finally, we use $\dot{\mathcal{T}}$ from Eq. (C15) and substitute the relative momentum for frequency, $\Omega = \hbar\kappa^2/m + \hbar Q^2/4m$, so that

$$\dot{\mathcal{E}}_{\mathrm{q}}^{(2)} = -\frac{\mu}{8\pi^6\hbar^4}\frac{\partial a}{\partial t}\frac{1}{a^2}\int_0^{2k_{\mathrm{F}}} dQQ^2 \times \int_0^{\sqrt{k_{\mathrm{F}}^2 - \frac{1}{4}Q^2}} d\kappa\kappa \mathrm{Im}[\mathcal{T}^2(\kappa, Q)]L(\kappa, Q), \quad \text{(C20)}$$

where

$$L(\kappa, Q) = \int_0^{k_{\mathrm{F}}} dkk^2\int_{-1}^1 dz\frac{1}{\kappa^2 - \frac{1}{4}Q^2 - k^2 + zQk} = -k_{\mathrm{F}} + \frac{1}{2Q}\left[k_{\mathrm{F}}^2 - \left(\frac{1}{2}Q - \kappa\right)^2\right]\ln\frac{\left|k_{\mathrm{F}} - \frac{1}{2}Q + \kappa\right|}{\left|k_{\mathrm{F}} + \frac{1}{2}Q - \kappa\right|} + \frac{1}{2Q}\left[k_{\mathrm{F}}^2 - \left(\frac{1}{2}Q + \kappa\right)^2\right]\ln\frac{\left|k_{\mathrm{F}} - \frac{1}{2}Q - \kappa\right|}{\left|k_{\mathrm{F}} + \frac{1}{2}Q + \kappa\right|}. \quad \text{(C21)}$$

Again, the two-dimensional integration in Eq. (C20) was done numerically in dimensionless variables in which

$$\dot{\mathcal{E}}_{\mathrm{q}}^{(2)} = \frac{\partial a}{\partial t}\frac{\hbar^2 n_\uparrow n_\downarrow}{\mu}\mathcal{Q}^{(2)} \quad \text{(C22)}$$

and

$$\mathcal{Q}^{(2)} = -\frac{18}{\vartheta^2}\int_0^2 d\tilde{Q}\tilde{Q}^2\int_0^{\sqrt{1-\frac{1}{4}\tilde{Q}^2}} d\tilde{\kappa}\tilde{\kappa}\mathrm{Im}[\tilde{\mathcal{T}}^2(\tilde{\kappa}, \tilde{Q})]\tilde{L}(\tilde{\kappa}, \tilde{Q}), \quad \text{(C23)}$$

where $\tilde{L} = L/k_{\mathrm{F}}$.

The total dimensionless energy transfer by the quasiparticle mechanism is the sum of both contributions

$$\mathcal{Q} = \mathcal{Q}^{(1)} + \mathcal{Q}^{(2)}. \quad \text{(C24)}$$

The quasiparticle mechanism of the energy transfer is specified.

APPENDIX D: REARRANGEMENT OF EXCHANGE INTEGRALS

The T-matrix and the energy gain are functions of amplitudes K and κ of the sum and relative momentum; see Appendix B. In addition, the θ functions depend on polar angles. We denote cosines of polar angles $z = (\mathbf{K}\cdot\kappa)/(K\kappa)$ and $z' = (\mathbf{K}\cdot\kappa')/(K\kappa')$. Except for these angles, the angular parts of momentum integrations in Eq. (18) can be performed analytically

$$\dot{\mathcal{E}}_{\mathrm{p}} = -\frac{4}{(2\pi)^5\hbar^3}\int_0^{2k_{\mathrm{F}}} dKK^2\int_0^{k_{\mathrm{F}}} d\kappa\kappa^2\Delta_E|\mathcal{T}(\kappa, K)|^2 \times \int_0^{k_{\mathrm{F}}} d\kappa'\kappa'^2\delta\left(\frac{\hbar^2\kappa^2}{m} - \frac{\hbar^2\kappa'^2}{m}\right)\int_{-1}^1 dz\int_{-1}^1 dz' \times\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - zK\kappa - \kappa^2\right) \times\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 + zK\kappa - \kappa^2\right) \times\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - z'K\kappa' - \kappa'^2\right) \times\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 + z'K\kappa' - \kappa'^2\right). \quad \text{(D1)}$$

The integration over κ' is trivial.

The integrand is odd in z and z', which allows us to restrict integration to their positive values

$$\dot{\mathcal{E}}_{\mathrm{p}} = -\frac{8m}{(2\pi\hbar)^5}\int_0^{2k_{\mathrm{F}}} dKK^2\int_0^{k_{\mathrm{F}}} d\kappa\kappa^3\Delta_E|\mathcal{T}(\kappa, K)|^2 \times\int_0^1 dz\int_0^1 dz'\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - zK\kappa - \kappa^2\right) \times\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - z'K\kappa - \kappa^2\right). \quad \text{(D2)}$$

In rearrangement, we have eliminated θ functions, which are trivially satisfied in the restricted integration region.

An additional reduction of θ functions follows from the symmetry with respect to the interchange $z \longleftrightarrow z'$, due to which we can restrict the integration by condition $z' < z$. The second θ function is then always unity, and therefore we can integrate over z':

$$\dot{\mathcal{E}}_{\mathrm{p}} = -\frac{16m}{(2\pi\hbar)^5}\int_0^{2k_{\mathrm{F}}} dKK^2\int_0^{k_{\mathrm{F}}} d\kappa\kappa^3\Delta_E|\mathcal{T}(\kappa, K)|^2 \times\int_0^1 dzz\,\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - zK\kappa - \kappa^2\right). \quad \text{(D3)}$$

Finally, we perform the z-integration

$$\dot{\mathcal{E}}_{\mathrm{p}} = -\frac{8m}{(2\pi\hbar)^5}\int_0^{2k_{\mathrm{F}}} dK\int_0^{k_{\mathrm{F}}} d\kappa\kappa\Delta_E|\mathcal{T}(\kappa, K)|^2 \times\theta\left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - \kappa^2\right) \times\left\{K^2\kappa^2\,\theta\left[k_{\mathrm{F}}^2 - \left(\frac{1}{2}K + \kappa\right)^2\right] + \left(k_{\mathrm{F}}^2 - \frac{1}{4}K^2 - \kappa^2\right)^2\theta\left[\left(\frac{1}{2}K + \kappa\right)^2 - k_{\mathrm{F}}^2\right]\right\}. \quad \text{(D4)}$$

The remaining two-dimensional integration has to be done numerically.

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